Water Resources Research

Supporting Information for

A climate model-informed nonstationary stochastic rainfall generator for design flood analyses in continental-scale river basins

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Additional Supporting Information (Files uploaded separately)

No additional supporting information.

Introduction

This file contains three text sections and one table. Text S1 provides details on the CDF-t method used for bias correcting the CESM2 data. Text S2 and Table S1 describe the overlapping ratio and parameter setting used in storm tracking method STARCH. Text S3 shows the mathematical derivation to transform a Generalized Gamma distribution to a Gamma distribution.

Text S1. CESM2 Bias Correction

The CDF-t method adjusts the CDF of an atmospheric variable from CESM2 to match the CDF from ERA5. Unlike quantile-quantile (QQ) mapping, it accounts for CDF changes between historical and future GCM simulations (Famien et al., 2018). A transformation T is obtained by linear interpolation to map the CDF of the atmospheric variable from CESM2 (F_{Gh}) to the CDF from ERA5 (F_{Sh}) over the calibration period (1979-2021). T is then applied to the CESM2 CDF for an early period (1951-1978) or future period (2022-2050), F_{Gf}, to generate an adjusted CDF F_{Sf}, which represents the "unobserved" ERA5 CDF for those periods. For the early and future periods, QQ mapping is performed between the CESM2 CDF F_{Gf} and adjusted CDF F_{Sf} to correct the CESM2 data. For the calibration period, QQ mapping is applied directly between F_{Gh} and F_{Sh}. This CDF-t method was applied to CESM2 precipitation, precipitable water, and IVT data for winter (Dec-Jan) and spring (Mar-May). More details on the CDF-t method can be found in Vrac et al. (2012).

Text S2. STARCH Overlapping Ratio and Parameter Setting

The overlapping ratio between two consecutive IVT objects was defined as follows (Liu & Wright, 2022):

$$R(t_1, i, j) = \frac{A}{A_i(t_1)} + \frac{A}{A_j(t_0)} \tag{S1}$$

where $R(t_1, i, j) \subset [0, 2]$ is the overlapping ratio, A is the overlapping area between IVT objects i and j, $A_i(t_1)$ is the area of IVT object i at time step t_1, $A_j(t_0)$ is the area of IVT object j at time step t_0; all the areas are measured by pixels.

The parameter setting in STARCH is shown in Table S1. More details about the parameters and their functions can be found in (Liu & Wright, 2022) and STARCH repository (https://github.com/lorenliu13/starch/tree/v1.0.1).

Dataset	**M**orph radius R_m (pixel)	**H**igh threshold (kg/m/s)	**L**ow threshold (kg/m/s)	**O**verlapping ratio threshold (-)
ERA5/CESM2	1	500	250	0.2

Table S1. Parameter setting in STARCH.

Text S3. Transformation of Generalized Gamma Distribution to Gamma Distribution

The probability density function (PDF) of a generalized gamma distribution can be written as:

$$f_X(x,a,c,b) = \frac{c}{b\Gamma(a)}(\frac{x}{b})^{ca-1}\exp(-(\frac{x}{b})^c) \quad \text{(S2)}$$

where shape parameters a > 0, c > 0 and scale parameter b > 0.

Applying a transformation $y = g(x) = x^c$, the PDF of y can be derived following the transformation of single random variable:

$$f_Y(y) = f_X(g^{-1}(y))\,|\frac{d}{dy}g^{-1}(y)| \quad \text{(S3)}$$

For the first term $f_X(g^{-1}(y))z$, we have:

$$f_X(g^{-1}(y)) = f_x(y^{1/c}) = \frac{c}{y^{1/c}\Gamma(a)}(\frac{y}{b^c})^a\exp(-\frac{y}{b^c}) \quad \text{(S4)}$$

For the second term, since $x = g^{-1}(y) = y^{1/c}$, we have:

$$|\frac{d}{dy}g^{-1}(y)| = \frac{1}{c}y^{1/c-1} \quad \text{(S5)}$$

Combining two terms and let $a' = a$ and $b' = b^c$, we will get:

$$f_Y(y,a',b') = \frac{1}{b'\Gamma(a')}(\frac{y}{b'})^{a'-1}\exp(-\frac{y}{b'}) \sim Gamma(a',b') \quad \text{(S6)}$$