Computing time dilation between inertial frames

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Abstract: The concepts underlying the Lorentz transform for time dilation and length contraction are presented. The notion of a rest frame is introduced and its usefulness discussed. The rest frame of the CMB (the Hubble frame) is proposed as the correct candidate for the rest frame in the Lorentz transform, and an equation developed to compute the relative time dilation between inertial frames based on this assumption.

Historical Background

In 1917, the American astronomer Vesto Slipher provided the first evidence that the light from many galaxies is strongly red-shifted, indicative of high recession velocities. (American Academy of Arts and Sciences, 2023) (Slipher, Nebulae, 1917) (Slipher, Radial velocity observations of spiral nebulae, 1917) Ten years later, Georges Lemaître observed that the recessional velocity of a galaxy increases with its distance from the Earth. (Lemaître, 1927) In 1929, astronomer Edwin Hubble combined his own measurements with those of Slipher to confirm this hypothesis, a property now known as the Hubble-Lemaître law. (Hubble, 1929) From this relationship, astronomers surmised that the universe is expanding. (Lemaître, 1927) Based on this assumption, Lemaître proposed the "Big Bang theory" of the origin of the universe, calling it the "hypothesis of the primeval atom" (Lemaître, 1948), and later "the beginning of the world". (Lemaître, 1931) The unperturbed motion of objects in the universe, due solely to the expansion of the universe, is now known as the "Hubble Flow". (Hubble Flow, Retrieved 2022)

Simultaneous with these developments, Albert Einstein was conceiving his special and general theory of relativity. (Einstein, 1905) (Einstein, 1916) This theory, based on the equivalence principle (Einstein, 1911) and the principle of general covariance, (Einstein, 1916) predicts that time and distance must vary as a function of velocity (concepts known as time dilation and length contraction), in order to accommodate the observation that the speed of light is constant. The theory also subsumes the proposition that all inertial motion is relative, i.e., that given two inertial frames in motion relative to each other, it is impossible to tell which one is moving. From this hypothesis, it is inferred that the laws of physics cannot accommodate a notion of absolute motion or absolute rest. (Einstein, 1905) (Marder, 1971, pp. 23, 24)

As a result of this inference, scientists conclude that given any two inertial frames in motion relative to each other, one may arbitrarily be assumed to be at rest, and the other to be in motion relative to it. (Marder, 1971, p.48f) This assumption is used to formulate the Lorentz transform equations for time dilation and length contraction (Yakovenko, 2019). Unfortunately, the reciprocal nature of this assumption leads to the paradoxical conclusion that meter sticks in relative inertial motion appear to be shorter than each other, (Marder, 1971, p. 56) and that clocks in inertial motion relative to each other each run slower than the other. (Marder, 1971, p. 58) (Milne, 1948, p. 31) The clock paradox has caused great furor in the scientific community, eliciting many attempts to explain it away. Various arguments have been presented: observers have different criteria for simultaneity (Greene, 1999, pp. 55-58); clocks and measuring rods cannot be synchronized/reconciled at a distance (Greene, 1999, pp. 43-46); special relativity cannot deal with acceleration; (Kak, 2007), special relativity only applies to empty space, not space filled with matter, (Unnikrishnan, 2005) etc. Even Einstein claimed the paradox did not apply because the situation invoked general relativity. (Einstein, 1918)

Despite this quandary, many authors and researchers use this assumption as the basis of their arguments, often identifying a particular frame of reference to be the one at rest. For example, Ernst Mach, to explain the effect of acceleration due to rotation, appealed to the stars of the universe as a backdrop. E. A. Milne, in his book "Kinematic Relativity", predicated his arguments on the galaxies as "natural frames of reference for the description of the motions of the contents of the universe." (Milne, 1948, pp. 2,3) J. C. Hafele and R. E. Keating, in their classic 1971 time-dilation experiment (Hafele & Keating, 1972, pp. 166-168) (Hafele & Keating, 1972, pp. 168-170) chose the non-rotating inertial frame of the earth as a rest frame of reference. S. Kak (Kak, 2007) suggested that the isotropy of the universe provides a suitable frame of reference.

