

# Axially Asymmetric Steady State Model of Jupiter's Magnetosphere-Ionosphere Coupling

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## Key Points:

- We develop an axially asymmetric model of Jovian magnetosphere-ionosphere coupling
- Comparison of model calculations with observed magnetodisc radial currents suggests an average radial mass transport rate of  $\sim 2000 \text{ kg s}^{-1}$
- Inclusion of a nightside partial ring current helps to explain the local time variation of the radial currents

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## Abstract

We present an axially asymmetric steady state model of Jupiter’s magnetosphere-ionosphere coupling with variable ionospheric conductivity dependent on the field-aligned current density. We use Juno and Galileo data to construct a simple model of the equatorial magnetic field, and develop a method for solving the system of partial differential equations describing magnetosphere-ionosphere coupling. Using this model we study the behavior of the system with different radial mass transport rates of magnetospheric plasma and the effect of additional field-aligned currents associated with Jupiter’s nightside partial ring current. We compare the model magnetodisc current intensities with those determined directly from magnetic field measurements in various local time sectors, and find that the value of mass transport rate of  $2000 \text{ kg s}^{-1}$ , larger than usually estimated, better accounts for the observed radial currents. We also find that the inclusion of field-aligned currents associated with Jupiter’s partial ring current helps to explain the local time variation of the radial currents, reducing the discrepancy between the model and the observations.

## Plain Language Summary

One of the key processes in Jupiter’s magnetosphere is the transfer of angular momentum from the planet to the magnetospheric plasma. It is a primary source of energy in the magnetosphere and is thought to be one of the main drivers of the auroral emissions. It was extensively studied using stationary force-balance models. As an approximation, these models assumed axial symmetry, but due to the solar wind influence the structure of Jupiter’s magnetosphere is different depending on local time. We present an improved model which partially accounts for this asymmetry. We use it together with Juno and Galileo spacecraft magnetic field measurements to study angular momentum transport at different local times. We found that the observations suggest an approximately twice larger plasma production rate by the moon Io, than is usually estimated. We also show how the asymmetries in the equatorial magnetospheric current can affect the angular momentum transfer.

## 1 Introduction

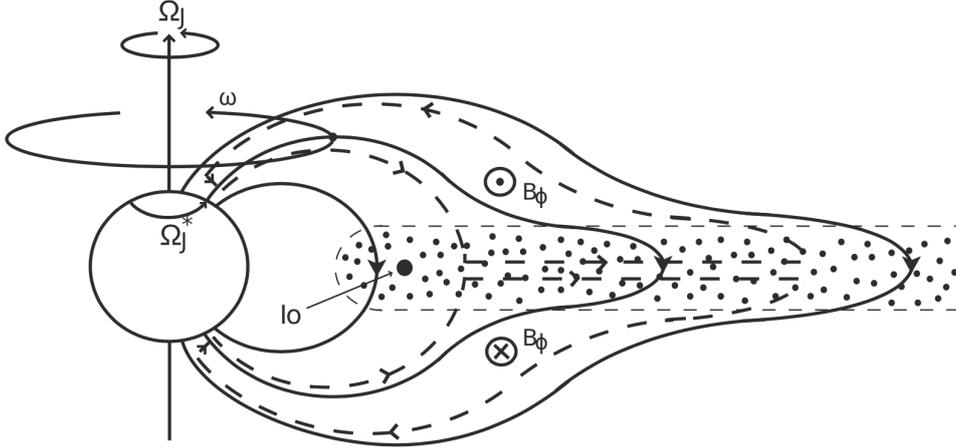
Jupiter has a powerful source of plasma deep within the magnetosphere, originating from the volcanic moon Io, which orbits at around  $6 R_J$ . Here  $R_J = 71,492 \text{ km}$  is Jupiter’s equatorial 1 bar radius (e.g Joy, 2002). Plasma is transported radially outward from the Io torus, and in the absence of a torque acting on the plasma its angular velocity would fall with radial distance  $r$  as  $1/r^2$  due to angular momentum conservation. However, the decrease in equatorial plasma angular velocity mapped into the ionosphere increases the collisional friction between ionospheric ions and atmospheric neutrals. In a steady state this torque is balanced by the  $\mathbf{j} \times \mathbf{B}$  force of the equatorward ionospheric Pedersen current, with the collisional torque being transferred to the equatorial magnetosphere by field-aligned currents. An equatorial outward radial current acts to enforce corotation via a  $\mathbf{j} \times \mathbf{B}$  force and closes the system (Hill, 1979). The effect of the resulting azimuthal field perturbations produced is often described in terms of frozen-in magnetic field lines being “bent back” by the lagging plasma flow. Figure 1 shows the overall current system enforcing corotation. The upward ionospheric field-aligned current region is associated with precipitating electrons and has been suggested to be the source of Jupiter’s main oval auroral UV emissions (Cowley & Bunce, 2001; Hill, 2001).

The key parameters of the Hill (1979) model of corotation enforcement are the equatorial magnetic field profile, the plasma mass outflow rate, and the Pedersen conductivity of the ionosphere. The model has been improved and built upon for many years. Pontius (1997) used a realistic magnetic field model that takes into account the current disc field, instead of the simple dipole field used by Hill (1979), to calculate the angular velocity

65 profile. Hill (2001) and Cowley and Bunce (2001) studied the current system correspond-  
 66 ing to the calculated angular velocity profiles and considered its connection to the au-  
 67 roral emissions. Nichols and Cowley (2003) studied the effect of different mass transport  
 68 rates and ionospheric conductivities on the angular velocity and the currents, while Nichols  
 69 and Cowley (2004) accounted for ionospheric conductivity modulations by precipitation  
 70 associated with upward field-aligned current regions. Nichols and Cowley (2005) self-consistently  
 71 took into account the field-aligned voltages, calculating the plasma angular velocity and  
 72 currents. They showed that for the Jovian magnetosphere, the difference between the  
 73 results of the model with and without field-aligned voltages are minor. This problem was  
 74 later analyzed by Ray et al. (2010), who, on the contrary, found that the influence of field-  
 75 aligned voltages is not negligible and that they increase the auroral field-aligned currents.  
 76 Smith and Aylward (2009) developed a self-consistent model of angular momentum trans-  
 77 port that includes both magnetosphere and thermosphere. Ray et al. (2015) studied the  
 78 magnetosphere-ionosphere-thermosphere system and found that the inclusion of field-  
 79 aligned voltages does not significantly change the dynamics of the thermosphere, while  
 80 variations in Pedersen conductivity have a strong influence on the system. Cowley et al.  
 81 (2007) studied the reaction of the Jovian magnetosphere-ionosphere system to a solar  
 82 wind pressure change and analyzed the variation in power of the accelerated electron pre-  
 83 cipitation. They found that for major compression the brightness increases, and super-  
 84 rotation arises, with the M-I coupling current system reversing. Yates et al. (2014) an-  
 85 alyzed the influence of solar wind pressure variations on the field-aligned currents, ther-  
 86 mospheric flows, heating, and auroral emissions. They concluded that in a steady state  
 87 the thermosphere becomes hotter as the magnetosphere becomes larger. Tao et al. (2009)  
 88 investigated the influence of neutral winds on Jovian magnetosphere-ionosphere coupling.  
 89 They developed an axisymmetric model that includes the thermospheric dynamics, con-  
 90 vection, and magnetosphere-ionosphere coupling. Tao et al. (2010) considered the influ-  
 91 ence of the diurnal variation of the ionospheric conductivity controlled by the solar EUV  
 92 radiation on the field-aligned currents. According to their results field-aligned currents  
 93 are strongest at noon, and weakest at dawn. Louarn et al. (2016) studied the empirical  
 94 relationship between auroral radio emissions and the radial mass transport rate. They  
 95 found that a larger radial outflow of plasma causes stronger field-aligned currents, and,  
 96 consequently, more intense auroral activity.

97 Azimuthal currents in the current disc are determined by radial force balance, and  
 98 hence depend on the angular velocity profile, while it, in turn, depends on the magnetic  
 99 field created by the current disc (e.g Arridge & Martin, 2018). Nichols (2011) used the  
 100 steady state model of radial force balance derived by Caudal (1986) to develop a self-  
 101 consistent model of the current disc and M-I coupling. Nichols et al. (2015) further de-  
 102 veloped this model by accounting for anisotropic plasma pressure. They found that the  
 103 anisotropic pressure component is a dominant or at least not negligible part of the force  
 104 balance from  $\sim 20 R_J$  to  $\sim 50 R_J$ . Using this model, Nichols et al. (2020) found that an  
 105 increase in the azimuthal and radial components of the magnetic field and the temper-  
 106 ature of the plasma correlates with enhanced brightness of the main auroral oval. They  
 107 concluded that a transient enhancement can be caused by an increase in the hot mag-  
 108 netospheric plasma pressure and iogenic plasma outflow.

109 Ray et al. (2014) studied local time (LT) asymmetries of M-I coupling at Jupiter  
 110 using the Vogt et al. (2011) empirical magnetic field model. While the field model used  
 111 was LT-dependent, they assumed that M-I coupling is locally axisymmetric. They used  
 112 an effective ionospheric Pedersen conductivity of 0.1 mho, constant in latitude and LT,  
 113 and a mass outflow rate of  $1000 \text{ kg s}^{-1}$ . Lorch et al. (2020) used magnetic field measure-  
 114 ments obtained from all the spacecraft that have visited Jupiter to map the average ra-  
 115 dial and azimuthal current intensities in the magnetodisc in radial distance and LT. Since  
 116 the radial currents are a key part of the M-I coupling current system, these observations  
 117 provide an important opportunity to study LT asymmetries at Jupiter.



**Figure 1.** Sketch showing a meridian cross-section through Jupiter's magnetosphere. Arrowed solid lines show magnetic field lines. Arrowed dashed lines show the currents.  $\Omega_J$ ,  $\Omega_J^*$  and  $\omega$  are the angular velocities of Jupiter, the upper neutral atmosphere in the ionospheric Pedersen layer, and the plasma in a given flux tube, respectively. The dotted region represents the current sheet plasma. Taken from Nichols (2011), adapted from Cowley and Bunce (2001).

