# Supporting Information for "Searching for partial ruptures in Parkfield"

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## Text S1: Scaling relations accounting for partial ruptures

We aim to determine how much slip is predicted to be in partial ruptures. To do so, we again consider repeating earthquakes and partial ruptures occurring on locked patches loaded by the surrounding creeping fault. The slip in each earthquake (S) should match the surrounding fault creep ($V_{creep}$) that has been loading the asperity in the time since the last event ($T_r$), e.g. $S = V_{creep}T_r$. We follow the arguments of Cattania (2019), that the behaviour of the repeating earthquakes is controlled by the ratio of the slip required to nucleate an event ($S_n$) and the slip required for the earthquake to rupture the full patch ($S_{full}$). $S_n$ increases as the nucleation size ($R_\infty$) increases, whereas $S_{full}$ increases with the size of the patch ($R$). If the slip required to nucleate an event is larger than the slip required for a full rupture, $S_n > S_{full}$, all ruptures are full ruptures. If the slip required to nucleate an event is smaller than the slip required for a full rupture, $S_n < S_{full}$, partial ruptures can occur. We define the ratio:

$$\frac{S_n}{S_{full}} \propto \sqrt{\frac{R_{inf}}{R}} \tag{1}$$

The ratio of the slip required to nucleate an event to the slip required for a full rupture is proportional to the square root of nucleation size divided by the asperity size. When this ratio is less than 1, partial ruptures can occur. The minimum slip for a partial rupture is $S_n$, although in each cycle, there may be more than one partial rupture, therefore more slip. Therefore, the ratio $\frac{S_n}{S_{full}}$, gives the minimum ratio of partial slip to full slip. Partial ruptures occur from $\sim$ R = $4.3R_\infty$ to $100R_\infty$, so the slip ratio varies from approximately 0.7 to 0.2. **Text S2: Scaling relations accounting for partial ruptures**

To find recurrence interval as function of $T$, we write:

$$M_0 = \mu\pi S_{eq}R^2 \sim \alpha(R)TV_{pl}R^2, \tag{2}$$

where where $K_c$ = fracture energy, $\phi$=geometrical factor accounting for rupture shape, $\mu'$ effective shear modulus (depending on mode II, III), $V_{pl}$ is the loading velocity, $S_{eq}$ is the average coseismic slip and $\alpha(R) = Seq/TV_{pl}$ is the fraction of slip deficit released in the earthquake. With $T \sim \sqrt{R}$,

$$M_0 \sim \alpha(R)R^{5/2} \sim \alpha(R)T^5 \tag{3}$$

Alternatively, we can write:

$$M_0 \sim \Delta\tau(R)R^3, \tag{4}$$

where $\Delta\tau(R) \sim \alpha(R) T V_{pl}/R \sim \alpha(R) R^{-1/2}$. Cattania and Segall (2019) assumed constant $\Delta\tau$, which gives: $T \sim M_0^{1/6}$, and implies:

$$\alpha(R) \sim R^{1/2}, \tag{5}$$

implying that larger repeaters take up a larger fraction of the slip deficit (consistent with the results from Appendix B). This is also similar to the interpretation from (Chen & Lapusta, 2009), in which a larger fraction of slip is taken up by aseismic slip for small events.

If instead we assumed constant $\alpha$ (for example $\alpha = 1$, which implies that all slip deficit is released during full ruptures), we would get $M_0 \sim R^{5/2}$ so that $T \sim M_0^{1/5}$. Note that this would also imply scale dependent stress drops: $\Delta\tau \sim R^{-1/2} \sim M_0^{-1/5}$.

## References

Cattania, C. (2019). Complex earthquake sequences on simple faults. *Geophysical Research Letters*, *46*(17-18), 10384–10393.

Cattania, C., & Segall, P. (2019). Crack models of repeating earthquakes predict observed moment-recurrence scaling. *Journal of Geophysical Research: Solid Earth*, *124*(1), 476–503.

Chen, & Lapusta, N. (2009). Scaling of small repeating earthquakes explained by interaction of seismic and aseismic slip in a rate and state fault model. *Journal of Geophysical Research: Solid Earth*, *114*(B1).

Waldhauser, F., & Schaff, D. P. (2021). A comprehensive search for repeating earthquakes in northern california: Implications for fault creep, slip rates, slip partitioning, and transient stress. *Journal of Geophysical Research: Solid Earth*, *126*(11), e2021JB022495.

