

A Novel Traveling-wave Direction Criterion for Hybrid Multi-terminal HVDC System

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Abstract: Fast and accurate fault direction criteria is crucial for efficient protection in hybrid multi-terminal HVDC transmission systems. The current variation method is commonly used to identify fault direction but is affected by fault resistance and distributed capacitance. This paper proposes a new direction criterion based on the backward fault traveling-wave, which has stronger resistance to interference from transition resistance and operates at a faster speed than other methods. Additionally, this paper provides the time-domain expression of the traveling wave after multiple reflections between the fault point and the converter station port, enabling the calculation of the threshold setting for the criterion. The new direction criterion is tested in PSCAD/EMTDC, and simulation results demonstrate that it can accurately identify fault direction and has a strong ability to withstand transition resistance.

1. Introduction

The hybrid multi-terminal HVDC system(MTDC) is developed from the dual-terminal HVDC system. It has the characteristics of high reliability, economical efficiency, flexible transmission. [1]-[2]. Compared with other HVDC systems, the hybrid MTDC system is more suitable for multi-power supply and multi-receiver power transmission, which has broader application prospects [3]-[4].

In the hybrid MTDC system, the converter station at the sending end generally adopts the line commutation converter (LCC) to achieve better economic efficiency and the converter station at the receiving end adopts modular multi-level converter (MMC) to avoid commutation failure [5]-[6]. Due to the existence of the MMC converter station, when a dc overhead line (OHL) fault occurs in the hybrid MTDC system, the current rises quickly. It is important to identify and cut off the fault OHL fast and accurately to ensure the safe operation and rapid recovery of the non-fault part of the system [7]-[8]. In order to achieve faster action speed, the main protection of hybrid MTDC system generally adopts non-unit protection [9]. Since the two OHLs are directly connected at the intermediate converter station of the system, the main protection at this position needs to be equipped with directional unit to distinguish the forward fault from the backward fault to ensure the selectivity of the action [10]. In the hybrid MTDC system, rapid and accurate direction criterion is critical for fault line identification.

The identification of fault direction through current variation is a widely used method. However, the accuracy of this method is influenced by factors such as fault resistance. [11-12] TW reflection and refraction will take place when the fault TW sent by the fault point spreads to the line ends. The reflection and refraction characteristics of TW can be used as the characteristic quantity of the direction identification. The direction criterion based on travelling waves (TWs) exhibits strong robustness to high-resistance faults. [13] identifies the fault direction by comparing the ratio of the high frequency components of the forward and backward TW. [14] proposes

the direction criterion by using the ratio of the integral of backward TW and forward TW. However, the TW direction criterion proposed in [13]-[14] are all based on the differences of TW characteristics obtained from the simulation, and meanwhile the setting value also depends on simulation.

In order to solve the problems that the TW protection criterion lacks of theoretical support and the setting value depends on simulation, some scholars have studied the analytic expression method of TW in the HVDC system, then applies it to design the new TW protection scheme and the calculation of the setting value of the criterion [15]. [16] concludes the analytic expression of the voltage TW at different line boundaries of the hybrid three-terminal HVDC system, and puts forward a dc line fast protection scheme using the time of the first extreme value of fault voltage. [17] analyzes the TW characteristics at the bus bar in the hybrid three-terminal HVDC system, derives the complex frequency domain expression of the fault initial TW, and then applies it to the setting value of the TW protection. [16]-[17] focuses on the analytical calculation of the first TW, and the influence of secondary or multiple TWs is not considered. In particular, when the fault point is close to the end of the line, the arrival time interval between the first TW and the subsequent TW will be very short, which is inseparable. At this time, if only the first TW is analyzed for fault characteristics, the influence of subsequent TWs may lead to the maloperation of the relevant protection schemes. As a result, it is necessary to carry out the study on the fault characteristics of TW after multiple reflections and refractions in a short time under the near and remote end OHL fault condition, to guarantee that the fault direction criterion can act correctly as well.

In this paper, the analytic calculation method of fault voltage TW at the protection of the intermediate converter station is studied. The time-domain expressions of TW after multiple reflections and refractions in a short time under the near and remote end fault of the OHL are derived emphatically. According to the analysis of the differences in TW characteristics between forward and backward faults, a novel direction criterion based on the amplitude characteristics of the backward TW is proposed, and the

theoretical calculation method of setting value is given. A simulation model of the Kun-Liu-Long hybrid three-terminal HVDC system is also built to verify the reliability and robustness of the direction criterion.

2. Analysis of TW Fault Characteristics

This paper takes the Kun-Liu-Long hybrid three-terminal HVDC system as the research reference. The basic topology of the system is depicted in Fig. 1. LCC converter station is used at the sending end of the system, and MMC is adopted at both converter stations at the receiving end.

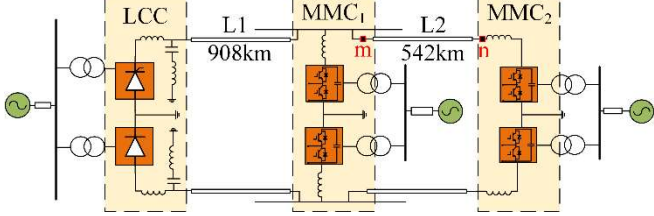


Fig. 1. The topology of the Kun-Liu-Long hybrid three-terminal DC system

LCC station connects the MMC₁ station with the OHL L1. MMC₁ station connects the MMC₂ station with the OHL L2, and the OHL L1 and L2 are connected to the MMC₁ station through the confluent bus. Table 1 shows the detailed system parameters [14].

Table 1 Parameters of System

Parameters	Value
Length of L1, L2(km)	908,542
Rated DC voltage(kV)	800
Rated AC voltage(kV)	525
Outlet inductance of MMC(H)	0.075
Bridge arm inductance of MMC (H)	0.1
Submodule capacitance of MMC (F)	0.015
Line-mode impedance of OHL (Ω)	232
Zero-mode impedance of OHL (Ω)	581
Attenuation coefficient of the OHL	6e-5

In a bipolar dc transmission system, electromagnetic coupling exists between OHLs after the line fault occurs. For purpose of simplifying the analysis, the phase-mode transformation is applied to achieve electromagnetic decoupling. The phase-mode transformation equation is shown as:

$$\begin{bmatrix} i_0 \\ i_1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} i_p \\ i_n \end{bmatrix} \quad \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u_p \\ u_n \end{bmatrix} \quad (1)$$

where i and u represent current and voltage, and the zero-mode and line-mode components are denoted by the subscripts 0 and 1, respectively.

