

Using Tidally-Driven Elastic Strains to Infer Regional Variations in Crustal Thickness at Enceladus

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Key Points:

- Variations in ice shell (i.e., crustal) thickness are crucial for understanding the thermodynamics and habitability of Enceladus.
- We develop a new method to infer spatially-varying crustal thickness using measurements of tidally-driven elastic strain.
- Using our method, we demonstrate recoveries of crustal thickness variations to within < 0.2 km error over length scales greater than 60 km.

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13 **Abstract**

14 Constraining the spatial variability of the thickness of the ice shell of Enceladus
 15 (i.e., the crust) is central to our understanding of its thermodynamics and habitability.
 16 In this study, we develop a new methodology to infer regional variations in crustal thick-
 17 ness using measurements of tidally-driven elastic strain. As proof of concept, we recover
 18 thickness variations from synthetic finite-element models of the crust subjected to di-
 19 urnal eccentricity tides. We demonstrate recovery of crustal thickness to within ~ 2 km
 20 of true values with < 0.2 km error over spherical harmonic degrees $l \leq 12$ (correspond-
 21 ing to half-wavelengths ≥ 60 km). Our computed uncertainty is significantly smaller than
 22 the inherent ~ 10 km ambiguity associated with inferring variations in crustal thick-
 23 ness solely from gravity and topography measurements. We therefore conclude that mea-
 24 suring elastic strain provides a relatively robust approach for probing crustal structure
 25 at Enceladus.

26 **Plain Language Summary**

27 Inferences of the thickness of Enceladus’s ice shell – or crust – can provide valu-
 28 able insights for our understanding the potential habitability and thermodynamics of this
 29 moon of Saturn. In this work, we develop a new method to infer regional variations in
 30 crustal thickness at Enceladus using measurements of deformation caused by tidal in-
 31 teractions with Saturn. Using models of Enceladus’s ice shell, we demonstrate that we
 32 can recover crustal thickness with a deviation of ~ 2 km relative to input values. Our
 33 approach to infer crustal thickness could complement traditional methods that rely solely
 34 on analyzing gravity and surface topography to constrain crustal structure at the satel-
 35 lite.

36 **1 Introduction**

37 Enceladus, a small moon of Saturn, is a geologically active and potentially habit-
 38 able ocean world (e.g., Porco et al., 2006; Postberg et al., 2009). Enceladus possesses both
 39 highly cratered landscapes and regions with active resurfacing (e.g., the South Polar Ter-
 40 rain or SPT) (Yin & Pappalardo, 2015; Schenk et al., 2018). Based on an incomplete
 41 spherical harmonic degree $l = 3$ gravity and topography fields derived primarily from
 42 three spacecraft flybys, the SPT is believed to have significantly thinner crust ($\sim 4 - 14$

43 km) relative to a mean crustal thickness ($\sim 20 - 40$ km) (Nimmo et al., 2011; Iess et al.,
44 2014; Hemingway et al., 2018). The SPT also possesses four large-scale fractures (infor-
45 mally known as ‘Tiger Stripes’; Porco et al., 2006). Cryovolcanic jets along the Tiger Stripes
46 are believed to supply material from a subsurface ocean (Thomas et al., 2016; Iess et al.,
47 2014) to a water-ice plume which exhibits diurnal variations in activity (e.g., Ingersoll
48 et al., 2020). Diurnal eccentricity tides may correspondingly regulate crustal dynamics
49 by cyclically deforming Enceladus over its 32.9 hr orbital period (Souček et al., 2016).

50 Characterizing the spatial variability of crustal thickness at Enceladus is crucial
51 for studying the satellite’s thermodynamics and habitability. It is believed that basal heat-
52 ing (and melting) of the ice shell maintains variations in crustal thickness over geologic
53 timescales (Čadek et al., 2019; Hemingway & Mittal, 2019). Additionally, for an ice shell
54 that exhibits Airy isostatic compensation of surface topography, the amplitudes of crustal
55 thickness variations are sensitive to the density of the ocean (Hemingway & Matsuyama,
56 2017). Determination of ocean density enables identification of differences between ocean
57 compositions predicted from analyzing plume material and actual ocean compositions
58 (Fifer et al., 2022). Knowledge of ocean composition (and in particular the abundances
59 of compounds NaCl, CO₂, H₂, NH₄, and CH₄) in turn constrain the pH, salinity, and
60 availability of chemical energy for metabolic reactions at Enceladus (Postberg et al., 2011;
61 Glein et al., 2018). Characterizing crustal thickness at Enceladus also constrains plau-
62 sible escape pathways of ocean material (e.g., local refreezing in thinned regions of the
63 ice shell; Čadek et al., 2019) and modes of intra-crustal processing of material sourced
64 from the ocean (Kite & Rubin, 2016; Ingersoll & Nakajima, 2016).

65 Several methods have been proposed to infer spatially variable crustal thickness
66 at Enceladus. Measurements of gravity and topography can provide constraints on vari-
67 ations in crustal thickness across regional spatial scales (i.e., $l = 2 - 20$) (e.g., Ermakov
68 et al., 2021). However, previous geodetic studies at Enceladus (Iess et al., 2014; McK-
69 innon, 2015; Hemingway & Mittal, 2019) predict a wide possible range of crustal thick-
70 ness values across regional spatial scales (e.g., 4 – 14 km, or ~ 10 km near the South Pole).
71 Ambiguity in determinations of crustal thickness from existing geodetic surveys arises
72 primarily from uncertain estimates of the impact of ocean and crustal densities on Ence-
73 ladus’s gravity field (Hemingway & Mittal, 2019). Efforts to more precisely determine
74 thickness using libration measurements (e.g., Thomas et al., 2016; Van Hoolst et al., 2016)
75 or observations of local lithospheric flexure induced by the presence of surface topogra-

76 phy (e.g., Giese et al., 2008) constrain values only at long ($l = 2$) or short ($l > 20$) spa-
 77 tial scales, respectively.

