Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

ARTICLE TYPE

Enclosing control for nonholonomic mobile agents with a moving target of unknown velocity†

Shuang Ju[1] | Jing Wang*[2] | Li-Ya Dou*[1]

[1]College of Information Science and Technology, Beijing University of Chemical Technology, Beijing, China
[2]School of Electrical and Control Engineering, North China University of Technology, Beijing, China

Correspondence
Jing Wang, School of Electrical and Control Engineering, North China University of Technology, Beijing 100144, China. Email: jwang@ncut.edu.cn; Li-Ya Dou, College of Information Science and Technology, Beijing University of Chemical Technology, Beijing 100029, China. Email: liyadou@mail.buct.edu.cn.

Summary

In this paper, an enclosing control problem is investigated for nonholonomic mobile agents with a moving target of unknown velocity. An adaptive observer containing two internal variables is first designed for each agent to compensate for the lack of the target velocity information. One variable is designed to estimate the unknown target velocity and further its estimation error is assessed by the other internal variable to subsequently guarantee the control performance. Then using the estimated information from the adaptive observer, a dynamic control law for circular formation of nonholonomic agents around the moving target is designed by a backstepping process. The global asymptotical stability of the closed-loop system is achieved under the proposed dynamic control law with the adaptive observer. Finally, a simulation is conducted to demonstrate the effectiveness of the proposed approach.

KEYWORDS:
target enclosing control, adaptive observer, nonholonomic mobile agents, unknown velocity

1 | INTRODUCTION

In recent years, enclosing control, also called circular formation control, of mobile agents has attracted considerable research interest due to its potential applications such as target pursuit and evasion[1], source localization[2], surveillance[3] and monitoring[4]. The enclosing control objective is that all the agents rotate around a static or moving target. A hybrid control approach has been proposed for static target enclosing control problem of the agents with limited sensing region[5]. A circular formation control without global information has been investigated, where the target is stationary while may change its position suddenly[6]. An optimization-based approach has been proposed for multiple nonholonomic mobile robots moving in circles around a static target with different radii[7]. The input disturbance has been considered for the enclosing control problem of a static target[8,9]. Note that the above control approaches have been proposed for the agents rotating around a static target. It is difficult to directly extend the results to the moving target case.

When the target is moving, the knowledge of the target current dynamics (e.g., the target position, velocity or acceleration in practice) plays a key role in the controller designing process[10,11]. Many excellent results have been presented based on the measurements of the target real-time velocity or acceleration. A decoupled approach has been given by making use of the relative velocity information of each agent to the target, as well as the target acceleration[12]. The proposed approach in Reference 12 can lead to a desired formation in almost all geometric patterns for the agents with double-integrator dynamics, which is an extension of the controller in Reference 13 for agents with single-integrator dynamics. Compared with the controller for single-integrators, the controller for double-integrators may be more easier to apply in some real robot[14]. Since many practical robots are subject

†Enclosing control for nonholonomic mobile agents with a moving target of unknown velocity.
[0]**Abbreviations:**

to nonholonomic constraints such as unmanned aerial vehicles and unmanned underwater vehicles, the enclosing formation control problem of multiple mobile agents under nonholonomic constraints is challenging but has great practical value[15]. A cooperative circumnavigation problem of a moving target has been studied based on backstepping design by using the target velocity information[16]. A local cooperative controller has been proposed based on the measurements of the target real-time velocity and acceleration in local coordinate frame[17]. An observer-based feedback linearization approach has been proposed for nonholonomic mobile agents[18]. While in some specific situations, the mobile agents can not get the target information due to their limited sensing ability and certain time-varying characteristics of the moving target. This leads to the lack of complete and accurate system dynamics, which makes it difficult for the mobile agents to handle the enclosing formation task[19].

A local control approach has been developed for single-integrator mobile agents to rotate around a moving target with a constant velocity which is unknown to any agent[20]. This result has been extended to solve the cooperative enclosing control problem of a moving target with an unknown time-varying velocity[21]. Also for the single-integrator mobile agents, a bearing-only method has been designed to exponentially solve the enclosing formation control problem by assuming that at least one agent can acquire the target information[22]. Additional collision avoidance challenge has been considered when none of the agents knows the amplitude of the target velocity[23]. Also lack of the amplitude information of the target velocity, a leader-following consensus problem has been investigated for Euler-Lagrange systems[24]. A distributed estimation and control problem has been studied for nonholonomic mobile robots with situation of leader-following formations when only a few members know leader's information[25]. For the case that the leader's information is not available to all the followers, when the leader velocity is constant, the formation error converges to zero asymptotically, while when the leader moves at a time-varying velocity, the formation error is globally uniformly ultimately bounded[26]. There have been other fruitful results of leader-following problem with unknown leader such as Reference 27 and the references therein. Most of the existing literatures consider that the leader's velocity is generated by some exogenous systems. It can be seen that there is still a lot of space for the enclosing control problem of nonholonomic mobile agents with a moving target of unknown time-varying velocity.

Inspired by the forgoing observations, we study the moving target enclosing control problem for multiple mobile agents subject to nonholonomic constraint. All agents do not know the target velocity information, and the target considered in this work is not described by a certain exogenous system or required to have the same model as the nonholonomic mobile agents. Each agent can sense the relative position information of the target and its neighbors in its own local coordinate frame. Therefore, the key idea is to design an adaptive observer to compensate for the lack of the target velocity information, based on which a dynamic control law can be designed for the agents to maintain a desired circular formation around the target. Main contributions of this work are concluded as follows.

i An adaptive observer is designed for each agent by introducing two internal variables to estimate the unknown information resulting from the unknown target velocity. Under the adaptive observer, the introduced internal variables can asymptotically converge to the actual target velocity and its estimation error respectively. Compared with the observers in References 23 and 25, which require that at least one agent can acquire the target information, the proposed adaptive observer in this work is applicable to the case that the target information is unknown to any agent.

ii Using the estimated information from the adaptive observer, a dynamic control law is designed for nonholonomic mobile agents by a backstepping process. By the dynamic control law with the adaptive observer, multiple nonholonomic mobile agents in a connected graph can asymptotically converge to the desired circular formation around the moving target. In References 22 and 26, which consider bearing measurement and communication limitation challenges, respectively, the formation tracking error is shown to be ultimately bounded when the target velocity is unknown to any agent.

The rest of this work is organized as follows. In Section II, we first introduce the problem formulation and preliminaries. In Section III, main results are given. Simulation results are given in Section IV. In Section V, we draw a conclusion of this work.

