

Ambiguous stability of glaciers at bed peaks

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Key Points:

- Many glaciers persist at peaks in bed topography while enduring considerable changes in climate.
- Bed peaks are locations of enhanced stability, not vulnerability, contrary to some prior theories which neglect the effect of bed slope.
- Persistence of glaciers at bed peaks may give way to sudden retreat without a concurrent climate change.

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Abstract

Increasing ice flux from glaciers retreating over deepening bed topography has been implicated in the recent acceleration of mass loss from the Greenland and Antarctic ice sheets. We show in observations that some glaciers have remained at peaks in bed topography without retreating despite enduring significant changes in climate. Observations also indicate that some glaciers which persist at bed peaks undergo sudden retreat years or decades after the onset of local ocean or atmospheric warming. Using model simulations, we show that glacier persistence may lead to two very different futures: one where glaciers persist at bed peaks indefinitely, and another where glaciers retreat from the bed peak suddenly without a concurrent climate forcing. However, it is difficult to distinguish which of these two futures will occur from current observations. We conclude that inferring glacier stability from observations of persistence obscures our true commitment to future sea-level rise under climate change.

Plain Language Summary

Glaciers that flow into the ocean may retreat when the ocean or atmosphere warms. These glaciers often sit on top of solid ground that is bumpy with sharp peaks. Satellites and geological indicators on the seafloor have recorded the retreat of glaciers in the past, and show correlation between the location of bed peaks and positions where glacier retreats have paused. Computer simulations confirm that sharp bed peaks may pause the retreat of a glacier, giving the appearance of stability. However, such a pause in glacier retreat may only be temporary. Some glaciers will suddenly retreat from a bed peak without a recent change in climate. This behavior may lead to mistaken interpretations of a glacier's future based on its current state or mistaken attributions of past glacier changes to changes in climate. Being able to predict whether a glacier will retreat or stay at these bed peaks is important for predicting future sea level rise from glacier change.

1 Introduction

Mass loss from the Greenland and Antarctic ice sheets has accelerated in recent decades, driven by increasing surface melt and discharge of ice from glaciers (Shepherd et al., 2018; Mouginit et al., 2019). The increase in glacier discharge is driven, in part, by glacier retreat over deepening (retrograde) bed topography, which may initiate a positive feedback known as the “marine ice sheet instability” (Weertman, 1974). However,

47 the climate forcing needed to initiate this positive feedback depends on a range of other
48 process including ice shelf buttressing and subglacial friction (Gudmundsson et al., 2012;
49 Robel et al., 2016; Haseloff & Sergienko, 2018; Pegler, 2018; Sergienko & Wingham, 2019),
50 which are not all represented accurately in theories of marine ice sheet stability and even
51 in complex ice sheet models which are used to project future ice sheet changes. In par-
52 ticular, bed topography that fluctuates on length scales of tens to hundreds of kilome-
53 ters leads to behaviors that are not accurately predicted using classical theories of ma-
54 rine ice sheet stability (Sergienko & Wingham, 2021).

55 As observations of subglacial bed topography and glacier retreat have improved,
56 we are learning that bed topography is bumpy at a wide range of length scales (Jordan
57 et al., 2017; Morlighem et al., 2017, 2020) and that many glaciers in Greenland and Antarc-
58 tica have undergone large retreats (Tinto & Bell, 2011; Smith et al., 2017; Catania et
59 al., 2018). Still, many glaciers have not retreated during the observational era, even while
60 nearby glaciers have retreated in response to regional warming of the ocean and atmo-
61 sphere. Notable examples includes Thwaites Glacier, West Antarctica, where geologi-
62 cal evidence recorded the persistence of the grounding line at a bed peak for hundreds
63 to thousands of years (Tinto & Bell, 2011), even amidst significant fluctuations in ocean
64 temperatures (Hillenbrand et al., 2017). Nearby, observations show that Pine Island Glacier
65 persisted at a bed peak until the 1970's, even though regional warming of the ocean be-
66 gan in the 1940's (Smith et al., 2017). As we will discuss further in section 2, large por-
67 tions of the Greenland coast have also been subject to incursions of warm ocean water,
68 though different glaciers have responded to these incursions in different ways (Catania
69 et al., 2018), with the presence of sharp bed peaks being a key factor both in Greenland
70 and in Antarctica.