78 We develop a new method for inferring crustal thickness at Enceladus using mea-
 79 surements of elastic strain across regional spatial scales. From Hooke’s law, strain along
 80 a loaded 1D system scales linearly with local stiffness (Figure 1). For an elastic layer,
 81 both layer thickness and elastic moduli influence the tendency for a medium to resist de-
 82 formation in response to an applied force. We therefore anticipate that changes in the
 83 strain field produced by diurnal tides at a given location in Enceladus’s crust will ex-
 84 hibit a linear relationship with variations in the local elastic thickness. We note that ice
 85 deformation on Enceladus is mostly elastic over 32.9 hr timescales (Shaw, 1985; Wahr
 86 et al., 2006; Neumeier, 2018). Thus, inferences of elastic thickness from diurnal tides closely
 87 approximate (to within $< 0.2\%$) true crustal thickness at Enceladus. Further details are
 88 described in the Supplementary text S1.5 of Berne et al. (2023).

89 Gradients in material properties, such as crustal thickness and elastic moduli, in-
 90 duce additional complexity in the response of a 2D layer to applied tractions (Hsu et al.,
 91 2011 cf. Equation 4). We refer to this phenomenon as the ‘gradient’ effect, which we il-
 92 lustrate in Figure 1. Crustal thickness gradients can lead to biased estimates of crustal
 93 thickness when using a linear (one-to-one) interpretation of strain fields, particularly when
 94 gradients are high (e.g., when variations in crustal thickness are present at short wave-
 95 lengths). Analytic models of diurnal tides at Enceladus are unable to predict deforma-
 96 tion caused by short-wavelength variations in crustal thickness (Beuthe, 2018; Rovira-
 97 Navarro et al., 2020). However, numerical Finite Element Models (FEMs) can accurately
 98 simulate deformation of ice shell geometries that incorporate variations in crustal thick-
 99 ness across a wide range of spatial scales (Souček et al., 2016; Behounkova et al., 2017;
 100 Souček et al., 2019; Berne et al., 2023).

101 Here, we introduce an approach to determine crustal thickness at Enceladus through
 102 the application of Hooke’s law to tidally-induced elastic strains (Section 2). Our approach
 103 utilizes numerical techniques to iteratively minimize differences between measured crustal
 104 strains and those predicted using FEMs of spatially heterogeneous ice shells subject to
 105 tidal loading. To account for potential gradient effects, our FEMs incorporate variations
 106 in crustal thickness at length scales down to ~ 25 km or spherical harmonic degree $l \sim$
 107 60. As proof of concept, we demonstrate recovering thickness using elastic strains from

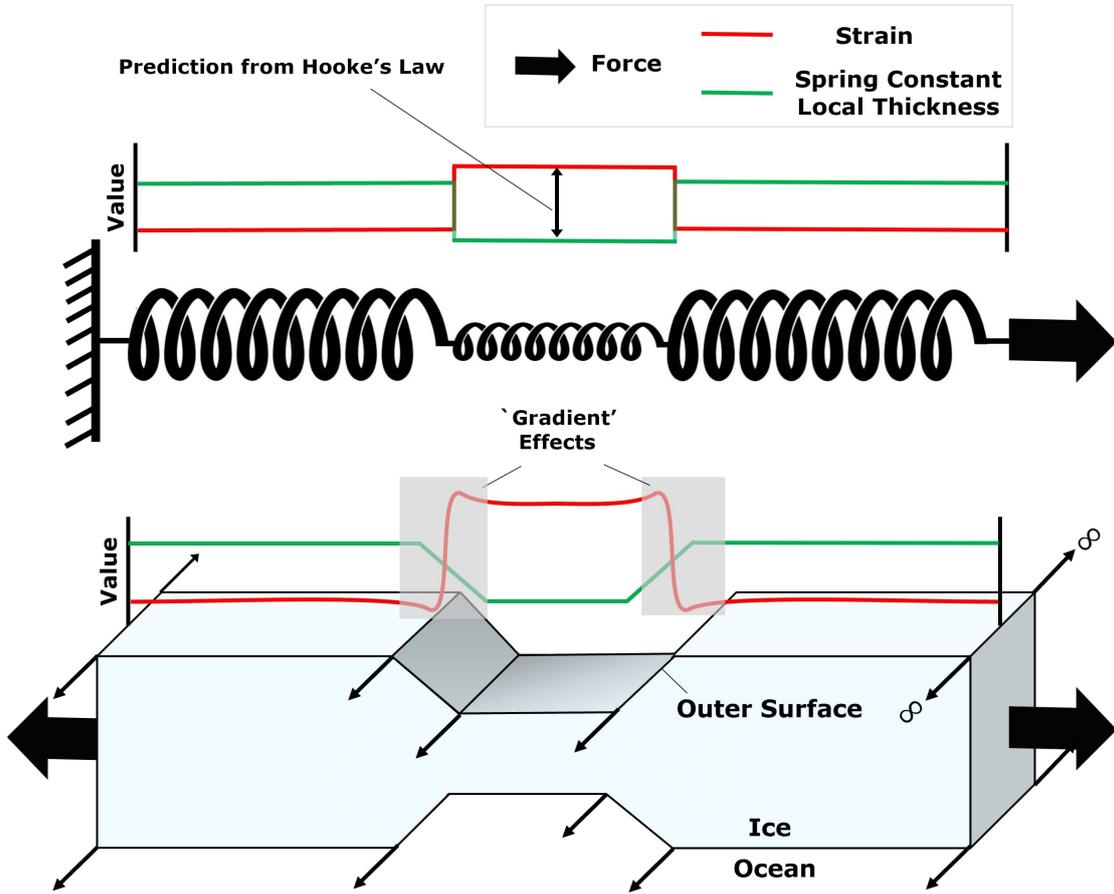


Figure 1. Crustal strain correlates with ice shell thickness. Top Panel: Example 1D elastic structure (i.e., springs in series) subject to an axial load. Hooke’s law predicts that strain is relatively higher where springs have lower stiffness (i.e., smaller spring constants). Bottom Panel: Analogous 2D elastic layer subject to a transverse load. In this case, local layer thickness modulates the effective stiffness (and strain) of the medium. We expect that measurements of strain at the outer surface of the crust (labelled) permit inferences of local thickness at Enceladus. Shaded regions denote locations where ‘gradient effects’ impact inferences of local thickness from strain fields.