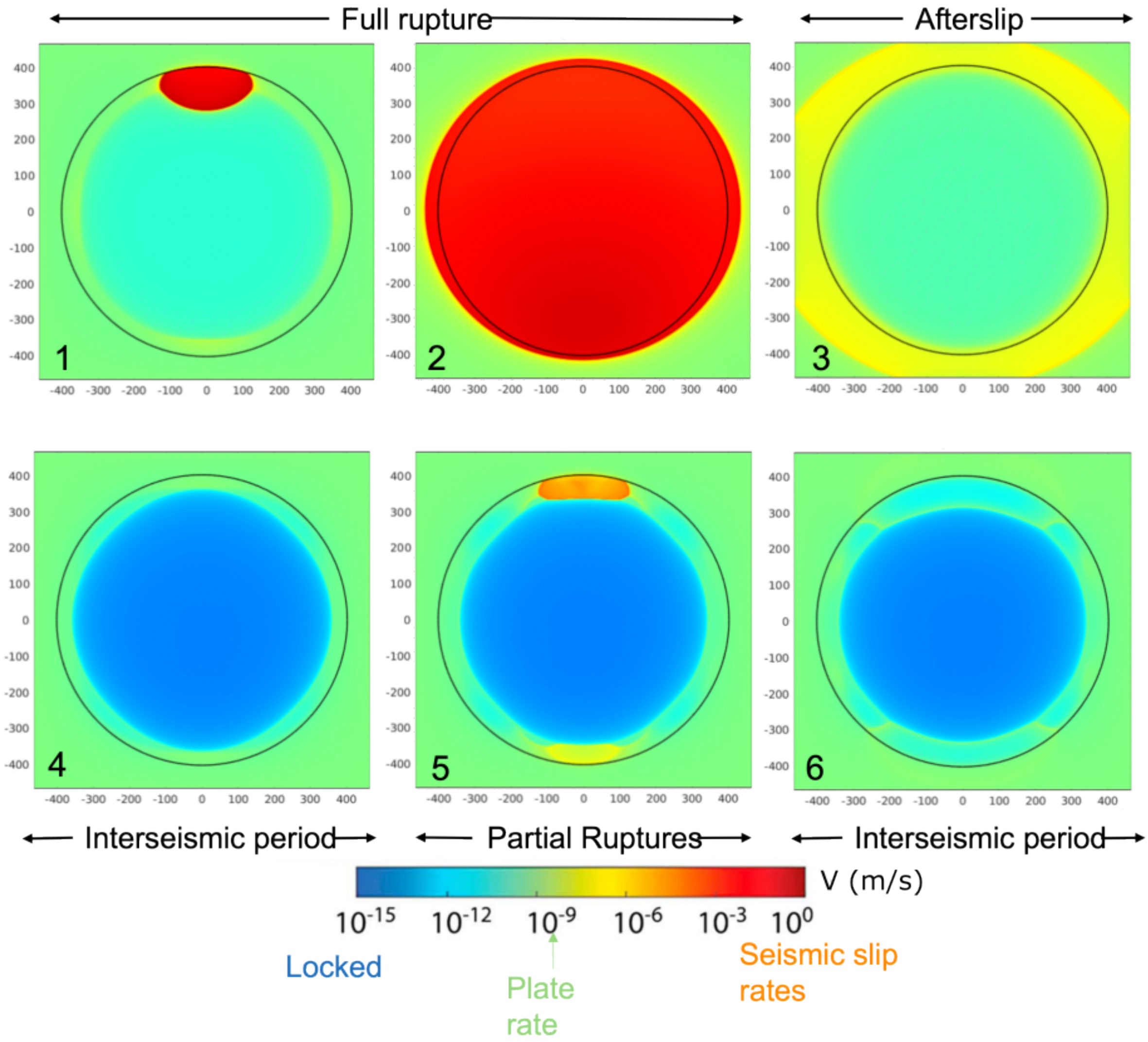



**Figure S1.** Example of a full rupture on an asperity of size R $= 8R_\infty$, from Cattania and Segall (2019).The colour indicates the slip speed. In this model, an event nucleates and ruptures the entire asperity. In the interseismic period, the asperity is locked. A creep front slowly erodes the asperity. In the bottom row, a partial rupture nucleates, and then the asperity is locked again.