71 Here we demonstrate both observationally and using model simulations that the
72 retreat of a marine-terminating glacier may pause at bed peaks for prolonged time pe-
73 riods even while the glacier continues to lose mass in response to a current or previous
74 climate forcing. The persistence of glaciers at bed peaks ultimately leads to one of two
75 very different futures: one in which the glacier continues to persist without losing mass
76 (section 3), and another where retreat occurs suddenly without a concurrent change in
77 climate and leads to a significant acceleration in mass loss. However, it is difficult to dis-
78 tinguish which of these two possible futures will occur from current observations of per-
79 sistent glaciers (section 4). We also discuss how glacier stability is not necessarily im-

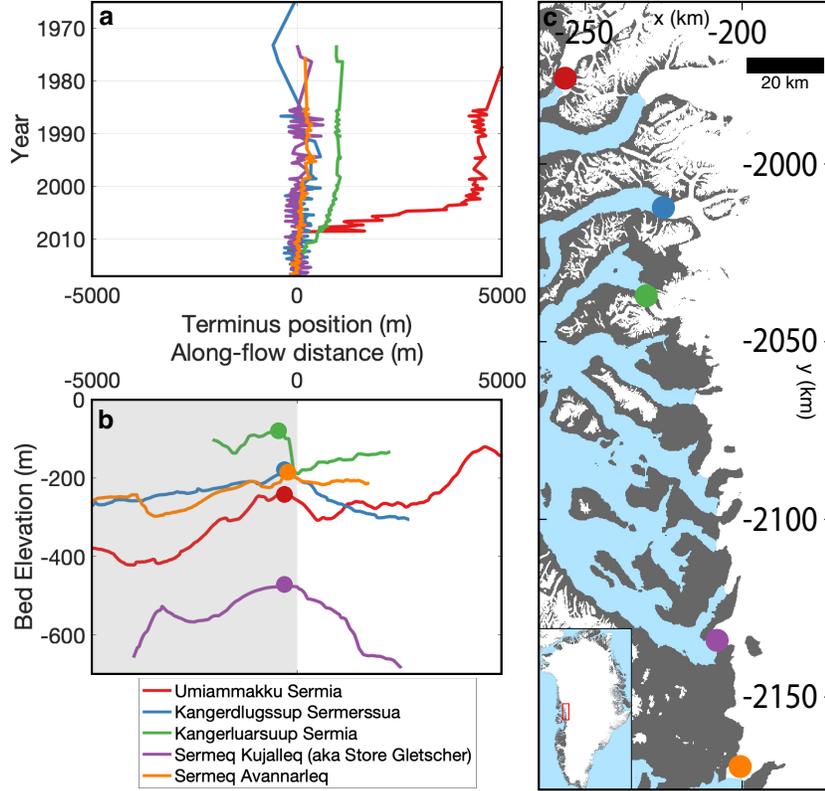


Figure 1. Observational evidence of terminus and grounding line persistence at bed peaks in Central West Greenland (CWG). (a) Terminus positions (x -axis) over time (y -axis) at five CWG glaciers derived from satellite-based sensors (Catania et al., 2018). (b) Along-flow bed topography at CWG glaciers in panel (a), with the x -axis is the along-flow distance relative to recent (2016) terminus position, with $x = 0$ representing the present position of the glacier termini and gray shading indicating where there is currently grounded glacier ice ($x < 0$). Nearest bed peaks upstream of the current terminus denoted by a filled circle in each case. For glaciers with strong cross-fjord variations in topography (Kangerluarsuup, Kujalleq), the minimum cross-fjord bed topography is used; for the others, mean cross-fjord topography is used. Bathymetry from BedMachine data compilation (panel b) (Morlighem et al., 2017). (c) Location of CWG glaciers in panels (a) and (b) on polar stereographic north projection (EPSG:3413).

80 plied by observations of glacier persistence (section 5). Ultimately, this ambiguous be-
 81 havior of seemingly “stable” glaciers obscures the true commitment to future sea level
 82 rise under anthropogenic climate change.