Notations: The n-dimensional Euclidean space is given by $\mathbb{R}^n$, the set of real number is $\mathbb{R}$. For a vector v, $\|v\|$ denotes the 2-norm of v. $\mathrm{diag}\{A_1, A_2, \cdots, A_n\}$ denotes the block diagonal matrix, where A_i, $i \in \{1, 2, \cdots, n\}$, is the main diagonal block matrix. For vector $x = [x_1; x_2] \in \mathbb{R}^2$, $\mathrm{atan2}(x) = \mathrm{atan2}(\frac{x_2}{x_1})$. I is the identity matrix. For $\forall \chi_i \in \mathbb{R}$, $i \in \{1, 2, \cdots, n\}$, $\mathrm{co}(\chi_i) = [\chi_1, \chi_2, \cdots, \chi_n]^T$.

2 | PROBLEM FORMULATION AND PRELIMINARIES

2.1 | Graph theory

An undirected graph $G = \{V, E\}$ is given to show the communication network of all agents, where $V = 1, 2, \cdots, n$ is the node set and $E \subseteq V \times V$ is the edge set. The edge denoted by (j, i) represents that node i and node j can send information to each other. Define $A = [a_{ij}]_{n \times n}$ as the adjacency matrix of graph G, where $a_{ii} = 0$ for all i, $a_{ij} > 0$ if $(i, j) \in E$ and $a_{ij} = 0$ otherwise. If there exists an edge between two agents, then the two agents are neighbors. For agent i, the neighbor set is defined as $N_i = \{j \in V | (i, j) \in E\}$. For the undirected graph G, there exists at least one path from node i to node j, $i, j \in V$, i.e., the undirected graph G is connected in this work.

2.2 | Problem Formulation

Consider n nonholonomic mobile agents moving freely in the plane. The kinematics of agent i is given as

$$\begin{cases} \dot{p}_i = h(\theta_i)v_i, \\ \dot{\theta}_i = \omega_i, \end{cases} \tag{1}$$

where $h(\theta_i) = [\cos\theta_i, \sin\theta_i]^T \in \mathbb{R}^2$, $p_i = [x_i \ y_i]^T \in \mathbb{R}^2$ is the position, $\theta_i \in (-\pi, \pi]$ is the bearing angle, $v_i \in \mathbb{R}$ is the linear velocity, $\omega_i \in \mathbb{R}$ is the angular velocity of agent i, $i = 1, 2, \cdots, n$, the control input of agent i is $u_i = [v_i \ w_i]^T$.

There exist a target moving in the plane at an unknown velocity, i.e.,

$$\begin{cases} \dot{p}_0 = v_0, \\ \dot{v}_0 = a_0, \end{cases} \tag{2}$$

where $p_0 = [x_0, y_0]^T \in \mathbb{R}^2$, $v_0 \in \mathbb{R}^2$, $a_0 \in \mathbb{R}^2$ are the position, the velocity and the acceleration, respectively. It is noted that the target acceleration a_0 is a piecewise constant and known by all agents. But the target velocity v_0 is unknown to any agent.

Assume that each agent occupies its own local coordinate frame, the orientation of which is consistent with that of a global coordinate system. In agent i's local coordinate frame, denote $\tilde{p}_i = p_i - p_0$ and $\tilde{p}_{ij} = p_i - p_j$ by the relative positions of the target and its neighbor j, respectively. We can obtain the relative information $\tilde{p}_i$ and $\tilde{p}_{ij}$ through measuring the relative distance and bearing angle to the target and agent j, $j \in N_i$, respectively. Using the direct measurements $\tilde{p}_i$ and $\tilde{p}_{ij}$, agent i can calculate the relative position information of its neighbor j to the target by $\tilde{p}_j = \tilde{p}_i - \tilde{p}_{ij}$. FIGURE 1 is depicted to understand the physical meanings of the above measurements.

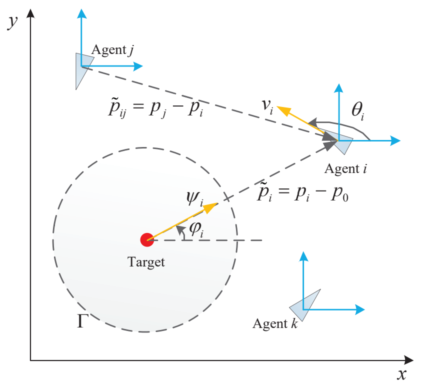

FIGURE 1 Enclosing a target with multiple nonholonomic mobile agents.

In this work, the enclosing control problem with a target moving with unknown velocity v_0 is concluded by the following two problems.

(i) Without the knowledge of the current target velocity v_0, the agents can not get the complete and accurate system dynamics. It is necessary to design an adaptive observer for each agent to estimate the target velocity v_0 by using the measurements of relative positions $\tilde{p}_i$ and $\tilde{p}_{ij}$. We introduce two internal variables in the adaptive observer. The one is to estimate the target velocity v_0, which is denoted by $\hat{v}_i$. Note that the accuracy of the variable $\hat{v}_i$, expressed as the estimation error $\delta_i = \hat{v}_i - v_0$, directly affects the control performance, which leads to the coupling of the nonlinearity of the agent model and the velocity estimation error. However, it is also impossible to obtain the actual estimation error δ_i without the knowledge of the current target velocity v_0. It challenges to evaluate whether the error δ_i converges to zero and further analyze the stability of system. Therefore, the other variable denoted by $\hat{\delta}_i$ is introduced to evaluate the accuracy of the variable $\hat{v}_i$. The control performance can be guaranteed by driving the internal variable $\hat{\delta}_i$ to asymptotically zero. The adaptive observer design problem with internal variables $(\hat{v}_i, \hat{\delta}_i)$ is described as follows.

Problem 1: Consider n nonholonomic mobile agents (1) and the target (2) moving with unknown velocity. For agent i, using the relative position information of its neighbors and the target, design an adaptive observer to estimate the target velocity information with internal variables

$$\begin{cases} \hat{v}_i = \mathbf{r}(\tilde{p}_i, a_0) \\ \hat{\delta}_i = \mathbf{b}(\hat{v}_i, \tilde{p}_i, \tilde{p}_{ij}) \end{cases}$$

for all $i = 1, 2, \cdots, n$, such that the following objectives are achieved:

- The internal variable $\hat{v}_i$ eventually converges to the target velocity v_0, i.e., $\lim_{t\to\infty} \hat{v}_i = v_0$.
- The internal variable $\hat{\delta}_i$ eventually converges to the estimation error δ_i, i.e., $\lim_{t\to\infty} \hat{\delta}_i = \delta_i$.