There exist line-mode components after faults occur for both pole-to-pole(P-P) and pole-to-ground(P-G) faults, and the attenuation and distortion of the line-mode components on OHLs are relatively small. Therefore, this study employs line-mode components for the identification of the direction criterion. Within 1 millisecond after the fault occurs, the system control link does not play a significant role, so the influence of control link can be ignored, and the system can be approximately regarded as a linear system. In order not to be affected by the load component, according to the

superposition theorem of the linear system, this paper only analyzes the faulty component.

The direction criterion of the measuring point m at the confluent bus of the MMC₁ converter station is studied in this paper. When the fault occurs on OHL L2, it is considered a forward fault, and when the fault occurs on OHL L1, it is considered a backward fault, as shown in Fig. 2. The fault location diagram is also shown in Fig. 2, where f_1 is located at any position on the OHL L1, f_2 is located near the head of the OHL L2, f_3 is located at the non-port position of the OHL L2, f_4 is located near the end of the OHL L2, and m is at the head of OHL L2. This section illustrates the backward fault TW (BFTW) characteristics of the point m after the fault happens on different positions of the OHL L1 and L2.

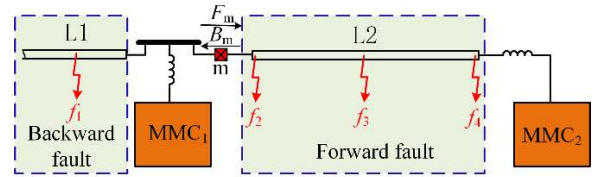


Fig. 2. The schematic diagram of the fault location

2.1. BFTW Characteristics at m in Case of the OHL L1 Fault

After a fault occurs at f_1 on OHL L1, the fault TW transmission process is shown in Fig. 3. According to Fig. 3, the forward fault TW sent from the point f_1 refracts after reaching the measuring point m to form the forward TW F_m^1 on OHL L2 at time t_0 . F_m^1 reflects at the end of OHL L2, then reaches m again forming the backward fault TW B_m^1 .

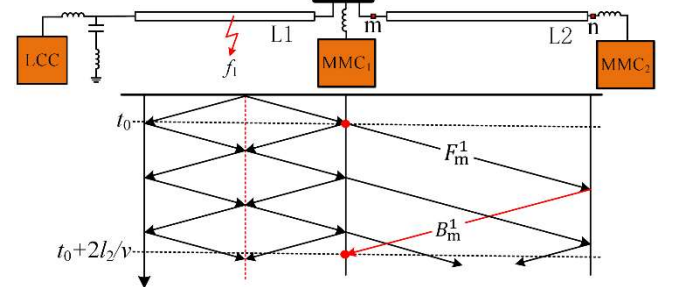


Fig. 3. The schematic diagram of fault TW transmission after f_1 fault

It can be seen that before the TW B_m^1 reaches the point m , the BFTW cannot be detected at m , so the expression of the TW B_m is shown as (2). v is the propagation velocity of line-mode TW, and l_2 is the length of OHL L2. (2) is valid for faults occurring at any position of OHL L1.

$$B_m = 0 \quad (t_0 \leq t \leq t_0 + \frac{2l_2}{v}) \quad (2)$$

2.2. BFTW Characteristics at m in Case of the OHL L2 Fault

This section analyzes the characteristics of TWs at the protection measurement point m after faults occur at various locations on line L2. Specifically, the faults occurring near the head of the OHL, those occurring away from the near end, and those occurring near the end of the OHL are studied separately.

2.2.1 BFTW characteristics at m in case of fault at point f_3 : The fault point f_3 is located at the non-end position, neither near the head nor the end of the OHL L2. When the fault

occurs at f_3 , the fault TW transmission process is shown in Fig. 4.

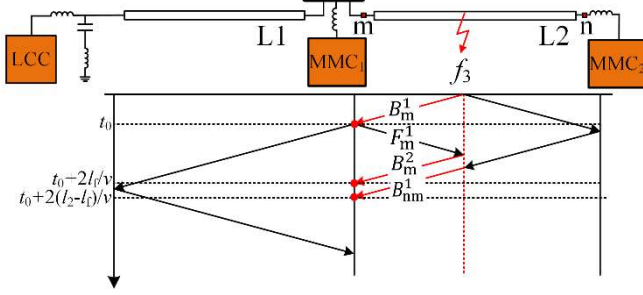


Fig. 4. The schematic diagram of fault TW transmission after f_3 fault

According to [18], the equivalent fault additional mode network at fault point f_3 can be obtained respectively after the P-G fault and the P-P fault occur on the line L2, as is shown in Fig. 5. In the figure, U_{dc} is the negative voltage of the line L2 under the condition of system normal operation. R_f is the transition resistance, and $Z_{1\Sigma}$ and $Z_{0\Sigma}$ are the equivalent line-mode and ground-mode impedance at the fault point. i_{f1} and i_{f0} are the line-mode and ground-mode current flowing into the fault point. u_{f0} and u_{f1} are the zero-mode and line-mode voltage at the fault point. Z_{c1} and Z_{c0} are the line-mode and zero-mode wave impedance of the OHL L2 respectively.

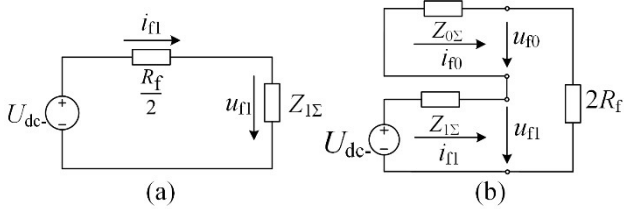


Fig. 5. The equivalent fault additional mode network (a) P-P fault (b) P-G fault

According to Fig. 5, the expression of the BFTW B_f^1 at the fault point after the P-P fault and P-G fault occur on the OHL L2 can be calculated and shown as:

$$B_f^1 = \begin{cases} \frac{2U_{dc}Z_{c1}}{R_f + Z_{c1}} & \text{P-P fault} \\ \frac{2U_{dc}Z_{c1}}{Z_{c1} + Z_{c1} + 4R_f} & \text{P-G fault} \end{cases} \quad (3)$$

At time t_0 , the TW B_m^1 reaches m at the head of the OHL L2, B_m^1 reflects at m, and forms the forward TW F_m^1 . F_m^1 arrives at the fault point f_3 again, and reflects at f_3 then the TW B_m^2 is formed. At the same time, the forward TW sent from the fault point reflects at n, then refracts at the fault point, and forms the TW B_{nm}^1 . Analyzing the TW transmission process, there is only the TW B_m^1 at m before the TW B_m^2 and B_{nm}^1 reach m.

