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ORIGINAL RESEARCH

A Reliability-Based Construction Approach for Wide-Area Backup Protection Systems Utilizing Phasor Measurement Units in Smart Grids

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Abstract

Reliability is a fundamental requirement of wide-area backup protection (WABP) systems, and it is one of the most crucial performance indicators for such a WABP system. Two definitions are introduced: the reliability of WABP systems and the undetectable probability of a power line. They are determined by the specific placement of communication links (CLs) and phasor measurement units (PMUs). A simultaneous optimization model is formulated to minimize the construction cost of WABP systems by optimizing the partitioning and placement of PMUs, protection centers (PCs), and CLs. The model takes into account several constraints, including latency, regional balance, and system reliability. To reduce computational complexity, a cluster-based genetic algorithm is developed to determine the optimal solution. Finally, numerical simulations are conducted using IEEE test cases. Results demonstrate that the proposed method can minimize the construction costs of WABP systems while increasing their reliability.

KEYWORDS
wide area networks, phasor measurement, reliability, optimisation, smart power grids

1 | INTRODUCTION

With the advent of high-speed communication, distributed computing, and modern sensors, phasor measurement units (PMUs) has been widely deployed in smart grids [1]. Time synchronized PMUs with the global positioning system can acquire voltage, current and frequency data of power grids [2]. The availability of time synchronized PMUs has facilitated the expanded use of wide-area backup protection (WABP) [3]. WABP can collect the global electrical quantities and accurately locate the faulted lines. It can overcome the disadvantages of traditional backup protection systems, such as poor selectivity and vulnerability to maloperation [4]. Therefore, PMU-based wide-area backup protection has been extensively investigated.

In WABP systems, faulted lines can be identified using electrical quantities such as current, voltage, impedance and phase acquired from PMUs. For example, a WABP scheme based on PMUs was presented to compare the magnitudes of bus sequence voltages at the system protection center to identify the bus closest to the fault [5]. A linear least squares method was used to determine the faulty line and fault location by utilizing the voltage and current phasors of backup protection zones [6]. A robust and efficient WABP scheme was proposed by utilizing available PMUs to identify asymmetrical faults in transmission lines, and the method does not require complete observability of power systems [7]. The concept of superimposed circuits was used for identifying faulty lines with spares PMU coverage in a WABP system [8]. Based on the allocation of PMUs, three types of subnetworks were evaluated, and the algorithm based on differential voltage was applied to each subnetwork in the system [9]. The impedance matrix of pre-fault bus was applied to identifying faulted lines [10]. It can be seen from above that the theory of WABP relies on PMU data. The availability of PMUs, which is determined by their placement, will influence the performance of a WABP system. Some research has focused on how to construct a PMU-based WABP system. For example, a PMU allocation scheme was investigated to detect faulted lines [11]. An integer linear programming model with considering the characteristics of synchrophasor devices (can also refer to PMUs) was proposed to to successively optimize partitioning, the number and location of synchrophasor devices in three stages [12].

In a PMU-based WABP system, the reliable operation of the wide-area protection system depends on the proper functioning of PMUs [13]. When a critical communication component or PMU fails, it can cause the entire wide-area protection system to become inactive, resulting in the loss of control over the entire power grid [14]. Therefore, reliability is a crucial performance index that cannot be overlooked in WABP systems [15]. However, the reliability of WABP systems has received little attention. In this paper, the reliability of WABP systems is presented and discussed in detail. A simultaneous optimization problem has been established that involves the placement of specific communication links and the allocation of PMUs and protection centers in order to minimize the construction costs of WABP systems. In addition, large-scale power grids are divided into different regions to ensure independent and real-time data transmission. The contribution of this paper is summarized as follows:

1) Two definitions, the reliability of WABP systems and the undetectable probability of a power line, are introduced, and they are determined by the specific placement of CLs and PMUs. The reliability of WABP systems is analyzed in detail and used as a constraint of the simultaneous optimization model.
2) A simultaneous optimization model of partitioning and the placement of PMUs, PCs and CLs is formulated to minimize the construction cost of WABP systems, subject to some constraints such as latency, region balance and reliability.
3) An improved genetic algorithm is developed to solve the simultaneous optimization model. A cluster algorithm is first employed to calculate the number and location of candidate PCs to reduce computational complexity.

The proposed method are evaluated on IEEE testing cases. Numerical simulations demonstrate the proposed method can minimize the construction costs of WABP systems while increasing reliability.

The other parts of the paper is arranged as follows. Section II summarizes the PMU-based WABP system. In Section III, reliability-based partitioning of the PMU-based WABP system is formulated subject to communication network requirements. The improved genetic algorithm is developed in Section IV. The IEEE testing cased are evaluated in Section V. Finally, conclusions can be provided in Section VI.

2 | SUMMARY OF PMU-BASED WABP SYSTEM

In WABP, a PMU can collect positive sequences from all bus voltages and line currents. A regional PC is installed at a substation and used to acquire and analyze data from the PMUs in a region, prior to sending action commands to the substation circuit breaker. In two-terminal lines, if both the voltage and current phasors are available for one terminal, the voltage of the other terminal is estimated from given line models, voltages and currents. This process is called the indirect measurement principle. The estimated voltage error will be small under no line failures or during the occurrence of an external fault. However, when a failure happens on the two-terminal line, the estimated voltage will differ significantly from the actual value. This principle is also valid for three-terminal lines and is commonly used by WABP in identifying faulted lines [12]. The number and placement of PMUs will affect the accuracy of fault line identification. Thus PMU placement and communication requirements are first discussed and evaluated.

2.1 | PMU Placement Principles

Power grids include substations connected to two-terminal lines, three-terminal lines, and independent substations connected to only one line. These are denoted by type-a, type-b, and type-c, respectively, as shown in Fig. 1.

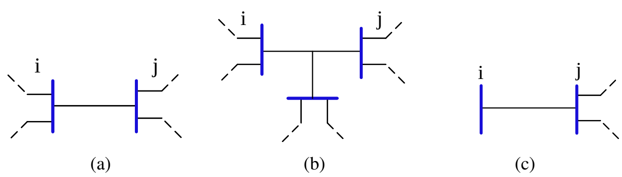

FIGURE 1 Three types of substations.

