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BDD Efficiency: Survey of BDD Edge Ordering Algorithms in Network Reliability

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Abstract

Network reliability analysis is vital for ensuring efficient and error-free communication within networking and communication applications. Binary Decision Diagrams (BDDs) have emerged as a powerful tool for analyzing and optimizing complex network infrastructures. The objective of this research paper is to conduct a comparative analysis of edge-ordering algorithms for network reliability using BDDs the study aims to evaluate and compare existing algorithms, providing valuable insights for selecting suitable edge-ordering algorithms that enhance network reliability. The paper concludes that snooker is outperforming among selected algorithms.

KEYWORDS:
Network reliability; Binary Design Diagram; Network efficiency; Edge ordering algorithm; complexity

1 | INTRODUCTION

Network science is a multidisciplinary field that studies the structure, behavior, and dynamics of complex networks[1] These networks can represent a wide range of systems, including social networks, transportation networks, biological networks, and communication networks. The study of network science plays a crucial role in enhancing our understanding of these intricate systems and facilitating the design of efficient and robust network infrastructures. Reliability analysis is of utmost importance in network infrastructures as it ensures efficient and error-free communication within various networking and communication applications. With the rapid advancement of technology, network reliability has become even more critical in modern applications such as the Internet of Things (IoT)[2] and Wireless Sensor Networks (WSN)[3]. These networks consist of numerous interconnected devices, and their reliability is essential for the seamless operation of various applications, such as smart homes, industrial automation, and healthcare monitoring systems, etc. In the field of network reliability, researchers[4], and[5] have developed various tools and methodologies to analyze and optimize complex networks. One such powerful tool is Binary Decision Diagrams (BDDs)[6] which have become increasingly important in the reliability analysis of network infrastructures. BDDs provide a structured and efficient approach to represent and manipulate Boolean functions, enabling the examination of network connectivity and reliability[7]. By utilizing BDDs, researchers can assess the resilience and robustness of network components, identify critical points of failure, and develop strategies to ensure uninterrupted communication. The study of network reliability involves analyzing the connectivity and performance of networks in the presence of link failures. By representing the network as a graph, where nodes and edges signify different network objects, the reliability analysis provides insights into the probability of successful communication between devices. However, the size of the Binary Decision Diagram encoding for a network depends on the ordering of links, which impacts the efficiency of reliability analysis. The optimal link ordering plays a crucial role in determining the size of the BDD encoding for a network. However, the complexity of finding the optimal link ordering has been

[0]**Abbreviations:** BDD, Binary Decision Diagrams; BFS, Breadth First Search; DFS, Depth-First Search; NDS, Network Driven Search; IoT, Internet of Things; WSN, Wireless Sensor Network

established as an NP-hard problem[8]. While traditional graph searching algorithms like Breadth-First Search (BFS) and Depth-First Search (DFS) have been employed for ordering the BDD, recent research in reliability analysis using BDD has introduced novel algorithms such as the Community Awareness Heuristic, Snooker, and Network Driven Search (NDS). These algorithms aim to improve the ordering and subsequently enhance network reliability. They take into consideration factors such as network topology, component importance, and community structure to determine the most effective link-ordering strategy. Motivated by the advancements in reliability analysis algorithms, performing a comparative analysis of edge ordering algorithms for network reliability using BDD is imperative. This analysis will facilitate the selection of the most appropriate algorithm for distributed network planning, thus addressing the reliability concerns of complex networking architectures, including heterogeneous networks and WSN networks. However, the main contribution of this research paper lies in conducting a comparative analysis of edge ordering algorithms for network reliability using Binary Decision Diagrams (BDDs). By evaluating and comparing existing algorithms, this study aims to provide valuable insights for selecting suitable edge-ordering algorithms that can enhance network reliability in resource-constrained networking environments. The paper reviews and analyzes previous works in the field, encompassing a range of edge ordering algorithms. This research paper contributes to the field by providing a comprehensive comparative analysis of edge ordering algorithms for network reliability using BDDs. The findings can aid researchers and practitioners in choosing the most suitable algorithm for their network environments, leading to improved network planning, enhanced reliability, and efficient resource utilization in complex architectures. The paper is structured with an introduction to network reliability in Section 1, an analysis of previous research in Section 2, fundamental concepts of Binary Decision Diagrams (BDDs) in Section 3, comparisons in Section 4, and conclusions in Sections 5.