108 a synthetic model of Enceladus’s ice shell using our methodology (Section 3). We assess
 109 uncertainty by comparing the discrepancy between thicknesses that are input into the
 110 synthetic models with thicknesses recovered from those models. We conclude by address-
 111 ing the utility of using imaging geodesy (e.g., Interferometric Synthetic Aperture Radar)
 112 to carry out the strain measurements required to recover crustal thickness variations at
 113 Enceladus (Section 4).

114 2 Methods

115 2.1 Input Model Specification

116 We first construct a spherically symmetric model geometry that is broadly consis-
 117 tent with the elastic structure of the crust of Enceladus. Building on the methodology
 118 discussed in Berne et al., (2023) (section 2.1), we start with a hollow shell with prescribed
 119 outer radius R and uniform thickness \tilde{D} (see Supplementary Table S1 for chosen values
 120 for parameters used throughout this work). Using the software package CUBIT (Skroch
 121 et al., 2019; CoreForm, 2020), we mesh our geometry with tetrahedral elements and as-
 122 sign a uniform shear modulus G and a bulk modulus value μ . We ignore the potential
 123 impact of viscous strain (Wahr et al., 2009). We also account for self-gravitational ef-
 124 fects and ignore inertial forces for our analysis. Elastic deformation of the core is expected
 125 to be several orders of magnitude smaller than elastic deformation of the ice shell (Schu-
 126 bert et al., 2007). We therefore treat the core as a rigid body for simulations.

127 For our analysis, we construct a synthetic ‘true’ crustal thickness model by mod-
 128 ifying the surface and ice-ocean boundary of our spherically symmetric geometry. $D^{true}(\mathbf{\Omega})$
 129 represents the spatially variable thickness of the outer ice shell of our synthetic model
 130 where $\mathbf{\Omega}$ is the position variable comprising the co-latitude longitude pair (θ, ϕ) in a body-
 131 fixed reference frame. Note that we can write the quantity $D^{true}(\mathbf{\Omega})$ as a sum over or-
 132 thonormal spherical harmonic basis functions $Y_{lm}(\mathbf{\Omega})$ scaled by coefficients d_{lm}^{true} (where
 133 l and m denote spherical harmonic degree and order):

$$134 \quad D^{true}(\mathbf{\Omega}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l d_{lm}^{true} Y_{lm}(\mathbf{\Omega}) \quad (1)$$

135 We generate d_{lm}^{true} up to $l = 60$ by compensating surface observed topography using a
 136 modified form of Airy isostasy (Hemingway & Matsuyama, 2017). Further details of our

137 procedure to generate thickness variations in the synthetic model are given in Supple-
 138 mentary S1.1.

139 2.2 Tidal Loading

140 Following the formulation described in Supplementary section S1.1 of Berne et al.,
 141 (2023), we apply forces associated with the driving potential produced by time-dependent
 142 diurnal eccentricity tides $V(r, \theta, \phi, t)$ (to the first order in eccentricity) to model geome-
 143 tries (Murray & Dermott, 1999):

$$144 \begin{aligned} V(r, \theta, \phi, t) = & r^2 \omega^2 e \cdot (\sin(\omega t) P_{22}(\mu) \sin 2\phi \dots \\ & \dots + \frac{3}{4} \cos(\omega t) (P_{22}(\mu) \cos 2\phi - 2 P_{20}(\mu))) \end{aligned} \quad (2)$$

145 In Equation 2, ω is Enceladus's orbital angular velocity, e is the body's orbital eccen-
 146 tricity, and r is radial position in a body-fixed reference frame. Time $t = 0, \frac{2\pi}{\omega}$ corre-
 147 sponds to orbital periape. $P_{20}(\mu)$ and $P_{22}(\mu)$ are associated Legendre Functions with
 148 the nested function $\mu = \cos(\theta)$. We use the 3D FEM code PyLith (Aagaard et al., 2007)
 149 for simulations. PyLith is a well-established geodynamic modelling tool which allows for
 150 complex bulk rheology and geometrical meshes. We have modified PyLith for modeling
 151 full spheres in a no-net-rotation/translation reference frame with central time-dependent
 152 body forces appropriate for eccentricity tides (See also section 2.2 of Berne et al., 2023).

153 2.3 Strain Computation

154 Following the methodology described in Tape et al. (2009) (cf. Equation 20), for
 155 a deforming quasi-spherical body with a linear isotropic elastic rheology, we can com-
 156 pute components of the horizontal strain rate tensor ϵ at the surface according to:

$$157 \epsilon_{ij} = \frac{1}{R} \begin{bmatrix} -\frac{3\mu-2G}{3\mu+4G} (2v_r + \frac{dv_\theta}{d\theta} + \dots & 0 & 0 \\ \dots v_\theta \cot\theta + \csc\theta \frac{dv_\phi}{d\phi} & & \\ 0 & v_r + \frac{dv_\theta}{d\theta} & \frac{1}{2}(-v_\phi \cot\theta + \csc\theta \frac{dv_\theta}{d\phi} + \frac{dv_\phi}{d\theta}) \\ 0 & \frac{1}{2}(-v_\phi \cot\theta + \csc\theta \frac{dv_\theta}{d\phi} + \frac{dv_\phi}{d\theta}) & v_r + v_\theta \cot\theta + \csc\theta \frac{dv_\phi}{d\phi} \end{bmatrix} \quad (3)$$

158 where quantities v_r, v_θ , and v_ϕ denote surface velocities in positive radial, co-latitude,
 159 and longitude directions. To compute v_r, v_θ and v_ϕ , we difference FEM displacement fields
 160 between 180 consecutive time points over the tidal cycle ($t=0$ to $t = \frac{2\pi}{\omega}$ in Equation 2).