118 In this paper we develop an axially asymmetric variation of the Hill (1979) model,  
 119 with variable ionospheric conductivity dependent on the field-aligned current density.  
 120 In section 2 we derive the model equations. In section 3 we describe the magnetic field  
 121 model and our approach to solving the differential equations describing M-I coupling.  
 122 We present the results of the modelling in section 4 and discuss them in section 5. Sec-  
 123 tion 6 summarises our conclusions.

## 124 2 Theoretical Background

### 125 2.1 Partial Differential Equation for Angular Velocity

126 In this section we derive the partial differential equation for the plasma angular ve-  
 127 locity profile, generalizing previous work to the case of axial asymmetry, thus forming  
 128 a two dimensional extension of the Hill-Pontius equation (Hill, 1979; Pontius, 1997; Cow-  
 129 ley et al., 2002). Calculation of the M-I coupling currents follows Cowley and Bunce (2001),  
 130 with an angular momentum balance equation derived analogously to that given by Cowley  
 131 et al. (2002), but now not assuming axial symmetry.

132 A simple way to map magnetically between the equatorial plane and the ionosphere  
 133 is provided by Euler's potentials. A magnetic field  $\mathbf{B}$  can be expressed in terms of such  
 134 potentials  $f$  and  $g$  as

$$135 \quad \mathbf{B} = \nabla f \times \nabla g. \quad (1)$$

136 Both  $f$  and  $g$  are constant along field lines because the magnetic field vector is perpen-  
 137 dicular to their gradients. In cylindrical coordinates  $g$  is usually chosen in such a way  
 138 that its isosurfaces are meridians of constant magnetic longitude. We assume  $g = \phi$  and  
 139  $f = F(\rho, \phi, z)$ . The model field is hence wholly poloidal with zero azimuthal compo-  
 140 nent. Function  $F$  is magnetic flux per unit azimuthal angle and is sometimes called the  
 141 flux function. With this assumption

$$142 \quad B_z = \frac{1}{\rho} \frac{\partial F}{\partial \rho}. \quad (2)$$

143 For purposes of modeling we consider the internal magnetic field of the planet to be dipolar,  
 144 for which function  $F_d$  is given by

$$145 \quad F_d = B_J \rho^2 \left( \frac{R_J}{r} \right)^3, \quad (3)$$

146 where  $\rho$  is the cylindrical distance from the magnetic axis,  $r$  is the distance from the cen-  
 147 ter of the planet, and  $B_J$  is Jupiter's equatorial magnetic field strength ( $B_J = 4.17 \times$   
 148  $10^5$  nT in the JRM09 internal field model of Connerney et al. (2018)). Near the plan-  
 149 etary surface the internal planetary field is dominant, so that assuming the planet is ap-  
 150 proximately spherical, the ionospheric  $F_i$  is

$$151 \quad F_i = \rho_i^2 B_J, \quad (4)$$

152 where  $\rho_i$  is the perpendicular distance from the magnetic axis in the ionospheric layer.  
 153 Since  $f$  is constant along a field line, we can map between the magnetospheric equator  
 154 and the ionosphere using

$$155 \quad F_e(\rho, \phi) = F_i(\rho_i, \phi). \quad (5)$$

156 From current continuity, the structure of the current system shown in Figure 1, and  
 157 the assumption of north-south symmetry it follows that on a given flux shell in a given  
 158 azimuthal sector

$$159 \quad \rho i_\rho = 2\rho_i i_P, \quad (6)$$

160 where  $i_\rho$  is the radial current intensity in the equatorial current disc integrated through  
 161 its north-south width, and  $i_P$  is the ionospheric height-integrated Pedersen current in-  
 162 tensity given by

$$163 \quad i_P = 2B_J \rho_i \Sigma_P^*(\Omega_J - \omega). \quad (7)$$

164 Here we have assumed that the polar planetary field is near-vertical and of strength  $2B_J$   
 165 (instead of the strict dipole formula  $B_r = 2B_J \cos \theta$ ).  $\Omega_J = 1.76 \times 10^{-4}$  rad s $^{-1}$  is  
 166 Jupiter's angular velocity,  $\omega$  is the ionospheric plasma angular velocity, and  $\Sigma_P^*$  is the  
 167 effective height-integrated ionospheric Pedersen conductivity. The effective conductiv-  
 168 ity accounts for rotational lagging of the neutral atmosphere relative to rigid corotation  
 169 due to ion-neutral collisions, and is reduced compared to the true value by an unknown  
 170 factor  $0 < (1 - k) < 1$ , taken to be equal 0.5 following Achilleos et al. (1998) and pre-  
 171 vious related works (e.g., Cowley & Bunce, 2001; Cowley et al., 2002; Nichols & Cow-  
 172 ley, 2004). From equations (4)–(7) we obtain

$$173 \quad \rho i_\rho = 4\Sigma_P^* F_e(\Omega_J - \omega). \quad (8)$$

174 We assume that the equatorial plasma is concentrated in a thin disc. The angu-  
 175 lar momentum per unit mass of the equatorial plasma is  $\rho^2 \omega(\rho, \phi)$ , where  $\omega$  is angular  
 176 velocity. The flux of angular momentum is caused by radial transport of the plasma and  
 177 its rotation around the planet. The change of angular momentum per unit time in the  
 178 volume between  $\rho$  and  $\rho + d\rho$  and inside the sector centered at azimuthal angle  $\phi$  with  
 179 angular width  $d\phi$  is

$$180 \quad dT_z = \frac{\partial(\dot{M}_\rho \rho^2 \omega)}{\partial \rho} \frac{d\phi}{2\pi} + \frac{\partial(\dot{M}_\phi \rho^2 \omega)}{\partial \phi} d\rho, \quad (9)$$

181 where  $\dot{M}_\rho$  is the radial mass transport rate per  $2\pi$  radians of azimuth (full equatorial  
 182 circle), and  $\dot{M}_\phi$  is the azimuthal mass transport rate per unit radial length. The Lorentz  
 183 force torque per unit volume about Jupiter's center is  $\mathbf{r} \times (\mathbf{j} \times \mathbf{B})$ , where  $\mathbf{r}$  is the posi-  
 184 tion vector,  $\mathbf{j}$  is the current density, and  $\mathbf{B}$  is the magnetic field. If  $B_\rho$  varies only slowly  
 185 with  $\rho$  on the scale of the sheet thickness, and  $B_\phi$  varies only slowly with  $\phi$ , then  $\text{div} \mathbf{B} =$   
 186  $0$  guarantees that  $B_z$  varies only slowly with  $z$  on the scale of the sheet thickness. Then  
 187 the  $z$ -component of the torque acting on the plasma inside the volume element consid-  
 188 ered is

$$189 \quad dT_z = -\rho^2 i_\rho B_z d\rho d\phi. \quad (10)$$

190 If the mass density of the plasma per unit area of the equatorial current sheet is  $D(\rho, \phi)$   
 191 then

$$192 \quad \dot{M}_\phi = \rho\omega D. \quad (11)$$

193 Substitution of equations (10) and (11) into the equation (9) gives

$$194 \quad \frac{\partial(\dot{M}_\rho \rho^2 \omega)}{\partial \rho} \frac{1}{2\pi} + \rho^3 \frac{\partial(\omega^2 D)}{\partial \phi} = -\rho^2 i_\rho B_z. \quad (12)$$

195 We then substitute the width-integrated radial current given by equation (8) into equa-  
 196 tion (12) to obtain the following partial differential equation for the angular velocity

$$197 \quad \frac{\partial(\dot{M}_\rho \rho^2 \omega)}{\partial \rho} \frac{1}{2\pi} + \rho^3 \frac{\partial(\omega^2 D)}{\partial \phi} = -4\Sigma_P^* \rho B_z F_e(\Omega_J - \omega). \quad (13)$$

198 Throughout the paper we will assume  $\dot{M}_\rho$  to be constant at all local times (we discuss  
 199 this assumption in section 5), which means that  $\frac{\partial(\dot{M}_\rho)}{\partial \rho} = 0$ . With this assumption from  
 200 continuity of the mass flow

$$201 \quad \text{div}(D\mathbf{v}) = 0, \quad (14)$$

202 where  $\mathbf{v}$  is plasma bulk velocity, we find

$$203 \quad \frac{\partial(\omega D)}{\partial \phi} = 0. \quad (15)$$

204 We then can simplify equation (13) to get

$$205 \quad \frac{\dot{M}_\rho}{2\pi} \frac{\partial(\rho^2 \omega)}{\partial \rho} + D \rho^3 \omega \frac{\partial \omega}{\partial \phi} = -4\Sigma_P^* \rho B_z F_e(\Omega_J - \omega). \quad (16)$$

206 Physically correct solutions must converge to almost rigid corotation close to the planet,  
 207 thus the boundary condition is

$$208 \quad \omega(\rho_{\text{inner}}, \phi) \approx \Omega_J, \quad (17)$$

209 where  $\rho_{\text{inner}}$  is any  $\rho$  inside the region of nearly rigid corotation. In this paper  $\rho_{\text{inner}}$  is  
 210 taken to be  $6 R_J$  (near the orbit of Io).

## 211 **2.2 Modulation of Ionospheric Conductivity by Field-Aligned Currents**

212 The ionospheric Pedersen conductivity is modulated by precipitation of electrons  
 213 accelerated by field-aligned voltages in the auroral region. Nichols and Cowley (2004),  
 214 using the results of Millward (2002), calculated how the height-integrated ionospheric  
 215 Pedersen conductivity depends on the outward field-aligned currents density, and pro-  
 216 vided the following analytic approximation

$$217 \quad \Sigma_P(j_{||i}) = 0.16j_{||i} + 2.45 \times \left( \frac{(j_{||i}/0.075)^2}{1 + (j_{||i}/0.075)^2} \right) \times \frac{1}{1 + \exp\left(-\frac{(j_{||i}-0.22)}{0.12}\right)}, \quad (18)$$

218 where  $j_{||i}$  is the field-aligned current density just above the ionospheric layer. The ef-  
 219 fective conductivity is then

$$220 \quad \Sigma_P^* = (1 - k)(\Sigma_P(j_{||i}) + \Sigma_{P0}), \quad (19)$$

221 where  $\Sigma_{P0}$  is the background height-integrated ionospheric Pedersen conductivity taken  
 222 to be 0.1 mho (Nichols & Cowley, 2004).