(ii) The objective of the target enclosing control problem is to design a cooperative control law u_i such that multiple nonholonomic mobile agents can move along a circular orbit centered at the target and simultaneously maintain a desired distribution on the orbit. The desired radius is given as $\rho_d > 0$ and the desired circular velocity is given as $w_d \in \mathbb{R}$. The desired angular spacing between agents i and j is denoted as $\phi_{ij} \in (-\pi, \pi]$ which is satisfied with $\phi_{ij} = -\phi_{ji}$. Note that the relative distance and the bearing angle between each agent and the target can be expressed as $\|\tilde{p}_i\| = \rho_i$ and $\varphi_i = \mathrm{atan2}(\tilde{p}_i)$, respectively. Then, using the estimated information from the adaptive observer, the dynamic controller design for the target enclosing problem can be formally defined as follows.

Problem 2: Based on the estimated information $\hat{v}_i$ and $\hat{\delta}_i$ from the adaptive observer and the local relative position information, design a dynamic control law for agent i, in the connected graph G in the form of

$$u_i = \mathbf{s}(\tilde{p}_i, \tilde{p}_{ij}, v_j, \hat{v}_i, \hat{\delta}_i, \hat{v}_j, \hat{\delta}_j, a_0),$$

for all $i = 1, 2, \cdots, n$, such that for any initial states $(p_i(0), \theta_i(0))$ the following objectives are achieved:

- The agents eventually move along the circular orbit Γ centered at the target with the desired radius ρ_d, i.e., $\lim_{t\to\infty} \rho_i = \rho_d$.
- The agents eventually move along the circular orbit Γ with the desired circular velocity w_d, i.e., $\lim_{t\to\infty} \dot{\varphi}_i = w_d$.
- The agents eventually perform a desired distribution on the circular orbit Γ, i.e., $\lim_{t\to\infty} \varphi_i - \varphi_j = \phi_{ij}, j \in N_i$.

3 | MAIN RESULTS

In this section, a dynamic control law with an adaptive observer is given first. Then a series of derivations are given to obtain a closed-loop system. Finally, the convergence of the closed-loop system is proved by a theorem.

3.1 | Dynamic controller design

Motivated by the backstepping method[16], the agent velocity v_i and the estimated target velocity $\hat{v}_i$ are designed to satisfy the following relation,

$$h(\hat{\theta}_i)v_i - \hat{v}_i = f_i, \tag{3}$$

where $f_i = [-k_1\tilde{\rho}_i I + \rho_d(w_d - g_i)\Lambda]\psi_i$ with the relative bearing of the target with respect to agent i

$$\psi_i = \frac{\tilde{p}_i}{\|\tilde{p}_i\|} = \frac{\tilde{p}_i}{\rho_i} = \begin{bmatrix} \cos\varphi_i \\ \sin\varphi_i \end{bmatrix},$$

$g_i = h_0 \tanh(\varepsilon_i)$, $\varepsilon_i = \sum_{j\in N_i} a_{ij}(\varphi_i - \varphi_j - \phi_{ij})$, $\Lambda = [0\ -1; 1\ 0]$, $\tilde{\rho}_i = \rho_i - \rho_d$, h_0 and k_1 are the positive constants, $\hat{\theta}_i$ represents the virtual signal of θ_i and noted that

$$\hat{\theta}_i = \mathrm{atan2}(f_i + \hat{v}_i). \tag{4}$$

The adaptive observer is designed first to estimate the target velocity v_0 and its estimation error δ_i as described in **Problem 1**, i.e.,

$$\begin{cases} \dot{\hat{v}}_i = -k_2\tilde{\rho}_i\psi_i + a_0, \\ \dot{\hat{\delta}}_i = -k_2\tilde{\rho}_i\psi_i - \tilde{\theta}_i c_i^T\sigma_i^T - \tilde{\theta}_i \sum a_{ij}(\lambda_i^T - \lambda_j^T)\epsilon_i^T\sigma_i^T, \end{cases} \tag{5}$$

where

$$\begin{aligned} &\tilde{\theta}_i = \theta_i - \hat{\theta}_i, \\ &c_i = [-k_1\tilde{\rho}_i I + \rho_d(w_d - g_i)\Lambda](I - \psi_i\psi_i^T)/\rho_i - k_1\psi_i\psi_i^T, \\ &\sigma_i = (f_i + \hat{v}_i)^T\Lambda^T/v_i^2, \\ &\Lambda = [0\ -1; 1\ 0], \\ &\lambda_i = \tilde{p}_i^T\Lambda^T/\rho_i^2, \\ &\epsilon_i = -\rho_d h_0(1 - \tanh^2(\varepsilon_i))\Lambda\psi_i, \end{aligned}$$

and k_2 is the positive constant.

Using the estimated information from the adaptive observer (5), we develop a dynamic control law for the enclosing control of nonholonomic agents with a moving target of unknown velocity as described in **Problem 2**, i.e.,

$$\begin{cases} v_i = \|f_i + \hat{v}_i\|, \\ \omega_i = -k_3\tilde{\theta}_i - k_2\tilde{\rho}_i\psi_i^T\tilde{h}_i v_i + \Pi(\hat{\delta}_i), \end{cases} \tag{6}$$

where k_3 is the positive constant, and $\Pi(\cdot)$ is the function,

$$\Pi(\hat{\delta}_i) = \sigma_i(\Xi(\hat{\delta}_i) + \dot{\hat{v}}_i) \tag{7}$$

with

$$\begin{aligned} &\Xi(\hat{\delta}_i) = b_i + c_i\hat{\delta}_i + \epsilon_i \sum a_{ij}(\lambda_i\hat{\delta}_i - \lambda_j\hat{\delta}_j), \\ &b_i = [-k_1\tilde{\rho}_i I + \rho_d(w_d - g_i)\Lambda](I - \psi_i\psi_i^T)\hbar_i/\rho_i + [-k_1\psi_i^T\hbar_i I - \rho_d\gamma_i\Lambda]\psi_i, \\ &\gamma_i = h_0(1 - \tanh^2(\varepsilon_i)) \sum a_{ij}(\lambda_i\hbar_i - \lambda_j\hbar_j), \\ &\hbar_i = \tilde{h}_i\tilde{\theta}_i v_i + f_i, \\ &\tilde{h}_i = (h(\theta_i) - h(\hat{\theta}_i))/\tilde{\theta}_i. \end{aligned}$$

In order to show the control mechanism intuitively, the data flow of multiple nonholonomic mobile agents in the control loop is depicted in FIGURE 2.