161 Subtracting the dilatation of the strain tensor ϵ_h from diagonal components of ϵ_{ij} in Equa-
 162 tion 3 permits computation of the horizontal deviatoric strain rate tensor $\tilde{\epsilon}_{ij}$:

$$163 \quad \tilde{\epsilon}_{ij} = \epsilon_{ij} - \epsilon_h \delta_{ij} \quad (4)$$

164 where δ_{ij} is the kronecker delta. We evaluate ϵ_h by averaging diagonal components of
 165 ϵ_{ij} in Equation 3:

$$166 \quad \epsilon_h = \frac{\epsilon_{11} + \epsilon_{22} + \epsilon_{33}}{3} \quad (5)$$

167 For convenience, we seek to parameterize strain rate at the surface of our models using
 168 a scalar quantity, here called $E(\mathbf{\Omega})$ (see section 2.1 for the definition of $\mathbf{\Omega}$). Using ma-
 169 trix components $\tilde{\epsilon}_{ij}$ in Equation 4, we define $E(\mathbf{\Omega})$ as the time-averaged 2nd invariant
 170 of the deviatoric horizontal strain rate:

$$171 \quad E(\mathbf{\Omega}) = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} |\tilde{\epsilon}_{11}\tilde{\epsilon}_{22} + \tilde{\epsilon}_{22}\tilde{\epsilon}_{33} + \tilde{\epsilon}_{11}\tilde{\epsilon}_{33} - \tilde{\epsilon}_{23}\tilde{\epsilon}_{32}| dt \quad (6)$$

172 By definition, the strain invariants do not depend on our coordinate system. The 2nd
 173 invariant of the horizontal deviatoric horizontal strain rate (i.e., as opposed to 1st or 3rd
 174 invariants, non-deviatoric strain rate, etc.) is especially sensitive to variations in elas-
 175 tic layer thickness and yields relatively high signal-to-noise ratios for recoveries of crustal
 176 thickness at Enceladus (see next section).

177 **3 Results**

178 **3.1 Initial Crustal Thickness Recovery**

179 We perform an iterative analysis to recover thickness from strain fields. $D^n(\mathbf{\Omega})$ and
 180 $E^n(\mathbf{\Omega})$ respectively denote thickness and strain fields evaluated at a given iteration num-
 181 ber n . To assess the discrepancy between thicknesses that are input and recovered from
 182 models for each iteration, we compute the mismatch $\delta D^n(\mathbf{\Omega})$ between $D^n(\mathbf{\Omega})$ and $D^{true}(\mathbf{\Omega})$:

$$183 \quad \delta D^n(\mathbf{\Omega}) = D^n(\mathbf{\Omega}) - D^{true}(\mathbf{\Omega}) \quad (7)$$

184 We initially (i.e., $n = 0$) recover crustal thickness $D^0(\mathbf{\Omega})$ using ‘observed’ strains (i.e.,
 185 $E^{obs}(\mathbf{\Omega})$) extracted from our FEM with thickness $D^{true}(\mathbf{\Omega})$. Hooke’s law predicts that

186 thickness is inversely proportional to strain in a 2D elastic medium subject to a trans-
 187 verse load (as previously shown in Figure 1). It is therefore reasonable to assume the fol-
 188 lowing linear relationship between $E^{obs}(\Omega)$ and $D^0(\Omega)$:

$$189 \quad \frac{D^0(\Omega)}{\tilde{D}} = \kappa(\Omega) \frac{E^{Base}(\Omega)}{E^{obs}(\Omega)} \quad (8)$$

190 where $\kappa(\Omega)$ is a linear transfer function and $E^{Base}(\Omega)$ is strain evaluated on a spher-
 191 ically symmetric Enceladus with uniform thickness \tilde{D} . $\kappa(\Omega)$ accounts for the differen-
 192 tial $E(\Omega)$ produced by thickness variations (of constant amplitude) located at different
 193 angular positions Ω across Enceladus. Since the amplitude of the tidal forcing varies over
 194 a predominantly long-wavelength $E(\Omega)$ pattern (i.e., $l = 2$; see Equation 2) we can ac-
 195 count for the impact of $\kappa(\Omega)$ by applying a high-pass filter to strain fields (for details,
 196 see Supplementary S1.2).

197 Figure 2 shows snapshots of $D^{true}(\Omega)$, $E^{obs}(\Omega)$, $D^0(\Omega)$, and $\delta D^0(\Omega)$ from our anal-
 198 ysis. For visualization, we plot the logarithm of $E^{obs}(\Omega)$ normalized by $E^{Base}(\Omega)$ (i.e.,
 199 $\hat{E}^{obs}(\Omega)$):

$$200 \quad \hat{E}^{obs}(\Omega) = \log \frac{E^{obs}(\Omega)}{E^{Base}(\Omega)} \quad (9)$$

201 As expected, patterns of $\hat{E}^{obs}(\Omega)$ correlate with patterns of $D^{true}(\Omega)$. Computed $\hat{E}^{obs}(\Omega)$
 202 fields reflect regional thinning at North and South poles, a relatively thicker crust at low
 203 latitudes, and the significant asymmetry in crustal thinning between northern and South-
 204 ern hemispheres visible in $D^{true}(\Omega)$. Recovered crustal thickness patterns, $D^0(\Omega)$, more
 205 closely match $D^{true}(\Omega)$ than do $\hat{E}^{obs}(\Omega)$. Slight differences between $D^{true}(\Omega)$ and $D^0(\Omega)$
 206 (i.e., $\delta D^0(\Omega)$ in Equation 7) appear to localize near regions with short-wavelength vari-
 207 ations in crustal thickness (i.e., high contour density) consistent with the influence of gra-
 208 dient effects on our analysis. In particular, we significantly overestimate crustal thick-
 209 ness ($\delta D^0(\Omega)$ values up to 25 km) along several prominent ridges over the Trailing and
 210 Southern Hemispheres.