The BFTW B_f^1 at f_3 is transmitted to m at the head of the OHL L2 and the BFTW B_m^1 is formed. During the process, the TW B_f^1 will be attenuated. According to the above analysis, there is only the first TW B_m^1 at the point m before the TW B_m^2 and B_{nm}^1 reach the point m. The expression of the TW B_m can be expressed as (4). l_f is the distance between f_3 and m, l_2 is the length of the line L2 and k_a is the attenuation coefficient of the OHL.

$$B_m = \begin{cases} \frac{2(1-k_a l_f)U_{dc}Z_{c1}}{R_f + Z_{c1}} & \text{P-P fault} \\ \frac{2(1-k_a l_f)U_{dc}Z_{c1}}{Z_{c1} + Z_{c1} + 4R_f} & \text{P-G fault} \end{cases} \quad (4)$$

$$(t_0 \leq t \leq \min\{t_0 + 2\frac{l_f}{v}, t_0 + 2\frac{(l_2 - l_f)}{v}\})$$

According to (4), if l_f is close to 0 or l_2 , that is, the fault point is near the head or end of the OHL L2, the time frame of (4) will become very small. However, it is necessary to analyze the data in the determined time window T_w in the actual fault line identification, so that the value range of fault distance l_f established by equation (4) can be obtained, as shown in (5).

$$\frac{vT_w}{2} \leq l_f \leq l_2 - \frac{vT_w}{2} \quad (5)$$

If the fault distance l_f does not meet the conditions shown in (5), that is, the fault point is considered to be near the head or the end of the OHL L2, these two conditions can be regarded as near and remote faults. (4) is not valid and further discussions are needed separately.

2.2.2 BFTW Characteristics at m in Case of Fault at f_2 : The fault point f_2 is located near the head of the OHL L2. The fault TW transmission process is shown in Fig. 6. The BFTW sent from f_2 will soon reach the measurement point m. Since f_2 is close to m, the BFTW will be reflected multiple times between f_2 and m in the time window T_s . Therefore, for the near end fault, when analyzing the BFTW B_m at m, the influence of the superposition of TWs after multiple reflections should be taken into account.

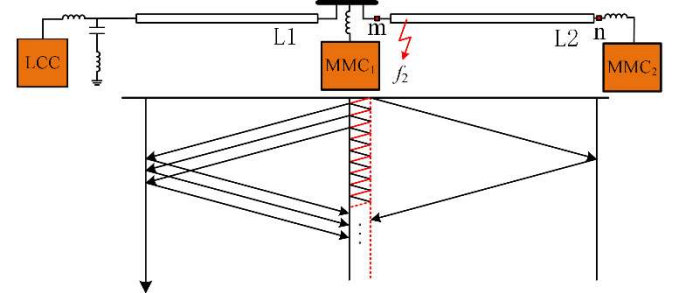


Fig. 6. The schematic diagram of fault TW transmission after f_2 fault

The expression of the first TW B_m^1 at m can be obtained according to (4). B_m^1 reflects at the confluent bus to form a forward TW F_m^1 . The equivalent circuit during the TW reflection is shown in Fig. 7 [19]. In the figure, C_{eq} is the equivalent capacitance of the MMC station, L_{eq} is the sum of the equivalent inductance of the MMC station and the inductance of the current limiting reactor at the outlet of the MMC station.

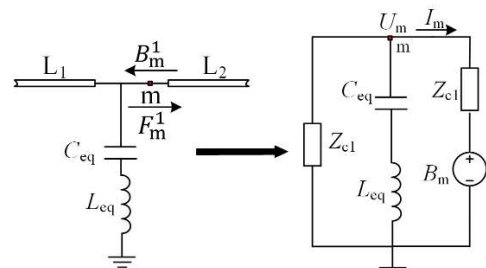


Fig. 7. The equivalent circuit diagram at the point m

On the basis of the equivalent circuit, the voltage U_m and current I_m at m in the complex frequency domain satisfy the relationship shown in (6).

$$\begin{cases} [\frac{U_m(s)}{Z_{c1}} + I_m(s)] \cdot (sL_{eq} + \frac{1}{sC_{eq}}) = U_m(s) \\ U_m(s) - Z_{c1}(s) \cdot I_m(s) = B_m^1(s) \end{cases} \quad (6)$$

The BFTW B_m^1 reflects at m and the forward TW F_m^1 is formed. F_m^1 can be expressed as:

$$F_m^1(s) = U_m(s) + Z_{c1}(s) \cdot I_m(s) \quad (7)$$

Combining (6) with (7), the expression of the forward TW F_m^1 can be deduced as:

$$F_m^1(s) = \frac{Z_{c1} \cdot B_m^1(s)}{2sL_{eq} + \frac{2}{sC_{eq}} - Z_{c1}} \quad (8)$$

The forward TW F_m^1 reflects at the fault point f_2 again and the BFTW B_m^2 is formed. Different fault types have different reflect conditions at the fault point. The expression of BFTW B_m^2 can be obtained, which can be expressed as:

$$B_m^2(s) = \begin{cases} \frac{Z_{c1}}{Z_{c1} + R_f} \cdot F_m^1(s) & \text{P-P fault} \\ \frac{Z_{c0} + 4R_f}{Z_{c1} + Z_{c0} + 4R_f} \cdot F_m^1(s) & \text{P-G fault} \end{cases} \quad (9)$$

The backward TW B_m^2 reaches the point m again and reflects, and thus forms the forward TW F_m^2 . According to (6) and (7), the expression of F_m^2 can be obtained as:

$$F_m^2(s) = \frac{Z_{c1} \cdot B_m^2(s)}{2sL_{eq} + \frac{2}{sC_{eq}} - Z_{c1}} \quad (10)$$

Based on the above analysis and combined with (8) and (9), the expression of backward TW B_m^i after the i th reflection can be expressed as follows:

$$B_m^i(s) = \begin{cases} (\frac{Z_{c1}}{Z_{c1} + R_f})^{i-1} \cdot (\frac{sZ_{c1}C_{eq}}{2s^2C_{eq}L_{eq} - sC_{eq}Z_{c1} + 2})^{i-1} \cdot B_m^1(s) & \text{P-P fault} \\ (\frac{Z_{c0} + 4R_f}{Z_{c1} + Z_{c0} + 4R_f})^{i-1} \cdot (\frac{sZ_{c1}C_{eq}}{2s^2C_{eq}L_{eq} - sC_{eq}Z_{c1} + 2})^{i-1} \cdot B_m^1(s) & \text{P-G fault} \end{cases} \quad (11)$$

The expression of backward TW $B_m^i(s)$ after each reflection in complex frequency domain is transformed to time-domain using the *Inverse Laplace Transform*, then the time-domain characteristics of TW B_m^i are analyzed. In the following analysis, the P-P fault is used as an illustration, and the condition of P-G fault is similar.