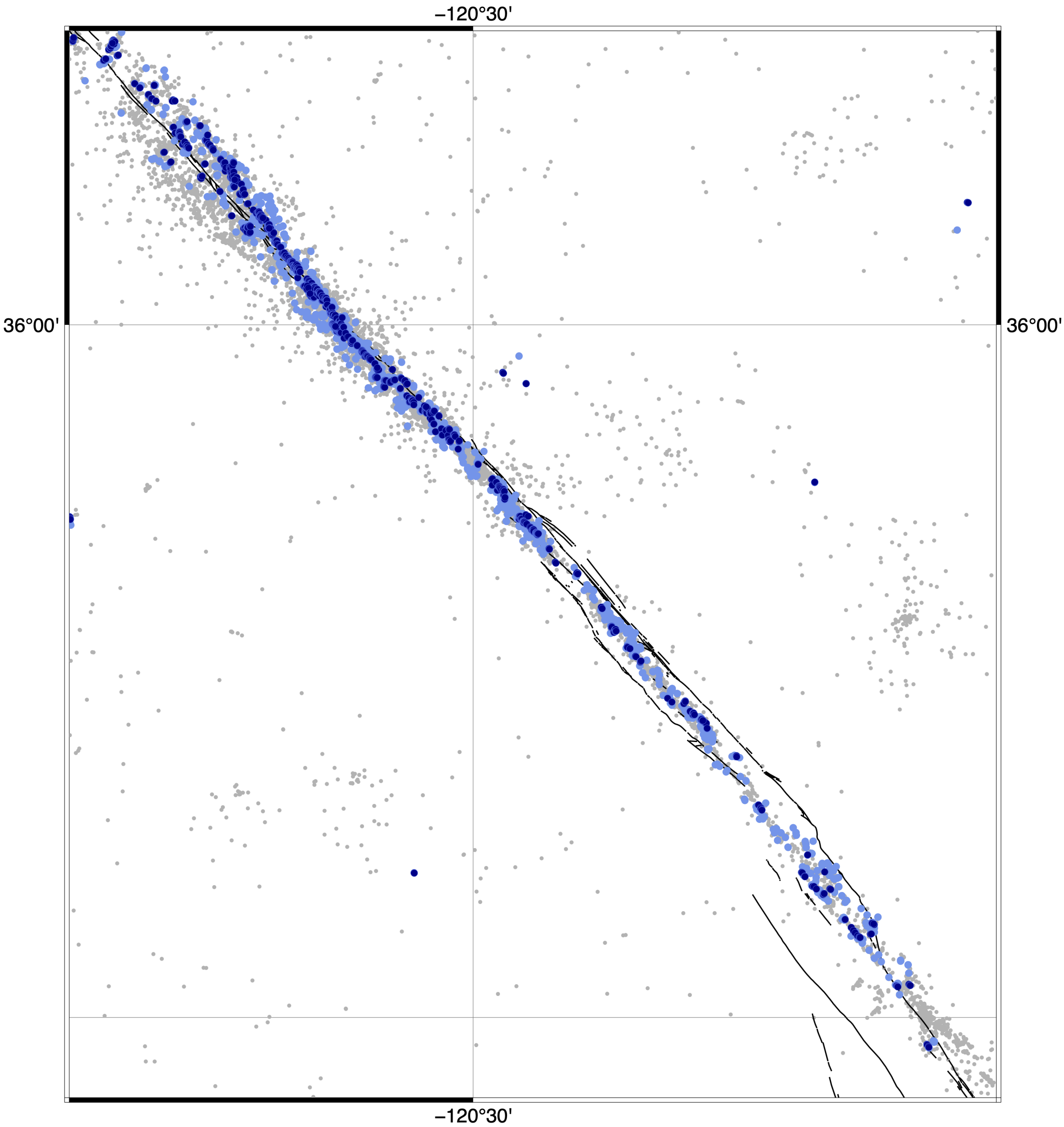



**Figure S2.** Seismicity of the Parkfield region. Grey dots show the events in the Northern California seismic network double-difference relocated catalogue. Dark blue events are repeating earthquakes identified by waldhauser2021comprehensive. Light blue events are repeating earthquakes identified in this study. Faults plotted from the USGS Quaternary faults and folds database.

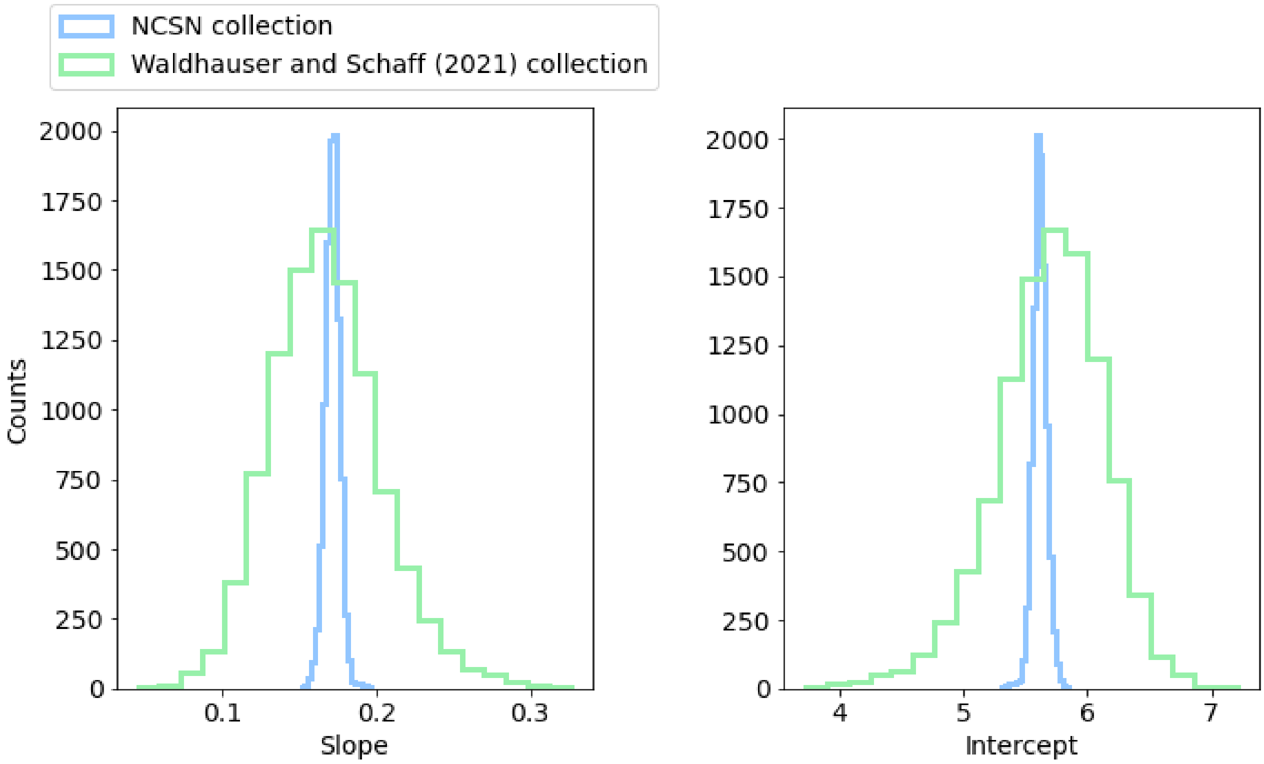



**Figure S3.** Bootstrapped values of the slope and intercept of the The best-fitting line of the moment-recurrence scaling.