2 Observations of glacier persistence

Central West Greenland provides a particularly well-observed laboratory for understanding glacier retreat over bumpy beds. As in most of Greenland, surface melting has been persistently intensifying since the 1970's (or potentially earlier; Trusel et al. (2018)). In the late 1990's an influx of warm water from the North Atlantic arrived in glacier fjords in this region (Holland et al., 2008). A compilation of terminus positions recorded by visible satellite imagery (Catania et al., 2018) show that many glaciers in Central West Greenland (CWG) retreated between the late 1990's and the early 2000's when ocean temperatures were warm. However, some glaciers in this region have not retreated during the observational record, despite experiencing the same influx of warm ocean water. Figure 1a shows observations of terminus positions at four such persistent glaciers, Kangerdlugssup Sermerssua (blue), Kangerluarsuup Sermia (green), Sermeq Kujalleq (purple, aka Store Gletscher), and Sermeq Avannarleq (orange). Figure 1b shows the along-flow bed topography at these same glaciers from the BedMachine v3 dataset (Morlighem et al., 2017). These glaciers have persisted less than one kilometer downstream of bed peaks, indicating the critical importance of peaks in bed topography in potentially delaying or preventing rapid glacier retreat. Glaciers in this region that did retreat following the ocean warming event in the late 1990's mostly retreated away from bed peaks (on which they had previously persisted) and all have since ceased retreat upon reaching a new bed peak. In Figure 1, we show one example of such a glacier, Umiammakku Sermia (red), which began rapidly retreating approximately 5 years after the arrival of warm waters in the region, before ceasing retreat at a bed peak around 2010.

Geological evidence from regions of past glacier retreat further demonstrates the importance of bed peaks in the response of glaciers to climate change. The bathymetry of the Ross Sea, Antarctica is composed of smooth, flat troughs separating large plateaus. Amid this smooth bathymetry, localized recessional moraines and grounding zone sediment wedges record locations where the deglacial retreat of glaciers in the Ross Sea embayment paused for prolonged time periods (Simkins et al., 2017; Greenwood et al., 2018). Figure 2 focuses on two particular locations in the Ross Sea where high-resolution multi-beam bathymetric observations show pervasive grounding zone wedges connecting, parallel to, and on top of seamounts on otherwise flat topography. Such evidence of grounding line persistence is not present in surrounding flat portions of the seafloor, indicating that these bed peaks (which are generated by non-glaciological processes) exert an

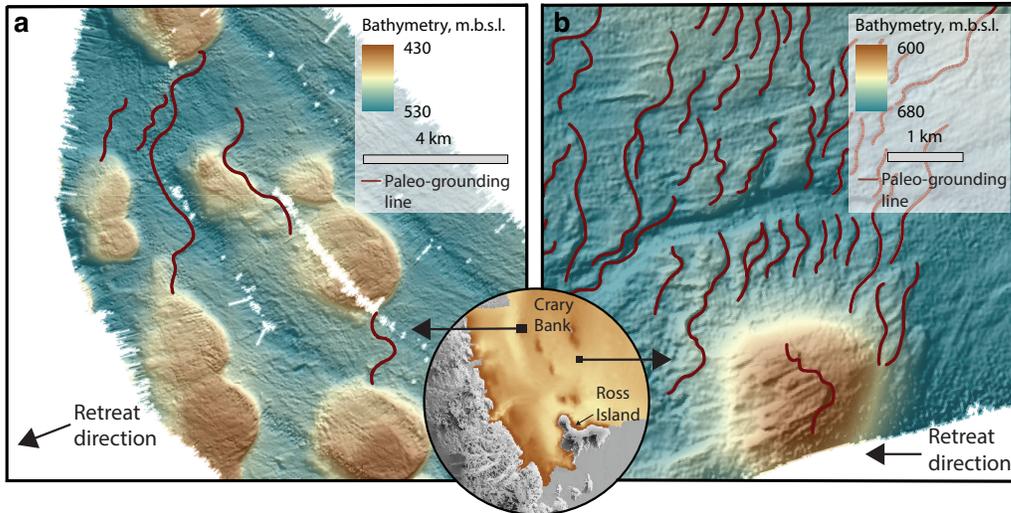


Figure 2. Bathymetry (in meters below sea level) of the south-western Ross Sea (inset bottom) with grounding zone wedges indicated by brown lines, which are generally perpendicular to the local retreat direction (indicated by black arrows). Multibeam echo sounding bathymetry collected on cruise NBP1502A (Simkins et al., 2017; Greenwood et al., 2018).