Based on (4), the first backward TW B_m^1 in time-domain can be obtained as follow:

$$B_m^1(t) = \frac{2U_{dc} \cdot Z_{c1}}{R_f + Z_{c1}} \quad (12)$$

The second backward TW $B_m^2(s)$ is converted into time-domain expression. According to Table 1, $Z_{c1}/16L_{eq}^2$ is much larger than $1/C_{eq}L_{eq}$, and $B_m^2(s)$ can be simplified as follow:

$$B_m^2(s) = \frac{U_{dc} \cdot Z_{c1}^3}{L_{eq}(Z_{c1} + R_f)^2} \cdot \frac{1}{s(s - \frac{Z_{c1}}{2L_{eq}})} \quad (13)$$

The time-domain expression of (13) can be obtained by the *Inverse Laplace Transform*:

$$B_m^2(t) = \frac{2U_{dc} \cdot Z_{c1}^2}{(Z_{c1} + R_f)^2} (e^{\frac{Z_{c1}}{2L_{eq}}t} - 1) \quad (14)$$

Similarly, the expression $B_m^3(t)$ can be obtained:

$$B_m^3(t) = \frac{2U_{dc} \cdot Z_{c1}^3}{(Z_{c1} + R_f)^3} (\frac{Z_{c1}}{2L_{eq}}te^{\frac{Z_{c1}}{2L_{eq}}t} - e^{\frac{Z_{c1}}{2L_{eq}}t} + 1) \quad (15)$$

Substituting the parameters in Table 1 into $B_m^2(t)$ and $B_m^3(t)$, when t is less than 0.2 ms, it can be calculated that B_m^2 is much larger than B_m^3 . Therefore, it is considered that B_m^3 can be ignored. It should be noted that although the specific parameters may differ in actual projects, the order of magnitude remains the same. The quantitative relationship between B_m^2 and B_m^3 differs by more than two orders of magnitude, therefore, B_m^3 can be ignored even the parameters change. Moreover B_m^n can be ignored when n is bigger than 2 and t is less than 0.2 ms, which can be proved as follows.

For the $n+1$ th backward TW $B_m^{n+1}(s)$ simplification:

$$B_m^{n+1}(s) = \frac{2Z_{c1}^{2n+1}U_{dc}}{(2L_{eq})^n(Z_{c1} + R_f)^n} \cdot \frac{1}{s(s - \frac{Z_{c1}}{2L_{eq}})} \quad (16)$$

Carry out the *Inverse Laplace Transform* on (16), it can be obtained:

$$B_m^{n+1}(t) = \frac{2Z_{c1}^{2n+1}U_{dc}}{(2L_{eq})^n(Z_{c1} + R_f)^n} \int_0^t \frac{t^n}{n!} \cdot e^{\frac{Z_{c1}}{2L_{eq}}t} dt \quad (17)$$

According to (17), it can be calculated that:

$$B_m^{n+1}(t) + \frac{Z_{c1}}{Z_{c1} + R_f} \cdot B_m^n(t) = \frac{2Z_{c1}^{2n}U_{dc}}{(2L_{eq})^{n-1}(Z_{c1} + R_f)^{n-1}} \cdot t^n e^{\frac{Z_{c1}}{2L_{eq}}t} \quad (18)$$

According to (18), the following inequality can be obtained:

$$\frac{2Z_{c1}^{2n}U_{dc}}{(2L_{eq})^{n-1}(Z_{c1} + R_f)^{n-1}} \cdot t^n e^{\frac{Z_{c1}}{2L_{eq}}t} < B_m^{n+1}(t) + B_m^n(t) < \frac{2Z_{c1}^{2n-1}U_{dc}}{(2L_{eq})^{n-1}(Z_{c1} + R_f)^{n-1}} \cdot t^n e^{\frac{Z_{c1}}{2L_{eq}}t} \quad (19)$$

According to (19), it can be obtained that:

$$B_m^{n+1}(t) < B_m^{n-1}(t) \quad (20)$$

According to (20), when n is greater than two, B_m^2 is much larger than B_m^n , so B_m^n can be ignored when t is less than 0.2ms. The expression of B_m is shown in equation (21), where t_0 is the time when the backward TW B_m^2 reaches m. When t is greater than zero, $\varepsilon(t)$ is one, otherwise $\varepsilon(t)$ is zero.

$$B_m = \sum_{i=1}^n B_m^i = B_m^1 + B_m^2 = \begin{cases} \frac{2U_{dc}Z_{c1}}{R_f + Z_{c1}} + \frac{2U_{dc}Z_{c1}^2}{(Z_{c1} + R_f)^2} \cdot [e^{\frac{Z_{c1}}{2L_{eq}}(t-t_0)} - 1]\varepsilon(t-t_0) & \text{P-P fault} \\ \frac{2U_{dc}Z_{c1}}{Z_{c1} + Z_{c0} + 4R_f} + \frac{2U_{dc}Z_{c1}Z_{c0}}{(Z_{c1} + Z_{c0} + 4R_f)^2} \cdot [e^{\frac{Z_{c1}}{2L_{eq}}(t-t_0)} - 1]\varepsilon(t-t_0) & \text{P-G fault} \end{cases} \quad (21)$$

To ensure that the theoretical analysis conclusion is correct, P-G fault and P-P fault are set at f_2 , which is 5km away from the head of the OHL L2. The fault resistance is 0Ω and 100Ω respectively, and the time when the fault occurs is set to 1.5s. Simulation and analytical waveforms of the BFTW B_m at m within 0.2ms after the fault can be obtained, as shown in Fig. 8.

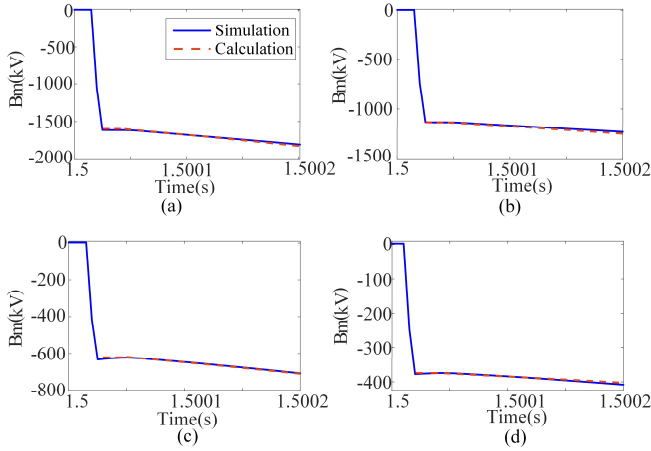


Fig. 8. Comparison between B_m simulation waveform and analytical waveform in case of f_2 fault (a) P-P fault with 0Ω (b) P-P fault with 100Ω (c) P-G fault with 0Ω (d) P-G fault with 100Ω

From Fig. 8, it can be found that the analytical waveform is basically consistent with the simulation waveform, which verifies the expression of B_m .