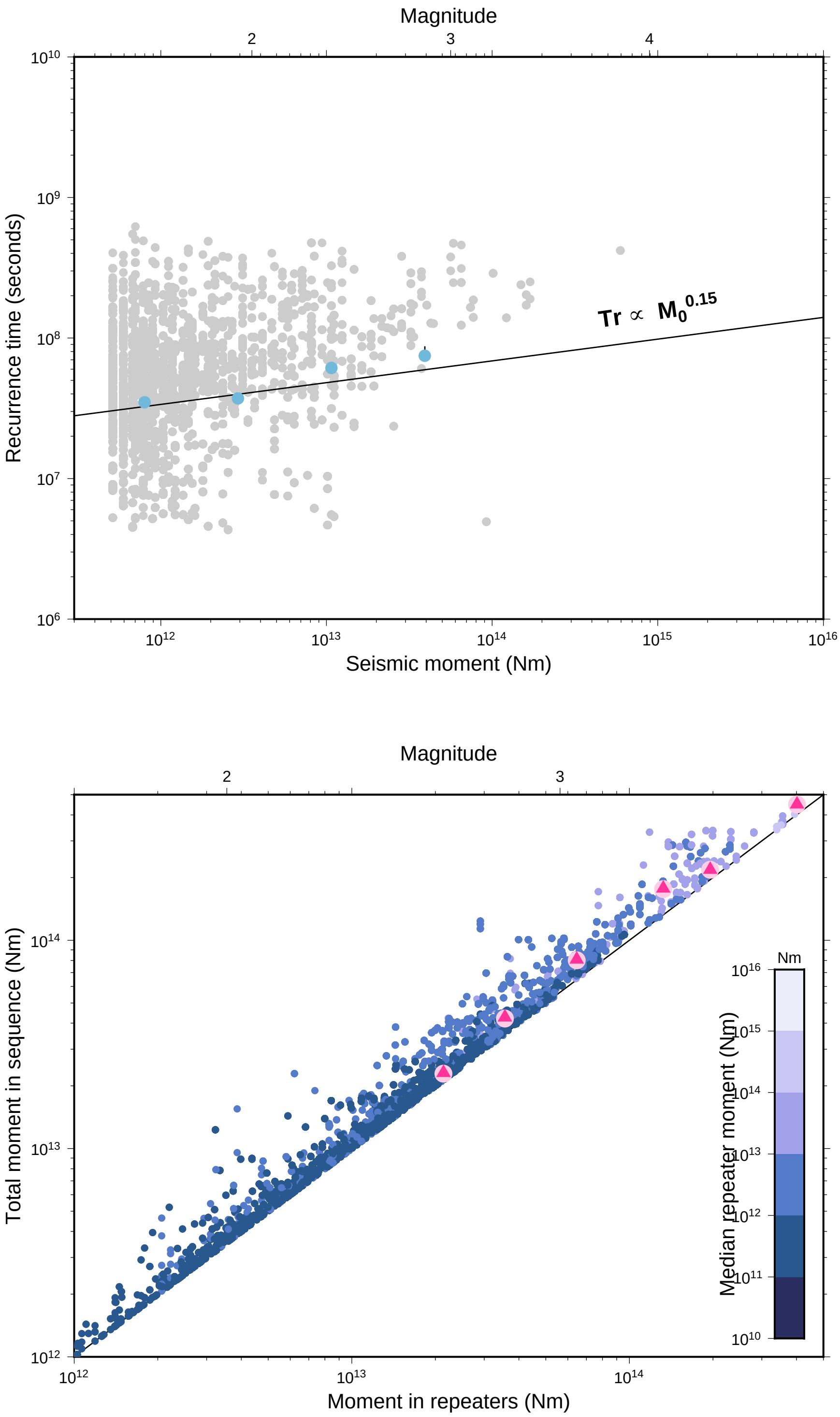



**Figure S4.** (a) Median recurrence interval versus median moment for pairs of repeating earthquakes from the relocated Northern California catalogue waldhauser2013real on a log-log scale assuming a stress drop of 3 MPa. Individual values are plotted as grey circles, with medians for moment bins shown as blue circles. The error bars on the medians indicate 95% confidence limits, which were estimated via bootstrapping (details in the text). The best-fitting line is plotted in solid black with a gradient of 0.15 (b) The total moment in repeating earthquakes compared to the total moment, including repeating earthquakes and partial ruptures. Each point is coloured by the median moment of the co-located repeating earthquakes. Light pink points are the binned means. The dark pink triangles are the binned means corrected for missing small events (see text for more details).