116 important control on glacier retreat. Other marine geophysical surveys of the seafloor
 117 in regions of past glacier retreat also reveal widespread geological evidence for prolonged
 118 periods of terminus persistence at bed peaks over a wide range of time periods and lo-
 119 cal conditions (Stoker et al., 2009; Greenwood et al., 2017; Todd & Shaw, 2012).

120 **3 Enhanced glacier stability at bed peaks**

121 To understand how bed peaks affect glacier stability, we consider first their effect
 122 on steady-state terminus positions, before considering their effect on transient evolution
 123 of the terminus position in the next section. To simulate a typical marine-terminating
 124 glacier near a bed peak, we use a one-dimensional flowline model of a marine-terminating
 125 glacier which solves the shallow stream/shelf approximation (SSA) and mass conserva-
 126 tion equations to determine the evolution of ice thicknesses, velocities, and terminus po-
 127 sition (as described in Schoof (2007a) and many other studies). The glacier is assumed
 128 to terminate where it goes afloat in seawater, and in the cases we consider in this study,
 129 ice flow is dominated by sliding over a moderately slippery bed (parameters listed in Ta-
 130 ble 1). We simulate the glacier velocity, thickness, and terminus position for prescribed
 131 surface mass balance (SMB, net annual snowfall and surface melt), which we assume to

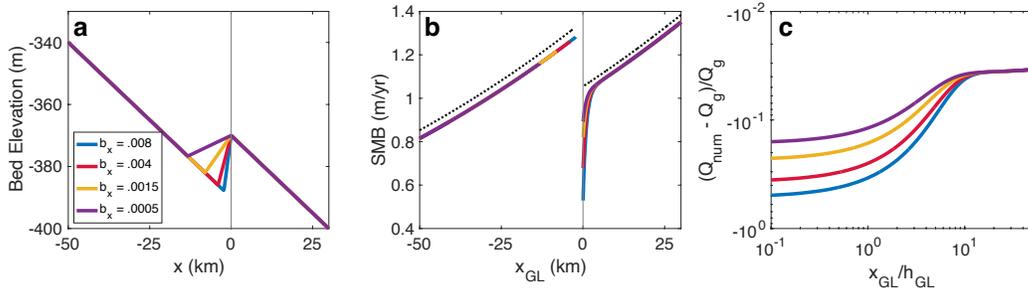


Figure 3. Simulated stable grounding line positions in the vicinity of a bed peak. (a) Four idealized bed topographies with differing bed slope just upstream of bed peak. (b) Bifurcation diagrams showing steady-state grounding line positions over a range of surface mass balance and initial grounding line positions. The dotted line is the stable grounding line positions predicted from an approximation for ice flux based on neglecting the effect of local slope, Q_g (Schoof, 2007b). (c) The proportional difference between the ice flux at the terminus predicted from our numerical solution, Q_{num} , and the ice flux that would be predicted on neglect of the effect of local slope, Q_g , as a function of distance from the bed peak (normalized by grounding line ice thickness).

132 be uniform over the surface of the glacier. The position of the terminus is accurately mod-
 133 eled using a refined moving mesh. Glacier steady-states are determined by numerically
 134 solving for glacier states with rates of change that are zero to within machine precision
 135 (and with transient perturbations to glacier state to confirm stability). We have also repli-
 136 cated the substance of the steady-state and transient results described hereafter using
 137 very high-resolution simulations of the Elmer/Ice Full-Stokes numerical glacier model
 138 (see Figure S1), indicating that the SSA simplification does not affect the substance of
 139 the conclusions in this study.