2.2.3 BFTW Characteristics at m in Case of Fault at f_4 :

The point n is at the end of the OHL L2, and the fault point f_4 is located near n. After fault occurs, the fault TW transmission process is shown in Fig. 9. Due to f_4 is close to n, the fault TW sent from f_4 will have multiple reflections between f_4 and n in a short period. Therefore, as for the remote fault at the line L2, when analyzing the TW B_m at m, the superposition result of the first backward TW from f_4 to m and the multiple refraction TWs should be considered.

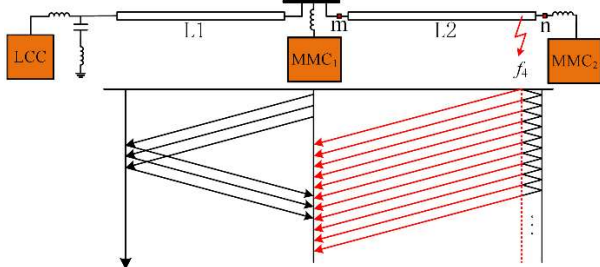


Fig. 9. The schematic diagram of fault TW transmission after f_4 fault

The equivalent circuit diagram at n after TW reflection can be derived, as shown in Fig. 10.

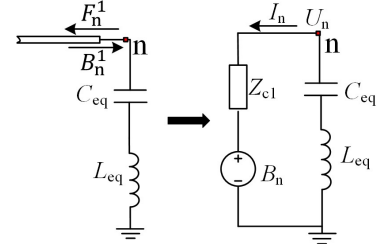


Fig. 10. The equivalent circuit diagram at the end of line L2

Due to f_4 is close to n, the influence of the OHL on the attenuation and distortion of the TW B_f^1 can be ignored, that is, B_f^1 and B_n^1 are approximately the same. The backward TW B_f^1 at fault point can be solved according to (4). Then, combined with the equivalent circuit at n, it can be expressed as:

$$\begin{cases} U_n(s) = -I_n(s) \cdot (sL_{eq} + \frac{1}{sC_{eq}}) \\ U_n(s) - Z_{c1}(s) \cdot I_n(s) = B_n^1(s) \end{cases} \quad (22)$$

The backward TW B_n^1 reflects at n and forms a forward TW F_n^1 . For F_n^1 :

$$F_n^1(s) = U_n(s) + Z_{c1}(s) \cdot I_n(s) \quad (23)$$

According to (22) and (23), the expression of forward TW F_n^1 at n is described as:

$$F_n^1(s) = \frac{(s - \frac{Z_{c1}}{L_{eq}}) \cdot B_n^1(s)}{s + \frac{Z_{c1}}{L_{eq}}} \quad (24)$$

The TW F_n^1 soon reaches the fault point f_4 again, and reflection will occur at the fault point to form a backward TW B_f^2 , and meanwhile refraction occurs to form the forward TW F_f^2 . The forward TW F_f^2 will propagate to m at the head of the OHL L2, and the backward TW B_f^2 will reach n again and reflect, thus the forward TW F_n^2 is formed. Fig. 11 shows the TW transmission process.

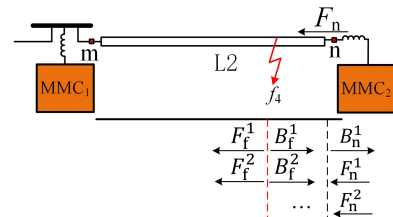


Fig. 11. The schematic diagram of fault TW reflection and refraction

According to (9) and (24), the forward TW F_n^i at n after the i th reflection of the line L2 fault can be obtained that:

$$F_n^i(s) = \begin{cases} (\frac{Z_{c1}}{Z_{c1} + R_f})^{i-1} \cdot (\frac{sL_{eq} - Z_{c1}}{sL_{eq} + Z_{c1}})^i \cdot B_n^1(s) & \text{P-P fault} \\ (\frac{Z_{c0} + 4R_f}{Z_{c1} + Z_{c0} + 4R_f})^{i-1} \cdot (\frac{sL_{eq} - Z_{c1}}{sL_{eq} + Z_{c1}})^i \cdot B_n^1(s) & \text{P-G fault} \end{cases} \quad (25)$$

(25) is firstly simplified to analyze the time-domain characteristics of the forward TW F_n^i after i th reflection.

Taking the P-P fault at f_4 of the OHL L2 as an example, the P-G fault is similar. On basis of (25), F_n^i can be simplified as:

$$F_n^i(s) = \left(\frac{Z_{cl}}{Z_{cl} + R_f}\right)^{i-1} \cdot \left(1 - \frac{\frac{2U_{dc}Z_{cl}}{s + \frac{Z_{cl}}{L_{eq}}}}{\frac{Z_{cl}}{L_{eq}}}\right)^i \cdot \frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} \cdot \frac{1}{s} \quad (26)$$

By binomial expansion of $F_n^i(s)$, it can be described as:

$$F_n^i(s) = (-1)^i \cdot \left(\frac{Z_{cl}}{Z_{cl} + R_f}\right)^{i-1} \cdot \frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} \cdot \frac{1}{s} + 2 \cdot (-1)^{i+1} \cdot \left(\frac{Z_{cl}}{Z_{cl} + R_f}\right)^{i-1} \cdot \frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} \cdot \frac{1}{s + \frac{Z_{cl}}{L_{eq}}} + o\left(\frac{1}{(s + \frac{Z_{cl}}{L_{eq}})^2}\right) \quad (27)$$

In (27), o represents a higher-order term. If the higher-order term is transformed by the *Inverse Laplace Transform*, the transformed time-domain expression contains the product term of time t , which is almost zero in the T_w time period selected in this paper, so it can be ignored in the analysis. The time-domain expression of the forward TW F_n^i can be obtained by applying the *Inverse Laplace Transform* to (28), and thus it can be obtained that:

$$F_n^i(t) = (-1)^i \cdot \left(\frac{Z_{cl}}{Z_{cl} + R_f}\right)^{i-1} \cdot \frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} + 2 \cdot (-1)^{i+1} \cdot \left(\frac{Z_{cl}}{Z_{cl} + R_f}\right)^{i-1} \cdot \frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} \cdot e^{-\frac{Z_{cl}}{L_{eq}}t} \quad (28)$$

According to (28), when a metallic fault occurs, for each TW arrival time, and $t=0$, it is described as:

$$F_n^i(0) = 2 \cdot (-1)^{i+1} U_{dc} \quad (29)$$

According to (29), after each backward TW is reflected at point n to form a forward TW, the amplitude of the forward TW should be the opposite of the previous amplitude, namely, a sudden change occurs. According to (28) and (29), it can be analyzed that the TW after each reflection has a tendency of changing suddenly at first and then it changes gradually. The forward TW F_n expression is shown in (30). Δt_f is the time interval of the two adjacent forward TWs after reflection.

$$F_n(t) = \sum_{i=1}^k F_n^i[t - (i-1)\Delta t_f] \varepsilon(t - (i-1)\Delta t_f) \quad (30)$$

When a metallic P-P fault occurs at 5km away from n at 1.5s, the TW F_n simulation waveform under this fault condition can be obtained, and the analytic waveform can be illustrated according to (28) and (30), as shown in Fig. 12.