In Fig. 1(a), substations i and j are both connected to a two-terminal line. Assuming a PMU is only installed in substation i and a failure occurs on line $i-j$, the voltage and current at substation j can be acquired from other PMUs (installed in adjacent substations) using the indirect measurement principle. A large difference in the calculated voltages at both terminals indicates there is at least one failure on the line. Similarly, when a PMU is installed only in substation j, the voltage and current at substation i can be obtained from other PMUs installed in adjacent substations. However, if PMUs are not installed in substation i and j, the voltage of the two substations cannot be determined using the indirect measurement principle if a failure happens on line $i-j$. As a result, this faulted line cannot be identified. Therefore, it is necessary for at least one substation on two-terminal lines to be installed with a PMU. This requirement is represented as follows:

$$\sum_{i \in N} w_i b_{fi} \geq 1 \quad f \in M, \tag{1}$$

$$b_{fi} = \begin{cases} 1 & \text{if } i \text{ is connected to line } f, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where M is the set of two-terminal lines. b_{fi} represents a binary variable between substation i and line f, w_i is a binary decision term equal to 1 if substation i is installed with a PMU (otherwise it is 0). N is the set of substations.

An example of a three-terminal line is shown in Fig. 1(b). Information acquired from all three substations is needed to identify internal failures in a system of this type. Therefore, the substations on three-terminal lines certainly need to be installed with PMUs:

$$w_i = 1 \quad \forall i \in O, \quad (3)$$

where O is the set of substations on three-terminal lines.

Fig. 1(c) shows an independent substation connecting a one-terminal line. Assuming that a PMU is installed in substation j and that no PMUs are installed in substation i. If a failure then happens on line $i-j$, the voltage at substation i can only be calculated from line $i-j$ and cannot be compared with other values. The faulted line $i-j$ is then said to be unobservable. Therefore, it is essential for all substations on one-terminal lines to be installed with PMUs. This requirement can be expressed as:

$$w_i + g_i \geq 2, \quad (4)$$

$$g_i = \sum_{f \in M} b_{fi} \quad \forall i \in N, \quad (5)$$

where g_i is the number of lines connecting substation i. When g_i is equal to 1, w_i should also equal 1 to satisfy the constraint represented by Eq. (4), necessitating that substation i be equipped with a PMU.

2.2 | PMU Communication Requirements

The series of communication rules listed below can be implemented to overcome system-wide WABP latency.

2.2.1 | Each PMU communicates with at least one PC

PMUs in each assigned region should send measurement information to a regional PC. This can be expressed as:

$$a_{ij} = \begin{cases} 1 & \text{if bus } i \text{ communicates with bus } j, \\ 0 & \text{otherwise,} \end{cases} \quad (6)$$

$$\sum_{j \in C} a_{ij} \geq 1 \quad \forall i \in Q, j \in C, \quad (7)$$

where Q is the set of PMU installation locations and C is the set of substations where PCs are located. The left side of Eq. (7) represents the number of PCs that communicate with the PMU of substation i.

2.2.2 | Substations adjacent to a substation without a PMU should communicate with at least one common PC

Previous placement rules require that all substations adjacent to a substation not equipped with an PMU should be equipped with PMUs. Under such conditions, faulted lines can be identified when PMUs at adjacent substations are connected to at least one common PC. This requirement can be expressed as follows:

$$\prod_{j \in C} \left(\sum_{i \in N} a_{ki} a_{ij} - \sum_{i \in N} a_{ki} \right) = 0 \quad \forall k \in N, k \notin Q, k \notin C, \quad (8)$$

here $\sum a_{ki} a_{ij}$ is the number of the PMU of substation k communicating with the protection center in substation j and $\sum a_{ki}$ represents the number of adjacent substations of substation k. The number of PMUs which can connect to PC j at substation k is then equal to the number of substations to which substation k is connected.

2.2.3 | PMUs on two-terminal lines should be connected to at least one common PC

Identifying failures on a line equipped with PMUs requires obtaining information from two PMUs and a PC. As such, both PMUs should communicate with at least one common PC. This condition can be represented as:

$$\prod_{j \in C} \left(\sum_{i \in N} b_{fi} a_{ij} - 2 \right) = 0 \quad \forall f \in M, \quad (9)$$

where $\sum_{i \in N} b_{fi} a_{ij}$ represents the number of PMUs on the two-terminal line f communicating with the protection center j.

2.2.4 | PMUs on three-terminal lines should be connected to at least one common PC

If three terminals on a line are equipped with PMUs, these PMUs should also be connected to at least one common PC, such that:

$$\prod_{j \in C} \left(\sum_{i \in N} b_{fi} a_{ij} - 3 \right) = 0 \quad \forall f \in P. \quad (10)$$

where P is the set of three-terminal lines.

3 | RELIABILITY-BASED CONSTRUCTION OF WABP SYSTEMS

Based on PMU data, a WABP system can calculate the voltage value for the bus using the principle of indirect measurement from various paths. It then identifies the faulty line by comparing the differences in calculated voltage values[12]. Therefore, the availability of PMUs directly affects the reliability of WABP systems. This section first analyzes the impact of communication network failures on the reliability of WABP systems. WABP has strict requirements for data transmission delay, so latency is also a constraint that must be considered when designing a WABP system. Finally, a mathematical model is formulated to minimize the construction cost of PMU-based WABP systems.

3.1 | The Reliability of WABP Systems

When PMUs and communication links are deployed, they will influence the reliability of WABP system. To further describe the reliability, the IEEE 5-bus test case is used as an example below, for calculating the reliability in various PMU and CL placement scenarios. In Scheme A, shown in Fig. 2(a), PMUs are placed at substations 2, 3, and 5. A PC is located at substation 5 and CLs are deployed along lines 2-5, 3-4, and 4-5. In scheme B, shown in Fig. 2(b), PMUs are installed at substations 1, 2, and 4. A PC is also located at substation 5 and CLs are deployed along lines 1-2, 2-5, and 4-5.

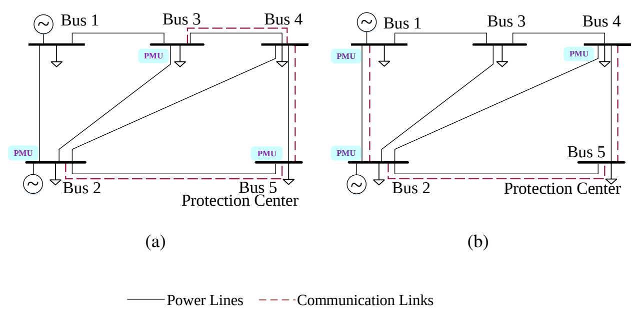

FIGURE 2 The proposed schemes applied to the IEEE 5-bus case.