211 3.2 Gradient Effect Correction

212 We iteratively adjust the amplitude of crustal thicknesses $D^n(\Omega)$ to minimize dif-
 213 ferences between strain produced by models with recovered crustal thickness fields $E^n(\Omega)$

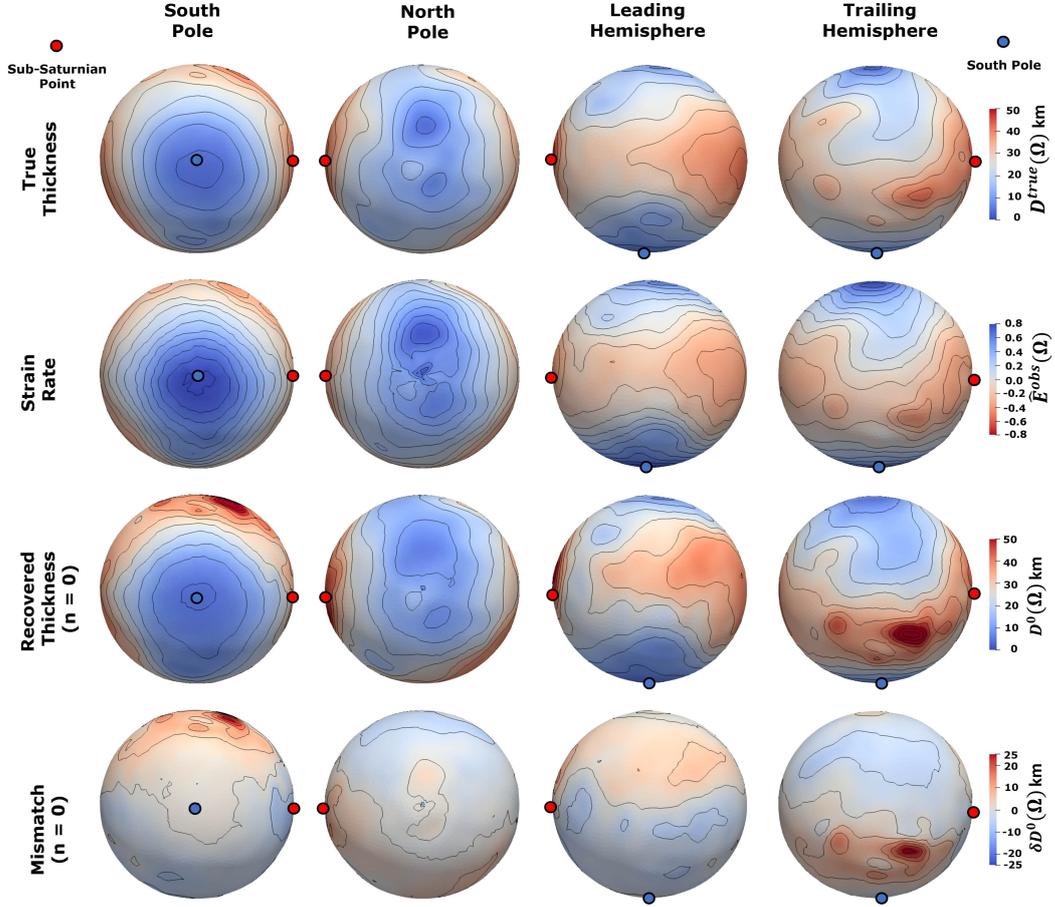


Figure 2. Snapshots of model input crustal thickness $D^{true}(\Omega)$ (first row), the simulated 2^{nd} invariant of time-averaged horizontal deviatoric strain rate $\widehat{E}^{obs}(\Omega)$ (see Equations 6 and 9) (second row), recovered crustal thickness $D^0(\Omega)$ evaluated from Equation 8 (third row), and mismatch between input and recovered thickness $\delta D^0(\Omega)$ (see Equation 7) for our initial recovery of crustal thickness ($n = 0$) viewed facing Southern, Northern, Leading, and Trailing hemispheres. See Supplementary S1.1 for description of how synthetic ‘true’ crustal thickness models were constructed. Plotted contours denote colorscale intervals of 0.05 (for $\widehat{E}^{obs}(\Omega)$ fields) and 5 km (for $D^n(\Omega)$ and $\delta D^n(\Omega)$ fields). Images are orthographic projections with labelled sub-Saturnian point and South Pole locations.

214 and ‘true’ strain $E^{obs}(\mathbf{\Omega})$ (see section 2.4). We define a cost function evaluated at a given
 215 iteration $\xi^E(n)$ as the integrated square of the difference between $E^{obs}(\mathbf{\Omega})$ and $E^n(\mathbf{\Omega})$:

$$216 \quad \xi^E(n) = \int (E^n(\mathbf{\Omega}) - E^{obs}(\mathbf{\Omega}))^2 \cdot d\mathbf{\Omega} \quad (10)$$

217 For comparison to $\xi^E(n)$, we additionally track the integrated square of the difference
 218 between true and recovered thicknesses $\xi^D(n)$:

$$219 \quad \xi^D(n) = \int (D^n(\mathbf{\Omega}) - D^{true}(\mathbf{\Omega}))^2 \cdot d\mathbf{\Omega} \quad (11)$$

220 We expect that the extent to which gradient effects distort strain fields at a given loca-
 221 tion scales with the magnitude of the local gradient in crustal thickness $\|\nabla D^n(\mathbf{\Omega})\|$. We
 222 therefore update $D^n(\mathbf{\Omega})$ to $D^{n+1}(\mathbf{\Omega})$ for iterations $n > 0$ following:

$$223 \quad \log\left(\frac{D^{n+1}(\mathbf{\Omega})}{D^n(\mathbf{\Omega})}\right) = \eta(n) \cdot \|\nabla D^n(\mathbf{\Omega})\| \cdot M(\mathbf{\Omega}) \quad (12)$$

224 where $\eta(n)$ is the learning rate and $M(\mathbf{\Omega})$ is a spatially variable prefactor defined as:

$$225 \quad M(\mathbf{\Omega}) = \kappa(\mathbf{\Omega})^{-1} \frac{E^n(\mathbf{\Omega}) - E^{obs}(\mathbf{\Omega})}{E^{Base}(\mathbf{\Omega})} \quad (13)$$

226 We incorporate $M(\mathbf{\Omega})$ into Equation 12 to ensure modifications to $D^n(\mathbf{\Omega})$ only correct
 227 for over- (under-) predictions of local thickness in locations with reduced (elevated) $E^n(\mathbf{\Omega})$
 228 relative to $E^{obs}(\mathbf{\Omega})$. We also update $\eta(n)$ between iterations following an adaptive al-
 229 gorithm to ensure $\xi^E(n)$ converges to a local minimum (Barzilai & Borwein, 1988). Note
 230 that we can expand $D^n(\mathbf{\Omega})$ for each iteration in Equation 12 into spherical harmonic func-
 231 tions:

$$232 \quad D^n(\mathbf{\Omega}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l d_{lm}^n Y_{lm}(\mathbf{\Omega}) \quad (14)$$