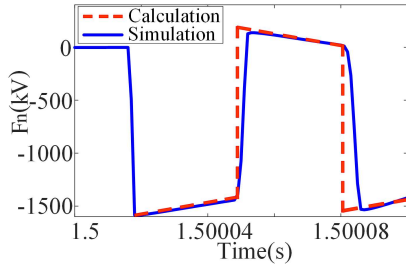


Fig. 12. The simulation and analytical waveform of the forward TW F_n at n

According to (28) and (30), the extreme value of TW F_n in the time window T_s after fault can be calculated, as shown in (31).

$$\min(F_n) = \begin{cases} \frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} & \text{P-P fault} \\ \frac{2U_{dc}Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} & \text{P-G fault} \end{cases}$$

$$\max(F_n) = \begin{cases} \frac{2U_{dc}Z_{cl}R_f}{(R_f + Z_{cl})^2} - \frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} \cdot 2e^{-\frac{Z_{cl}}{L_{eq}}T_w} & \text{P-P fault} \\ \frac{2U_{dc}Z_{cl}^2}{(Z_{cl} + Z_{c0} + 4R_f)^2} - \frac{2U_{dc}Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \cdot 2e^{-\frac{Z_{cl}}{L_{eq}}T_w} & \text{P-G fault} \end{cases} \quad (31)$$

The forward TW F_n at n firstly propagates to f_4 , and then refracts at f_4 to form the TW F_{nf} , which continues to spread to m at the head of the OHL L2. According to the TW refraction formula, F_{nf} can be expressed as:

$$F_{nf} = \begin{cases} \frac{R_f}{R_f + Z_{cl}} \cdot F_n & \text{P-P fault} \\ \frac{Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \cdot F_n & \text{P-G fault} \end{cases} \quad (32)$$

The TW B_m received at m is composed of two parts, the backward TW B_f directly generated from f_4 and the forward TW F_{nf} reflected at the end of OHL L2 and then refracted by f_4 . Ignoring the attenuation and distortion effects of the OHL on the TW, the TW B_m can be expressed as:

$$B_m = B_f + F_{nf} \quad (33)$$

To ensure that the theoretical analysis conclusion is correct, P-G fault with a resistance of 100Ω is set at f_4 , which is 5km away from the end of the OHL L2. The TW B_m simulation waveform and analytical waveform at m can be obtained, as shown in Fig. 13. According to Fig. 13, the simulation waveform is basically consistent with the analytical waveform in the variation trend and value range.

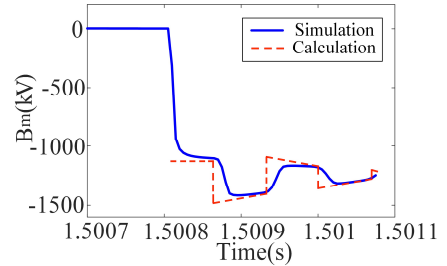


Fig. 13. The simulation and analytical waveform of the TW B_m in case of the OHL L2 remote fault

3. Fault Direction Criterion

The starting time at protection m after line fault is set as t_0 , and the criterion of protection starting unit is:

$$\left| \frac{dU_m}{dt} \right| > \Delta_1 \quad (34)$$

In (34), U_m denotes the voltage at point m, and Δ_1 is the setting threshold. During the setting process, the abrupt change of voltage rate caused by voltage fluctuation during normal system operation should be avoided.

According to the four fault conditions analyzed in Section II, the value range of TW B_m in the time window T_s can be obtained.

When the fault occurs on the OHL L1, B_m is 0 before the reflected TW reaches m.

When the fault occurs at f_2 , according to (21), the value range of B_m within T_s can be obtained as follow:

$$\frac{2U_{dc}Z_{cl}}{R_f + Z_{cl}} + \frac{2U_{dc}Z_{cl}^2}{(Z_{cl} + R_f)^2} \left(e^{\frac{Z_{cl}}{2T_{eq}}} - 1 \right) \leq B_m \leq \frac{2U_{dc}Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \quad (35)$$

When the fault occurs at f_3 , according to (4), the value range of B_m can be obtained as follow:

$$\frac{2(1 - k_a I_f)U_{dc}Z_{cl}}{R_f + Z_{cl}} \leq B_m \leq \frac{2(1 - k_a I_f)U_{dc}Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \quad (36)$$

When the fault occurs at f_4 , according to (3),(31),(32),(33) the value range of B_m within T_s can be calculated as follow:

$$\frac{2U_{dc}(1 - k_a I_f)(Z_{cl} + 2R_f)Z_{cl}}{(R_f + Z_{cl})^2} \leq B_m \leq \frac{2U_{dc}(1 - k_a I_f)Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \quad (37)$$

$$\left[1 + \frac{Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \cdot 2e^{\frac{Z_{cl}}{2T_{eq}}} + \frac{Z_{cl}^2}{(Z_{cl} + Z_{c0} + 4R_f)^2} \right]$$

From the above analysis, in the time period of $t_0 < t < t_0 + T_w$, if the fault occurs on the OHL L1, the TW B_m is 0. While, if the fault occurs on line L2, it can be seen from (35), (36) and (37) that B_m is negative and has a maximum value. The maximum value is obtained, as shown in (38).

$$\max(B_m) = \frac{2U_{dc}(1 - k_a I_f)Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \left[1 + \frac{Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \cdot 2e^{\frac{Z_{cl}}{2T_{eq}}} + \frac{Z_{cl}^2}{(Z_{cl} + Z_{c0} + 4R_f)^2} \right] \quad (38)$$