In this section, p_d and q_d represent the reliability and failure probability of a given CL[16], respectively. These terms satisfy $p_d + q_d = 1$ and can be used to quantify the failure probability (r) for a communication network as follows:

$$r = \prod_{d \in S} p_d \prod_{d \in K} q_d, \tag{11}$$

where K is the set of damaged CLs and S is the set of normal CLs.

Since it is highly uncommon for more than three CLs to be damaged simultaneously, this situation is not considered here. The probability that all CLs are normal is given by:

$$r_0 = \prod_{1}^{V} p_d, \tag{12}$$

where V is the total number of CLs. The probability for a single damaged CL can be expressed as:

$$r_d = (\prod_{1}^{V-1} p_d) q_d, \tag{13}$$

and that of two damaged CLs is given by:

$$r_{d_1 d_2} = (\prod_{1}^{V-2} p_d) q_{d_1} q_{d_2}. \tag{14}$$

Equations (1)-(10) suggest all power lines are observable under these two scenarios (i.e., if some power lines fails, the failure can be detectable). The calculation is simplified by setting the reliability probability p_d for each CL to 0.99952[17]. Equations (11)-(14) are used to calculate r_0, r_d, and $r_{d_1 d_2}$. To further quantify the reliability of the WABP system, the undetectable probability of a power line is introduced.

TABLE 1 Probabilities of various failure states.

DCLs	NO	2-5	3-4	4-5	3-4 & 2-5	2-5 & 4-5	3-4 & 4-5
r	0.9986	$4.795e^{-4}$	$4.795e^{-4}$	$4.795e^{-4}$	$2.303e^{-7}$	$2.303e^{-7}$	$2.303e^{-7}$

Note: DCLs — Damaged Communication Links

Definition 1 - *the undetectable probability of a power line*
The term “undetectable probability of a power line” refers to the cumulative probability of the power line being undetectable in the event of communication link failures, taking into account the known PMUs and the deployment scheme of the corresponding communication links. Fault events included both single- and double-link failures, which are calculated as described above. The probability of a power line being undetectable can be expressed as follows:

$$W_f = \sum_{\varepsilon \in E} r_\varepsilon u_\varepsilon \tag{15}$$

where E represents all the fault events of communication links, as shown in the first row of Table I for the network shown in Fig. 2 (a). r_ε represents the probability of event ε, as shown in the second row of Table I for the network shown in Fig. 2 (a). u_ε is a binary variable equal to 1 if a power line f cannot be detected in the failure event ε; otherwise, it is 0. To calculate the undetectable probability of power lines, the following inferences can be drawn according to the PMU placement principle described in Eq.(1)-(5).

For a node N that has not deployed a PMU, assume that node N is connected to other m nodes, as shown in Fig. 3(a).

Inference 1: *If PMUs are deployed at all m nodes, all power lines can be detected.*

Inference 2: *If there are at least $m-1$ PMUs unavailable, all m power lines connected to node N become undetectable.*

Inference 3: *If fewer than $m-1$ PMUs are unavailable at the same time, only the power lines connected to the failed PMUs are undetectable.*

In Fig. 3(b), for a node P that deploys a PMU, the set of power lines connected to node P with only one side having a PMU is defined as set A, and the set of power lines with PMUs on both sides is defined as set B. The probability of power lines being undetectable in set A depends solely on the operational status of the PMU at node P.

Inference 4: *If any communication link failure event causes the PMU at node P to be unavailable, the power lines in set A will be undetectable.*

Inference 5: *When the PMU is unavailable at node P, the power lines in set B degrade to those in inference 1. Consequently, the undetectable probability of the power lines in set B is calculated based on inference 1.*

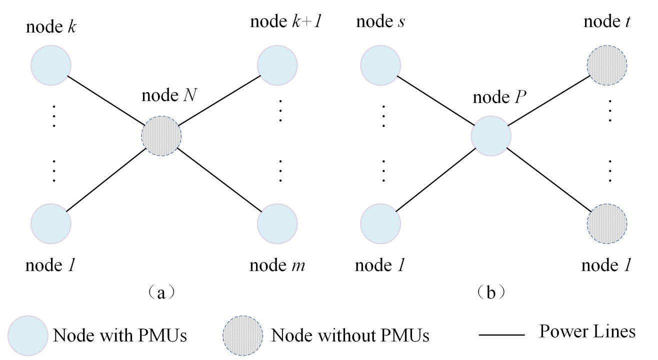

FIGURE 3 The node connection cases in the above inferences

According to inferences (1)-(5), in the IEEE 5-bus test case shown in Fig. 2, the possible communication link failure events for both schemes and the unavailable PMUs under each failure event, as well as the undetectable power lines, are shown in the Table II and Table III.

TABLE 2 Scheme A

Faulty communication links	The unavailable PMUs	The undetectable power lines
3-4	bus-3	1-2, 1-3, 2-3, 3-4
2-5	bus-2	1-2, 1-3, 2-4
4-5	bus-3	1-2, 1-3, 2-3, 3-4
3-4, 4-5	bus-3	1-2, 1-3, 2-3, 3-4
3-4, 2-5	bus-2, bus-3	all power lines
2-5, 4-5	bus-2, bus-3	all power lines

TABLE 3 Scheme B

Faulty communication links	The unavailable PMUs	The undetectable power lines
1-2	bus-1	1-2, 1-3
2-5	bus-1, bus-2	all power lines
4-5	bus-4	3-4, 2-4, 2-5, 4-5
1-2, 2-5	bus-1, bus-2	all power lines
1-2, 4-5	bus-1, bus-4	all power lines
2-5, 4-5	bus-1, bus-2, bus-4	all power lines

Definition 2 - *the reliability of the WABP system*

If a line fault cannot be detected, the entire WABP system will not function. The reliability of the WABP system refers to the probability that the system functions correctly. The reliability of the WABP system is determined by:

$$R = 1 - \sum_{f \in F} W_f \tag{16}$$

where F represents the set of all power lines in the WABP system, and f represents a power line.

