

The Effects of Non-Normal Flow Rules on Fracture Angles in Viscous-Plastic Sea Ice Models

The Effects of Non-Normal Flow Rules on Fracture Angles in Viscous-Plastic Sea Ice Models
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1 - Context

- Sea ice is an important component of the climate system. Sea ice mechanical models need to be accurate for climate predictions as well as for navigation in ice-infested waters.
- Sea ice rheologies are most commonly modeled with a Viscous-Plastic (VP) rheology (Miles, 1970).
- Stress lines of deformation, as LRFs, determine the sea ice motion.
- Recent high-resolution simulations use the VP rheology and assume (2D):
 - The stress follows by Mohr-Coulomb (MC) and Gurtin-Maxwell (GM) and Gurtin-Maxwell (GM) stress the creation of leads and the evolution of deformation with the most commonly used VP rheology.

2 - A new rheology

- A new VP rheology has two components:
 - (1) A yield curve P :
 - Defines the stress at fracture.
 - (2) A plastic potential D :
 - Defines the deformation after fracture. The flow rule is perpendicular to the plastic potential.
- To study the flow rule's effect on the fracture angles, we created a new rheology with an elliptical yield curve and a normal (1) flow rule by defining a different elliptical plastic potential.

3 - Idealized Experiment

Prescribed Strain

Flow line

Open Boundary

4 - Theory

Three theoretical positions for the angle compare (Miles, 1970):

- (1) Coulomb Angle θ_C depends on the slope of the yield curve (Coulomb, 1773):

$$\theta_C = \frac{\pi}{2} - \frac{\sigma}{\tau}$$
- (2) Rankine Angle θ_R depends on the slope of the plastic potential (Rankine, 1870):

$$\theta_R = \frac{\pi}{2} - \frac{\sigma}{\tau}$$
- (3) Beltrami Angle θ_B with $\theta_B = \theta_C$ (Beltrami et al., 1977):

$$\theta_B = \frac{\pi}{2} - \frac{\sigma}{\tau}$$

5 - Results

Example of fracture with three values of θ_C for $\sigma/\tau = 2.2$. The shear stress rate $\dot{\gamma}$ is shown here.

6 - Conclusion

- The (1) flow rule interaction sets the fracture angles in agreement with the predictions of the Rankine Angle.
- (2) By introducing an independent plastic potential, we change the fracture angle and the stress concentration.
- The new rheology can also create fracture angles smaller than 30°.
- (3) The (1) flow rule has the advantages of being observable with remote sensing methods, unlike the others.
- Observations could be used to design new VP rheologies, for

Just high-resolution observations of sea ice deformation and sea ice stresses could help to design new VP rheologies, for

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PRESENTED AT:



1 - CONTEXT

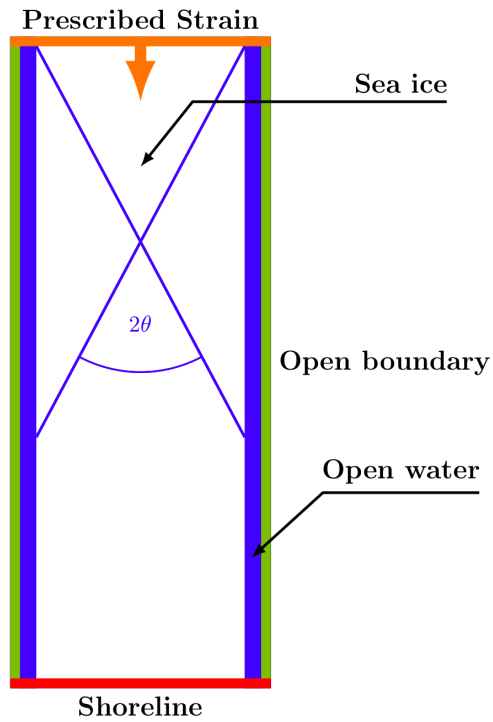
- Sea ice is an important component of the climate system. Sea ice numerical models need to be accurate for climate predictions as well as for navigation in ice-infested water.
- Sea ice dynamics are most commonly modeled with a Viscous-Plastic (VP) rheology (Hibler, 1979).
- Narrow lines of deformation, or LKFs, dominate the sea ice motion.
- Recent high-resolution simulations use the VP rheology and feature LKFs.
 - The video below, by Nils Hutter (AWI) and Dimitris Menemenlis (JPL), shows the creation of leads and the evolution of deformation with the most commonly used VP sea ice rheology.