For the measuring point m, the fault on the OHL L1 is regarded as a backward fault, while the fault on the OHL L2 is regarded as a forward fault. In order to identify the fault direction, the threshold K_{set} can be set. If B_m is less than K_{set} , it is determined as a forward fault, otherwise it is a backward fault. K_{set} can be set as the maximum value of the backward TW B_m after the OHL L2 (forward) fault. B_m is always negative after the OHL L2 (forward) fault. To improve the sensitivity of the fault direction identification, the threshold K_{set} can be demonstrated as follows:

$$K_{set} = 0.2 \max(B_m) = \frac{0.4U_{dc}(1 - k_a I_f)Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \left[1 + \frac{Z_{cl}}{Z_{cl} + Z_{c0} + 4R_f} \cdot 2e^{\frac{Z_{cl}}{2T_{eq}}} + \frac{Z_{cl}^2}{(Z_{cl} + Z_{c0} + 4R_f)^2} \right] \quad (39)$$

The fault direction criterion is shown as (40), where $\Delta i_{m1}(k)$ and $\Delta u_{m1}(k)$ are the fault components of line-mode current and voltage at m.

$$\begin{cases} \Delta u_{m1}(k) - Z_{cl}\Delta i_{m1}(k) \geq K_{set} & \text{backward fault} \\ \text{Otherwise} & \text{forward fault} \end{cases} \quad (40)$$

According to the fault analysis in Section II. B, the time window T_w is set as 0.2ms. The criterion (40) is to detect whether the BFTW B_m exceeds the threshold K_{set} within 0.2ms after the fault start-up unit trips.

According to (38), after a forward fault, $\max(B_m)$ is negative. The derivative of $\max(B_m)$ expression shows that it increases with the rise of transition resistance. Therefore, in the K_{set} setting calculation, the transition resistance R_f is set as 500 Ω to ensure the accuracy of identification under high resistance fault conditions.

4. Simulation Verification

Taking the Kun-Liu-Long hybrid three-terminal HVDC system as the referent, the simulation model is established in PSCAD/EMTDC. Table 1 displays the specific system parameters. The fault point f_1 is located at the midpoint of the OHL L1, the fault points f_2, f_3 and f_4 are respectively located 10km from the head of the OHL L2, in the middle of the OHL L2 and 10km from the end of the OHL L2. The fault location can be seen in Fig. 2. The transmission line mode is frequency-dependent and the tower model is shown as Fig. 14.

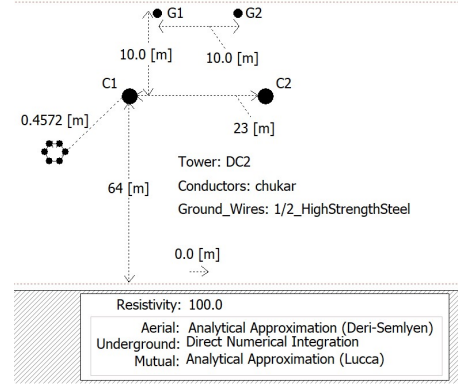


Fig. 14. The tower model of the OHL

According to the fault analysis in Section II. B, the time window T_w is set as 0.2ms and the threshold K_{set} can be calculated as -22.3 kV by bringing the parameters in Table 1 into (39). The threshold Δ_1 of fault start-up unit is set as 25kV/ms. The fault time is set as 1.5s and the sampling frequency of the protection device is 50 kHz.

4.1. BFTW Characteristics at m in Case of the OHL L2 Fault

When a fault occurs at f_1 on the OHL L1, the simulation waveform and threshold curve of backward TW B_m at m are shown in Fig. 15. The fault types are P-P fault and P-G fault. The time shown by the red dotted line in the figure is the start time of the protection, and the time window is 0.2ms.

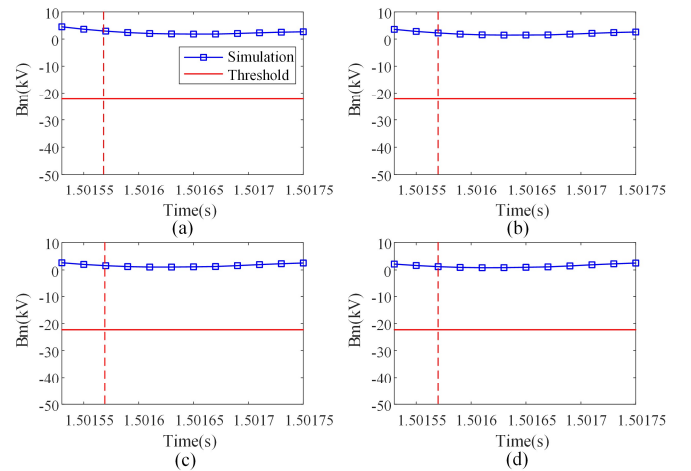


Fig. 15. The simulation waveform and threshold curve in case of f_1 fault (a) P-P fault with 0 Ω (b) P-P fault with 100 Ω (c) P-G fault with 0 Ω (d) P-G fault with 100 Ω

Fig. 15 demonstrates that the TW B_m after the OHL L1 fault is approximately 0 and is greater than the threshold K_{set} within the time window. Therefore, it can be accurately obtained that the faults occur on the OHL L1, and determine it as a forward fault.

4.2. Verification of Judgment in Case of the OHL L2 Fault

When a fault occurs on the OHL L2, the simulation waveform and threshold curve of the backward TW B_m at m after the fault at f_2, f_3 and f_4 on the OHL L2 can be obtained, as shown in Fig. 16, 17 and 18 separately.

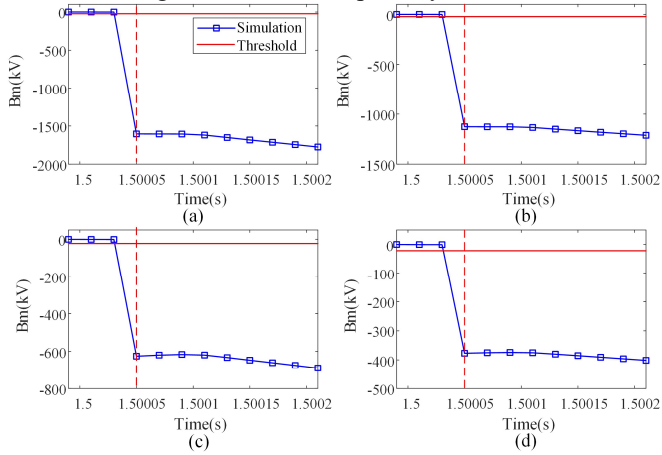


Fig. 16. The simulation waveform and threshold curve in case of f_2 fault (a) P-P fault with 0Ω (b) P-P fault with 100Ω (c) P-G fault with 0Ω (d) P-G fault with 100Ω