The undetectable probabilities for all power lines and the reliability of the WABP system are calculated for the two schemes shown in Fig. 2, the results are provided in Table IV.

TABLE 4 Calculation results for the two presented schemes.

Schemes		Scheme A	Scheme B
undetectable probability of power lines	line 1-2	$1.473e^{-3}$	$9.957e^{-4}$
	line 1-3	$1.473e^{-3}$	$9.957e^{-4}$
	line 2-3	$9.957e^{-4}$	$4.982e^{-4}$
	line 2-4	$4.980e^{-4}$	$9.957e^{-4}$
	line 2-5	$4.606e^{-7}$	$9.957e^{-4}$
	line 3-4	$9.957e^{-4}$	$9.957e^{-4}$
	line 4-5	$4.606e^{-7}$	$9.957e^{-4}$
the reliability of the system		0.994523	0.993528

As seen in the table, the reliability of the WABP system for Scheme A is 0.994523, which is higher than that of Scheme B. Therefore, the placement of PMUs and CLs will affect the reliability of WABP system.

3.2 | Communication Latency

The communication latency between a PC and its associated PMUs can be expressed as follows[18]:

$$T = t_f + t_p + \frac{L}{R} + \theta, \tag{17}$$

where T is the total link delay, t_f is the fixed delay associated with transducer usage, DFT processing, data concentration, and multiplexing, t_p is the link propagation delay (the propagation time for electromagnetic waves in the channel), L is the transmitted data quantity, R is the data transmission rate for the link (considered to be unlimited for fiber-optic channels[19]), and θ is the associated random delay jitter. In this study, only t_p is considered for path hops. As a result, real-time calculations in a region are determined from the maximum hop between a PMU and its regional PC.

It is possible for more than one path to exist between a PMU and its corresponding PC. As such, Dijkstra's K-shortest path algorithm is adopted to calculate routing strategies using the hop constraint X_0[20]. In each region, the real-time propagation is determined from the maximum hop of all paths. For example, the maximum hop in the r^{th} region is given by:

$$X^r_{max} = \underset{1<s^r<M^r}{Max} X_{s^r t^r}. \tag{18}$$

In the r^{th} region, s^r represents a PMU, t^r denotes the PC, and M^r is the number of PMUs . The real-time operation of the WABP system can then be determined from the maximum hop in all regions, expressed as:

$$X_{\max} = \underset{1\leq r\leq N_{PC}}{Max} X^r_{max}, \tag{19}$$

where $X_{\max}$ is the real-time index of the WABP system. N_{PC} is the number of regions.

3.3 | Construction Costs

The total construction cost of a WABP system includes the cost of PMUs, PCs, and CLs, represented as follows:

$$E_{PMU} = e_{pmu} \times N_{PMU}, \tag{20}$$

$$E_{PC} = e_{pc} \times N_{PC}, \tag{21}$$

$$E_L = e_l \times L. \tag{22}$$

Here, E_{PMU} is the total cost of deploying PMUs, e_{pmu} is the cost of a single PMU, and N_{PMU} is the number of PMUs. E_{PC} is the cost of all PCs, e_{pc} is the cost of a single PC (including investment, operation, and maintenance costs). E_L is the cost of deploying CLs with a total length of L, and e_l is the CL cost for one kilometer.

3.4 | Formulation

The proposed WABP system can be developed using a series of conditions described as follows:

$$Min \quad Z = E_{PC} + E_{PMU} + E_L, \tag{23}$$

such that

$$R \geq 0.99, \tag{24}$$

$$X_{max} \leq X_0, \tag{25}$$

$$B_{ij} \leq {B_{ij}}^{max}, \tag{26}$$

$$\sum_{i\in Q} a_{ij} \leq h_{max} \quad \forall j \in C. \tag{27}$$

where Z is the total construction cost of a WABP system. The reliability constraint, represented by Eq. (25), assumes a reliability threshold of 0.99. The latency constraint is given by Eq. (26), where X_0 is the maximum hop. The bandwidth constraint for each CL is provided in Eq. (27), where B_{ij} and B^{max}_{ij} are the required bandwidth and maximum capacity of link $i-j$, respectively. Finally, a constraint limiting the number of PMUs communicating with the same PC (i.e. the balance requirement between regions) is represented by Eq. (28), where h_{max} is the maximum number of PMUs that communicate with the PC in each region.

4 | THE PROPOSED METHOD

Genetic algorithms have been used as adaptive techniques for solving various practical scientific and engineering problems[21 22]. Previous studies have demonstrated that such techniques can converge to global optimum for large-scale NP-hard problems[23]. However, since the placement of PMUs and CLs is implicitly adjusted with variations in PC placement, it is necessary to first determine the set of candidate PCs to reduce computational complexity. Therefore, the first step of the proposed method is to use the clustering algorithm to generate multiple power system partitioning schemes based on the set of candidate PCs. Next, the minimum number and location of PMUs are determined according to Eq.(1-5) for each partitioning scheme. Finally, we propose an improved genetic algorithm to minimize construction costs for WABP systems.

4.1 | Clustering substations for Partitioning

Substations of a higher degree exhibit a higher probability of being a PC (i.e., the center of a cluster). The degree of a substation is calculated by,

$$D_i = \sum_{i,j\in N} a_{ij}, \tag{28}$$

All substations are sorted in descending degrees and the set of the sorted substations is represented by D. The substations that are ranked higher in D will be selected as the preferred candidate PC. In large-scale power grids, the number of clusters can be automatically determined to minimize construction costs for WABP systems. If the number of nodes is given by n, the minimum number clusters is 1 and the maximum number is $\sqrt{n}$. consequently, to ensure diversity of partitioning, the number of candidate PCs was set to $N_J = (1.5 \sim 2)\sqrt{n}$[24]. In addition, candidate PCs were not intentionally adjacent to ensure a uniform distribution.

Once the location of PCs is determined, a K-Means clustering algorithm can be used for partitioning a power grid into multiple regions. The following equation is used for measuring the distance between a substation and all the candidate PCs.

$$d_{i\hat{r}} \leftarrow \arg\min_{r} d_{ir}, i \in Q. \tag{29}$$

where $d_{i\hat{r}}$ is the minimum distance between the substation and all PCs, and d_{ir} is the distance between the substation and the rth PC. v_r and h_r are defined to represent the center and number of substations in the rth region. v_r is updated according to the calculated results in the rth region[25]. The pseudo-code for partitioning substations can be expressed in Algorithm 1.