223 If we assume the absence of electric currents perpendicular to the magnetic field  
 224 lines in the region between the current disc and the ionosphere, then  $j_{||}/B$  is constant

225 along the field lines. Following Cowley et al. (2002) we then find the equatorial field-aligned  
 226 current density from the divergence of the radial currents

$$227 \quad \frac{j_{\parallel}}{B} = \frac{j_z}{B_e} = -\frac{1}{2B_z} \frac{1}{\rho} \frac{d}{d\rho} (4\Sigma_P^* F_e(\Omega_J - \omega)). \quad (20)$$

228 If there exists a partial ring current in Jupiter’s magnetosphere, we should add the di-  
 229 vergence of the azimuthal magnetodisc currents  $\nabla_{\phi} i_{\phi}$  to the equation to obtain

$$230 \quad \frac{j_{\parallel}}{B} = \frac{j_z}{B_e} = -\frac{1}{2B_z} \left( \frac{1}{\rho} \frac{d}{d\rho} (4\Sigma_P^* F_e(\Omega_J - \omega)) + \nabla_{\phi} i_{\phi} \right). \quad (21)$$

231 We will later define these additional field-aligned currents explicitly as an input param-  
 232 eter for the model. The ionospheric field-aligned current density is then

$$233 \quad j_{\parallel i} = -\frac{B_J}{B_z} \left( \frac{1}{\rho} \frac{d}{d\rho} (4\Sigma_P^* F_e(\Omega_J - \omega)) + \nabla_{\phi} i_{\phi} \right). \quad (22)$$

234 Equations (16) and (22) constitute a system of partial differential equations (PDEs)  
 235 for  $\omega$  and  $j_{\parallel i}$ . Their solution requires a second boundary condition at some distance  $\rho_{j0}$

$$236 \quad j_{\parallel i}(\rho_{j0}, \phi) = j_{\parallel i0}(\phi). \quad (23)$$

237 Ideally we would like to set  $\rho_{j0} = \rho_{\text{inner}}$  and solve system of equations (16) and (22)  
 238 radially outward from  $\rho_{\text{inner}}$ . But this system is unstable in the near-rigid corotation re-  
 239 gion and is nearly impossible to solve this way. In the next section we will discuss the  
 240 reasons for this in detail and describe our approach to obtaining approximate solutions.

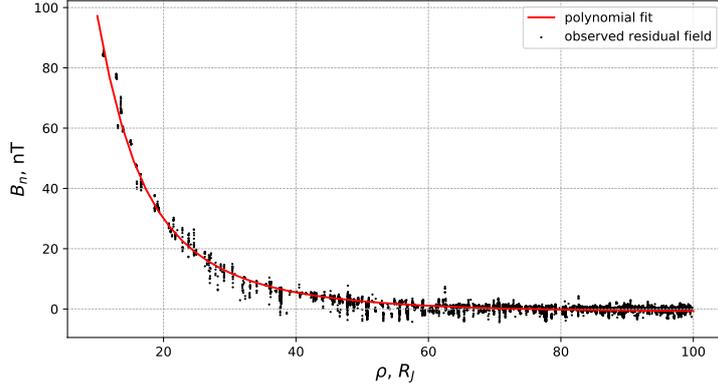
### 241 3 Modeling Approach

#### 242 3.1 Magnetic Field Model

243 The model equatorial magnetospheric magnetic field employed has been derived  
 244 from Galileo and Juno magnetometer data, using data from all Galileo orbits and from  
 245 Juno perijoves 0–22 that are currently available. As in Lorch et al. (2020), we split the  
 246 data into eight 3 h wide LT sectors, yielding sufficient data to cover radial distances of  
 247 interest, and allowing us to readily compare our results with those of Lorch et al. (2020).  
 248 For our purposes we are interested only in the equatorial magnetic field. To determine  
 249 whether or not a data point is inside the current disc we used two conditions. The first  
 250 one requires spacecraft to be closer than  $4 R_J$  to the center of the sheet according to the  
 251 Khurana and Schwarzl (2005) model. We use a large half-width of  $4 R_J$  instead of the  
 252 commonly used value of  $2\text{--}2.5 R_J$  because sheet crossings predicted by Khurana and Schwarzl  
 253 (2005) are often shifted from the observed ones. The second condition requires the  $\rho$ -  
 254 component of the magnetic field to be smaller than 2 nT. Its purpose is to select the real  
 255 crossings from the broad intervals picked by the first condition. In each LT sector we fit-  
 256 ted the polynomial

$$257 \quad B_n(\rho) = \frac{a}{\rho} + \frac{b}{\rho^2} + \frac{c}{\rho^3} \quad (24)$$

258 to the component of the residual magnetic field, that is normal to the local current sheet  
 259 surface. The residual field was obtained by subtracting the JRM09 model of the inter-  
 260 nal planetary field (Connerney et al., 2018) from the data. (We note that for the mod-  
 261 elling we still used a dipolar internal magnetic field because of the assumed north-south  
 262 symmetry and the neglect of dipole tilt). Because local time sector 9 h is not covered  
 263 by Juno and Galileo trajectories beyond  $40 R_J$  we were unable to get a valid fit for it.  
 264 We instead used an average of two neighboring sectors (6 h and 12 h) fits. Figure 2 shows  
 265 the data and the fit for the midnight sector.



**Figure 2.** Black dots show the component of the residual magnetic field normal to the current sheet for the 3 h LT sector centered on 00/24 h. The residual magnetic field is obtained by subtracting the internal field according to the JRM09 model of the internal planetary field (Connerney et al., 2018) from the data. The red line shows the polynomial fit given by equation (24) with coefficients listed in Table 1.

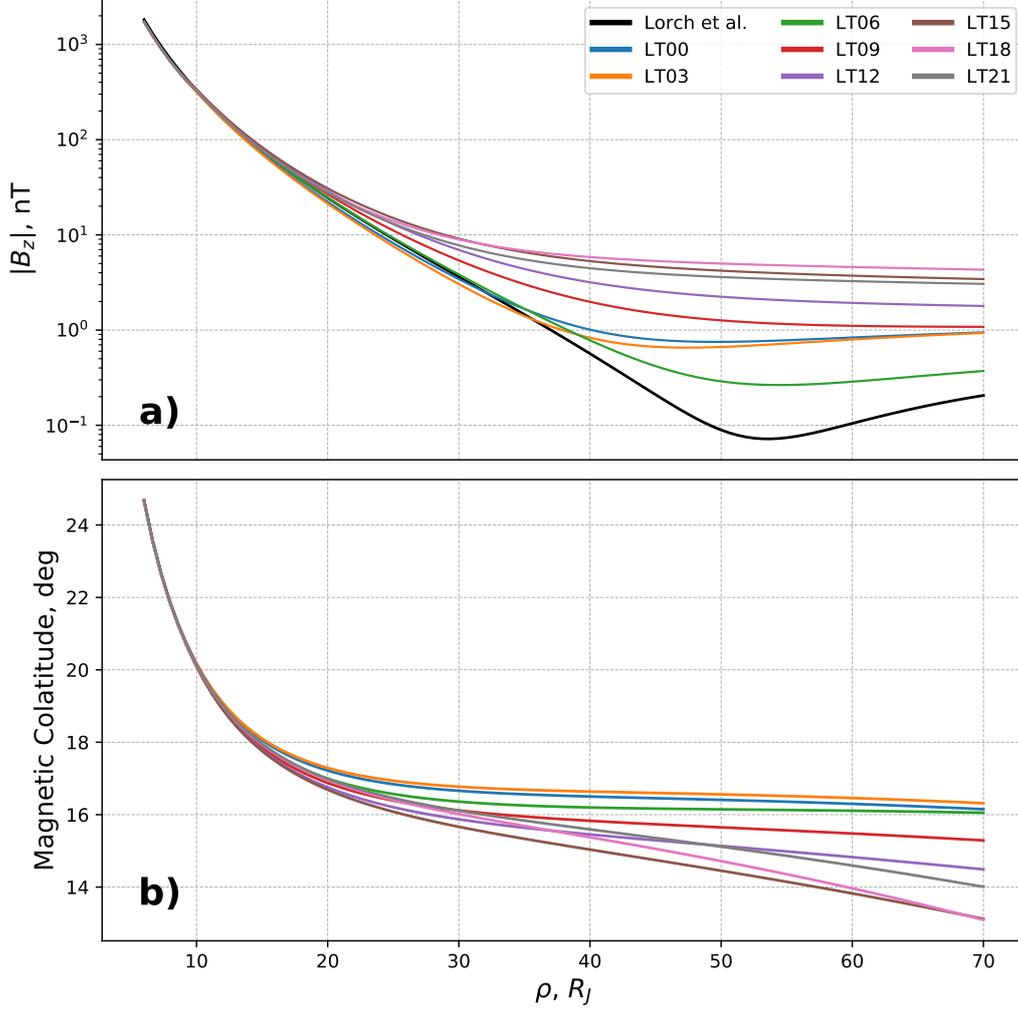
**Table 1.** Coefficients of the polynomial given by equation (24), fitted to the residual  $z$  component of the equatorial magnetic field in LT sectors 00 to 21 h. Coefficients used by Lorch et al. (2020) are shown for comparison.