[VIDEO] <https://www.youtube.com/embed/uj1spaWExy8?rel=0&fs=1&modestbranding=1&rel=0&showinfo=0>

- However, the sea ice VP rheology overestimates the intersection angles between LKFs when compared to observations (Hutter and Losch, 2020).
- The fracture angles are linked to the elliptical yield curve with a normal flow rule, and cannot be below 30° in uniaxial compression (Ringeisen et al., 2019).
- The effects of using a non-normal flow rule are still unknown, even though non-normal flow rules have been used in past studies (Ip et al. 1991, Hibler and Schulson 2000).

In our work, we investigate the effects of the flow rule on the fracture angles by using an idealized compression experiment.

3 - IDEALIZED EXPERIMENT

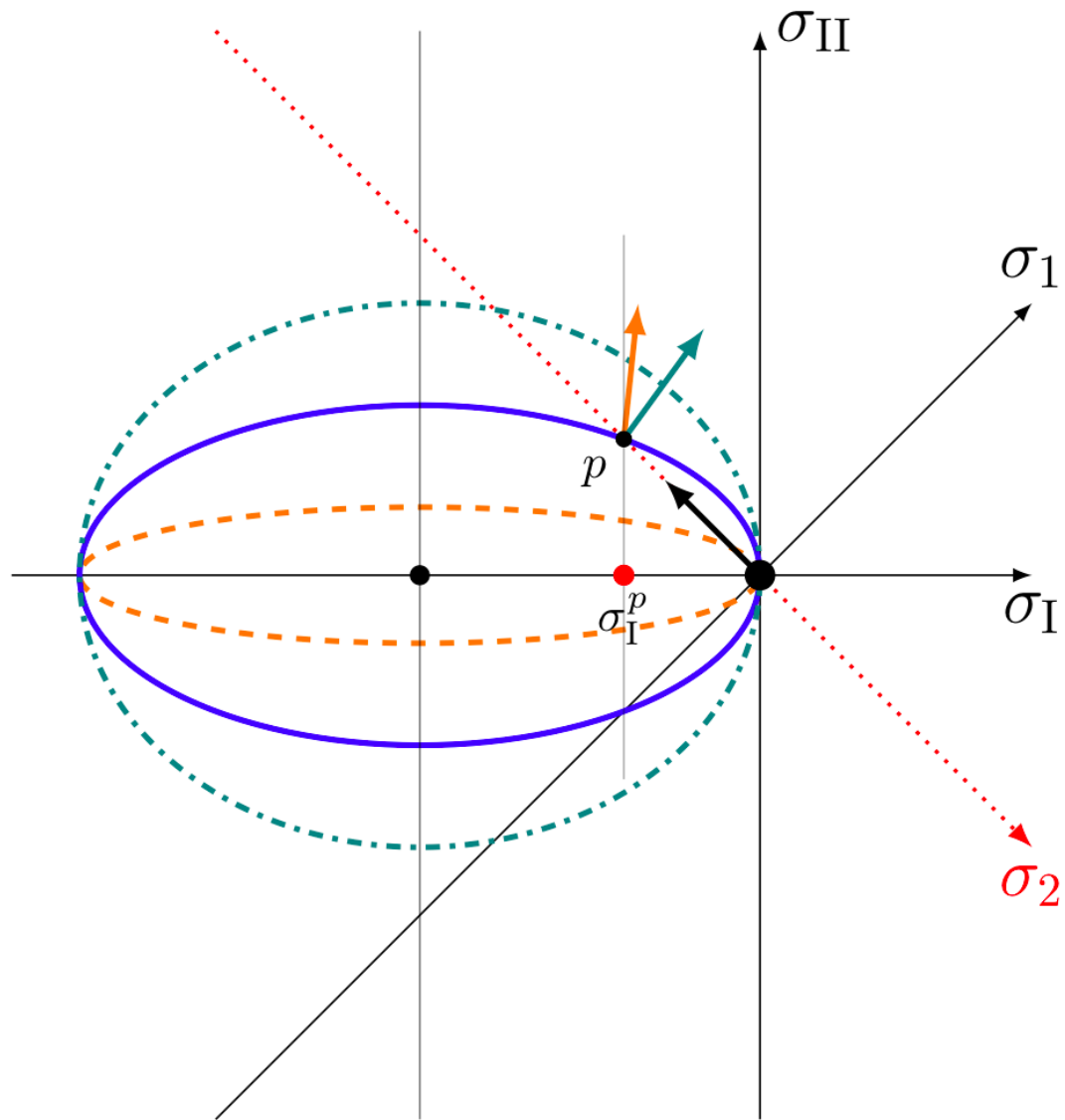


Experimental setup for uniaxial compression.

Below is a video of a toy model of the experiment:

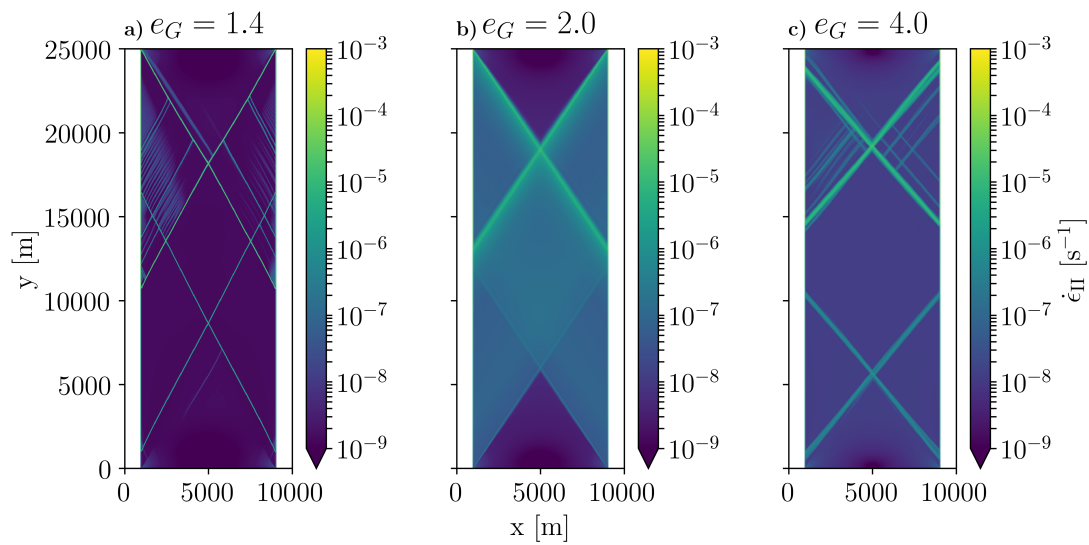
[VIDEO] <https://www.youtube.com/embed/AB2LzQ9K9jk?rel=0&fs=1&modestbranding=1&rel=0&showinfo=0>

(thanks to Nils Hutter for the toy model)