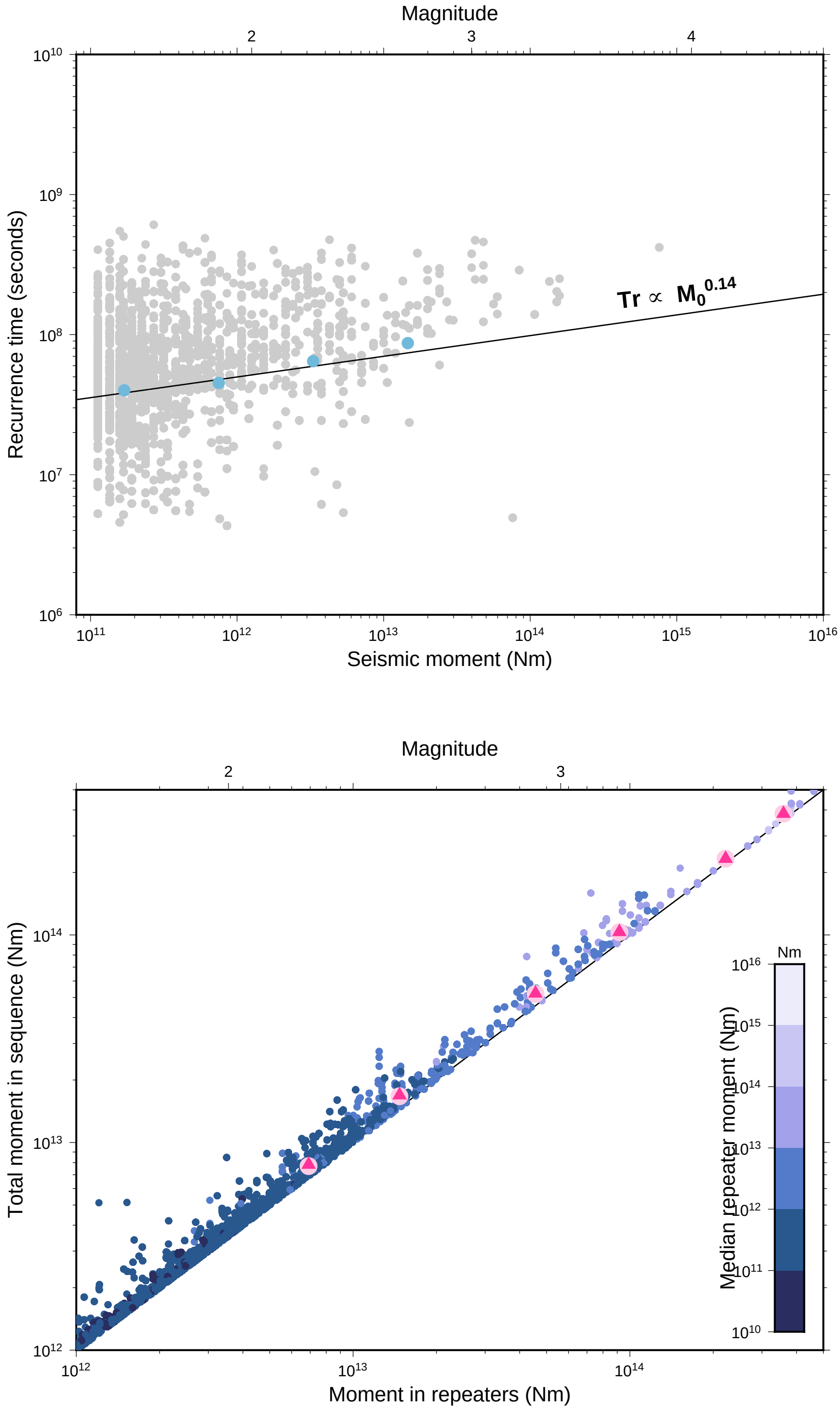
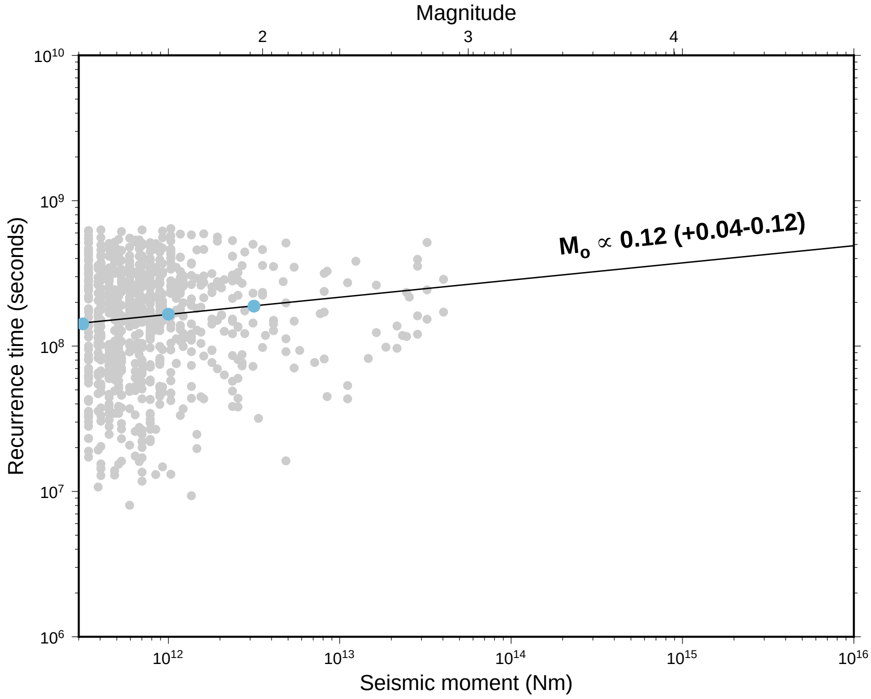



**Figure S5.** Same as Figure S.2, but assuming a stress drop of 10 MPa, and a local magnitude scaling of $M_0 = 10^{1.51M+16.1}$ .