LT	$a, 10^2 \cdot \text{nT}$	$b, 10^4 \cdot \text{nT}$	$c, 10^4 \cdot \text{nT}$
00	-2.859	2.280	-10.217
03	-2.944	2.351	-10.529
06	-1.983	1.923	-8.492
09	-2.504	1.941	-8.540
12	-3.025	1.958	-8.588
15	-4.653	2.327	-10.894
18	-5.876	2.783	-13.039
21	-4.450	2.352	-9.776
Lorch et al.	-1.825	1.893	-8.441

266 Because of the sparsity of data in the inner region and to avoid divergence for  $\rho <$   
 267  $10 R_J$  we used the current disc field model developed by Pensionerov et al. (2019) in-  
 268 stead of the polynomial given by equation (24), smoothing the transition between the  
 269 two field regimes by linear interpolation. Table 1 lists the coefficients of the polynomial  
 270 used in the LT sectors 00 to 21 (labeled by their central local time value), while Figure 3  
 271 shows the resulting approximation combined with the dipolar magnetic field and its mag-  
 272 netic colatitude mapping. Function  $F$  for this field model was then obtained by integrat-  
 273 ing equation (2) from the small non-zero value of  $\rho = 0.01 R_J$  to the local  $\rho$  value. Be-  
 274 cause we use the magnetic field model as an input for the system of equations (16) and (22),  
 275 we solve it on a fine  $\rho$  grid, but in 3 h wide LT sectors. The outer boundary for our so-  
 276 lutions is set to be at  $70 R_J$ .

### 277 3.2 Method for Obtaining Numerical Solutions

278 Most of the Hill-Pontius differential equation solutions quickly diverge to very large  
 279 positive or negative values of angular velocity in the inner magnetosphere. The phys-



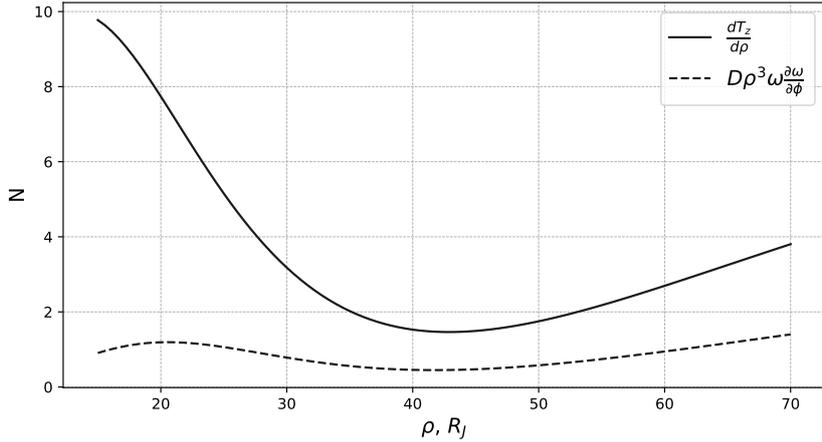
**Figure 3.** Panel (a) shows the absolute value of the  $z$  component of the equatorial magnetic field obtained by combining the dipolar field and the polynomial approximation of the residual magnetic field given by equation (24) with the coefficients used in the present model for LT sectors 00 to 21 h (Table 1). We also show the field profile derived by Lorch et al. (2020). Within  $10 R_J$  the Pensionerov et al. (2019) current disc field model is used. Panel (b) shows the magnetic mapping of ionospheric colatitude as a function of equatorial radial distance for the model shown.

280 ically correct solution converges to  $\sim \Omega_J$ . In the one-dimensional case with an explic-  
 281 itly defined ionospheric conductivity function  $\Sigma_P^*(\rho)$ , the solution can easily be obtained  
 282 by solving the equation radially outward with the boundary condition  $\omega(\rho_{\text{inner}}) = \Omega_J$ .  
 283 This is because solutions with slightly different boundary conditions near the planet quickly  
 284 converge to the one solution we are interested in as shown in the appendix of Cowley and  
 285 Bunce (2003).

286 This approach cannot be applied to the Hill-Pontius equation combined with the  
 287 equation for ionospheric conductivity modulation by field-aligned currents, because it  
 288 becomes unstable in the inner region. Since we cannot start the solution from the rigid  
 289 corotation region, we cannot ensure the fulfillment of  $\omega(\rho_{\text{inner}}) = \Omega_J$  by solving the equa-  
 290 tions radially outward. Nichols and Cowley (2004) deal with this problem by solving the  
 291 equations radially inward. A solution obtained in this way eventually diverges to large  
 292 negative or positive angular velocity, with the physically correct solution sitting in-between.  
 293 This family of solutions maps to a range of boundary conditions, where larger bound-  
 294 ary angular velocities correspond to the solutions diverging upward and the smaller ones  
 295 to the solutions diverging downward. It allows to use binary search to find the bound-  
 296 ary value that corresponds to rigid corotation near the planet. Nichols and Cowley (2004)  
 297 fixed the field-aligned current at a distance of  $100 R_J$  and binary searched the physically  
 298 correct angular velocity. Tracing the solution deep inside the near-rigid corotaion region  
 299 requires the boundary value to be specified to a large number of digits, quickly exceed-  
 300 ing the 64-bit float accuracy. Nichols and Cowley (2004) traced the solution to  $10\text{--}20 R_J$   
 301 and used an approximate iterative solution in the inner region. The same can be done  
 302 with a fixed angular velocity and binary-searched field-aligned current boundary con-  
 303 dition.

304 The method for solving the one dimensional Hill-Pontius equation with an explicit  
 305 conductivity can easily be adapted to the two dimensional case. However, the Nichols  
 306 and Cowley (2004) method for the equation with variable conductivity cannot, because  
 307 the binary search becomes impossible due to the influence of the azimuthal sectors on  
 308 each other. The crux of the issue lies in the second term of equation (16) that accounts  
 309 for the net azimuthal transport of angular momentum  $D\rho^3\omega\frac{\partial\omega}{\partial\phi}$ . It is useful, therefore,  
 310 to estimate the significance of this term in comparison with the other terms. For this  
 311 purpose, as well as later for obtaining the solutions, we need an estimate of the plasma  
 312 mass density. We employed the profile of cold plasma concentration per unit magnetic  
 313 flux from Nichols (2011), which together with our magnetic field model yields the num-  
 314 ber density per unit area, and hence the mass density per unit area assuming an aver-  
 315 age ion mass of 20 amu.

316 To estimate the significance of azimuthal transport we used the Hill (1979) ana-  
 317 lytical angular velocity profile applicable in case of a purely dipolar magnetic field, be-  
 318 cause it is generally representative of the plasma angular velocity behavior at Jupiter.  
 319 We also assumed an upper bound for the azimuthal derivative of  $\omega$  to be  $0.5 \times (\Omega_J -$   
 320  $\omega)$ , consequent on the fact that as the angular velocity converges towards rigid corota-  
 321 tion, its azimuthal derivative should converge towards zero. The choice of the coefficient  
 322 0.5 is based on estimates of the derivative obtained by solving the one-dimensional sys-  
 323 tem of equations for each LT separately. Using these assumptions, we compared the elec-  
 324 tromagnetic torque (the right hand side of equation (16)) with the azimuthal transport  
 325 term. Figure 4 shows a comparison using Hill's solution with a characteristic distance  
 326  $\rho_H = 25 R_J$  (the distance at which the angular velocity starts to deviate significantly  
 327 from rigid corotation). The azimuthal transport term becomes less significant the closer  
 328 we get to the planet. This means that in the region with  $\rho$  less than some boundary  $\rho_B$   
 329 we can neglect the second term in equation (16) and solve the system of equations (16)  
 330 and (22) for each LT sector using the Nichols and Cowley (2004) method. Ideally we would  
 331 like  $\rho_B$  to be as small as possible to better justify the neglect of the azimuthal transport  
 332 term. But we need  $\rho_B$  to be large enough for the boundary conditions search to be pos-



**Figure 4.** Comparison of terms in equation (16), where the solid line shows the electromagnetic torque term while the dashed line shows an estimate of the net azimuthal transport of angular momentum term.

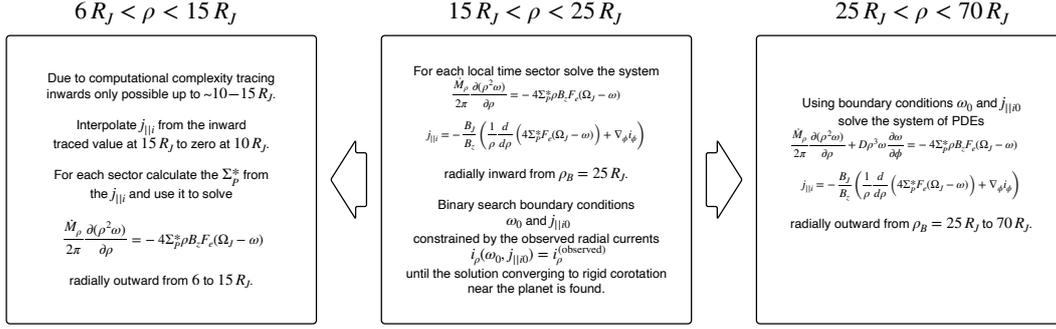
333 sible. We picked  $\rho_B = 25 R_J$  as it was the smallest that worked reliably for the search.  
 334 The result varies with the plasma density and  $\rho_H$ , but in most cases the azimuthal ang-  
 335 ular momentum transport effect is at least several times less than the electromagnetic  
 336 torque effect in the region  $\rho < 25 R_J$ .

337 From  $\rho_B$  the solutions are traced inwards to  $15 R_J$ . Tracing to the region closer  
 338 to the planet becomes too computationally intensive. To obtain the solution in the re-  
 339 gion from  $6$  to  $15 R_J$  we interpolate the field-aligned currents at the  $15 R_J$  boundary  
 340 to zero at  $10 R_J$  using a cubic spline. We then calculate the corresponding ionospheric  
 341 conductivity and use it explicitly to solve the simple Hill-Pontius equation radially out-  
 342 ward in the  $6$  to  $15 R_J$  region. Because the field-aligned currents at  $15 R_J$  are compar-  
 343 atively small, the combined solution is practically continuous.

344 Finally, we use the values of  $\omega$  and  $j_{\parallel i}$  at  $\rho_B$  which for each LT sector correspond  
 345 to a solution that converges to approximately  $\Omega_J$  in the inner region as boundary con-  
 346 ditions to solve the system of equations (16) and (22) at  $\rho > \rho_B$ . We solve it radially  
 347 outward on a fine  $\rho$  grid in the same  $3$  h wide LT sectors. We no longer neglect the net  
 348 azimuthal transport term, using the full left hand side of equation (16). This approach  
 349 usually yields a valid solution, though, depending on the magnetic field profile and the  
 350 chosen fixed boundary condition, it can produce angular velocities and field-aligned cur-  
 351 rents diverging to large positive values.