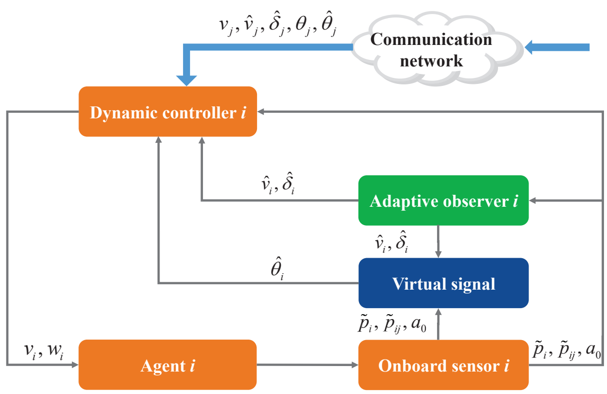

FIGURE 2 Data flow of multiple nonholonomic mobile agents in the control loop.

3.2 | Closed-loop system

According to equations (1)-(2), the relative position dynamics between the target and agent i is

$$\dot{\tilde{p}}_i = h(\theta_i)v_i - v_0.$$

According to (3), it yields

$$\dot{\tilde{p}}_i = \hbar_i + \delta_i.$$

It follows that the dynamics of the distance between the target and agent i is

$$\dot{\tilde{\rho}}_i = \psi_i^T \dot{\tilde{p}}_i = \psi_i^T (\tilde{h}_i \tilde{\theta}_i v_i + f_i + \delta_i) = \psi_i^T \tilde{h}_i \tilde{\theta}_i v_i - k_1 \tilde{\rho}_i + \psi_i^T \delta_i, \tag{8}$$

where $\psi_i^T \Lambda \psi_i = 0$ is used.

According to the definition of $\hat{\theta}_i$ in (4), the derivatives of $\hat{\theta}_i$ is calculated by $\dot{\hat{\theta}}_i = \Pi(\hat{\theta}_i)$, where $\Pi(\cdot)$ is the same function as in (7). Note that $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, one has

$$\dot{\tilde{\theta}}_i = \omega_i - \dot{\hat{\theta}}_i. \tag{9}$$

Substituting the angular velocity w_i of (6) into (9), it obtains

$$\dot{\tilde{\theta}}_i = -k_3 \tilde{\theta}_i - k_2 \tilde{\rho}_i \psi_i^T \tilde{h}_i v_i + \Pi(\hat{\delta}_i) - \Pi(\delta_i).$$

Letting $e_i = \hat{\delta}_i - \delta_i$, one has

$$\Pi(\hat{\delta}_i) - \Pi(\delta_i) = \sigma_i(\Xi(\hat{\delta}_i) - \Xi(\delta_i)) = \sigma_i[c_i e_i + \epsilon_i \sum a_{ij}(\lambda_i e_i - \lambda_j e_j)].$$

Then it yields

$$\dot{\tilde{\theta}}_i = -k_3 \tilde{\theta}_i - k_2 \tilde{\rho}_i \psi_i^T \tilde{h}_i v_i + \sigma_i[c_i e_i + \epsilon_i \sum a_{ij}(\lambda_i e_i - \lambda_j e_j)]. \tag{10}$$

In order to show that the conditions required in both **Problem 1** and **Problem 2** are satisfied, rewrite the closed-loop system in a compact form. According to (8) and (10), one has

$$\dot{\tilde{\rho}} = D(\psi_i^T \tilde{h}_i \tilde{\theta}_i)v - k_1 \tilde{\rho} + D(\psi_i^T)\delta \tag{11}$$

and

$$\dot{\tilde{\theta}} = -k_3 \tilde{\theta} - k_2 D(\tilde{\rho}_i \psi_i^T \tilde{h}_i)v + [D(\sigma_i c_i) + D(\sigma_i \epsilon_i) L D(\lambda_i)]e, \tag{12}$$

where $D(\chi_i) = \mathrm{diag}\{\chi_1, \chi_2, \cdots, \chi_n\}$, $i = 1, 2, \cdots, n$, $\tilde{\rho} = \mathrm{co}(\rho_i)$, $v = \mathrm{co}(v_i)$, $\tilde{\theta} = \mathrm{co}(\tilde{\theta}_i)$, $e = \mathrm{co}(e_i)$.

Note that the dynamics of the estimation error of the target velocity is calculated by $\dot{\delta}_i = \dot{\hat{v}}_i - a_0$ which can be written in the following compact form

$$\dot{\delta} = \dot{\hat{v}} - \tilde{a}, \tag{13}$$

where $\delta = \mathrm{co}(\delta_i)$, $\tilde{a} = \mathrm{co}(a_0)$ and $\hat{v} = \mathrm{co}(\hat{v}_i)$. Furthermore, one obtains that $\dot{e}_i = \dot{\hat{\delta}}_i - \dot{\delta}_i$ which can also be written in the compact form as

$$\dot{e} = \dot{\hat{\delta}} - \dot{\hat{v}} + \tilde{a}, \tag{14}$$

where $\hat{\delta} = \mathrm{co}(\hat{\delta}_i)$.

3.3 | Convergence analysis

Based on the analysis in the above subsections, the following theorem is given to conclude the main results.

Theorem 1. Under the dynamic control law (6) with adaptive observer (5), nonholonomic mobile agents (1) in a connected graph G can asymptotically converge to a desired circular formation around the moving target (2) of unknown velocity if the control parameters satisfying $h_0, k_1, k_2, k_3 > 0$, that is, both **Problem 1** and **Problem 2** are solved.

Proof. We first show that the conditions required in **Problem1** hold by the adaptive observer (5).