Algorithm 1 Clustering substations for partitioning

Input: N_Num: the number of substations;
Output: J: candidate set of PCs; S_r: set of substations in the r^{th} region;
$S_r \leftarrow \emptyset$, $i \leftarrow 0$, $r \leftarrow 0$, $v_r \leftarrow 0$, $h_r \leftarrow 0$;
repeat
  $d_{i\hat{r}} \leftarrow \arg\min d_{ir}$;
  $S_{\hat{r}} \leftarrow i \cup S_{\hat{r}}$;
  update v_r;
  $h_r \leftarrow h_r + 1$;
  $i \leftarrow i + 1$;
until ($i \leq N\_Num$);

4.2 | Placing PMUs

To satisfy the observation of all power lines, the PMU placement principle described in Eq.(1-5) suggests that the minimum number of PMUs and their location should be determined. A specific pseudo-code for this approach is as follows:

Algorithm 2 Placing the minimum number of PMUs

Input: N_Num: number of substations;
Output: $\bar{N}$: set of substations with PMUs;
Initialization $i \leftarrow 1; \bar{N} \leftarrow \phi$;
Substations are divided into type-a, type-b, and type-c: t_a, t_b, and t_c;
Place PMUs at each substation;
repeat
  if $i \in t_a$ **then**
    delete i from $\bar{N}$;
    if any line cannot be observed **then**
      $\bar{N} \leftarrow i \bigcup \bar{N}$;
    end if
  end if
until $i <= N\_Num$;

4.3 | Genetic Algorithm

4.3.1 | Encoding

In the proposed technique, a candidate set of PCs is used as a chromosome, which consists of a number of gene instances given by $\gamma = (\gamma_1, ...\gamma_{N_J})$. The value of a gene will be 1 if a substation is equipped with a PC (otherwise it is 0). Here, the IEEE bus-14 testing case is used as an example to explain the encoding. The description above can be used to determine $J = \{1, 2, 4, 6, 10, 14\}$. Bus-4 and bus-10 are chosen for original protection centers. Thus, the number of regions is 2 and $\gamma = (0, 0, 1, 0, 1, 0)$.

4.3.2 | Population

The initial population consists of partitioning schemes (i.e., chromosomes or individuals). Combinations of these chromosomes, from the initial population, must cover all possible partitioning schemes. This ensures that reproduction operators will produce as new chromosomes that define all possible partitions from one generation to the next. However, it is possible that a candidate scheme may not satisfy the reliability requirement. As such, a substation will be randomly selected to be equipped with an PMU and the corresponding CL placed until the reliability condition is satisfied.

4.3.3 | Fitness Function

According to the characteristics of genetic algorithm "survival of the fittest", the larger the fitness function, the easier it is for individuals with low cost to survive. Therefore, the fitness value of each chromosome is calculated by the following equation, given by:

$$f = \frac{1}{Z}. \tag{30}$$

In the iterative process of genetic algorithm, a candidate set of PCs with an optimal fitness value is preserved for the next generation. A flowchart of this improved genetic algorithm is provided in Fig. 4.

5 | NUMERICAL SIMULATIONS

The simulations are organized as follows. First, the proposed approach is compared with the model developed by[12]. Second, the technique is evaluated using the IEEE 14-bus, 30-bus, 39-bus, 57-bus, and 118-bus test cases. Finally, the sensitivity is analyzed using variations of certain constraints. In power systems, optical power ground wires (OPGWs) including one or more optical fibers are often used as communication channels.

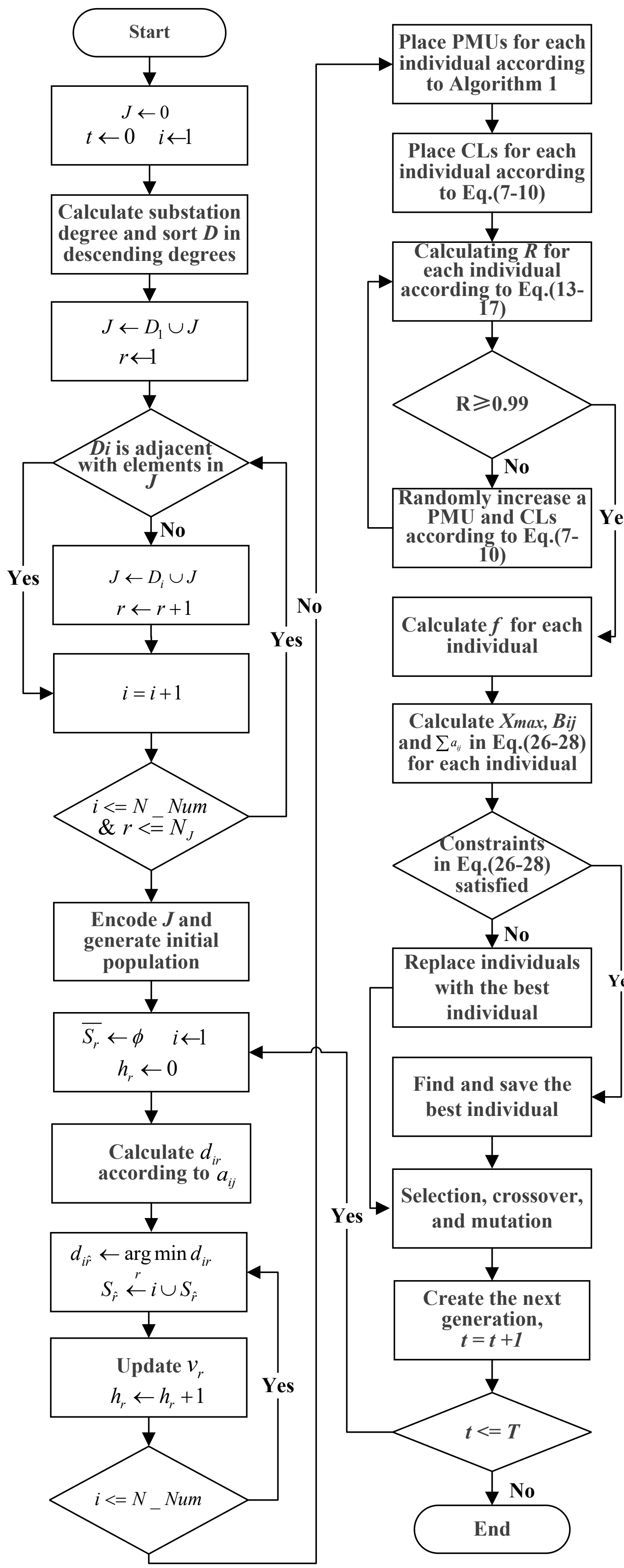

FIGURE 4 A flowchart for the improved genetic algorithm.