140 The four idealized bed topographies we consider (Figure 3a) all have a single bed
 141 peak, but with different reverse bed slopes just upstream of the peak, and otherwise the
 142 same forward-sloping bed (i.e. shallowing towards the interior). Simulations show that
 143 over a wide range of SMB, glacier termini persist indefinitely (i.e. reside at a stable steady-
 144 state) near bed peaks (Figure 3b). We find that at the sharp bed peaks we consider in
 145 this study, which entail a rapid spatial transition from a forward-sloping bed to a suf-
 146 ficiently reverse sloped bed (i.e. over a horizontal length scale of several kilometers), lead
 147 to glacier stability over a wider range of SMB than what is predicted in prevailing the-

Table 1. Parameter values for steady-state and transient retreat simulations (unless otherwise specified in text)

Parameter	Description	Value
A_g	Nye-Glen Law coefficient ($\text{Pa}^{-n} \cdot \text{s}^{-1}$)	4.2×10^{-25}
b_x	Prograde bed slope	1×10^{-3}
C	Basal friction coefficient ($\text{Pa} \cdot \text{m}^{-1/n} \cdot \text{s}^{1/n}$)	1×10^6
g	Acceleration due to gravity ($\text{m} \cdot \text{s}^{-2}$)	9.81
m	Weertman friction law exponent	1/3
n	Nye-Glen Law exponent	3
Δt	Time step (yr)	1
ρ_i	Ice density ($\text{kg} \cdot \text{m}^{-3}$)	917
ρ_w	Seawater density ($\text{kg} \cdot \text{m}^{-3}$)	1028

148 ories of terminus stability (dotted line in Figure 3b, reproduced from Schoof (2012)). The
 149 steeper the reverse sloped bed upstream of the bed peak, the wider the range of SMB
 150 over which the glacier will remain stable. For the steepest reverse slope (shown in blue),
 151 the glacier remains stable a short distance downstream of the topographic high for a sig-
 152 nificant range of SMB from from 0.5 to 1.0 m yr^{-1} .

153 The cause of enhanced glacier stability at bed peaks can be explained by examin-
 154 ing how ice flux out of the glacier changes as the glacier gets close to the bed peak. As
 155 the terminus approaches within ~ 10 ice thicknesses of the bed peak, the ice flux (Fig-
 156 ure 3c) decreases much more rapidly than is predicted under the assumption of negli-
 157 gible bed slope near the terminus. Indeed the magnitude of this reduction in ice flux near
 158 the bed peak (10-50% in these examples) is comparable to the effect of ice shelf buttress-
 159 ing on grounding line ice flux (Reese et al., 2018; Mitcham et al., 2021). The cause of
 160 this rapid decline in ice flux is a lowered driving stress on the ice flowing uphill to the
 161 bed peak, which lowers ice velocity just upstream of the bed peak and influences termi-
 162 nus ice flux through longitudinal viscous stresses. This reduced terminus ice flux is in
 163 balance with the total ice flux arriving at the terminus from upstream, maintaining a
 164 stable terminus position, even under dramatically lower SMB.

165 The enhanced stability of simulated termini near bed peaks, as compared to prior
166 theory, explains counterintuitive aspects of observations. There is a wide range of ex-
167 ternal forcing over which a terminus will persist at a bed peak, explaining why so many
168 glacier termini are observed at bed peaks on bumpy bed topography. Indeed, repeating
169 these steady-state simulations for corrugated bed topography (a regular series of peaks
170 and troughs) indicates stable glacier terminus positions exist almost exclusively at bed
171 peaks (Figure S2). The reduced glacier sensitivity to climatic changes at bed peaks also
172 explains why many glaciers are observed to persist, seemingly on the precipice of insta-
173 bility, even while experiencing substantial fluctuations in local climate (Tinto & Bell, 2011;
174 Hillenbrand et al., 2017; Catania et al., 2018). Such enhanced stability of glaciers at bed
175 peaks is in contrast to the prevailing idea that glaciers at bed peaks are particularly “vul-
176 nerable” to fluctuations or trends in climate due to their proximity to reverse-sloping beds
177 over which the marine ice sheet instability occurs (Gladstone et al., 2012; Ross et al.,
178 2012; Morlighem et al., 2020).