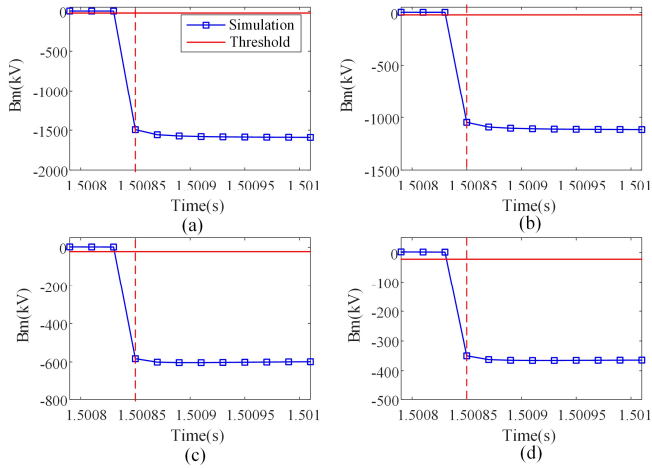


Fig. 17. The simulation waveform and threshold curve in case of f_3 fault (a) P-P fault with 0Ω (b) P-P fault with 100Ω (c) P-G fault with 0Ω (d) P-G fault with 100Ω

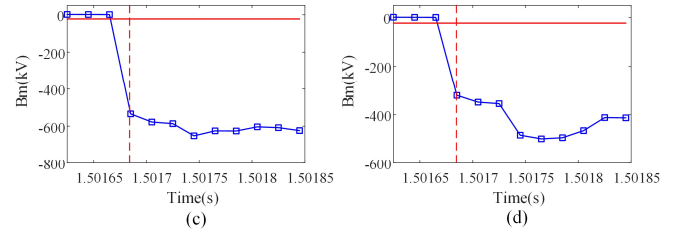
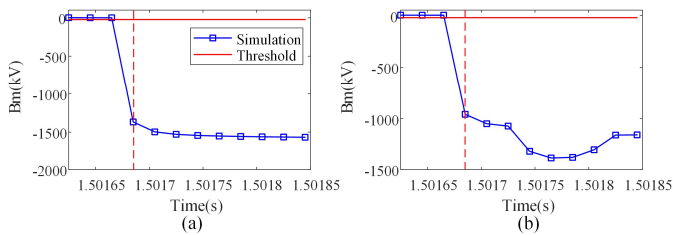


Fig. 18. The simulation waveform and threshold curve in case of f_4 fault (a) P-P fault with 0Ω (b) P-P fault with 100Ω (c) P-G fault with 0Ω (d) P-G fault with 100Ω

It can be seen from Fig. 16, 17 and 18 that the TW B_m after the OHL L2 fault is always negative and significantly less than K_{set} within the time window. Therefore, this method can accurately identify the fault occurs on OHL L2 and determine it as a forward fault.

4.3. Robustness Test of High Resistance Fault

In order to test the robustness to high-resistance fault of the criterion proposed in this paper, the transition resistance is set as 500Ω . The minimum value of TW B_m in case of backward fault and the maximum value of TW B_m in case of forward fault are obtained, as shown in Table 2.

Table 2 Robustness test of high resistance fault

Fault type	Fault point	Max(B_m) (kV)	Min(B_m) (kV)	Fault direction
P-G fault	f_1	7.3	-1.2	Backward
	f_2	-146	-213	Forward
	f_3	-141	-159	Forward
	f_4	-199	-249	Forward
P-P fault	f_1	6.9	-1.4	Backward
	f_2	-515	-697	Forward
	f_3	-505	-536	Forward
	f_4	-672	-815	Forward

According to Table 2, in the case of high-resistance fault, $\min(B_m)$ after backward fault is higher than K_{set} . After the forward fault, $\max(B_m)$ is less than K_{set} . It is demonstrated that the method proposed in this paper is still applicable in the case of a P-G fault with a transition resistance of 500Ω and has a strong robustness to high transition resistance.

4.4. Robustness Test of Noise

Gaussian white noise with a signal-to-noise ratio of 20dB is superimposed on the TW B_m to evaluate the anti-noise capability of the criteria established in this paper. As shown in Table 3, the minimum value of the TW B_m in case of the backward fault and the maximum value of the TW B_m in case of the forward fault is obtained.

Table 3 Robustness test of noise

Fault type	Fault point	Max(B_m) (kV)	Min(B_m) (kV)	Fault direction
P-G fault	f_1	5.3	-1.5	Backward
	f_2	-245	-323	Forward
	f_3	-255	-271	Forward
	f_4	-337	-436	Forward
	f_1	7.4	-1.2	Backward

P-P fault	f_2	-772	-893	Forward
	f_3	-749	-761	Forward
	f_4	-887	-973	Forward

According to Table 3, $\min(B_m)$ is much larger than K_{set} in the case of backward fault, and $\max(B_m)$ is much smaller than K_{set} in the case of forward fault, which proves that the criterion is still applicable under the condition of noise interference has a strong robustness to noise interference.

4.5. Comparison with Other Direction Criterion

[20] employs the current variation Δi as the characteristic quantity for the direction criterion. The fault direction criterion can be expressed as:

$$\Delta i > \Delta_1 \quad (\text{Forward fault}) \quad (41)$$

To avoid the influence of current fluctuations under the normal system operation and load power change, the threshold Δ_1 is selected as 0.35kA. The direction criterion in [20] and the criterion proposed in this paper are respectively tested, and the fault type is P-G fault. Table 4 displays the test result.

Table 4 Comparison between Siemens criterion

Fault resistance(Ω)	Fault point	Δi (kA)	Fault direction	Max (B_m) (kV)	Fault direction
100	f_2	1.67	Forward	-403	Forward
	f_3	0.85	Forward	-378	Forward
	f_4	0.69	Forward	-369	Forward
500	f_2	0.62	Forward	-146	Forward
	f_3	0.31	/	-141	Forward
	f_4	0.23	/	-199	Forward

According to Table 4, the direction criterion proposed by [20] can identify the fault direction when the fault resistance is 100 Ω . But the judgment fails when the fault resistance is 500 Ω , while the criterion proposed in this study can still correctly judge.

5. Conclusion

The analytical calculation method of voltage TW after the fault occurs at OHL in the hybrid three-terminal HVDC system is studied and a direction criterion based on the characteristics of the backward TW is proposed in this paper. The contributions and innovations of this paper are as follows:

1) For the first time, the time-domain expression of the backward TW after multiple reflections and refractions is derived when the fault occurs at the near and remote end of the OHL, and the difference of the TW between the forward fault and the backward fault is revealed.

2) A new direction criterion based on the amplitude of the backward TW is proposed, which realizes accurate and quick direction identification and does not rely on the line boundary element. The setting value is directly calculated through system parameters instead of depending on the simulation.

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