2 | RELATED WORK

Existing research in the field of network reliability analysis using Binary Decision Diagrams (BDDs) has been discussed as follows. Here, it is focused on summarizing the key findings, methodologies, and approaches adopted by previous researchers in this area. Previous studies have investigated various edge ordering algorithms for BDDs in the context of network reliability analysis. Y. Mo. et. al.[8] discussed the use of graph searching algorithms, such as Breadth First Search (BFS) and Depth First Search (DFS), etc. for edge ordering. These algorithms have been explored to determine the order in which edges are considered in the BDD representation. The use of BDDs for network reliability analysis can be traced back to the pioneering work of the author[9], who introduced BDDs as a representation technique for Boolean functions. Authors in[10], and[11] further advanced the field by introducing efficient algorithms for constructing and manipulating BDD structures. Subsequently, researchers[12] [13] [14] [15] explored the application of BDDs in reliability analysis, particularly in assessing fault trees and evaluating the reliability of complex systems. In the specific context of network reliability analysis, researchers in[7] demonstrated the functional construction of BDDs for analyzing network reliability. Their work focused on representing and analyzing the various components and connections within networks using BDDs, enabling a comprehensive assessment of reliability. In addition to traditional approaches, recent research has explored alternative edge-ordering algorithms. For example,[16] proposed a randomized graph algorithm for BDD edge order, which was utilized by[17]. This randomized algorithm randomly orders the edges in the BDD, providing a simple and fast approach. While it may not always produce optimal results, it is suitable for applications where speed is prioritized. Another edge ordering algorithm discussed in[18] is topological edge ordering. This algorithm orders the edges in the BDD based on their topological relationships, giving priority to edges connected to nodes with higher topological order. This approach offers efficiency advantages and is suitable for applications where both speed and size are important. Furthermore, dynamic variable ordering of BDDs was explored in[19], presenting a method for adaptively rearranging the variables in the BDD during the analysis process. Additionally,[8], and[20] discussed snooker edge ordering algorithms and edge ordering for community structures of IoT networks. Various edge ordering algorithms for BDDs in the context of network reliability analysis have been explored in this section. The literature encompasses traditional approaches such as BFS and DFS, as well as alternative algorithms like randomized graph algorithms, topological edge ordering, snooker edge ordering, and edge ordering for community structures of IoT networks. These studies have significantly contributed to the understanding and advancement of BDD-based techniques for network reliability analysis, offering insights into their strengths, limitations, and applicability in practical scenarios. In patent[21] presents a method to optimize logic circuit manipulation using ordered binary decision diagrams and breadth-first manipulation, improving memory access efficiency. It organizes memory access and maintains level-specific queues for efficient processing. Also, the patent[22] describes accessing sensor data samples using BDD, converting them into min-terms, and generating a characteristic function to determine min-term membership in the set. This aids in analyzing sensor measurements efficiently. A patent[23] proposes network coding-based architectures to boost data transfer reliability and efficiency in next-gen wireless networks, using thread-based and intra-session random linear coding techniques for network reliability. The patent[24] introduces a method for building decision diagrams in a digital circuit's structural network representation efficiently, utilizing dynamic depth-first and breadth-first schedules with size limits for binary decision representations. It adjusts size limits and techniques for comprehensive logic function description.

3 | PRELIMINARY CONCEPT

The BDD structure provides compact representations of Boolean expressions. The BDD associated with a Boolean expression is a compact encoding of the truth table of this expression. A BDD is a DAG (Directed Acyclic Graph) based on Shannons decomposition.

3.1 | Shannons Decomposition Theorem

Shannon discussed a decomposition theorem in the Boolean circuit theory[25]. The Shannon decomposition theorem is the base of BDD. Shannon decomposition depends on the if-then-else(ite) format[7]. For instance, let f be a Boolean expression on, and x be a variable of X, then the function can be expressed as shown in equation 1.

$$f = x \cdot f_{x=1} + \neg x \cdot f_{x=0} \tag{1}$$

Where f valued at $x = v$ is denoted by $f_{x=v}$, $\neg x$ is the negation of the $\cdot$ represents the conjunction of two binary variables and $+$ represents the disjunction of two binary variables x, the if-then-else (ite) format is (equation 2):

$$f = ite(x, F_1, F_2) = x \cdot f_{x=1} + \neg x \cdot f_{x=0} \tag{2}$$

Where F_1 is at $x = 1$ and F_2 is at $x = 0$.