Take the following Lyapunov function

$$V_1 = \frac{1}{2}k_2\tilde{\rho}^T\tilde{\rho} + \frac{1}{2}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2}\delta^T\delta + \frac{1}{2}e^Te.$$

According to equations (11)-(14), the derivative of V_1 is calculated as

$$\begin{aligned}\dot{V}_1 =& k_2\tilde{\rho}^T[D(\psi_i^T\tilde{h}_i\tilde{\theta}_i)v - k_1\tilde{\rho} + D(\psi_i^T)\delta] + \tilde{\theta}^T[-k_3\tilde{\theta} - k_2D(\tilde{\rho}_i\psi_i^T\tilde{h}_i)v] + \tilde{\theta}^T[D(\sigma_ic_i) + D(\sigma_i\epsilon_i)LD(\lambda_i)]e+\\ &\delta^T(\dot{\hat{v}} - \tilde{a}) + e^T(\dot{\hat{\delta}} - \dot{\hat{v}})\\ =& -k_1k_2\tilde{\rho}^T\tilde{\rho} - k_3\tilde{\theta}^T\tilde{\theta} + [k_2\tilde{\rho}^TD(\psi_i^T) + \dot{\hat{v}}^T - \tilde{a}^T]\delta + [\tilde{\theta}^TD(\sigma_ic_i) + \tilde{\theta}^TD(\sigma_i\epsilon_i)LD(\lambda_i) + (\dot{\hat{\delta}} - \dot{\hat{v}} + \tilde{a})^T]e.\end{aligned}$$

According to the adaptive observer (5), there exist

$$k_2\tilde{\rho}^TD(\psi_i^T) + \dot{\hat{v}}^T - \tilde{a}^T = 0$$

and

$$\tilde{\theta}^TD(\sigma_ic_i) + \tilde{\theta}^TD(\sigma_i\epsilon_i)LD(\lambda_i) + (\dot{\hat{\delta}} - \dot{\hat{v}} + \tilde{a})^T = 0.$$

The derivative of V_1 is obtained as

$$\dot{V}_1 = -k_1k_2\tilde{\rho}^T\tilde{\rho} - k_3\tilde{\theta}^T\tilde{\theta} \leq 0.$$

It indicates that $\lim_{t\to\infty}\hat{\theta}_i = \theta_i$, $\lim_{t\to\infty}\hat{\delta}_i = \delta_i = 0$ and $\lim_{t\to\infty}\hat{v}_i = v_0$ hold. The adaptive observer (5) can asymptotically solve **Problem 1**.

Besides, it is also obtained that $\lim_{t\to\infty}\rho_i = \rho_d$ holds. The first objective of **Problem 2** is satisfied. Then we will show that the last two objectives in **Problem 2** can be satisfied through the dynamic control law (6).

According to the definition of φ_i, one has

$$\begin{aligned}\dot{\varphi}_i =& \frac{\tilde{p}_i^T\Lambda^T\dot{\tilde{p}}_i}{\rho_i^2}\\ =& \frac{\tilde{p}_i^T\Lambda^T(\hbar_i + \delta_i)}{\rho_i^2}\\ =& \frac{\tilde{p}_i^T\Lambda^Tf_i}{\rho_i^2} + \frac{\tilde{p}_i^T\Lambda^T\tilde{h}_i\tilde{\theta}_iv_i}{\rho_i^2} + \frac{\tilde{p}_i^T\Lambda^T\delta_i}{\rho_i^2}\\ =& \frac{\rho_d(w_d - g_i)}{\rho_i} + \frac{\tilde{p}_i^T\Lambda^T\tilde{h}_i\tilde{\theta}_iv_i}{\rho_i^2} + \frac{\tilde{p}_i^T\Lambda^T\delta_i}{\rho_i^2}\\ =& w_d - g_i + \xi_i,\end{aligned}$$

where

$$\xi_i = -\frac{\tilde{\rho}_i(w_d - g_i)}{\rho_i} + \frac{\tilde{p}_i^T\Lambda^T\tilde{h}_i\tilde{\theta}_iv_i}{\rho_i^2} + \frac{\tilde{p}_i^T\Lambda^T\delta_i}{\rho_i^2}. \tag{15}$$

Note that ξ_i will converge to zero as $t\to\infty$ due to the above results $\lim_{t\to\infty}\tilde{\rho}_i = 0$, $\lim_{t\to\infty}\tilde{\theta}_i = 0$ and $\lim_{t\to\infty}\delta_i = 0$.

Let $\tilde{\varphi}_i = \varphi_i - w_dt - \phi_i$, with $\phi_i - \phi_j = \phi_{ij}$. One has

$$\dot{\tilde{\varphi}}_i = -g(\varepsilon_i) + \xi_i,$$

where $g(\varepsilon_i) = g_i$. Note that $\varepsilon_i = \sum_{j\in N_i}a_{ij}(\varphi_i - \varphi_j - \phi_{ij}) = \sum_{j\in N_i}a_{ij}(\tilde{\varphi}_i - \tilde{\varphi}_j)$. There exists

$$\dot{\varepsilon}_i = -\sum_{j\in N_i}a_{ij}[g(\varepsilon_i) - g(\varepsilon_j) - (\xi_i - \xi_j)].$$

Take the Lyapunov candidate as

$$V_2 = \sum_i\int_0^{\varepsilon_i}g(\tau)d_\tau.$$

Then it is obtained that

$$\begin{aligned}\dot{V}_2 &= \sum_i g(\varepsilon_i)\dot{\varepsilon}_i \\ &= -\sum_i \sum_{j\in N_i} a_{ij} g(\varepsilon_i)[g(\varepsilon_i) - g(\varepsilon_j)] + \sum_i \sum_{j\in N_i} a_{ij} g(\varepsilon_i)(\xi_i - \xi_j) \\ &= -\frac{1}{2}\sum_i \sum_{j\in N_i} a_{ij}[g(\varepsilon_i) - g(\varepsilon_j)]^2 + \sum_i \sum_{j\in N_i} a_{ij} g(\varepsilon_i)(\xi_i - \xi_j).\end{aligned}$$

Note that $|g(\varepsilon_i)| \leq \mu$, one has

$$\dot{V}_2 \leq -\frac{1}{2}\sum_i \sum_{j\in N_i} a_{ij}[g(\varepsilon_i) - g(\varepsilon_j)]^2 + \sum_i \sum_{j\in N_i} a_{ij}\mu(|\xi_i| + |\xi_j|).$$

Integrating the inequality from 0 to t, it yields

$$V_2(t) + \frac{1}{2}\int_0^t \sum_i \sum_{j\in N_i} a_{ij}[g(\varepsilon_i) - g(\varepsilon_j)]^2 d_\tau \leq V_2(0) + \int_0^t \sum_i \sum_{j\in N_i} a_{ij}\mu(|\xi_i| + |\xi_j|)d_\tau. \quad (16)$$