### 352 3.3 Boundary Conditions

353 As mentioned above, the Nichols and Cowley (2004) method for selecting bound-  
 354 ary conditions requires one of them to be fixed. The specific value for it is dictated by  
 355 the observations. At the moment we do not have detailed time-averaged measurements  
 356 of the angular velocity at  $\rho_B = 25 R_J$ . Recently, Lorch et al. (2020) derived the diver-  
 357 gence of the observed magnetodisc currents, which we could use for our boundary con-  
 358 ditions. However it is calculated as a finite difference of the observed currents, and while  
 359 the statistical errors of the currents themselves are relatively small, for the divergence  
 360 they become significant. Instead of the divergence, we opted to use Lorch et al. (2020)  
 361 radial current measurements directly. Using equations (8) and (19) we can constrain the



**Figure 5.** Diagram outlining the algorithm for obtaining solutions to the system of equations (16) and (22).

boundary conditions by the observed radial current

$$i_\rho(\omega_0, j_{||i|0}) = i_\rho^{(observed)}. \quad (25)$$

The binary search works slightly differently than in the case when one of the values is fixed. We conducted the search for  $j_{||i}$ , while the corresponding  $\omega$  was calculated from equation (25) on each iteration. Not every value of the radial current has a solution converging to near-rigid corotation. We found that one of the important factors determining whether or not such a solution exists for a given boundary radial current is the value of  $\dot{M}_\rho$ . For example, with  $\dot{M}_\rho = 1000 \text{ kg s}^{-1}$  the observed radial currents are typically too large and the equation has no physically correct solutions. In such cases we iteratively decreased the boundary radial currents from the observed value, each time conducting a new search, until a valid solution became possible. Figure 5 shows a diagram, outlining the algorithm for for obtaining the solutions.

## 4 Results

### 4.1 Solutions Without Field-Aligned Currents from the Partial Ring Current

We now examine the solutions obtained using the method described in section 3, and compare the model magnetodisc radial current intensities with those determined by Lorch et al. (2020). The key model parameter is the radial mass transport rate  $\dot{M}_\rho$ . For simplicity we assume it to be symmetrical in LT and constant with  $\rho$  (we discuss these assumptions in section 5). Here we compare solutions for the canonical  $\dot{M}_\rho$  value of  $1000 \text{ kg s}^{-1}$ , and an increased value of  $2000 \text{ kg s}^{-1}$ .

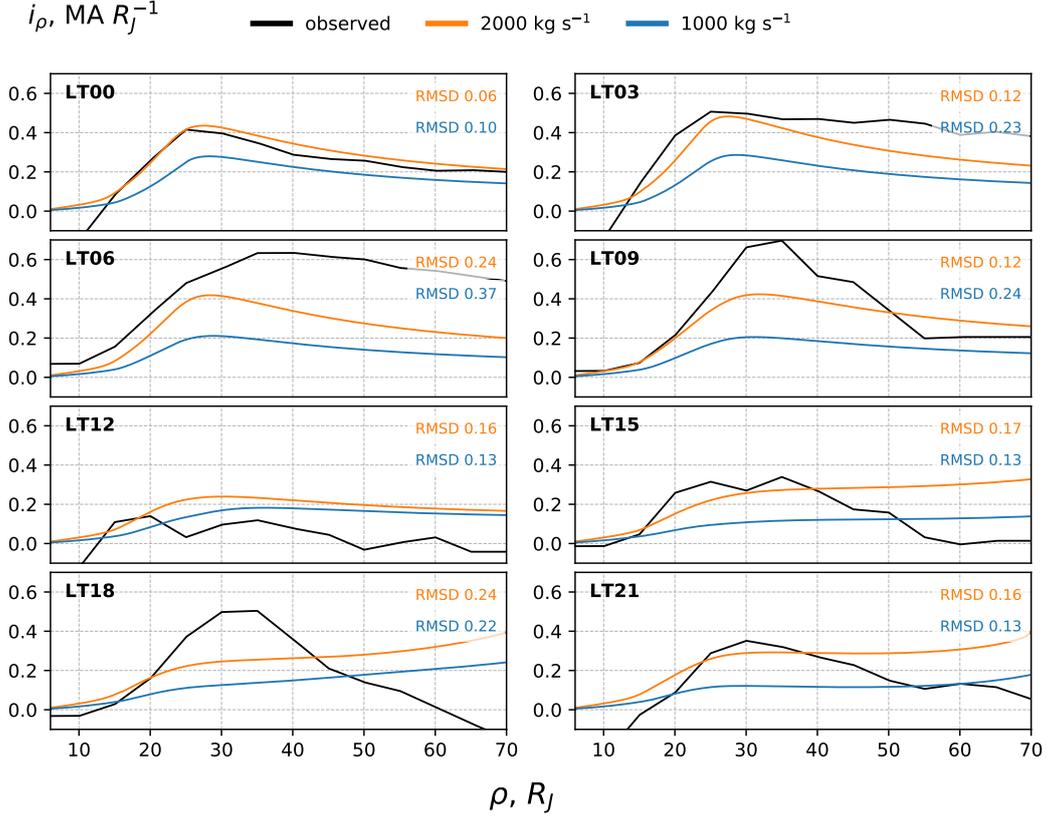
In Figure 6 we compare the width-integrated radial current intensities calculated from the model results for  $\dot{M}_\rho = 1000 \text{ kg s}^{-1}$  (blue lines) and  $2000 \text{ kg s}^{-1}$  (orange) with the observed current intensities from Lorch et al. (2020) (black) in the LT sectors from 00 to 21 h. For each sector the root-mean-square deviation (RMSD) of the model currents from the observed ones is given. The major observation here is that the  $1000 \text{ kg s}^{-1}$  model systematically underestimates the observed currents within  $\sim 40 R_J$  in all local times except noon. The  $2000 \text{ kg s}^{-1}$  model, on the other hand, comes closer to the observed values in this region, especially in sectors 00/24, 03, 15 and 21 h. However it still underestimates the currents in the 06, 09 and 18 h sectors. The noon sector likely has a radial mass transport rate lower than  $1000 \text{ kg s}^{-1}$ , as both models overestimate the currents there.

In the region closer than  $\sim 15 R_J$  in some sectors the observed radial currents become negative, most prominently at 21, 00/24, and 03 h. Radial currents in our model

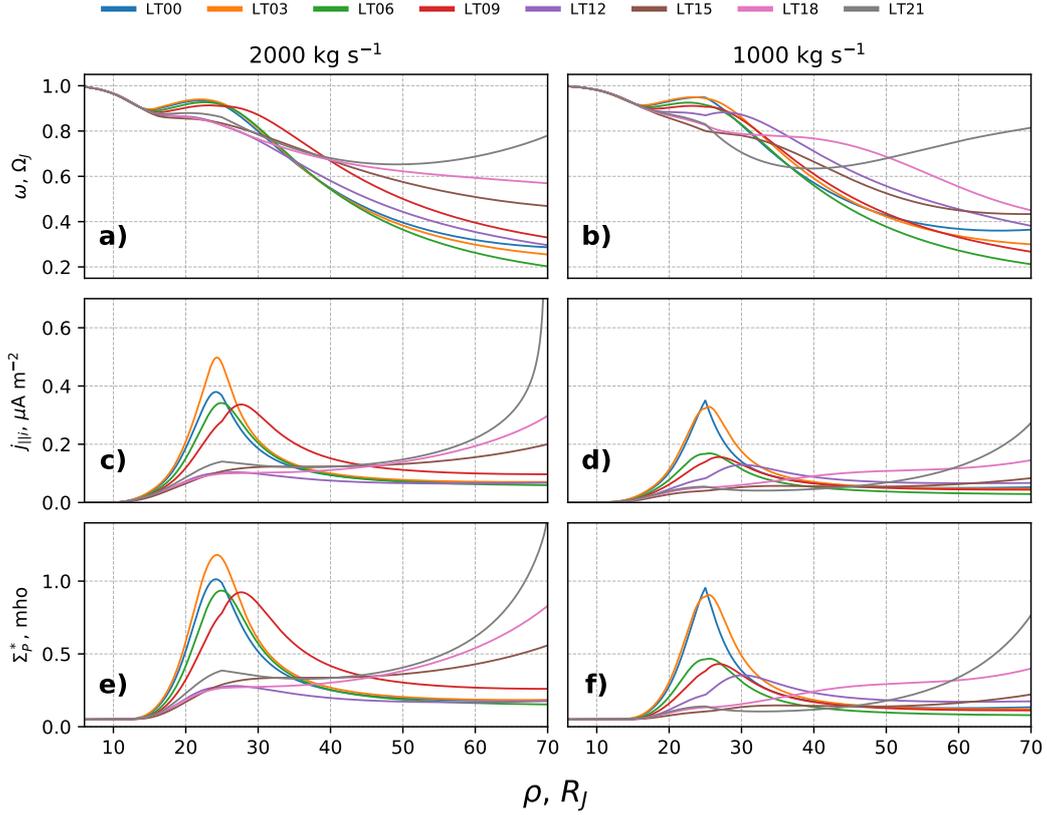
396 always converge to zero near the planet and so it cannot explain these observations. Be-  
 397 yond  $40 R_J$ , the behavior of the observed currents changes depending on LT. In the midnight-  
 398 dawn sector the radial current intensities tend to decrease with radial distance very slowly,  
 399 while in the dusk sector and at 09 h they decrease significantly faster. The model cur-  
 400 rents are generally more similar in behavior to the observed currents in the dawn sec-  
 401 tor than in the dusk sector. In the LT sectors 00–09 h the  $2000 \text{ kg s}^{-1}$  model has 40-  
 402 50% smaller RMSD than the  $1000 \text{ kg s}^{-1}$  model. In sectors 12–21 h the situation is op-  
 403 posite, though less pronounced: the  $1000 \text{ kg s}^{-1}$  model has 10-25% smaller RMSD than  
 404 the  $2000 \text{ kg s}^{-1}$  model.