179 The enhanced stability near points of destabilization and non-smooth changes in
180 bed topography is also a hallmark of a “crossing-sliding bifurcation” (di Bernardo et al.,
181 2008). The system behavior in the vicinity of such bifurcations is different from the canon-
182 ical “saddle-node bifurcation”, which has previously been identified as the route through
183 which grounding lines lose stability (Mulder et al., 2018; Pegler, 2018). In a saddle-node
184 bifurcation, the loss of stability occurs smoothly and suddenly upon approach of the sys-
185 tem state to the bifurcation point. In a crossing-sliding bifurcation, the loss of stabil-
186 ity instead occurs due to an abrupt (i.e., non-smooth) change in the system properties.
187 In the case of a sharp bed peak, smooth parameter variations causes the stable glacier
188 state to reach the bifurcation point and then remains there, before eventually crossing
189 the bed peak and initiating a large change in glacier state. This distinction in type of
190 bifurcation can be important because it leads to larger “jumps” in the system state (i.e.,
191 ice volume loss) upon crossing the bifurcation. In other words, as the forcing gradually
192 changes (i.e., SMB decreases), the onset of rapid ice loss is delayed, leading to a higher
193 rate of ice loss if and when the terminus crosses the bed peak. In Figure 3b, this amounts
194 to the difference between a 30 km jump in grounding line position for a relatively smooth
195 bump (i.e. the dotted or purple lines), compared to a jump of 50-100 km for sharper peaks
196 (yellow, red, blue lines). As we will show next, even when climate trends or fluctuations

197 exceed the threshold for instability identified in Figure 3b, the onset of rapid glacier re-
198 treat may be substantially delayed.

199 **4 Distinguishing glacier stability from transient persistence**

200 In transient simulations of terminus retreat over idealized bed peaks (the same as
201 in the previous section), a glacier is initialized at a steady state with its terminus just
202 downstream of a bed peak, and is then subjected to a 40% step reduction in SMB uni-
203 formly over the glacier catchment. Figure 4a shows that some of the simulated glaciers
204 retreat up to, then transiently persist, just downstream of the bed peak for a period of
205 time spanning decades to centuries (yellow and red lines), before eventually crossing the
206 bed peak and rapidly retreating over the reverse sloping bed. There are also cases where
207 there is merely a brief slowdown in the rate of retreat at the bed peak (purple line), and
208 other cases where the persistence continues indefinitely (blue line). We define such an
209 indefinitely persistent case as “stable” in the mathematical sense, where a system state
210 persists forever with no change in forcing. Similar behaviors of transient and indefinite
211 persistence of grounding lines at bed peaks also occur in equivalent full-Stokes simula-
212 tions of grounding line retreat over bed peaks (Figure S3) and in SSA simulations of glacier
213 retreat with different types of forcing and smoothed bed peaks (Figures S4-S6). These
214 transient simulations show that even for SMB values that do not correspond to a sta-
215 ble glacier configuration (shown in Figure 3b), there may still be prolonged periods of
216 transient terminus persistence. Longer periods of transient persistence lead to more rapid
217 subsequent retreat, which continues even as the terminus encounters forward-sloping bed
218 topography.

219 Glaciers that persist at bed peaks continue to lose mass through thinning upstream
220 of the grounding line following changes in climate forcing, as seen in observations of re-
221 cent thinning upstream of the terminus at persistent glaciers throughout Greenland (Kjeldsen
222 et al., 2015; Mouginot et al., 2019; Shepherd et al., 2020; Felikson et al., 2017). At per-
223 sistent glaciers in CWG, this thinning is mostly being driven by negative surface mass
224 balance anomalies, which are largely offset by dynamic thickening bringing ice from up-
225 stream portions of glacier catchments (Felikson et al., 2017). Ultimately, this upstream-
226 intensified thinning leads a decrease in ice surface slope and upstream slowing, which is
227 captured in observations of persistent CWG glaciers (Joughin et al., 2010) and our sim-
228 ulations (Figure 4b-c). Though such thinning is less than that occurring at retreating