They are built along with power systems, which will reduce maintenance and construction costs. Partitioning calculations can be simplified by assuming that information is transmitted over OPGWs. Hence, the topology of power communication networks is same to that of power grids. Link expenses for installing communication cables are also avoided, resulting in a link laying cost of e_l = \$600 per kilometer[25]. The cost of a PMU is then \$5,000 and the cost of each PC is set to e_{pr} = \$8,000[25]. The distance between substations is set using a technique developed by Shahraeini et al.[26]. Calculations are simplified by assuming that all power lines exhibit the same configuration. The bandwidth B_{ij}^{max} of each CL is randomly generated in the range [$1Mbit/s$, $40Mbit/s$] and the traffic for each PMU is randomly distributed, ranging from $1Mbit/s$ to $4Mbit/s$.

5.1 | Performance comparison

In the experiments, the maximum number of PMUs allowed to communicate with the same PC is set to 30, with a hop of X_0=4. A comparison of simulated results is given in Table V.

TABLE 5 A comparison of simulated results.

Cases	The proposed method			the reference [12]		
	NR	NL	PMUs	NR	NL	PMUs
IEEE 14	1	13	10	1	8	8
IEEE 30	1	25	21	1	17	17
IEEE 57	6	63	37	2	39	30
IEEE 118	11	136	89	3	86	64

Note: NR — Number of Regions NL — Number of Links

Table V suggests that in test cases with a small number of nodes (e.g., IEEE 14-bus and 30-bus), partitioning results are similar to those of Zare al.[12]. It is then highly probable that one region is sufficientto satisfy the delay constraints for all PMUs. As the number of nodes increases, the proposed approach produces more regions than the comparative model developed by[12]. This evidence suggests that a single region cannot satisfy PMU delay constraints. In this study, the proposed method includes the actual costs of PCs, CLs, and PMUs. However, numerical simulations of Zare's model tested the effects of varying weights and the difference between the costs of PMUs and PCs (the cost of CL placement was not considered). These simulations produce differing region quantities. In addition, the number of PMUs and placed communication links exceeds that of the comparison study[12], due to the consideration of reliability constraints that require the placement of additional PMUs. The number of placed CLs also exceeds that of Zare et al., since this quantity is related to the number of PMUs[12].

Comparison results for the reliability tests are discussed in Fig.5. As seen from the figure, the reliability of the proposed approach is higher than that of the comparison study [12]. This is because the reliability of a WABP system is used as a constraint in the optimization model, while the reliability was not considered in reference [12]. This issue becomes more prominent as the number of nodes and CLs increases. The reliability reported by Zare et al. for the IEEE 30-bus case is 0.96246 [12], while the reliability of the proposed technique is 0.994. In addition, the gap between these two techniques becomes larger with an increasing number of nodes. For example, the reliability for the IEEE 118-bus case is 0.9914 using the proposed method and only 0.6241 in the comparison study [12]. Furthermore, the current study includes more PMUs and CLs, thereby increasing practicality and reliability.

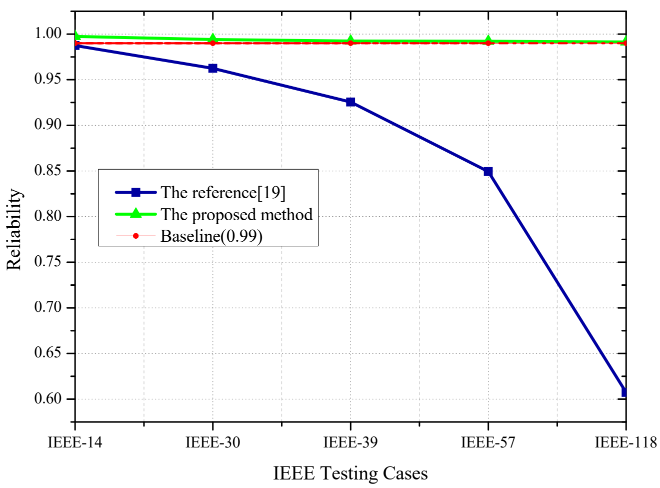

FIGURE 5 Reliability comparison results.

5.2 | Evaluation of the proposed method

In this section, we evaluate the proposed technique using various IEEE test cases. The maximum number of PMUs used to communicate with the same PC is set to 30 and X_0 is set to 4. Results for the IEEE 39-bus case are shown in Fig. 6.

The test case shown in Fig. 6 only includes type-a and type-c substations, there are no three-terminal lines. For example, two substations on line 1-39 belong to type-a, substation 1 is placed with a PMU, and substation 39 does not need to be placed with a PMU. In contrast, since substation 35 is connected to only one link, line 22-35 should be placed with a PMU. As such, the IEEE 39-bus case can be divided into three regions. In order to comply with the principle of independent protection, these regions include multiple repeated substations. For example, substation 17 belongs to two regions as it can be used to observe both line 17-27 and 16-17. However, two PCs are required to observe failures in the two transmission lines. Thus, measurement data for substation 17 must be sent to the PCs located at substations 16 and 25. Specific PMU placement and partitioning results for other IEEE cases are shown in Table VI.