The Hubble Frame

Recent studies of the Cosmic Microwave Background (CMB) and Stochastic Gravitational Wave Background (SGWB) have, for the first time in history, provided an experimental method to identify the inertial frame of the Hubble Flow. For brevity, we shall call this inertial frame the Hubble frame. Researchers theorize that the CMB and SGWB, having originated in the early moments of the development of the universe, provide a way to identify the peculiar motion of the earth-sun system. (Aurich & Reinhardt, 2021) Measurements show a unique and identifiable dipole in the CMB that supports this assumption. (Lineweaver, et al., 1996)(Odenwald, retrieved 2022) It is believed that when other contributing factors are accounted for, the SGWB will do the same. (Kuroyanagi, Chiba, & Takahashi, 2018) (Christensen, 2019) (Bartolo, et al., 2022)

Although the measurements of the CMB and SGWB profiles are taken from our unique perspective (near earth), the same measurement could hypothetically be performed at any point in space. Such a measurement would produce a unique frame of reference at each point in space (the local Hubble Frame), whose velocity relative to our own would vary according to its distance from us, as prescribed by the Hubble-Lemaître law. The Hubble frame, available at every point in space, seems the most likely candidate for the universal rest frame for which scientists have been searching. Indeed, many researchers have chosen the rest frame of the CMB as their basis. (Maglara & Tsagas, 2022) (Nadolny, Durrer, Kunz, & Padmanabhan, 2021) (Dalang, Millon, & Baker, 2023) (Green, 2004, pp. 234-235)

The Hubble frame is universal in the sense that one exists at every point in space. Note that each such inertial frame is different from every other, in that it is in motion relative to the other at a velocity directly proportional to the distance separating the points at which they are defined. (Davis & Scrimgeous, 2014) An object at rest in the Hubble frame may be considered to be at absolute rest in that frame of reference. This notion does not violate the laws of physics, and general covariance still holds firm.

Computing Time Dilation

Since an object at rest in the Hubble frame is at absolute rest, it stands to reason that it would not experience time dilation. Time, therefore, proceeds at its maximum pace in the inertial frame of the Hubble flow, and flows at the same pace in every Hubble frame. This is what was called by Edward Harrison "cosmic time." (Soter & Tyson, 2001, p. 139) All time dilation and length contraction, therefore, are relative to the local Hubble frame (i.e., the Hubble frame at the point where the measurements are being made). An observer travelling through that point at a non-zero peculiar velocity v will experience time dilation of

$$\frac{t}{t_0} = \sqrt{1 - \frac{v^2}{c^2}} \quad \text{so that} \quad t_0 = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} = t * \gamma(v) \tag{1}$$

where t is the elapsed time as experienced by the traveler, t_0 is the corresponding elapsed proper time as experienced in the Hubble frame, and γ(v) is the traditional Lorentz factor. This is the correct way to calculate time dilation.

For two travelers crossing the same point at velocities v_1 and v_2

$$t_0 = t_1 * \gamma(v_1) = t_2 * \gamma(v_2) \quad \text{so that} \quad t_1 = t_2 * \frac{\gamma(v_2)}{\gamma(v_1)} \tag{2}$$

Thus, the relative time dilation between our two travelers may be greater than, less than, or equal to one, depending on the velocity of the travelers relative to the Hubble Frame. But it is never the case that each traveler perceives their time to be slower than the other's. This is quite different from the conventional computation which considers only the relative velocity of the two travelers, resulting in the clock paradox described above.

References

American Academy of Arts and Sciences. (2023, February 24). *Vesto Melvin Slipher*. Retrieved from Member Directory: https://www.amacad.org/person/vesto-melvin-slipher

Aurich, R., & Reinhardt, D. (2021). Determining our peculiar velocity from the aberration in the cosmic microwave background. *Mon.Not.Roy.Astron.Soc. 506*, 3259-3265.

Bartolo, N., Bertacca, D., Caldwell, R., Contaldi, C. R., Cusin, G., De Luca, V., . . . Sakellariadou, M. (2022). Probing anisotropies of the Stochastic Gravitational Wave Background with LISA. *JCAP11*.

Christensen, N. (2019). Stochastic Gravitational Wave Backgrounds. *Reports on Progress in Physics, Vol. 82*.

Dalang, C., Millon, M., & Baker, T. (2023). Peculiar velocity effects on the Hubble constant from time-delay cosmography. *arXiv, unpublished*.