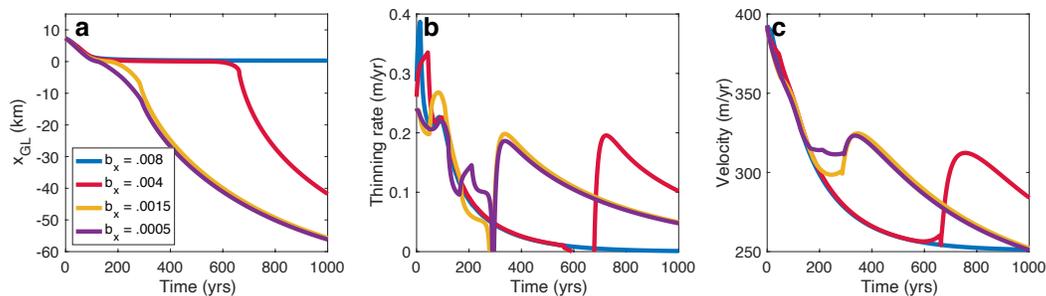


Figure 4. Simulated terminus retreat in the vicinity of a bed peak. (a) Evolution of a terminus from steady state, in response to an instantaneous 40% reduction in surface mass balance over the glacier catchment, for a variety of upstream bed slopes. Grounding line position (y -axis) is relative to bed peak location as in Figure 3a. (b) Thinning rate 50 km upstream of grounding line in transient simulations. (c) Ice velocity 50 km upstream of grounding line in transient simulations.

229 glaciers through dynamic thinning, it nonetheless shows that persistence of a glacier ter-
 230 minus is not necessarily indicative of a glacier in mass equilibrium.

231 We can compare our simulated glaciers which stabilize at bed peaks to those which
 232 merely pause at bed peaks to ascertain whether observations of persistent glaciers may
 233 provide evidence of their eventual fate. We find that, regardless of their eventual fate
 234 (remaining or retreating), the glaciers we simulate which persist at bed peaks have up-
 235 stream thinning rates within millimeters/year of each other (Figure 4b and Figure S8),
 236 and ice velocities within meters/year of each other (Figure 4c and Figure S7). It would
 237 thus be exceedingly difficult to observationally distinguish glaciers that are merely paused
 238 from those that have stabilized indefinitely at bed peaks. Other studies have also found
 239 that, in realistic simulations of the future retreat of glaciers away from bed peaks, small
 240 uncertainties in the observed glacier state, bed topography, or the climate forcing pro-
 241 duce large uncertainties in the timing of the onset of rapid glacier retreat which is then
 242 amplified by the divergence of retreat predictions due to marine ice sheet instability (Gladstone
 243 et al., 2012; Robel et al., 2019). Ultimately, the delicate balance between advection and
 244 thinning at persistent glaciers makes it exceedingly difficult to project retreat of glaciers
 245 over bumpy bed topography, and further emphasizes the need for more accurate obser-
 246 vational constraints on glacier state and rate of change, bed topography, and local cli-
 247 mate change.

5 Discussion and Conclusions

We have shown that glaciers observed at bed peaks have two possible futures: they may remain at the bed peak indefinitely (i.e., stabilize) or initiate retreat, potentially long after the onset of climate change. Glaciers persisting at bed peaks may continue to lose mass in response to a previous or sustained climate change, though there will be an increasing “disequilibrium” between this mass loss and the total committed glacier mass loss implied by contemporaneous climate forcing (Christian et al., 2018). If the terminus does eventually cross the bed peak, terminus retreat and total glacier mass loss accelerates rapidly, relaxing the glacier disequilibrium between instantaneous and total committed mass loss. Eventually, the total sea level contribution from non-persistent and transiently persistent glaciers may be similar, though the timing and rate of peak mass loss may be very different (e.g., Figure 4).

In attempting to infer the future behavior of glaciers persisting at bed peaks, observations can be deceptive. We have shown that ice flux and thickness may change considerably with relatively little change in the terminus position. Thus, interpreting observations of terminus change requires accurate measurements of bed topography and the critical context of changes in other aspects of glacier state (particularly interior thickness and velocity) to assess whether the glacier is in balance. Additionally, the slow response time scale of glaciers, particularly those that have encountered bed peaks, indicates that the utility of “stability” as a tool for categorizing observed glacier changes is limited without the critical context of multi-centennial (or millennial) glacier changes, and the climate forcing over that time period. The scope of these challenges and potential impacts indicate that we should direct a similar degree of attention and resources to closely observing and carefully simulating persistent glaciers as we do to rapidly changing glaciers, as it is possible and perhaps likely that they will eventually contribute just as much to future sea level rise.

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 283 available on publication. Model codes used for conducting numerical experiments are also
 284 available as persistent Zenodo repositories (MATLAB SSA model: <https://doi.org/10.5281/zenodo.5245271>,
 285 Julia SSA model: <https://doi.org/10.5281/zenodo.5245331>). Full-stokes simulations con-
 286 ducted with Elmer/Ice which is openly available at: <http://elmerice.elmerfem.org/>.

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