From the definition of ξ in (15), it is noted that ξ_i converges to zero as $t \to \infty$. Then one has $\int_0^t \sum_i \sum_{j\in N_i} a_{ij}\mu(|\xi_i| + |\xi_j|)d_\tau$ is bounded. According to (16), both $V_2(t)$ and $\int_0^t \sum_i \sum_{j\in N_i} a_{ij}[g(\varepsilon_i) - g(\varepsilon_j)]^2$ are bounded. Therefore, by Barbalat's lemma, one has

$$\lim_{t\to\infty} \sum_i \sum_{j\in N_i} a_{ij}[g(\varepsilon_i) - g(\varepsilon_j)]^2 = 0. \quad (17)$$

It indicates $\lim_{t\to\infty} g(\varepsilon_i) = \lim_{t\to\infty} g(\varepsilon_j)$ for all i, j. It yields $\lim_{t\to\infty} \varepsilon_i = \lim_{t\to\infty} \varepsilon_j$ due to the monotonicity of function $g(*)$. Note that $\sum_i \varepsilon_i = 0$ holds all the time. Then one has $\lim_{t\to\infty} \varepsilon_i = 0$. It follows $\lim_{t\to\infty} \varphi_i - \varphi_j = \phi_{ij}$ and then $\lim_{t\to\infty} \dot{\varphi}_i = w_d$. The dynamic control law (6) can asymptotically solve **Problem 2**. The proof is completed. □

Remark 1. The proposed dynamic control law does not require the target velocity information and is applicable for more situations. While in the existing results, it is commonly assumed that the target velocity is known to all agents [12,16] or at least one agent [23]. Besides, we model each mobile agent by considering its nonholomomic constraint, which is more implementable for some real robots than the controllers proposed for single-integrator agents [20,21]. Furthermore, we consider the case that the target is moving with constant piecewise acceleration, and finally the agents achieve the global asymptotical tracking of a moving leader of unknown velocity with the desired circular formation by the proposed approach. In Reference 14, the target moves at unknown velocity with continuous time-varying acceleration, and local asymptotical stability of the multiple mobile agents is achieved finally.

4 | SIMULATION

In order to illustrate the validity of the proposed dynamic controller, we conduct the following simulation in this section.

Consider $n = 3$ mobile agents moving in the plane. The adjacency matrix is $A = [0, 1, 0; 1, 0, 1; 0, 1, 0]$. The initial states of the agents and the target are (-6,-4,1), (-7,3,0), (5,-4,0) and (0,0), respectively. The radius of the desired circular orbit is $\rho_d = 3$m and the desired circular velocity of each agent is set by $w_d = 1$rad/s. The desired angular spacing is prescribed as $\phi_{12} = 2\pi/3$, $\phi_{23} = 2\pi/3$ and $\phi_{13} = -2\pi/3$. The controller parameters are set as $h_0 = 0.2$, $k_1 = 5$, $k_2 = 1$, $k_3 = 0.2$. The target acceleration is given by $a_0 = [0.01; 0.02]$ as $t < 20$s and $a_0 = [-0.03; -0.02]$ as $20\text{s} \leq t \leq 30$s and $a_0 = [0; 0]$ as $t > 30$s.

Applying the dynamic control law (6) with the adaptive observer (5), the simulation results are shown in the following figures. FIGURE 3 shows the trajectories of all agents, where $+$ and $*$ stand for the initial position and the final position of the target, respectively, $\circ$ is the initial position of each agent, and $\bullet$ is the final position of each agent. From FIGURE 3, we can see that the agents perform a desired circular formation around the target even when the motion direction of the target abruptly changes. Furthermore, FIGURE 4 and FIGURE 5 show that the distance between each agent and the target ρ_i and the angular spacing between neighbor agents $\varphi_i - \varphi_j$ converges to the desired ones. FIGURE 6 shows the convergence of angular spacing between neighbor agents $\varphi_i - \varphi_j$, which indicates that the agents perform a desired distribution. FIGURE 7 shows that the virtual signal $\hat{\theta}_i$ converges to the actual one. In FIGURE 8 and FIGURE 9, the velocity υ_i and the angular velocity ω_i of each agent are depicted respectively. The estimation errors δ_i and e_i are shown in FIGURE 10 and FIGURE 11, respectively. All the estimation errors converge to zero eventually, which implies that the adaptive observer is effective. Therefore, the simulation results above validate our theoretical result even when the motion direction of the target has an abrupt change.

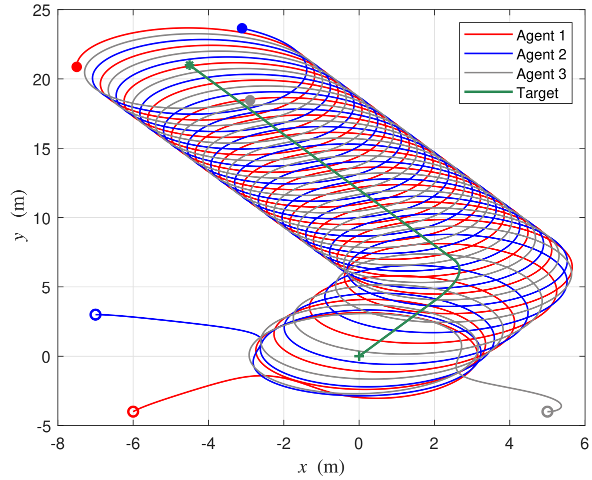

FIGURE 3 Trajectories of the agents.

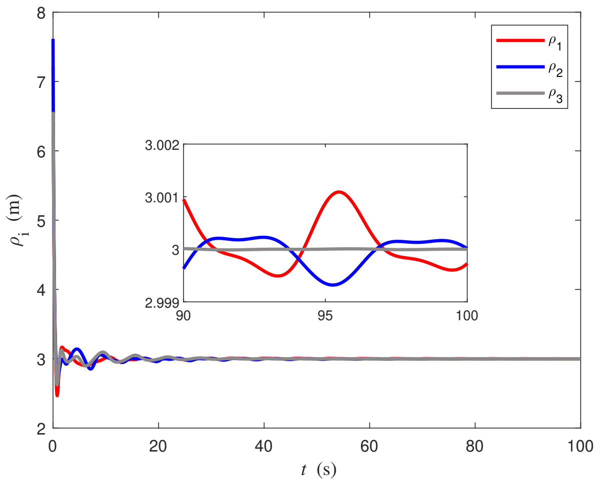

FIGURE 4 Distance between each agent and the target ρ_i.