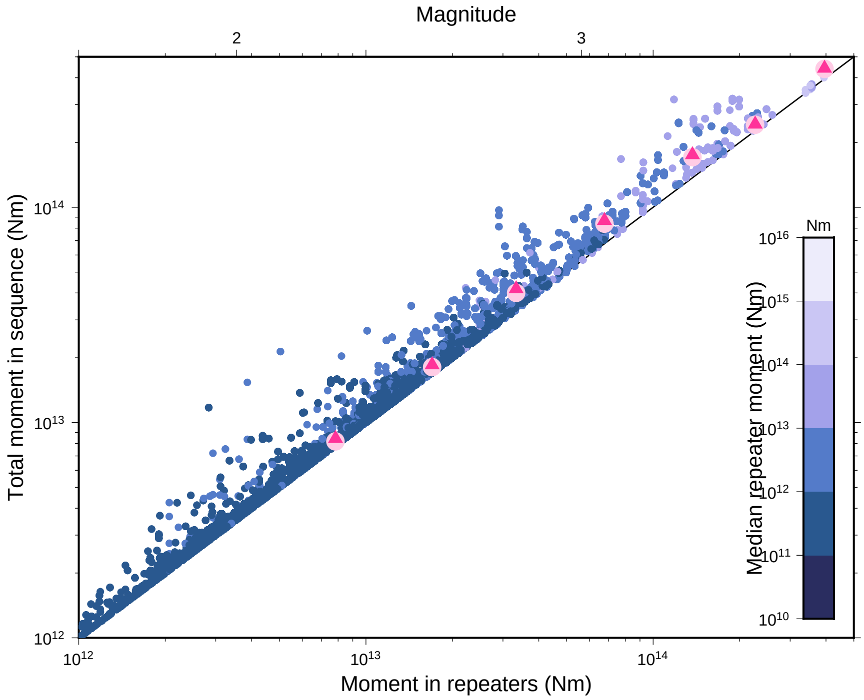




**Figure S6.** Same as Figure S.2 but using the location error ellipse reported in the NCSN catalog. Repeating earthquakes and partial ruptures are only identified when the maximum error is located within the rupture radius.

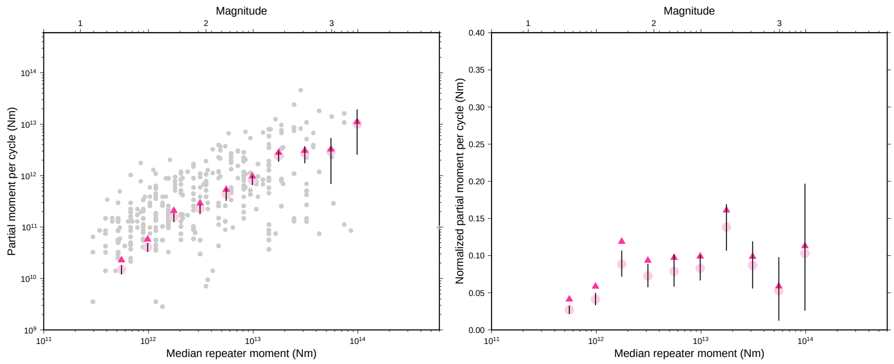



**Figure S7.** left: Total moment in repeating earthquakes to the total moment to partial ruptures, normalised by the number of repeating earthquakes of each repeater and its co-located events, identified using the 95th percentile error ellipse in the NCSN catalogue (See main text for further description of method). Light pink dots show the binned averages from bootstrapping, following the same bootstrapping procedure described in the text. The grey lines show the 5th and 95th percentiles of these binned medians. The dark pink triangles are the corrected bin values, following the same correction described in the text. Right: Ratio of median repeater moment to the normalised sum of the partial moment.

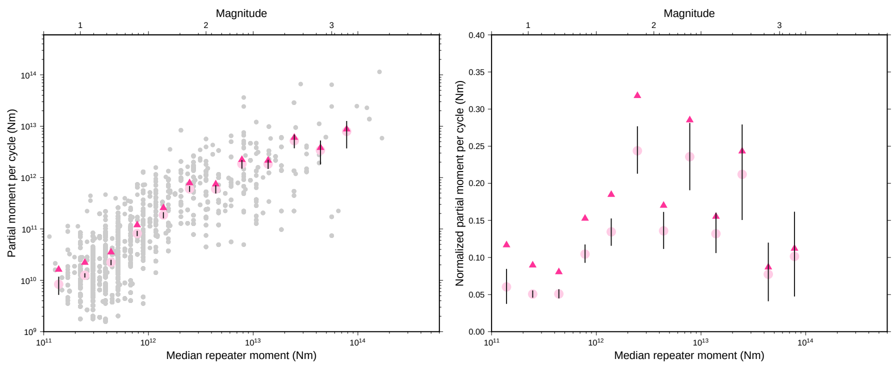



**Figure S8.** Same as figure S7, but using partial ruptures of repeating earthquakes identified by Waldhauser and Schaff, 2021.