233 where d_{lm}^n are spherical harmonic coefficients. We can examine mismatch in the spec-
 234 tral domain (i.e., spectral power) by evaluating the root-mean-square (RMS) of thick-
 235 ness coefficients d_{lm}^n in Equation 14 and d_{lm}^{true} in Equation 1 over l (i.e., d_l^n and d_l^{true} re-
 236 spectively) as well as the RMS percentage difference between coefficients d_{lm}^n and d_{lm}^{true}
 237 (i.e., δd_l^n):

$$d_l^n = \left(\frac{1}{2l+1} \sum_m (d_{lm}^n)^2 \right)^{1/2} \quad (15a)$$

$$d_l^{true} = \left(\frac{1}{2l+1} \sum_m (d_{lm}^{true})^2 \right)^{1/2} \quad (15b)$$

$$\delta d_l^n = \left(\frac{1}{2l+1} \sum_m \left(\frac{d_{lm}^n - d_{lm}^{true}}{d_{lm}^{true}} \right)^2 \right)^{1/2} \cdot 100\% \quad (15c)$$

238 Figure 3 shows snapshots of $D^{true}(\mathbf{\Omega})$, $D^1(\mathbf{\Omega})$, $\delta D^1(\mathbf{\Omega})$, $D^{12}(\mathbf{\Omega})$, and $\delta D^{12}(\mathbf{\Omega})$ (i.e.,
 239 see Equations 7 and 12). In addition, Figure 4 shows $\delta D^n(\mathbf{\Omega})$, d_l^n , d_l^{true} , and δd_l^n for it-
 240 erations $n = 0, 1$, and 12 (see Equation 15). The non-zero $\delta D^0(\mathbf{\Omega})$ observed in Figure
 241 3 drives broad differences between recovered and true spectral powers d_l^0 and d_l^{true} as
 242 well as non-zero values of δd_l^0 across all wavelengths. Iterating through our analysis once
 243 ($n = 1$) reduces $\delta D^1(\mathbf{\Omega})$ to < 20 km along prominent ridges over the Trailing and South-
 244 ern Hemispheres, lessens the mismatch between d_l^1 and d_l^{true} curves, and diminishes δd_l^1
 245 values across all wavelengths. Further changes to crustal thickness for iterations $n = 1$
 246 $- 12$ reduces δd_l^{12} across longer wavelengths (i.e., $l \leq 12$) and reduces $\delta D^{12}(\mathbf{\Omega})$ to ~ 2
 247 km (1σ confidence).

248 Figure 4 presents the strain mismatch cost function, $\xi^E(n)$, and integrated thick-
 249 ness mismatch, $\xi^D(n)$, for iterations $n = 0 - 15$. Our results demonstrate a slight increase
 250 in $\xi^E(n)$ from iterations $n = 12 - 15$, while $\xi^D(n)$ (as well as $\delta D^n(\mathbf{\Omega})$ and δd_l^n) values
 251 remain largely unchanged after $n \sim 12$. Bayesian approaches (e.g., Cawley & Talbot,
 252 2007) enable determination of a suitable cutoff iteration number to avoid over-fitting strain
 253 fields (i.e., in the absence of knowledge of 'true' thickness) but are beyond the scope of
 254 the current study.

255 4 Discussion and Conclusion

256 We examine the relationship between tidally-driven elastic strains and spatially vari-
 257 able crustal thickness at Enceladus. Results show a broad correlation between strain fields
 258 and crustal thickness across the moon (see Figure 2). Gradient effects modulate strain
 259 patterns (see Figures 3 and 4) with diminishing impact at longer wavelengths (see Fig-
 260 ure 4). Our approach permits final recoveries of crustal thickness to within ~ 2 km of
 261 input values (1σ confidence) with minimal error (< 0.2 km) across spatial wavelengths
 262 $l \leq 12$ (i.e., corresponding to a half-wavelength of ~ 60 km; see Figure 4, iteration n
 263 $= 12$).