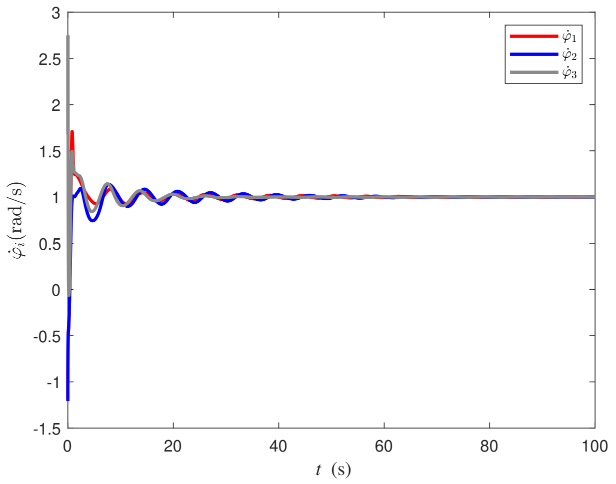

FIGURE 5 The circular velocity of each agent $\dot{\varphi}_i$.

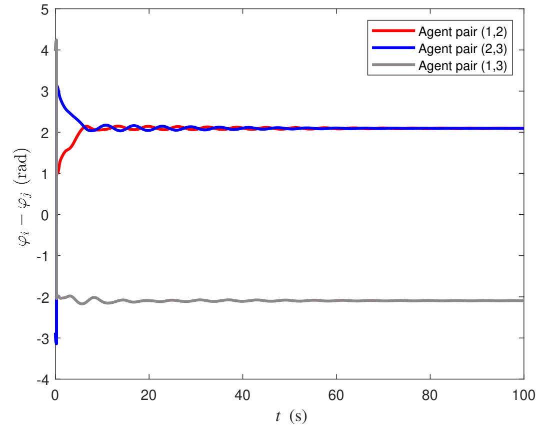

FIGURE 6 The angular spacing between each pair of neighbor agents $\varphi_i - \varphi_j$.

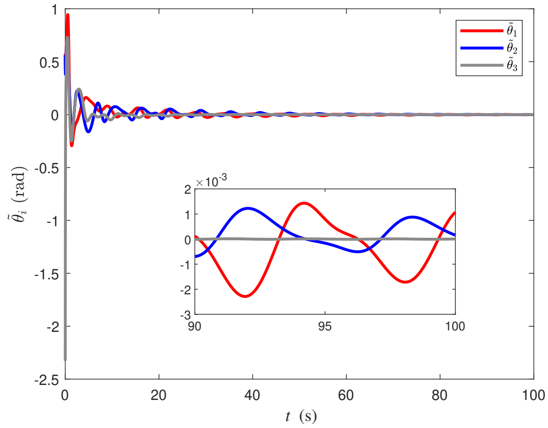

FIGURE 7 Estimation error between $\hat{\theta}_i$ and θ_i.

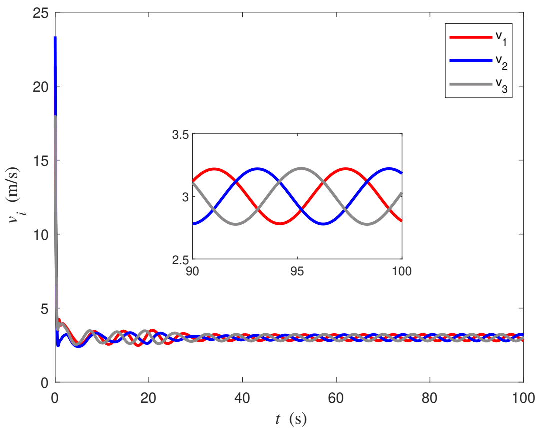

FIGURE 8 Velocity of the agents v_i.

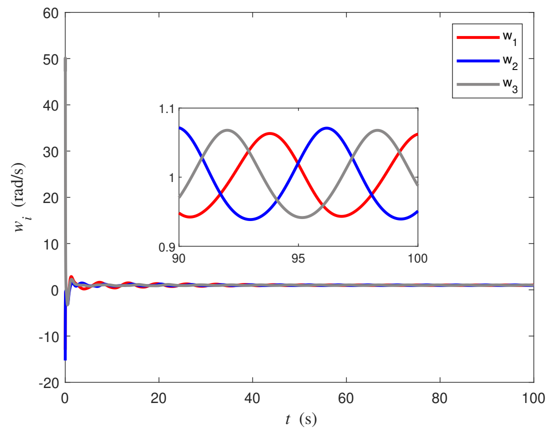

FIGURE 9 Angular velocity of the agents ω_i.

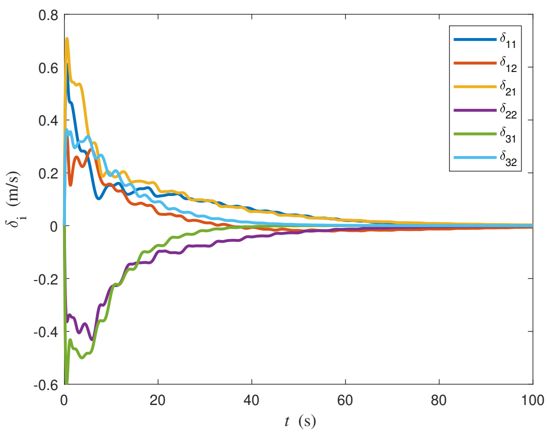

FIGURE 10 Estimation error between v_0 and $\hat{v}_i$.

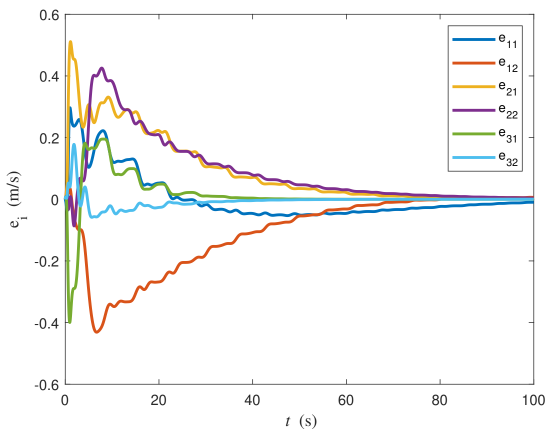

FIGURE 11 Estimation error between $\hat{\delta}_i$ and δ_i.