3.2 | Binary Decision Diagram and Network Reliability

A BDD consists of three types of nodes a root node, a decision node, and a constant node. Decision nodes are non-sink nodes that are labeled by Boolean variable and these nodes have two edges that lead to child nodes. These edges are called 0-edge (or else-edge) and 1-edge (or then-edge). 0-edge led to a Boolean expression when $x = 0$ which represents $f_{x=0}$, while the other 1-edge led to an expression when $x = 1$ which represents $f_{x=1}$. An ordered BDD is defined as a BDD with the constraint that the variables are ordered, and every path from the root non-sink decision node to a constant node visits the variables in ascending order. Further, a reduced ordered BDD is an ordered BDD where each node represents a distinct logic expression. BDDs provide a powerful tool for analyzing and optimizing network systems and can be used in a variety of applications in the field of networking. BDD for network reliability is discussed by the author in[8] using a top-down recursive manner. Using equation 1 on the BDD model that consists of all paths between the source and destination node we can calculate the reliability. So, the modified equation is shown in equation 3.

$$Pr(BDD) = p_e \cdot Pr(BDD.T) + q_e \cdot Pr(BDD.E) \tag{3}$$

Where $Pr()$ is a function to calculate probability having a BDD model as input. p_e is the reliability of e^{th} link, BDD.T and BDD.E are the child nodes of the current BDD node. q_e is unreliability i.e. $q_e = 1 - p_e$ for e^{th} link.

3.3 | Construction of BDD

In this section, we are discussing two methods to generate a BDD. To construct a Binary Decision Diagram (BDD) for a Boolean function, one method is to manually calculate it using the Shannon decomposition theorem. The first step is to choose a variable ordering. Let's consider a Boolean function f that depends on three variables a, b, and c (shown in equation in equation 4):

$$f = ac + bc + \neg a \neg b \neg c \tag{4}$$

We can choose any variable ordering for a, b, and c. In this case, let's choose the ordering of a, b, and then c. The next step is to decompose the function into smaller sub-functions using the Shannon decomposition theorem. This process is shown in Fig.

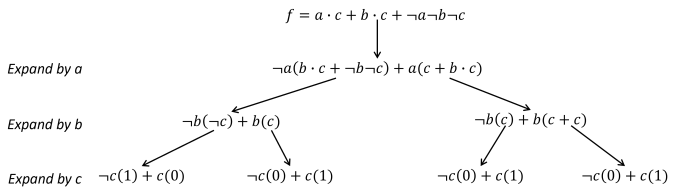

FIGURE 1 Expansion of equation (4) using equation (1).

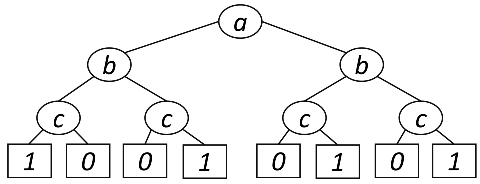

FIGURE 2 BDD generated.

(1). We repeat this step for all variables in the ordering to construct the BDD. The resulting BDD will have nodes representing the sub-functions and edges representing the variables. This graph can then be used to efficiently evaluate the Boolean function for different input combinations. The resplendent decision diagram of this calculation is shown in Fig (2).

4 | COMPARISON OF ALGORITHMS

In this section, we explore various algorithms for edge ordering in Binary Decision Diagrams. The size of a BDD is greatly affected by the specific order in which edges are selected. However, finding the optimal ordering for a network has been proven to be a challenging NP-hard problem[8]. As a result, these techniques have been employed to obtain favorable edge orderings in BDD generation. In this paper, we are comparing BFS, DFS, NDS, and snooker edge ordering algorithms.

4.1 | BFS

Breadth-First Search (BFS) is a commonly used algorithm for edge ordering in Binary Decision Diagrams (BDDs). It explores the graph level by level, starting from the root node and gradually moving to its neighboring nodes before proceeding to the next level. BFS follows a First-In-First-Out (FIFO) approach, visiting nodes in the order they were discovered.

4.2 | DFS

When it comes to Binary Decision Diagram (BDD) edge ordering, the Depth-First Search (DFS) algorithm plays a crucial role. DFS starts at the root node and explores as far as possible along each branch before backtracking. In the context of BDD edge ordering, DFS determines the order in which the edges of the BDD will be considered. The DFS ordering follows the Last-In-First-Out (LIFO) principle, where the algorithm explores the deepest unvisited nodes first and backtracks only when there are no more unvisited nodes. The choice of DFS ordering can significantly impact the resulting BDD size. Different DFS orders can yield distinct BDD structures and sizes. By exploring deeper levels before backtracking, DFS may produce different BDD structures compared to other algorithms. DFS is one of the fundamental algorithms for BDD edge ordering.