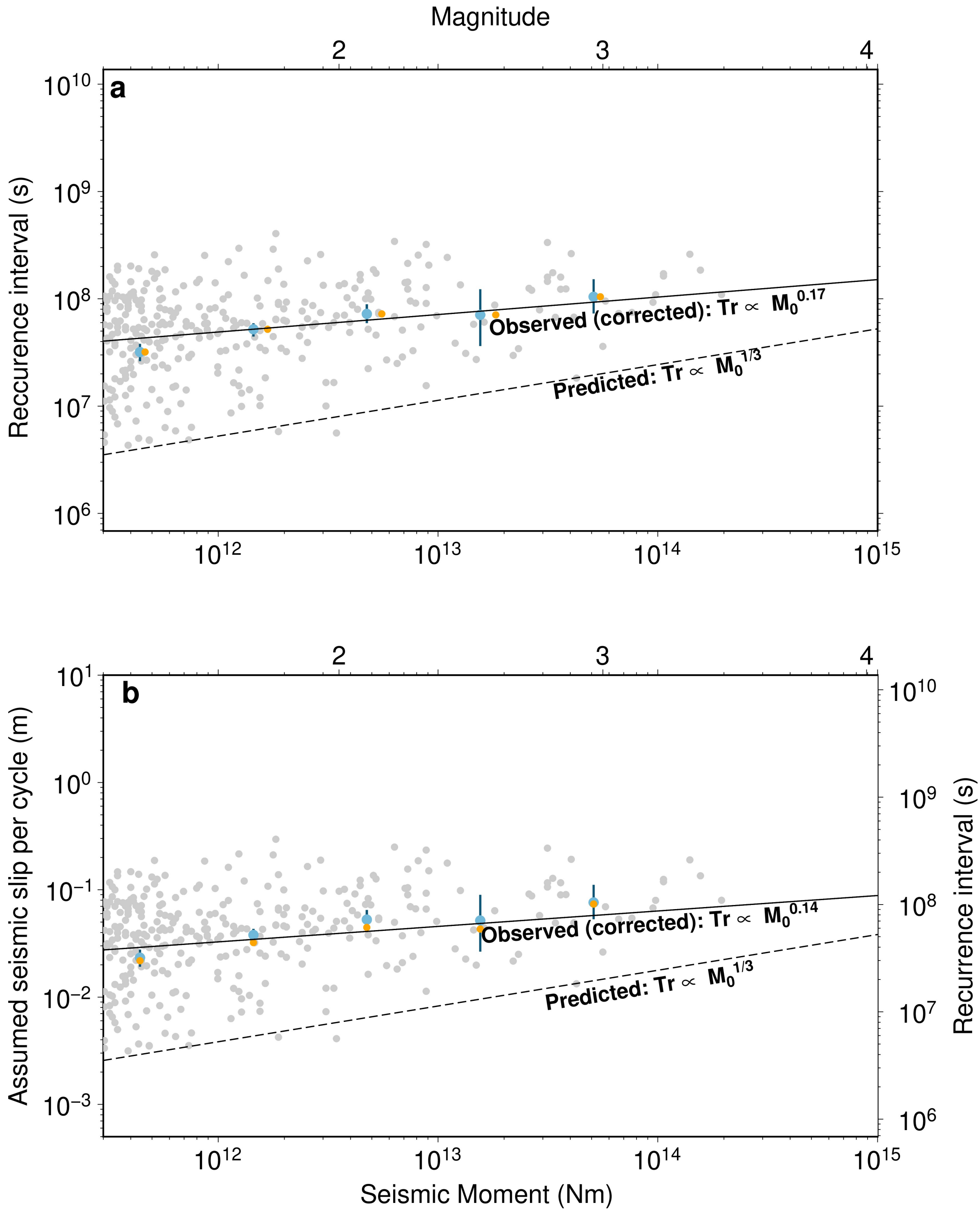



**Figure S9.** Log slip per repeating cycle versus median moment for sequences, assuming a long-term fault slip rate of 3mm/year for Repeating earthquakes in the Waldhauser and Schaff (2021) catalogue. Individual values are plotted as grey circles, with medians for moment bins shown as blue circles. The error bars on the medians indicate 95% confidence limits, which were estimated via bootstrapping (details in the text). The orange circles are the medians for moment bins corrected for the moment in the partial ruptures.

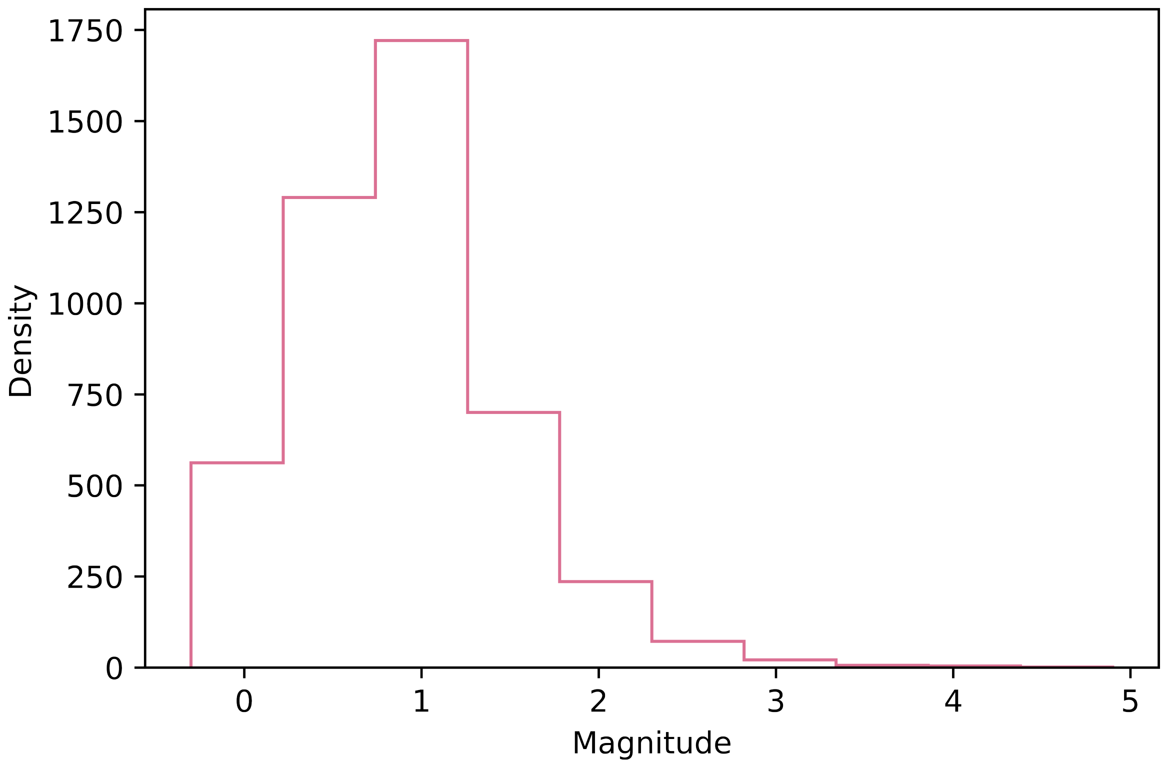



**Figure S10.** Distribution of the magnitudes of events from the Waldhauser repeating catalogue that were missed in our search for repeating earthquakes.