405 Figure 7 demonstrates the angular velocities, ionospheric field-aligned currents, and  
 406 effective conductivity for the  $2000 \text{ kg s}^{-1}$  and  $1000 \text{ kg s}^{-1}$  cases. Model field-aligned cur-  
 407 rents in LT sectors 15, 18 and 21 h rapidly increase beyond  $\sim 50 R_J$ , while the angular  
 408 velocity falls very slowly or even trends back towards rigid corotation. This is not sup-  
 409 ported by observations and is likely an artefact of the model, caused by the assumption  
 410 of constant radial mass outflow and the absence of additional field-aligned currents. These  
 411 features disappear when we incorporate field-aligned currents from the partial ring cur-  
 412 rent into the calculations in the next section. At some local times there is a noticeable  
 413 derivative discontinuity at  $\rho_B$  in all of the variables. This is the result of neglecting az-  
 414 imuthal transport for  $\rho < \rho_B$ . These discontinuities are not severe, which indicates that  
 415 our approximation was justified.

416 Angular velocities in the case of  $2000 \text{ kg s}^{-1}$  start to deviate significantly from rigid  
 417 corotation slightly closer to the planet than for the case of  $1000 \text{ kg s}^{-1}$ , thus producing  
 418 stronger radial currents. The ionospheric field-aligned currents and conductivity behave  
 419 similarly for both radial mass transport values. Field-aligned currents are typically in  
 420 the range  $0.1\text{--}0.2 \mu\text{A m}^{-2}$ , with peaks in the dawn sector at  $20\text{--}30 R_J$  reaching  $0.4\text{--}0.5 \mu\text{A m}^{-2}$ .  
 421 The corresponding effective conductivities range from 0.1 to 1.3 mho.



**Figure 6.** Equatorial width-integrated radial current intensities plotted versus distance  $\rho$  from the planetary magnetic axis in 3 h wide LT sectors centered on 00–21 h. Black lines show the empirical currents derived from magnetic field data by Lorch et al. (2020), while the blue and orange lines show model currents for  $\dot{M}_p=1000$  and  $2000 \text{ kg s}^{-1}$ , respectively. The root-mean-square deviation (RMSD) of the model current from the observed current is shown for both model values.



**Figure 7.** Panels (a) and (b) show angular velocities, (c) and (d) ionospheric field-aligned current densities, and (e) and (f) effective height-integrated ionospheric conductivities, for each LT sector. Panels on the left correspond to a mass transport rate of  $2000 \text{ kg s}^{-1}$ , while the panels on the right correspond to  $1000 \text{ kg s}^{-1}$ . All parameters are plotted versus equatorial distance from the planetary rotation axis, mapped along field lines in the case of the ionospheric parameters in panels (c)–(f).

**Table 2.** Parameters  $A$  and  $\rho_c$  of the approximate form for the ionospheric field-aligned currents associated with the partial ring current given by equation (26), as employed in LT sectors 00 to 21 h for the models with radial mass outflow rates of  $2000 \text{ kg s}^{-1}$  and  $1000 \text{ kg s}^{-1}$

LT	$2000 \text{ kg s}^{-1}$		$1000 \text{ kg s}^{-1}$	
	$A, \mu\text{A m}^{-2}$	$\rho_c, R_J$	$A, \mu\text{A m}^{-2}$	$\rho_c, R_J$
00	0.00	30	0.00	30
03	-0.10	30	-0.10	30
06	-0.20	30	-0.10	30
09	0.00	30	0.00	30
12	0.00	30	0.00	30
15	0.08	30	0.05	35
18	0.10	31	0.05	35
21	0.10	30	0.05	35

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## 4.2 Solutions with Field-Aligned Currents from the Partial Ring Current

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While Lorch et al. (2020) provide the divergence of the observed azimuthal equatorial current, this divergence, as discussed above, comes with significant statistical errors. The solutions of the Hill-Pontius equation with variable conductivity are very sensitive not only to the magnitude of the ionospheric field-aligned currents, associated with the divergence of the azimuthal currents ( $(\nabla_{\phi} i_{\phi})_i$  for brevity), but also to their radial derivatives. The variation of the observed divergence from one bin to the next caused by the said errors, usually renders the equations unsolvable. Thus, instead of using the observed divergences to calculate  $(\nabla_{\phi} i_{\phi})_i$  directly, we employed a simple parametric equation.

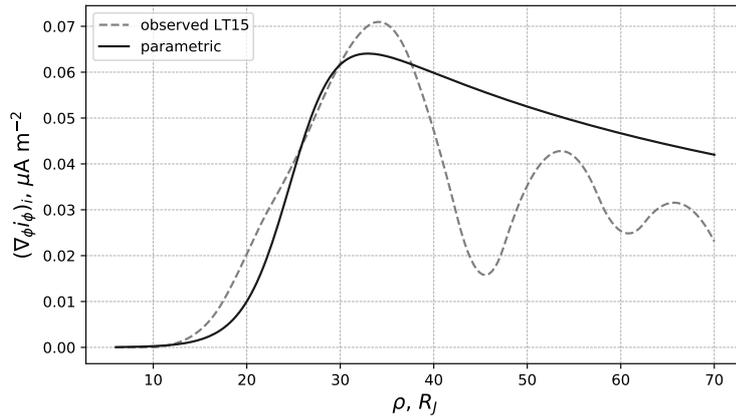
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We found that the radial currents tend to increase when the radial derivative of  $(\nabla_{\phi} i_{\phi})_i$  is positive and to decrease when its negative. This is, to a certain degree, expected, because the ionospheric conductivity increases with the density of the field-aligned currents, and the radial currents are proportional to the conductivity. According to Lorch et al. (2020), the azimuthal current is removed from the magnetodisc in the dawn sector and added back to it in the dusk sector. Removal and addition correspond to negative and positive  $(\nabla_{\phi} i_{\phi})_i$ , respectively. If we now assume that  $(\nabla_{\phi} i_{\phi})_i$  decreases in magnitude with distance from the planet, the sign of its derivative becomes consistent with the observed behavior of the radial currents at dawn and dusk. In the dusk sector the derivative of  $(\nabla_{\phi} i_{\phi})_i$  becomes negative, which can explain the faster decrease of the observed radial currents there. The opposite is true at dawn, where the positive derivative slows the decrease of radial currents with distance. On these grounds, we use the following equation for  $(\nabla_{\phi} i_{\phi})_i$

$$(\nabla_{\phi} i_{\phi})_i = A\rho_c \frac{\tanh\left(\frac{\rho - \rho_c + d}{d}\right) + 1}{\rho + \rho_c}. \quad (26)$$

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Parameter  $d$  was set to be  $5 R_J$ , while the chosen values of  $\rho_c$  and  $A$  for each LT are presented in Table 2. This equation smoothly interpolates to zero at distances smaller than  $\rho_c$ , with the smoothness of interpolation controlled by  $d$  (as we assume the azimuthal currents become symmetrical in the inner magnetosphere) and its absolute value falls as  $1/\rho$  at greater distances. The specific values of  $\rho_c$  and  $A$  were chosen to best fit the observed radial currents with the model ones. The form of the approximation as well as its parameters are somewhat arbitrary, so we only aim to qualitatively study the effects

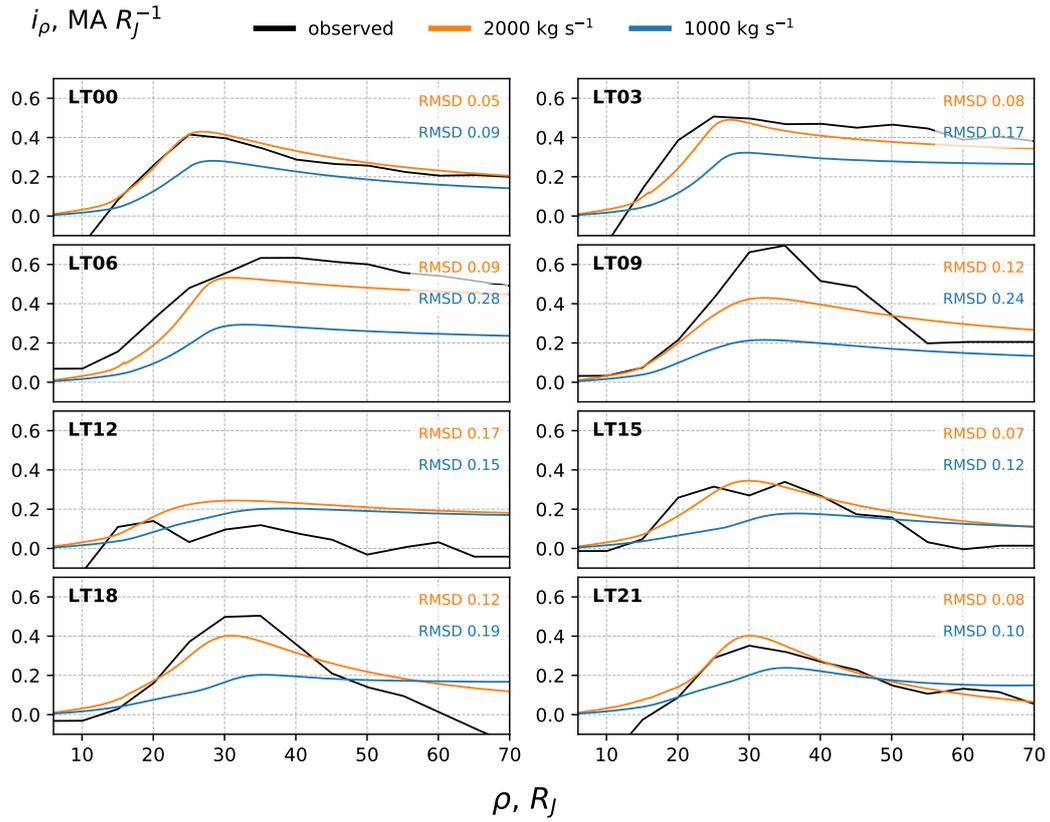


**Figure 8.** Field-aligned current from the partial ring current versus distance from the magnetic axis for 15 h LT. The solid line shows the parametric approximation given by equation (26) with  $A = 0.08 \mu\text{A m}^{-2}$  and  $\rho_c = 30 R_J$ , corresponding to the  $2000 \text{ kg s}^{-1}$  case in Table 2, while the dashed line shows the field-aligned current calculated from the observed currents from Lorch et al. (2020). Inside  $20 R_J$  the observed azimuthal current divergence was not taken into account due to relatively large errors, and was linearly interpolated to zero from the boundary value. Beyond  $20 R_J$  the observed divergence was interpolated by quadratic splines.