4.3 | Network-Driven Search

Network-driven search (NDS) is an alternative approach to edge ordering in Binary Decision Diagrams (BDDs). Unlike Depth-First Search and Breadth-First Search, which focus solely on graph structure, NDS incorporates additional network-related information. It considers metrics like edge weights and connectivity patterns to guide the search. NDS aims to optimize BDD structure based on specific network requirements, evaluating different edge-ordering strategies. These strategies prioritize aspects like minimizing BDD size or improving efficiency for certain operations. NDS leverages network-specific knowledge, potentially yielding more compact and efficient BDD representations compared to generic search algorithms like DFS or BFS. It is a heuristic approach tailored to the characteristics and requirements of the given network.

4.4 | Snooker

The Snooker edge ordering heuristic, proposed by Wu in[20], enhances network reliability analysis using Binary Decision Diagrams (BDDs). Unlike traditional algorithms like Breadth-First Search (BFS) and Depth-First Search (DFS), Snooker introduces

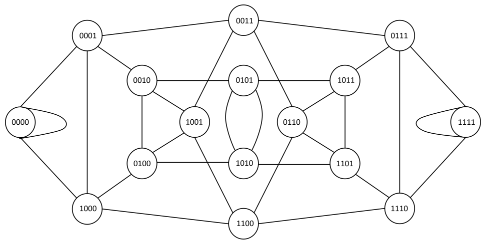

FIGURE 3 De Bruijn networks of order 4

Root Node	Terminal pair	BFS	DFS	NDS	Snooker
1	(0,5)	1626	4507	874	582
1	(12,14)	895	4944	385	287
10	(1,13)	2856	7976	2124	1830
10	(14,15)	2123	2788	2049	1078

TABLE 1 Size Comparison for De Bruijn networks of order 4.

a unique approach by considering node degrees. It prioritizes nodes with lower degrees during BDD generation, aiming to minimize the size of the boundary set. Empirical studies in the paper show that Snooker consistently produces favorable edge orderings, improving BDD efficiency for network reliability analysis. Wu's research makes a valuable contribution by addressing the challenge of minimizing BDD size in network reliability analysis, providing insights and evidence on the effectiveness of the Snooker heuristic.

4.5 | Comparison for De Bruijn Network

De Bruijn networks, named after mathematician Nicolaas Govert de Bruijn, are directed graphs widely used in computer science with diverse applications. They consist of $2n$ nodes labeled with binary strings of length n, representing all possible combinations. The connections between nodes are determined by left or right shifts and variations in the first or last bit, creating a cyclic structure. In the context of network reliability analysis, De Bruijn networks provide a promising approach to enhancing the reliability of networks. By analyzing connectivity patterns, identifying vulnerabilities, and implementing fault-tolerant measures, these networks optimize network design and bolster overall reliability and resilience, ensuring the uninterrupted operation of networking systems. We have considered De Bruijn networks of order 4 as shown in Fig. (3). To evaluate different algorithms, we randomly selected source and terminal nodes, and then executed four algorithms: Breadth First Search, Depth First Search, Network Driven Search, and Snooker. The objective was to assess the performance of these algorithms based on the size of Binary Decision Diagrams associated with a particular ordering. The results of running these algorithms on the order 4 De Bruijn network is summarized in Table 1. The table provides insights into the sizes of the BDDs associated with each algorithm and ordering. By comparing the sizes of the BDDs, we can assess the efficiency and effectiveness of each algorithm in terms of the space required to represent the Boolean functions associated with the De Bruijn network. Table 1 demonstrate that the Snooker algorithm outperforms other algorithms, such as BFS, DFS, and NDS, in reducing the size of Binary Decision Diagrams for the order 4 De Bruijn network. This indicates that the Snooker algorithm's edge ordering strategy is particularly effective in generating more efficient BDDs.

5 | CONCLUSION

This paper conducts a thorough comparative analysis of edge ordering algorithms for network reliability. It addresses challenges in network reliability, emphasizing its importance in complex networks and Wireless Sensor Networks (WSN). The study highlights Binary Decision Diagrams (BDDs) as a powerful tool for evaluating network connectivity. The findings showcase the effectiveness of various edge ordering algorithms for BDD, with the Snooker algorithm emerging as the top performer. This research provides valuable insights for researchers and practitioners to enhance network planning and resource utilization in network environments, emphasizing the need for continuous innovation in network reliability.

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