5 | CONCLUSION

In this work, an enclosing control problem has been investigated for nonholonomic mobile agents in the absence of the velocity information of the moving target. An adaptive observer is first designed for each agent to estimate the target velocity and its corresponding estimation error. Using the estimated information from the adaptive observer, a dynamic control law has been further developed by a backstepping process, which drives the agents to a desired circular formation centered at the target. Simulation results have verified the effectiveness of the proposed approach which is even effective when the motion direction of the target has an abrupt change. In the future, we will consider the saturation constraint of control quantities in practical applications.

ACKNOWLEDGMENTS

The authors want to express their acknowledgment for the support from the National Natural Science Foundation of China under Grants 61973023 and 62103031, the Natural Science Foundation of Beijing Municipality under Grant 4202052.

Conflict of interest

The authors declare no potential conflict of interests.

References

1. Meng DY. Feedback of control on mathematics: Bettering iterative methods by observer system design. *IEEE Transactions on Automatic Control*. DOI: 10.1109/TAC.2022.3174650.

2. Lin C, Lin ZY, Zheng RH, Yan GF, Mao GQ. Distributed source localization of multi-agent systems with bearing angle measurements. *IEEE Transactions on Automatic Control*. 2016;61(4):1105-1110.

3. Ahmed SA, Popov VL, Topalov AV, Shakev NG. Perimeter detection and surveillance of polluted areas by robotized agents in a hybrid wireless sensor network. *IFAC-PapersOnLine*. 2016;49(29):247-252.

4. Deng K, Chen YS, Belta C. An approximate dynamic programming approach to multiagent persistent monitoring in stochastic environments with temporal logic constraints. *IEEE Transactions on Automatic Control*. 2017;62(9):4549-563.

5. Lan Y, Yan GF, Lin ZY. Distributed control of cooperative target enclosing based on reachability and invariance analysis. *Systems and Control Letters*. 2010;59(7):381-389.

6. Yu X, Liu L. Distributed circular formation control of ring-networked nonholonomic vehicles. *Automatica*. 2016;68:92–99.

7. Qin JH, Wang S, Kang Y, Liu QC. Circular formation algorithms for multiple nonholonomic mobile robots: An optimization-based approach. *IEEE Transactions on Industrial Electronics*. 2019;66(5):3693-3701.

8. Yu X, Ma J, Ding N, Zhang AD. Cooperative target enclosing control of multiple mobile robots subject to input disturbances. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*. 2021;51(6):3440-3449.

9. Ju S, Wang J, Dou LY. MPC-based cooperative enclosing for nonholonomic mobile agents under input constraint and unknown disturbance. *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2022.3164713.

10. Wang TQ, Huang J. Time-varying formation control with attitude synchronization of multiple rigid body systems. *International Journal of Robust and Nonlinear Control*, 2022;32(1):181-204.

11. Guo MC, Xu DB, Liu L. Adaptive nonlinear ship tracking control with unknown control direction. *International Journal of Robust and Nonlinear Control*. 2018;28(7):2828-2840.

12. Wang C, Xia WG, Xie GM. Limit-cycle-based design of formation control for mobile agents. *IEEE Transactions on Automatic Control*. 2020;65(8):3530-3543.

13. Wang C, Xie GM. Limit-cycle-based decoupled design of circle formation control with collision avoidance for anonymous agents in a plane. *IEEE Transactions on Automatic Control*. 2017;62(12):6560-6567.

14. Shi LL, Zheng RH, Liu MQ, Zhang SL. Distributed circumnavigation control of autonomous underwater vehicles based on local information. *Systems and Control Letters*. 2021;148(3):104873.

15. Yang HJ, Li Q, Zuo ZQ, Zhao H. Event-triggered model predictive control for multi-vehicle systems with collision avoidance and obstacle avoidance. *International Journal of Robust and Nonlinear Control*. 2021;31(11):5476-5494.

16. Miao ZQ, Wang YN, Fierro R. Cooperative circumnavigation of a moving target with multiple nonholonomic robots using backstepping design. *Systems and Control Letters*. 2017;103:58-65.

17. Yu X, Liu L. Cooperative control for moving-target circular formation of nonholonomic vehicles. *IEEE Transactions on Automatic Control*. 2017;62(7):3448-3454.

18. Peng XH, Guo KX, Li X, Geng ZY. Cooperative moving-target enclosing control for multiple nonholonomic vehicles using feedback linearization approach. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*. 2021;51(8):4929-4935.

19. Shen D, Xu JX. Robust learning control for nonlinear systems with nonparametric uncertainties and nonuniform trial lengths. *International Journal of Robust and Nonlinear Control*. 2019;29(5):1302-1324.

20. Guo J, Yan GF, Lin ZY. Local control strategy for moving-target-enclosing under dynamically changing network topology. *Systems and Control Letters*. 2010;59(10):654-661.

21. Ju S, Wang J, Dou LY. Enclosing control for multi-agent systems with a moving target of unknown bounded velocity. *IEEE Transactions on Cybernetics*. DOI: 10.1109/TCYB.2021.3072031.

22. Dou LY, Song C, Wang X, Liu L, Feng G. Target localization and enclosing control for networked mobile agents with bearing measurements. *Automatica*. DOI: 10.1016/j.automatica.2020.109022.

23. Dou LY, Yu X, Liu L, Wang XF, Feng G. Moving-target enclosing control for mobile agents with collision avoidance. *IEEE Transactions on Control of Network Systems*. DOI: 10.1109/TCNS.2021.3078120.

24. Lu MB, Liu L. Leader–following consensus of multiple uncertain Euler–Lagrange systems with unknown dynamic leader. *IEEE Transactions on Automatic Control*. 2019;64(10):4167-4173.

25. Miao ZQ, Liu YH, Wang YN, Yi G, Fierro F. Distributed estimation and control for leader-following formations of nonholonomic mobile robots. *IEEE Transactions on Automation Science and Engineering*. 2018;15(4):1946-1954.

26. Liang XW, Wang HS, Liu YH, Liu Z, Chen WD. Leader-following formation control of nonholonomic mobile robots with velocity observers. *IEEE/ASME Transactions on Mechatronics*. 2020;25(4):1747-1755.

27. Long MK, Su HS, Zeng ZG. Distributed observer-based leader-follower consensus of multiple Euler-Lagrange systems. *IEEE Transactions on Neural Networks and Learning Systems*. DOI: 10.1109/TNNLS.2022.3172484.