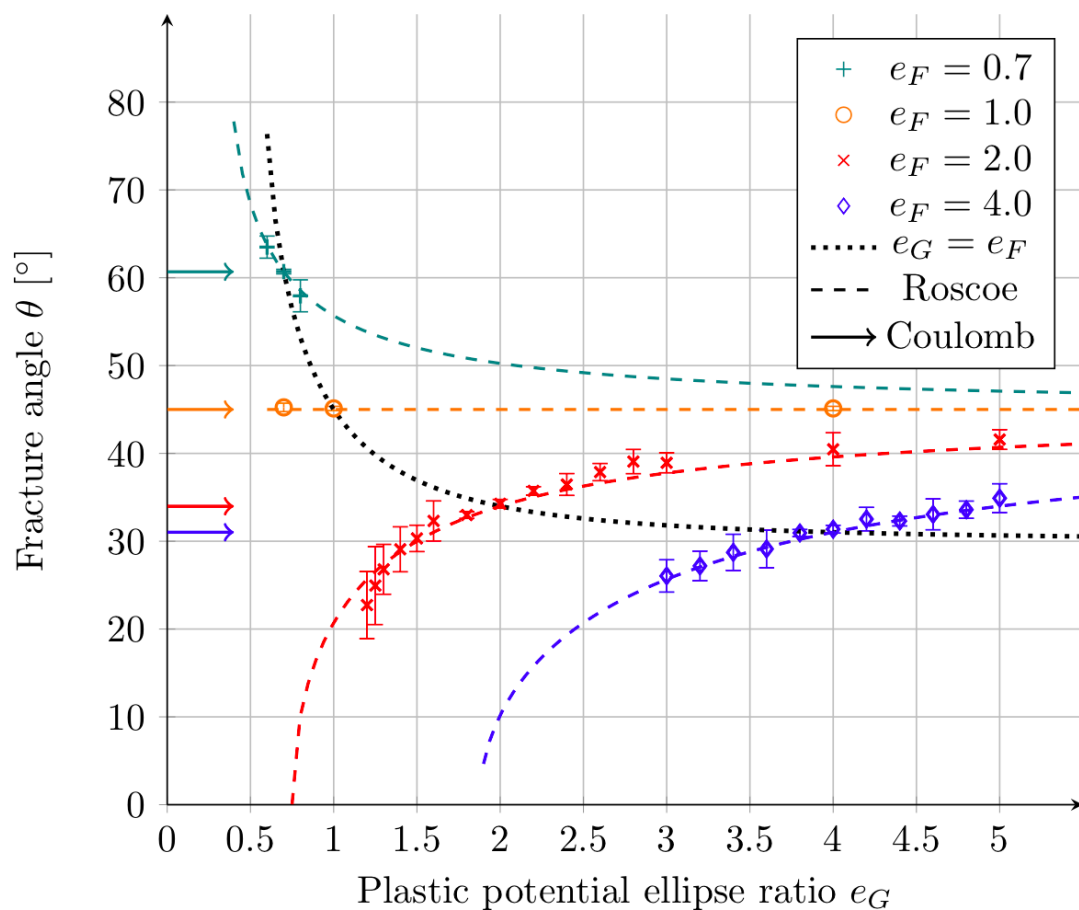


Schematic of stresses and flow rules with a yield curve (blue) and two different plastic potentials (orange and teal). In uni-axial compression, the stresses are along the σ_2 axis (red). For the same stress state p , the flow rule can be changed by changing the plastic potential ellipse ratio (orange and teal arrows).

5 - RESULTS



Example of fracture with three values of e_G for $e_F=2.0$. The shear strain rate ϵ_{II} is shown here.



Fracture angles as a function of the ellipse ratio of the plastic potential e_G for 4 values of e_F . The dashed lines show the predictions of the Roscoe angles. Arrows show the constant prediction from the Coulomb angles.

4 - THEORY

Three theoretical predictions for the angle compete (Vermeer, 1990):

- Coulomb Angle θ_C depends on the slope of the yield curve (Coulomb, 1773).