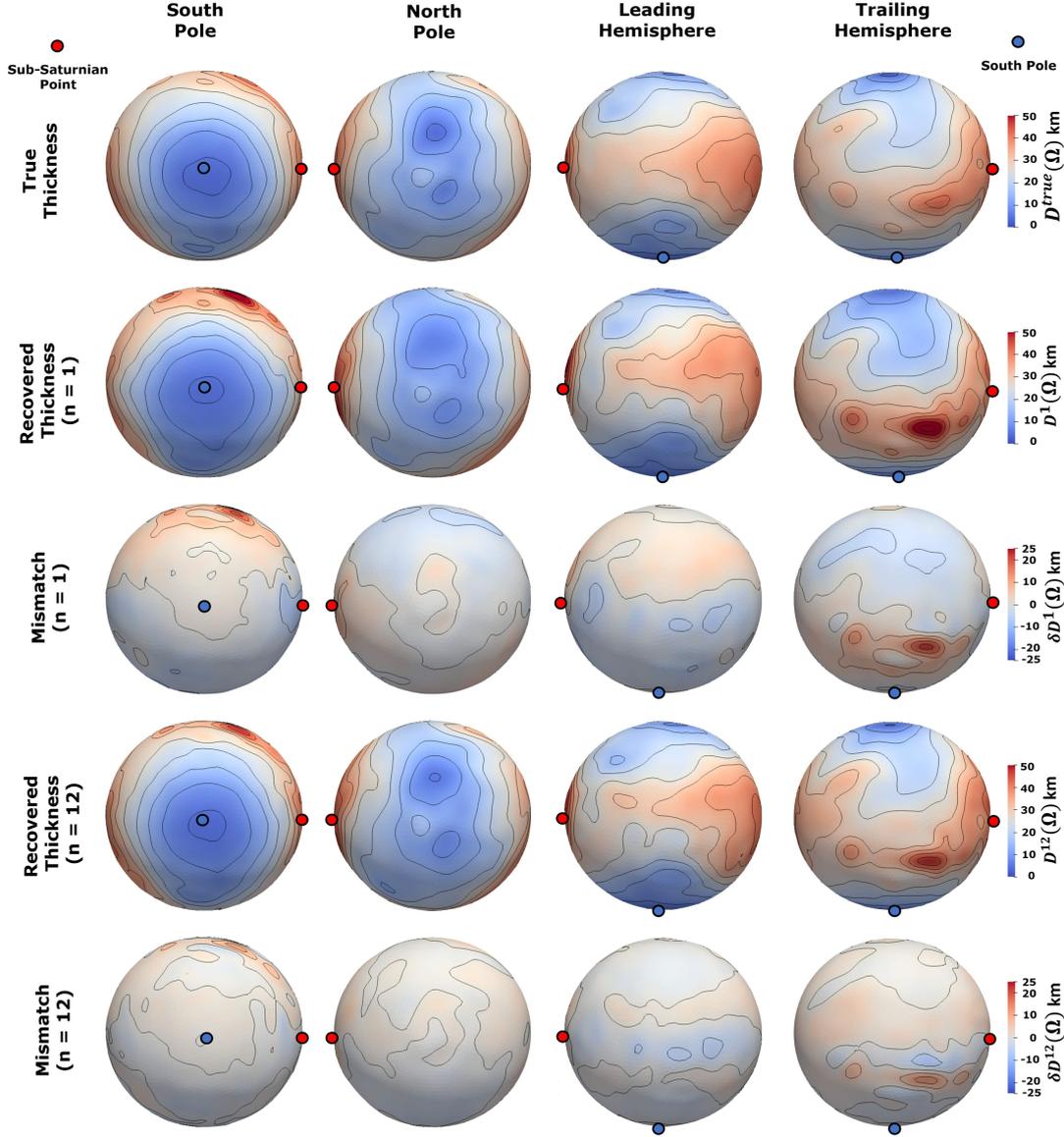


Figure 3. Snapshots of input crustal thickness $D^{true}(\Omega)$ (first row), recovered crustal thickness following $n = 1$ iteration $D^1(\Omega)$ (second row), mismatch between model and input crustal thickness following $n = 1$ iteration $\delta D^1(\Omega)$ (see Equations 7) (third row), recovered crustal thickness following $n = 12$ iterations $D^{12}(\Omega)$ (fourth row), and mismatch between model and input crustal thickness following $n = 12$ iterations $\delta D^{12}(\Omega)$ (fifth row) viewed facing the Southern, Northern, Leading, and Trailing hemispheres. Plotted contours denote colorscale intervals of 5 km for $\delta D^n(\Omega)$ and $D^n(\Omega)$ fields. Images are orthographic projections with labelled sub-Saturnian point and South Pole locations.