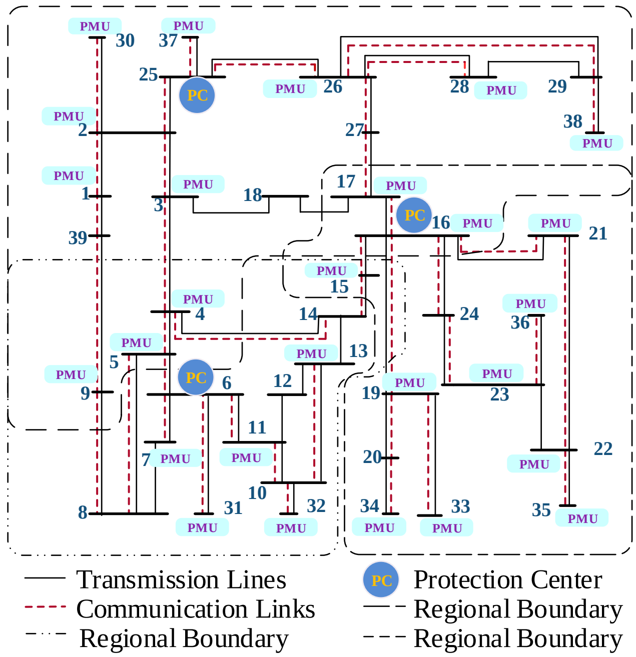

FIGURE 6 Results for the IEEE 39-bus system.

5.3 | Sensitivity Analysis

In this section, the results of simulation experiments are evaluated for IEEE test cases in which h_{max} and X_0 vary. First, h_{max} is set to 15, 30, 45, and 60, while other parameters remain unchanged. Corresponding results are given in Table VII.

As seen in the table, the number of regions in large-scale systems decreases as h_{max} increases. For example, the number of regions is 11 when h_{max} is set to 15 and 30 in the IEEE 118-bus case. As h_{max} increases to 60, the total number of regions decreases 5. However, the number of regions remains unchanged for the IEEE 14-bus case. This is because partitioning is limited by the maximum number of PMUs that communicate with the same PC, even as the number of PMUs becomes larger. Thus, increasing h_{max} will only reduce the number of regions.

Second, X_0 is set to 3, 4, 5, and 6, while other parameters remains unchanged. Corresponding results for IEEE test cases are shown in Table VIII.

As shown in the table, the value of X_0 inversely affects the number of regions, which decreases as X_0 increases. Taking the

TABLE 6 Simulated results using various IEEE test cases.

System	Cost	R	Number of Regions	Number of Links	Protection Center	PMUs
IEEE14	52	0.9975	1	13	4	Region 1: 1 2 3 4 6 8 9 10 12 13
IEEE30	93.5	0.9940	1	25	6	Region 1: 1 2 4 5 6 8 10 11 12 13 14 16 18 19 21 22 23 24 26 27 29
IEEE39	151.5	0.9944	3	38	16	Region 1: 15 16 17 19 20 21 22 23 24 33 34 35 36
					6	Region 2: 4 5 6 7 8 9 10 11 12 13 14 15 31 32
					25	Region 3: 1 2 3 4 5 9 16 17 18 25 26 27 28 29 30 37 38 39
IEEE57	255.5	0.9901	6	63	13	Region 1: 1 3 4 9 11 12 13 14 15 16 17 41 44 45 46 47 49
					19	Region 2: 9 10 12 13 20 21 22 23 24 36 37 38 39 40 44 47 48 49 50 51 56
					37	Region 3: 6 7 8 9 10 11 12 13 39 40 41 42 43 54 55 56 57
					26	Region 4: 24 25 30 31 32 33 34 35 36
					32	Region 5: 6 7 24 26 27 28 29 52 53
					8	Region 6: 1 2 3 4 5 6 7 18 19 20
IEEE118	540.5	0.9916	11	136	12	Region 1: 12 1 2 3 4 6 7 11 14 117 16 5 13 15
					30	Region 2: 17 5 8 9 10 16 26 30 15 18 31 113 3 4 6 11 25 28 31 38
					37	Region 3: 37 33 34 35 36 39 40 41 42 43 44 38 15 49
					59	Region 4: 59 54 55 56 57 58 60 61 62 63 49 53 64 67
					89	Region 5: 85 83 84 86 87 88 89 90 91 92 93 102 82 94 100 101
					32	Region 6: 32 22 23 24 25 27 28 31 113 114 115 70 26 17
					80	Region 7: 82 77 78 79 80 81 94 95 96 97 98 99 118 83 69 75 76 68 92 100
					103	Region 8: 110 100 101 103 104 105 106 107 108 109 111 112 92 94 98 99
					19	Region 9: 19 13 15 18 20 21 34 11 14 17 33 17 22
					49	Region 10: 46 45 47 48 49 50 51 52 53 44 69 54 69
					69	Region 11: 65 38 64 66 67 68 69 70 71 72 73 74 75 76 116 37 61 63 49 62 47 24

TABLE 7 The number of regions for varying h_{max} values.

System	Number of regions			
	h_{max}=15	h_{max}=30	h_{max}=45	h_{max}=60
IEEE14	1	1	1	1
IEEE30	2	1	1	1
IEEE39	3	3	2	2
IEEE57	6	6	4	3
IEEE118	11	11	7	5

TABLE 8 The number of regions for varying values of X_0.

System	Number of regions			
	X_0=3	X_0=4	X_0=5	X_0=6
IEEE14	2	1	1	1
IEEE30	3	1	1	1
IEEE39	3	3	2	2
IEEE57	9	6	4	3
IEEE118	15	11	7	7

IEEE 118-bus as an exmaple, when X_0 is set to 3 and 4, the number of regions is 15 and 11, respectively. As X_0 increases to 5 and 6, the total number of regions decreases to 7. When X_0 is set to 3 in the IEEE 14-bus case, the number of regions is 2. As X_0 increases to 4, 5, and 6, the total number of regions decreases to 1. This suggests that when the delay constraint X_0 is small, the number of regions cannot satisfy real-time PMU constraints for the PC, making it necessary to increase the number of regions. However, as X_0 increases, the number of regions remains constant. It may then be reasonable to suppose that as X_0 becomes larger, h_{max} and the reliability constraints will dominate the partitioning results.

6 | CONCLUSION

In this paper, the partitioning of large-scale power grids is formulated to decrease WABP system construction costs by simultaneously optimizing the placement of specific CLs and the partitioning of PMUs. In this optimization problem, the locations and quantities of PMUs, PCs, and CLs are simultaneously determined. An improved genetic algorithm is then proposed to identify an optimal solution. A clustering model is first adopted to calculate candidate numbers and locations for PCs, thereby reducing the computational complexity. Numerical simulations are then conducted using IEEE test cases. Experimental results demonstrate the practicality of the proposed technique. While only link failures are considered in this paper, no limits are imposed on the number or origin of failures. As such, other equivalent failure types can easily be incorporated into the optimization model.