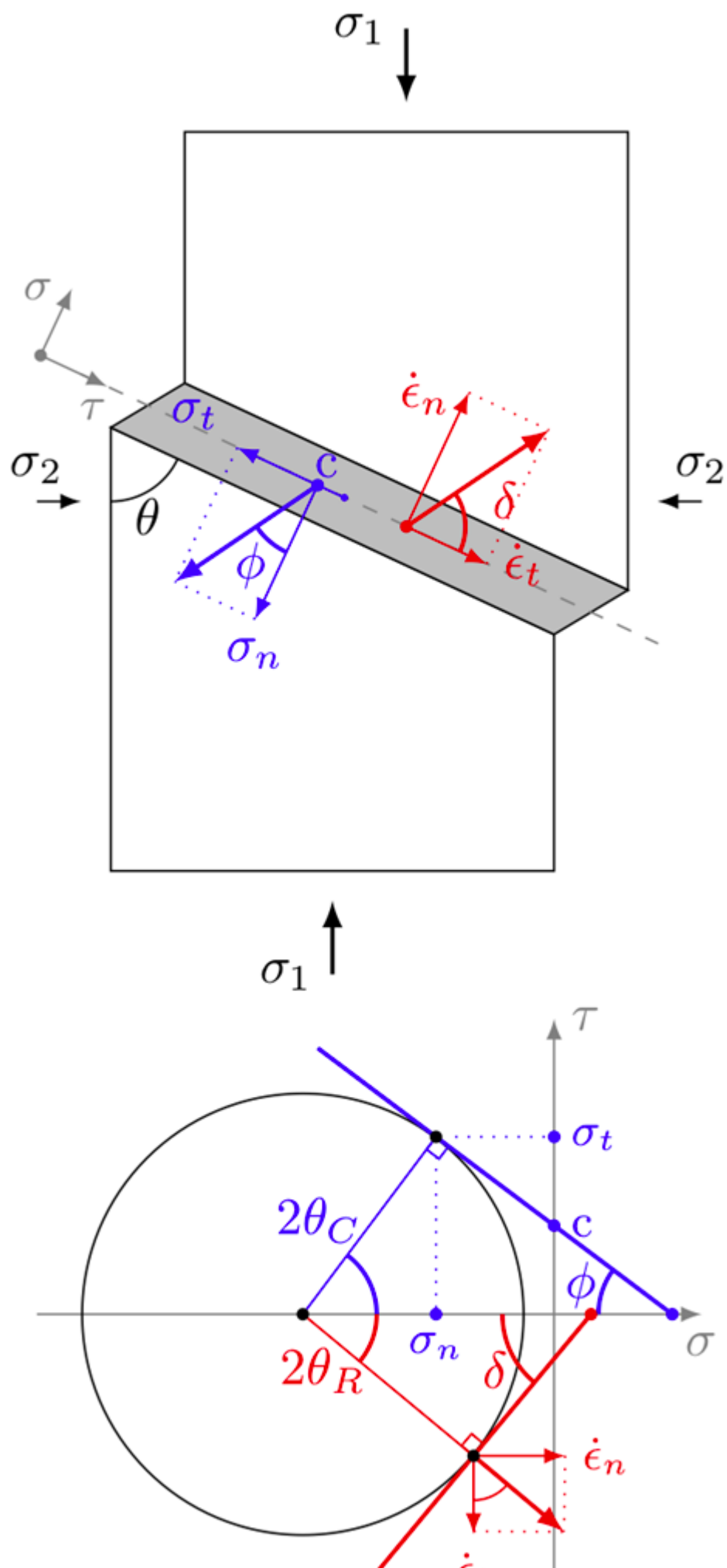
$$\theta_C = \frac{\pi}{4} - \frac{\phi}{2}$$

- Roscoe Angle θ_R depends on the slope of the plastic potential (Roscoe, 1970).

$$\theta_R = \frac{\pi}{4} - \frac{\delta}{2}$$

- Arthur Angles, $\theta_A = \theta_R + \theta_C$ (Arthur et al., 1977).