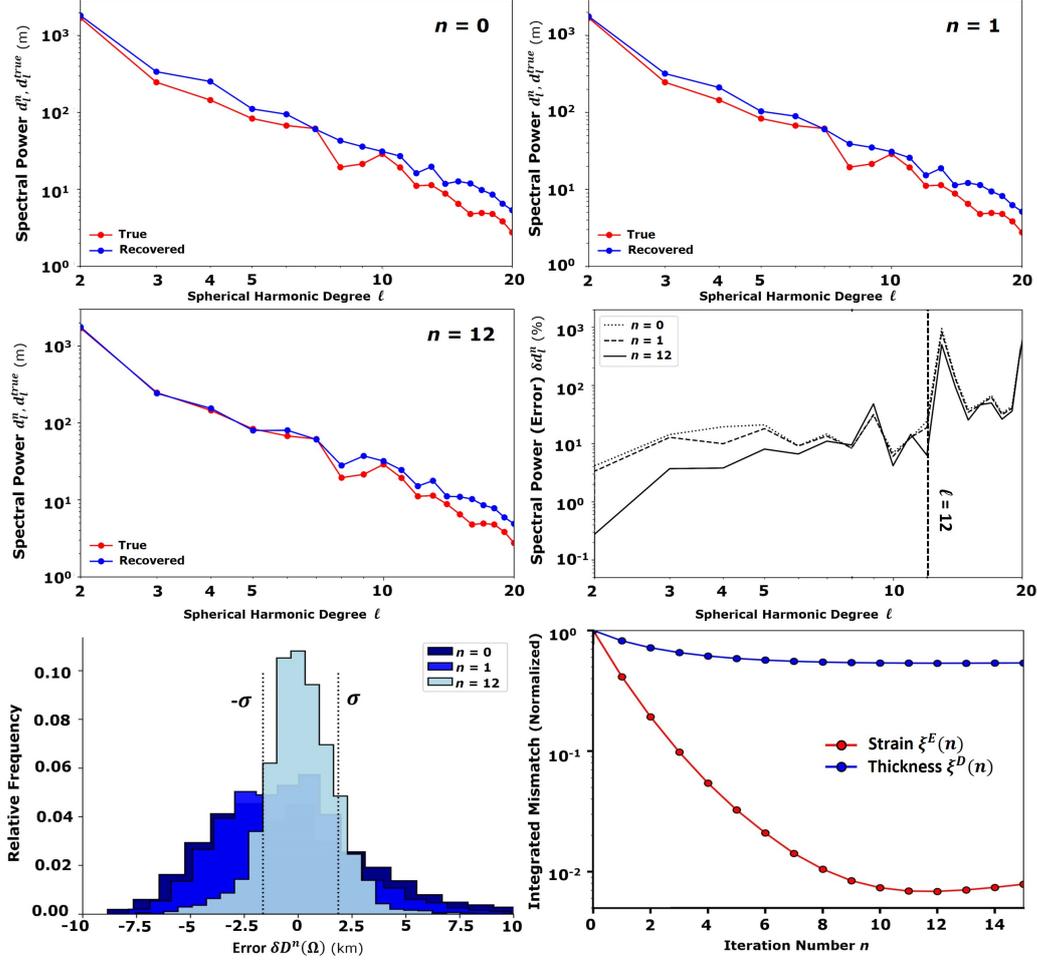


Figure 4. Analysis of mismatch between thickness fields that are input and recovered from models at a given iteration n using our analysis. Upper and center left panels show d_l^n and d_l^{true} evaluated for spherical harmonic degrees $l = 2 - 20$. d_l^n and d_l^{true} denote the spectral power of input and recovered thicknesses (see Equation 15ab; $d_l^n = d_l^{true}$ denotes a perfect recovery of crustal thickness). Note that the difference between d_l^n and d_l^{true} decreases (i.e., error decreases) after several iterations (i.e., increasing values of n). Center right panel shows δd_l^n evaluated for spherical harmonic degrees $l = 2 - 20$. δd_l^n is the spectral power of mismatch between input and recovered thicknesses at spherical harmonic degree l (see Equation 15c; $\delta d_l^n = 0$ denotes a perfect recovery of crustal thickness). Note that δd_l^n decreases (i.e., error decreases) after several iterations (i.e., increasing values of n). Vertical dash-dot lines at $l = 12$ marked for reference. Lower left panel shows a histogram of $\delta D^n(\Omega)$ values (evaluated at FEM node locations) across recovered models for $n = 0, 1$, and 12 . 1σ for the $n = 12$ case plotted as vertical dash-dot lines for reference. Lower right panel shows the cost function $\xi^E(n)$ (see Equation 10) and integrated thickness mismatch $\xi^D(n)$ (see Equation 11), normalized relative to the maximum value, for iterations $n = 0 - 15$. X-axes of upper and center panels are plotted in \log_{10} scale.

264 Our approach to correct for gradient effects minimally reduces error in crustal thick-
 265 ness estimates for $l > 12$ (see Figure 4). Note that length scales $l = 12 - 20$ ($\sim 60 - 40$
 266 km) approach the upper limit of input crustal thickness values for models (~ 50 km,
 267 see Figures 2 and 3). We therefore suspect that tidal strain exhibits a generally more
 268 complex relationship with the amplitude of thickness variations than that considered by
 269 our approach (see Equations 8 and 12) at length scales that are comparable to the crustal
 270 thickness. To further examine the scaling relationship between crustal thickness and the
 271 accuracy of recoveries of crustal thickness, we repeat our analysis with mean crustal thick-
 272 ness $\tilde{D} = 50$ km corresponding with maximum thickness values ~ 100 km. In this case,
 273 crustal thickness estimates become significantly less accurate (i.e., exhibit $> 10\%$ er-
 274 ror) across wavelengths $l \geq 8$ (i.e., ≤ 90 km; for further details see Supplementary S2.1).

275 Our analysis assumes a-priori knowledge of \tilde{D} (mean crustal thickness). \tilde{D} is a cru-
 276 cial parameter for acquiring an initial estimate of variations in crustal thickness (i.e., $D^0(\Omega)$,
 277 see Equation 8). Measuring the amplitude of long-wavelength diurnal deformation (e.g.,
 278 Love numbers k_{20} and h_{20}) at Enceladus (Beuthe, 2018; Berne et al., 2023) likely enables
 279 determinations of \tilde{D} (see Figure 4 in Berne et al., 2023) to within $< 20\%$ of the true value,
 280 which is comparable to errors for estimates of spatially variable crustal thickness using
 281 our approach at long wavelengths (see Figure 4). Moreover, our iterative analysis is ex-
 282 pected to correct for over-(under-) predictions of strain due to excessively low (high) ini-
 283 tial estimates of \tilde{D} (see Equation 13).

284 While fractures are assumed to have no impact on the recovery of crustal thick-
 285 ness in our initial analysis, faults such as Tiger Stripes and other potential weak zones
 286 (i.e., chasma and Circum-Tectonic Boundaries; Yin & Pappalardo, 2015) may concen-
 287 trate strain under tidal loading. FEMs enable us to compute the deformation caused by
 288 both fractures and variations in crustal thickness, allowing for easy future modification
 289 of our methodology to examine the impact of fractures on strain fields at Enceladus (For
 290 more information, see Berne et al. (2023) on the ‘weak zone’ formulation).

291 In this study, we treat the single case of a crustal structure with Airy isostatic com-
 292 pensation of surface topography (Hemingway & Matsuyama, 2017). However, future in-
 293 vestigations could employ Monte Carlo methods to examine whether the derived ~ 2
 294 km error from our analysis holds for recoveries of thickness on a range of different sce-
 295 nario crustal models of Enceladus. In addition, modifying our approach to consider the

296 full surface strain field (Tape et al., 2009) or static gravity and topography data (Hem-
297 ington & Mittal, 2019) could yield estimates of crustal thickness that are more precise
298 than results presented in Figures 2 – 4. Although beyond the scope of this study, such
299 analyses are crucial to fully evaluate the reliability of our methodology for recovering 3D
300 crustal structure from real geodetic measurements at Enceladus.

301 In the future, geodetic imaging techniques such as Interferometric Synthetic Aperture
302 Radar (InSAR) measurements of ground deformation from orbiting platforms (e.g.,
303 Simons & Rosen, 2015) could enable the analysis described in this work. Our results in-
304 dicate that the presence of crustal thickness variations generates maximum peak-to-peak
305 horizontal and radial displacements of approximately ± 1 -10 cm over the tidal cycle (see
306 also Figures 2 and 3 of Berne et al., 2023). These values exceed the demonstrated sen-
307 sitivity of InSAR measurements to ground displacement (e.g., Simons & Rosen, 2015).
308 Moreover, our analysis can be extended to calculate the relationship between strain and
309 thickness at discrete time points, such as during repeat InSAR passes. As such, we ex-
310 pect that InSAR measurements would provide key insights into the interior structure of
311 Enceladus and thus to our assessment of the moon’s habitability.

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323 **Open Research**

324 The data used in this study were generated using the software package PyLith (Aa-
325 gaard et al., 2007; Aagaard et al., 2022). PyLith is an open-source finite element code
326 for modeling geodynamic processes and is available on GitHub and Zenodo repositories

327 (Aagaard et al., 2022). The specific PyLith version used in this study was v2.2.2. The
 328 mesh geometries utilized in this study were created using CUBIT (v15.2), a node-locked
 329 licensed software which is available through the developer Sandia National Laboratories
 330 (Skroch et al., 2019; CoreForm, 2020).

331

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