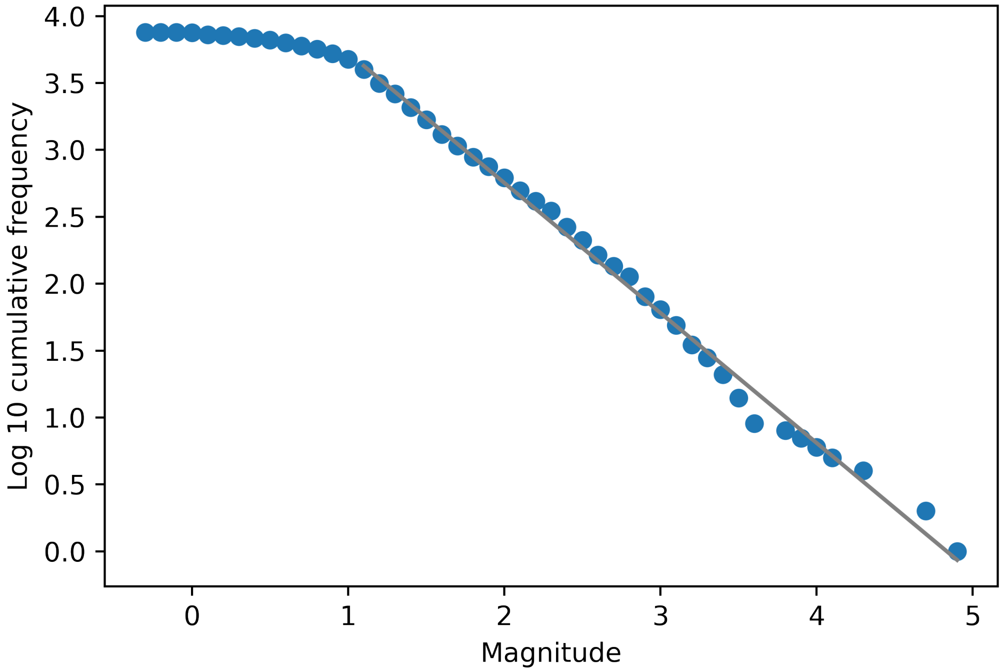



**Figure S11.** Linear Gutenberg-Richter distribution for earthquakes in Parkfield.

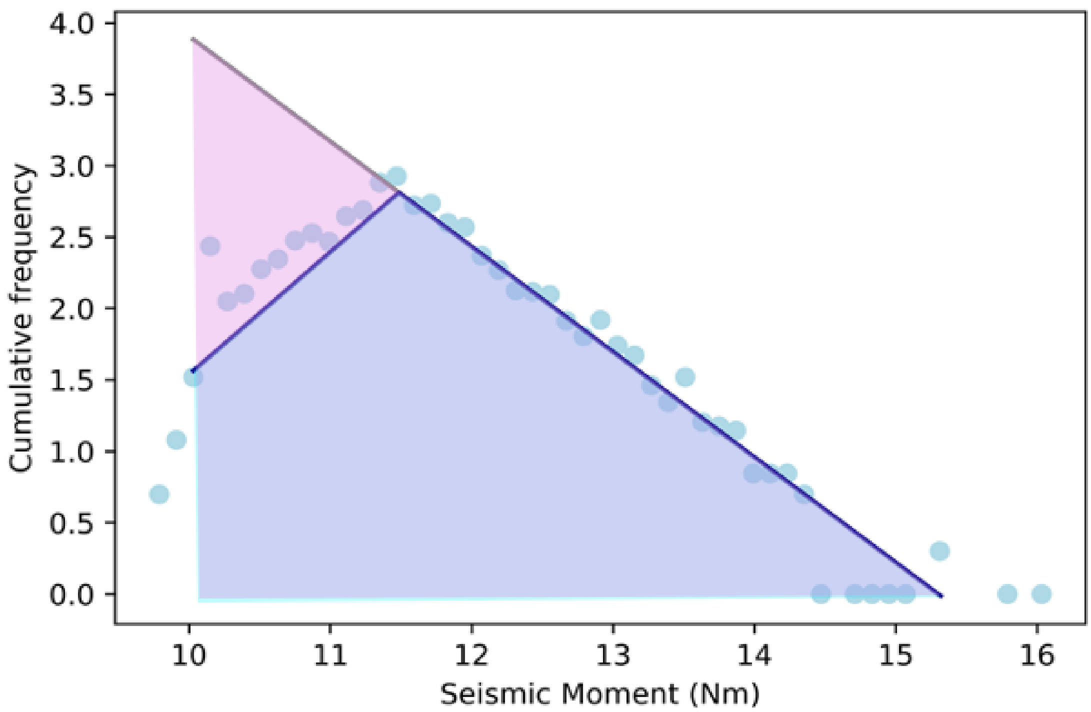



**Figure S12.** Illustration of how the correction of small magnitude missed events is carried out. Blue dots show the cumulative frequency of events in each magnitude. The blue line is the observed magnitude-frequency distribution, which has a positive slope below the magnitude of completeness. The grey line is the theoretical distribution – the distribution is extended to smaller magnitudes. To calculate the correction, for each magnitude repeater, we take the ratio of the area beneath the observed curve (blue) to the area below the theoretical curve (pink).