REFERENCES

1. Koloushani, S.M., Taher, S.A.: Dynamic wide-area cooperative protection: A new approach. *IET Gener. Transm. Distrib.* 00, 1-14 (2023).
2. Lin, X., Ma, S., Jin, N., Tong, N., Chen, L., Li, Z., Rong, Z. and Zhang, P. (2020), Compensation voltage moduli comparison-based wide-area protection algorithm independent of data synchronism and integrity. *IET Gener. Transm. Distrib.*, 14: 6682-6689.
3. M. M. Eissa, M. E. Masoud and M. M. M. Elanwar, "A Novel Back Up Wide Area Protection Technique for Power Transmission Grids Using Phasor Measurement Unit," *IEEE Trans. Power Del.*, vol. 25, no. 1, pp. 270-278, Jan. 2010.
4. J. Ma, C. Liu and J. S. Thorp, "A wide-area backup protection algorithm based on distance protection fitting factor," *IEEE Trans. Power Del.*, vol. 31, no. 5, pp. 2196-2205, Oct. 2016.
5. P. K. Nayak, A. K. Pradhan and P. Bajpai, "Wide-area measurement-based backup protection for power network with series compensation," *IEEE Trans. Power Del.*, vol. 29, no. 4, pp. 1970-1977, Aug. 2014.
6. M. K. Neyestanaki and A. M. Ranjbar, "An Adaptive PMU-Based Wide Area Backup Protection Scheme for Power Transmission Lines," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1550-1559, May 2015.
7. S. Azizi, G. Liu, A. S. Dobakhshari and V. Terzija, "Wide-Area Backup Protection Against Asymmetrical Faults Using Available Phasor Measurements," *IEEE Trans. Power Del.*, vol. 35, no. 4, pp. 2032-2039, Aug. 2020.
8. M. Rezaei Jegarluei, P. Aristidou, W. Fernandes and S. Azizi, "Wide-Area Backup Protection Using Sparse Synchronized/Unsynchronized PMU Measurements," *IEEE Trans. Power Del.*, doi: 10.1109/TPWRD.2023.3249822.
9. Shalini and S. R. Samantaray, "A Differential Voltage-Based Wide-Area Backup Protection Scheme for Transmission Network," *IEEE Syst. Journal*, vol. 16, no. 1, pp. 520-530, Mar. 2022.
10. M. R. Jegarluei, A. S. Dobakhshari and S. Azizi, "Reducing the Computational Complexity of Wide-Area Backup Protection in Power Systems," *IEEE Trans. Power Del.*, vol. 37, no. 3, pp. 2421-2424, Jun. 2022.
11. M. H. Rezaeian Koochi, P. Dehghanian and S. Esmaeili, "PMU Placement With Channel Limitation for Faulty Line Detection in Transmission Systems," *IEEE Trans. Power Del.*, vol. 35, no. 2, pp. 819-827, April 2020.
12. J. Zare, F. Aminifar and M. Sanaye-Pasand, "Communication-Constrained Regionalization of Power Systems for Synchrophasor-Based Wide-Area Backup Protection Scheme," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1530-1538, May 2015.
13. Yu Hai, Yu Yue, Yao Qigui and Yin Haiqing, "Analysis on the Reliability of Wide Area Protection Communication System," *2012 IEEE International Conference on Communication Technology (ICCT)*, Chengdu, 2012, pp. 329-333.
14. Z. -H. Dai, Z. -P. Wang and Y. -J. Jiao, "Reliability Evaluation of the Communication Network in Wide-Area Protection," *IEEE Trans. Power Del.*, vol. 26, no. 4, pp. 2523-2530, Oct. 2011.
15. R. He, S. Yang, J. Deng, T. Feng, L. L. Lai and M. Shahidehpour, "Reliability Analyses of Wide-Area Protection System Considering Cyber-Physical System Constraints," *IEEE Trans. Smart Grid*, vol. 12, no. 4, pp. 3458-3467, July 2021.
16. Y. Wang, C. Wang, W. Li, J. Li and F. Lin, "Reliability-Based Incremental PMU Placement," *IEEE Trans. Power Syst.*, vol. 29, no. 6, pp. 2744-2752, Nov. 2014
17. C. Grigg et al., "The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010-1020, Aug. 1999.
18. B. Naduvathuparambil, M. C. Valenti and A. Feliachi, "Communication delays in wide area measurement systems," *Proc. the Thirty-Fourth Southeastern Symposium on System Theory*, Huntsville, AL, USA, 2002, pp. 118-122.
19. Radio Subcommittee of IEEE Power System Communications Committee, "Analog/digital microwave considerations for electric/gas utilities," *IEEE Trans. Power Del.*, vol. 8, no. 3, pp. 798-815, Jul. 1993.
20. Chen, Y.L., "An algorithm for finding the K-quickest paths in a network", *Computing Operation Research*, vol. 20, pp. 59-65, 1993.
21. L. Davis, *Handbook of Genetic Algorithms.* Van Nostrand Reinhold, 1991.
22. J.J. Grefenstette, "Genetic Algorithms for Changing Environments," *Proc. Second Int'l Conf. Parallel Problem Solving Nature*, pp. 137-144, 1992.
23. B. Lorenzo and S. Glisic, "Optimal Routing and Traffic Scheduling for Multihop Cellular Networks Using Genetic Algorithm," *IEEE Trans. Mobi. Comp,*, vol. 12, no. 11, pp. 2274-2288, Nov. 2013.
24. X. Wang and W. Sheng, "Adaptive usage of K-means in evolutionary optimized data clustering," *2017 International Conference on Machine Learning and Cybernetics (ICMLC)*, Ningbo, 2017, pp. 15-20.
25. H. Jia, C. Liu, Y. Gai and J. Ma, "Simultaneous Optimization of Optical Communication Link Placement and Partitioning of Wide-Area Protection System in Smart Grids," *IEEE Trans. Power Del.*, vol. 36, no. 5, pp. 3140-3149, Oct. 2021.
26. M. Shahraeini, M. S. Ghazizadeh and M. H. Javidi, "Co-Optimal Placement of Measurement Devices and Their Related Communication Infrastructure in Wide Area Measurement Systems," *IEEE Trans. Smart Grid*, vol. 3, no. 2, pp. 684-691, Jun. 2012.