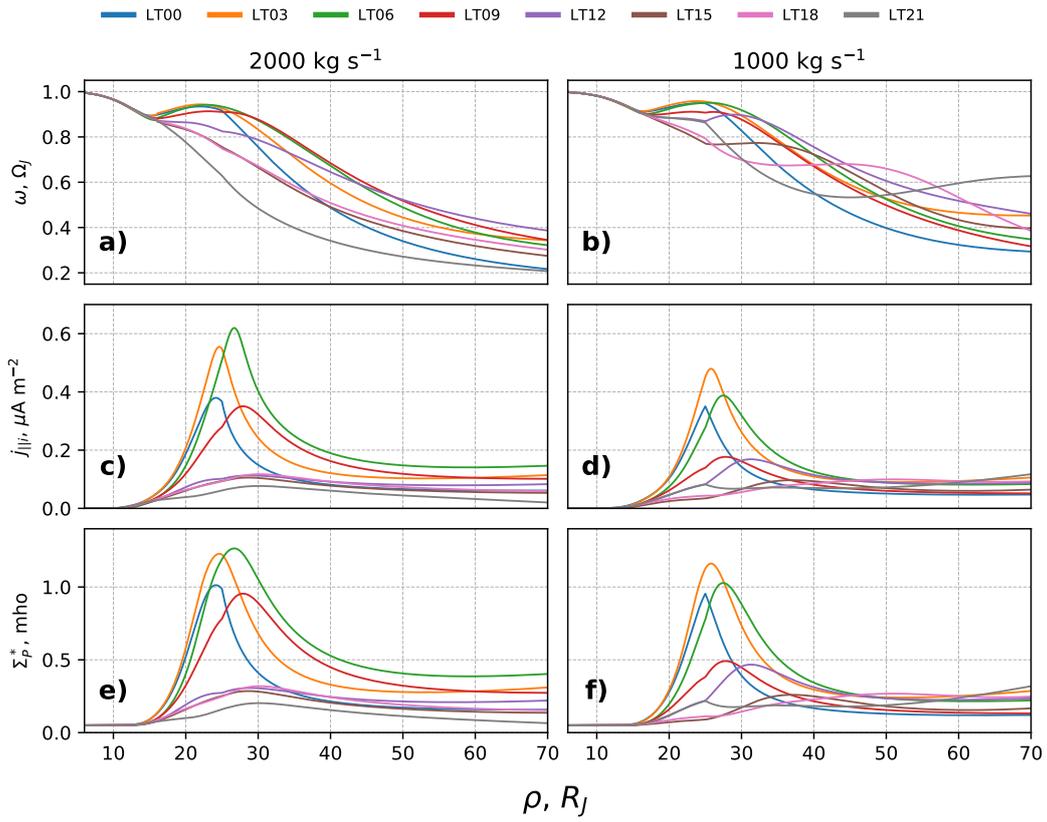
454 of  $(\nabla_\phi i_\phi)_i$ . Figure 8 shows an example of  $(\nabla_\phi i_\phi)_i$  at 15 h LT calculated using equation (26)  
 455 (solid line), together with  $(\nabla_\phi i_\phi)_i$  calculated from observations (dashed line).

456 Figure 9 shows the radial currents in the same format as Figure 6, but with  $(\nabla_\phi i_\phi)_i$   
 457 included. In sectors 12 and 00/24 h LT the addition of  $(\nabla_\phi i_\phi)_i$  did not improve the fit,  
 458 so we didn't include  $(\nabla_\phi i_\phi)_i$  in the final calculations. For the midnight and noon sec-  
 459 tors such an assumption seems plausible, because for a nightside partial ring current we  
 460 expect  $(\nabla_\phi i_\phi)_i$  to be present mostly at dawn and dusk.  $(\nabla_\phi i_\phi)_i$  also didn't improve the  
 461 fit at 09 h LT. In this sector the observed  $(\nabla_\phi i_\phi)_i$  is strongly negative, so if its magni-  
 462 tude decreases with distance its derivative is positive. As stated above, this leads to the  
 463 model radial current falling slower than for the case without  $(\nabla_\phi i_\phi)_i$ , while the observed  
 464 current at 09 h, on the contrary, decreases very sharply. This behavior might be a re-  
 465 sult of a change in the radial mass transport rate with distance, unaccounted for in the  
 466 present model. It also should be noted that, as indicated above, the 09 h sector has sub-  
 467 stantially less spacecraft coverage, which means that our magnetic field model and the  
 468 Lorch et al. (2020) results might be inaccurate. Because of this we set  $(\nabla_\phi i_\phi)_i$  to zero  
 469 in this sector as well.

470 In the rest of the sectors  $(\nabla_\phi i_\phi)_i$  significantly decreases the RMSD of the model  
 471 with both radial mass transport values. For  $\dot{M}_\rho = 2000 \text{ kg s}^{-1}$  RMSD is 40–60% lower.  
 472 For  $\dot{M}_\rho = 1000 \text{ kg s}^{-1}$  the change is less dramatic with RMSD 10–40% lower than in  
 473 the case without  $(\nabla_\phi i_\phi)_i$ . Although the addition of  $(\nabla_\phi i_\phi)_i$  generally increases the model  
 474 radial currents in the  $1000 \text{ kg s}^{-1}$  case, this value still significantly underestimates the  
 475 observations within  $\sim 40 R_J$ . For  $2000 \text{ kg s}^{-1}$  the improvement in RMSD mostly comes  
 476 from the region outside  $40 R_J$ . With  $\dot{M}_\rho = 2000 \text{ kg s}^{-1}$  our model describes the Lorch  
 477 et al. (2020) results very well in sectors 00/24, 03, 06 and 21 h. While the fit in sectors  
 478 15 and 18 h was significantly improved (reducing the RMSD twofold), the model still does  
 479 not fit the observations beyond  $40 R_J$  well. Overall, the  $2000 \text{ kg s}^{-1}$  model has signif-  
 480 icantly lower RMSD than the  $1000 \text{ kg s}^{-1}$  model at most local times.



**Figure 9.** Same as Figure 6, but for the models including field-aligned currents from the partial ring current.



**Figure 10.** Same as Figure 7, but for the models including field-aligned currents from the partial ring current.

481 Figure 10 demonstrates the angular velocities, ionospheric field-aligned currents,  
 482 and effective conductivity in the same format as Figure 7, but for the model including  
 483  $(\nabla_{\phi} i_{\phi})_i$ . The anomalies in the dusk sector with diverging angular velocities are no longer  
 484 present. Although the results changed for the individual sectors, the ranges and the be-  
 485 havior of all variables remain generally the same, with the dawn sector still having larger  
 486 field-aligned currents peaks at 20–30  $R_J$  than the dusk sector.

## 487 5 Discussion

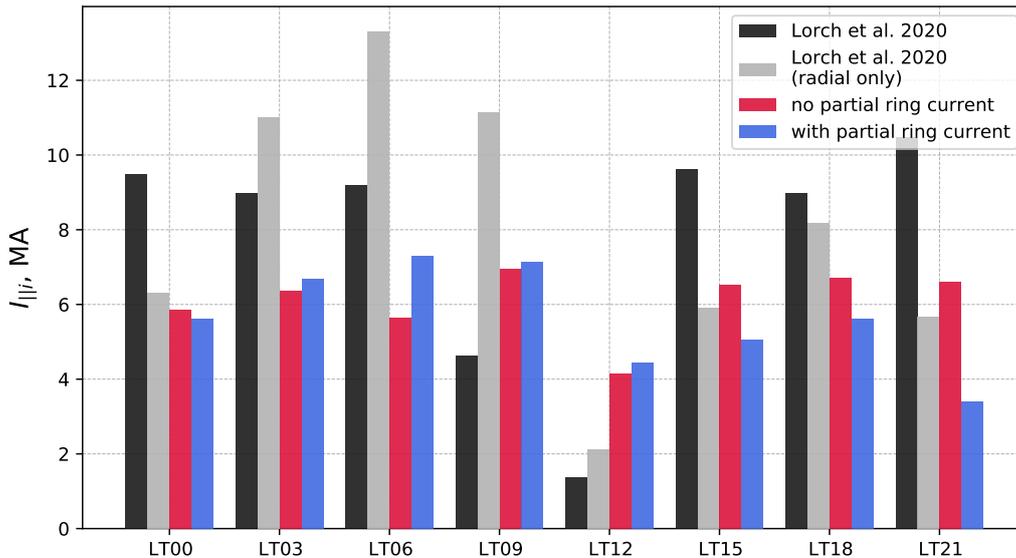
488 A key feature of steady state M-I coupling models is their relative simplicity, com-  
 489 pared to full 2D or 3D MHD modelling. This simplicity allows one to test the response  
 490 of the system to different values of various parameters with low iteration time and com-  
 491 puting power requirements, and to compare the results with observations. Here we have  
 492 developed a variation of this model which is asymmetrical in LT. This allows us to com-  
 493 pare the radial equatorial current intensities calculated using the model with those de-  
 494 termined from magnetic field measurements by Lorch et al. (2020) in eight 3 h wide LT  
 495 sectors centered on 00 to 21 h.