Davis, T., & Scrimgeous, M. (2014). Deriving accurate peculiar velocities (even at high redshifts). *Mon. R. Astron. Soc.*

Einstein, A. (1905). Zur Elektrodynamik bewegter Körper. *Ann. d. Phys., 17*, 891.

Einstein, A. (1911). Über den Einfluss der Schwerkraft auf die Aus breitung des Lichtes. *Ann. d. Phys., 35*, 898.

Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. *Ann. d. Phys., 49*, 767.

Einstein, A. (1918). Dialog über Einwände gegen die Relativitätstheorie. *Die Naturwissenschaften 6*, 697-702.

Green, B. R. (2004). The Fabric of the Cosmos. In B. R. Green, *The Fabric of the Cosmos* (pp. 132-139). New York: Vintage Books.

Greene, B. (1999). *The Elegant Universe.* New York: W. W. Norton.

Hafele, J., & Keating, R. E. (1972, March 24). Around-the-World Atomic Clocks: Observed Relativistic Time Gains. *Science, Vol. 177*, 168-170.

Hafele, J., & Keating, R. E. (1972). Around-the-World Atomic Clocks: Predicted Relativistic Time Gains. *Science, Vol. 177*, 166-168.

Hubble Flow. (Retrieved 2022). Retrieved from Cosmos - The Swineburne Astronomy Online Encyclopedia of Astronomy: https://astronomy.swin.edu.au/cosmos/h/hubble+flow

Hubble, E. (1929). A Relation Between Distance and Radial Velocity Amoung Extra-Galactic Nebulae. *Proc. N. A. S.*, 168-173.

Kak, S. (2007). Moving Observers in an Isotropic Universe. *International Journal of Theoretical Physics*, 1424-1430. Retrieved from arXiv: https://link.springer.com/article/10.1007/s10773-006-9281-2

Kuroyanagi, S., Chiba, T., & Takahashi, T. (2018). Probing the Universe through the Stochastic Gravitational Wave Background. *JCAP*.

Lemaître, G. (1927). Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Annales de la Société Scientifique de Bruxelles, A47*, 49-59.

Lemaître, G. (1931). The Beginning of the World from the Point of View of Quantum Theory. *Nature, Volume 127, Issue 321*, 706ff.

Lemaître, G. (1948). L'Hypothèse de l'Atome Primitif. *Publications du Laboratoire d'Astronomie et de Geodesie de l'Universite de Louvain, vol. 12*, D1-D19.

Lineweaver, C. H., Tenorio, L., Smoot, G. F., Keegstra, P., Banday, A. J., & Lubin, P. (1996). The Dipole Observed in the COBE DMR Four-Year Data. *Astrophys.J. 470*, 38-42.

Maglara, M., & Tsagas, C. (2022). Peculiar velocities in the early universe. *Phys. Rev. D 106*.

Marder, L. (1971). *Time and the Space-Traveller.* Philadelphia: University of Pennsylvania Press.

Milne, E. A. (1948). *Kinematic Relativity.* Oxford: Clarendon Press.

Nadolny, T., Durrer, R., Kunz, M., & Padmanabhan, H. (2021). A new way to test the Cosmological Principle: measuring our peculiar velocity and the large scale anisotropy independently. *JCAP 11*.

Odenwald, D. S. (retrieved 2022, November 24). *How fast is the Earth moving with respect to the rest of the universe?* Retrieved from NASA Image Science Center, Image Education and Public Outreach (POETRY): https://image.gsfc.nasa.gov/poetry/ask/a10552.html

Slipher, V. M. (1917). Nebulae. *Proceedings of the American Philosophical Society vol. 56*, (pp. 403-409).

Slipher, V. M. (1917). Radial velocity observations of spiral nebulae. *The Observatory, Vol. 40*, 304-306.

Soter, S., & Tyson, N. E. (2001). *Cosmic Horizons: Astronomy at the Cutting Edge.* New York: The New Press.

Unnikrishnan, C. (2005). On Einstein's resolution of the twin clock paradox. *Current Science 89*, 2009-2015.

Yakovenko, V. M. (2019, December 2). *Derivation of the Lorentz Transformation.* Retrieved from UMD Department of Physics: https://physics.umd.edu/~yakovenk/teaching/