496 In this work we compared the equatorial radial currents produced by the model us-  
 497 ing radial mass outflow rates of 1000 and 2000  $\text{kg s}^{-1}$ , and found that the model cur-  
 498 rents are in significantly better agreement with observations when a transport rate of  
 499 2000  $\text{kg s}^{-1}$  is used. Currents produced in the 1000  $\text{kg s}^{-1}$  case are systematically lower  
 500 than those observed. This result is unexpected, with most estimates of radial mass out-  
 501 flow rate being lower. In Jupiter’s magnetosphere the transport rate is generally assumed  
 502 to be equal to the Io plasma production rate. Various empirical estimates of the plasma  
 503 production rate have been made, ranging from 150 to 2000  $\text{kg s}^{-1}$  (Broadfoot et al., 1981;  
 504 Vasyliunas, 1983; Bagenal, 1997; Bagenal & Delamere, 2011), and the canonical value  
 505 of 1000  $\text{kg s}^{-1}$  has been used in many previous related works (Cowley & Bunce, 2001;  
 506 Cowley et al., 2002; Nichols, 2011; Ray et al., 2014; Nichols et al., 2015). Nichols et al.  
 507 (2020) used the canonical value as representative of the typical outflow rate, while us-  
 508 ing 2350  $\text{kg s}^{-1}$  for an enhanced plasma production case. Hill (2001) used the value of  
 509 2000  $\text{kg s}^{-1}$  in his calculations, while Nichols and Cowley (2003) studied solutions of the  
 510 Hill-Pontius equation for various outflow rates from 100 to 10000  $\text{kg s}^{-1}$ .

511 The magnetic field observations used by Lorch et al. (2020) to calculate the cur-  
 512 rents, and those employed by us to construct our model of the magnetospheric equato-  
 513 rial field, are taken from observations made on the trajectories of several spacecraft, thus  
 514 representing a time-averaged picture. We then take the mass transport rates used in our  
 515 modelling to correspond to the average mass transport rate in the system and hence can-  
 516 not explain the 2000  $\text{kg s}^{-1}$  value as temporarily enhanced.

517 Another potential explanation comes from our neglect of the changes of mass out-  
 518 flow rate with distance from the planet. This can change the results in many different  
 519 ways, as both the radial derivative of the outflow and the extra azimuthal flow created  
 520 are a part of the differential equations. Our model fits the data better in the nightside  
 521 magnetosphere, where the approximation of constant outflow is probably closer to re-  
 522 ality. The inclusion of a variable outflow rate is one of the prime avenues for further re-  
 523 search. However, the model with the canonical outflow rate of 1000  $\text{kg s}^{-1}$  underesti-  
 524 mates the observations even within 20  $R_J$ , where the variability of radial outflow is prob-  
 525 ably much less pronounced, than in the outer regions. Thus we find the outflow variabil-  
 526 ity with distance unlikely to remove the discrepancy.

527 We also neglected  $B_{\phi}$  in our calculations. It does not directly affect the angular  
 528 momentum balance, but can “bend” the azimuthal sectors, changing the effective mag-  
 529 netic field profile. Because the model with 1000  $\text{kg s}^{-1}$  underestimates the observed cur-



**Figure 11.** Upward field-aligned currents integrated over the ionosphere in one hemisphere for each 3 h wide LT sector. Black and gray bars show the Lorch et al. (2020) observations with and without azimuthal currents divergence, while the red and blue bars show results without and with field-aligned currents from the partial ring current, respectively. A  $2000 \text{ kg s}^{-1}$  mass outflow rate is used for the model results.

530 rents across almost all of the local times, we find this change unlikely to affect our con-  
 531 clusions and leave it for future work.

532 Nichols and Cowley (2004) compared their calculated radial currents with those  
 533 derived from Galileo azimuthal magnetic field data obtained in the midnight LT sector,  
 534 and found that the model currents better fit the observed values with  $\dot{M}_\rho$  set to a larger  
 535 value of 2000 or 3000  $\text{kg s}^{-1}$ . We used the same approximation for the conductivity de-  
 536 pendence on the field-aligned ionospheric current density as Nichols and Cowley (2004),  
 537 as well as the same atmosphere slippage coefficient of 0.5, which affects the effective con-  
 538 ductivity. Preliminary tests with a lower slippage coefficient and hence higher effective  
 539 conductivity did not show an increase in the model radial currents, while tests with a  
 540 higher coefficient and consequent lower effective conductivity showed a decrease in cur-  
 541 rents. From this it follows that changes in the slippage coefficient do not alter the sys-  
 542 tematic underestimation of the observed currents by the  $1000 \text{ kg s}^{-1}$  model. However,  
 543 more rigorous study is needed on the behavior of the solutions with different approxi-  
 544 mations for the conductivity dependence on the field-aligned current.

545 We also considered the effect of field-aligned currents from the partial ring current  
 546 on the solutions. Because they are sensitive to the radial derivatives of  $(\nabla_{\phi^i \phi^j})_i$ , we were  
 547 unable to use the observed divergences directly. Instead we used a simple analytic form  
 548 for the resulting ionospheric field-aligned currents with parameters individually selected  
 549 for each LT sector. The direction of the field-aligned currents we used is in agreement  
 550 with the Lorch et al. (2020) observations. The inclusion of these currents in the model  
 551 allowed us to significantly improve the agreement between the observed and model equa-  
 552 torial radial currents, reducing the root-mean-square deviation by 40–60% in most lo-  
 553 cal times. However, both the form of the approximation and the specific parameters are  
 554 to an extent arbitrary, with only the sign of the currents being directly tied to the ob-  
 555 servations. So while the inclusion of such currents *can* help to explain the variation of

556 radial currents behavior between dawn and dusk sector, it not necessarily *does*. At lo-  
 557 cal times 09, 15 and 18 h the inclusion of extra field-aligned currents was not sufficient  
 558 to explain the sharp decrease in observed radial currents. As stated above, a possible way  
 559 to improve the model in this regard would be to account for mass transport rate vari-  
 560 ations with the distance from the planet.

561 Bonfond et al. (2015) used HST images to estimate the brightness asymmetry of  
 562 the main oval of Jupiter’s UV aurora. They found that in the southern hemisphere the  
 563 dusk sector emission is on average  $\sim 3$  times brighter than in the dawn sector, while in  
 564 the northern hemisphere the dusk sector is only  $\sim 1.1$  times brighter (possibly due to  
 565 the northern magnetic anomaly complicating the analysis). As a possible explanation  
 566 of this asymmetry, Bonfond et al. (2015) suggested the presence of a partial ring cur-  
 567 rent in the nightside magnetosphere, whose field-aligned currents would strengthen the  
 568 main oval aurora at dusk while weakening it at dawn. The calculations by Ray et al. (2014)  
 569 are inconsistent with the Bonfond et al. (2015) results, since they predict stronger field-  
 570 aligned currents in the dawn sector. Our calculations also show stronger field-aligned cur-  
 571 rents in the dawn sector. However, the total upward field-aligned current is not larger  
 572 in the dawn sector, because in the dusk sector it covers a significantly wider latitude range.  
 573 Figure 11 shows the upward field-aligned current integrated over the ionosphere in one  
 574 hemisphere for each of the 3 h wide LT sectors. This figure shows our results for cases  
 575 with and without  $(\nabla_{\phi} i_{\phi})_i$  as well as the Lorch et al. (2020) observations with and with-  
 576 out azimuthal current divergence. We integrated over the ionospheric colatitudes that  
 577 correspond to the equatorial radial distances range  $6 R_J < \rho < 55 R_J$ , to avoid the  
 578 diverging model currents in the case without  $(\nabla_{\phi} i_{\phi})_i$ . The model current is 20–50% smaller  
 579 than the observed current at all local times other than noon. For the model currents in  
 580 both cases and for the full observed current there is no strong dawn-dusk asymmetry.  
 581 From these results it follows that the additional field-aligned currents from the partial  
 582 ring current do not necessarily affect the aurora in the simple way suggested by Bonfond  
 583 et al. (2015). Additional field-aligned currents change the conductivity of the ionosphere,  
 584 which in turn changes the angular velocity profile, and hence the field-aligned currents  
 585 from the divergence of the radial currents. In the LT sectors 15, 18 and 21 h, which have  
 586 positive  $(\nabla_{\phi} i_{\phi})_i$ , the total positive field-aligned current is less than in the case without  
 587  $(\nabla_{\phi} i_{\phi})_i$ , while in the 03 and 06 h LT sectors, which have negative  $(\nabla_{\phi} i_{\phi})_i$ , the total is  
 588 larger. The resulting field-aligned currents depend strongly on the magnitude and ra-  
 589 dial derivative of the additional currents.

## 590 6 Conclusions

591 We have presented an axially asymmetrical variant of the steady state M-I coupling  
 592 model for the Jovian magnetosphere. We have compared the radial magnetodisc currents  
 593 calculated using this model with those derived by Lorch et al. (2020) from in situ mag-  
 594 netic field observations.

- 595 1. We found that the observed radial current magnitudes require an average radial  
 596 mass transport rate of  $2000 \text{ kg s}^{-1}$ , significantly higher than the value typically  
 597 used of  $1000 \text{ kg s}^{-1}$ .
- 598 2. We considered the effect of field-aligned currents associated with the nightside par-  
 599 tial ring current on the M-I coupling system. We found that their inclusion allows  
 600 a partial explanation of the diurnal variations of the magnetodisc radial currents,  
 601 reducing the discrepancy between the model and observations by 10–60%, depend-  
 602 ing on the local time.

603 A notable simplification in the present model is the assumption of a constant radial mass  
 604 transport rate with the distance from the planet. Accounting for changes in the mass  
 605 transport rate with distance is one of the directions for future work.

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1. Files fgm\\_jno\\_13\\_YYYYDDDpc\\_r60s\\_v01 from the JNO-J-3-FGM-CAL-V1.0 dataset (<https://doi.org/10.17189/1519711>)
2. Files ORBXX\_SYS3.TAB from the GO-J-MAG-3-RDR-MAGSPHERIC-SURVEY-V1.0 dataset (<https://doi.org/10.17189/1519668>).

Ancillary calculations were made with the SPICE Toolkit via SpiceyPy (<https://doi.org/10.21105/joss.02050>).

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