$$\theta_A = \frac{\pi}{4} - \frac{\delta + \phi}{4}$$



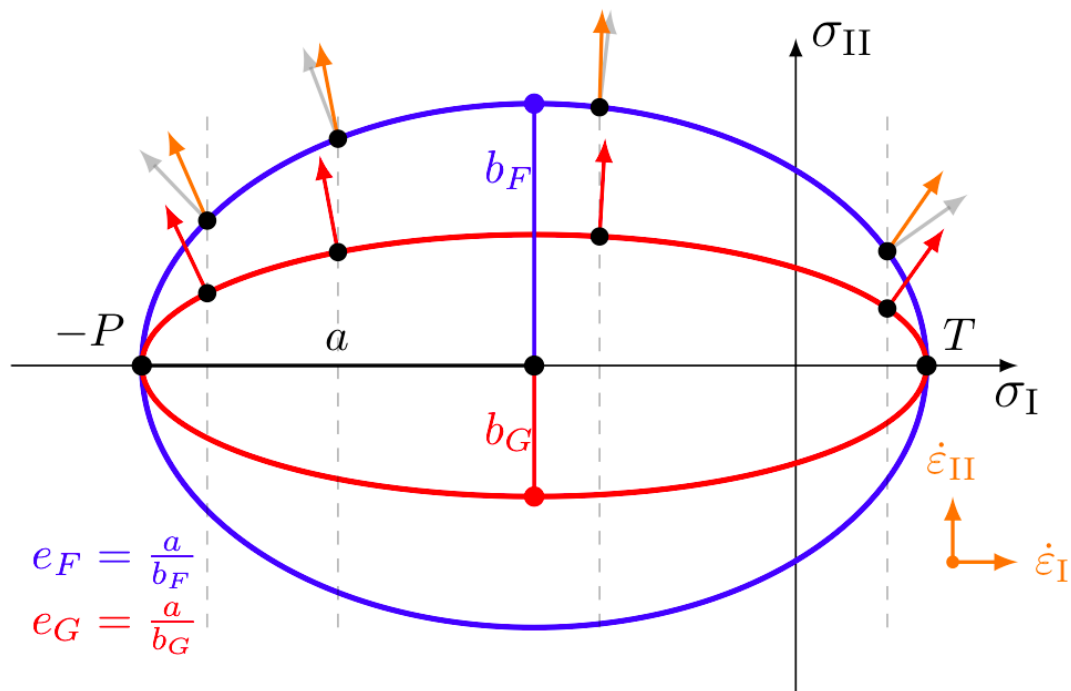
Schematics of the differences between the Coulomb and Roscoe angles, θ_C and θ_R , and how they depend respectively from ϕ and δ . The top panel shows the stresses σ and strain rates $\dot{\epsilon}$ in physical space. The bottom panel shows the same in the stress-space oriented along the fracture line, as shown on the first panel. On the bottom panel, the axis τ is mirrored to increase readability. The top panel is derived from Vermeer (1990).

2 - A NEW RHEOLOGY

A sea ice VP rheology has two components

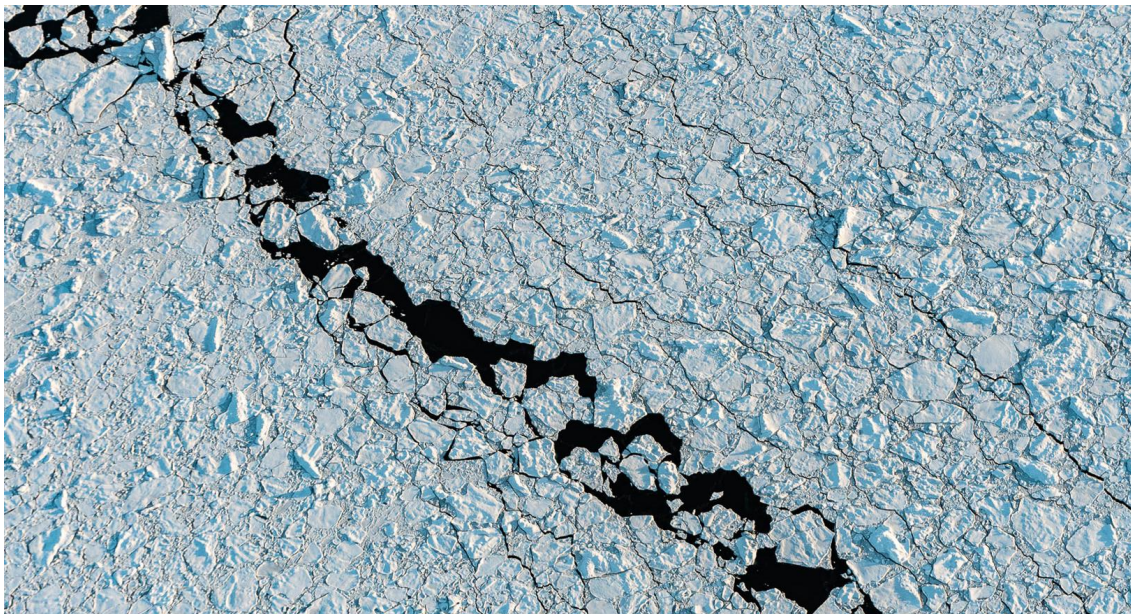
- A yield curve F :
 - Defines the stress at fracture.
- A plastic potential G :
 - Defines the deformation after fracture: the flow rule is perpendicular to the plastic potential.

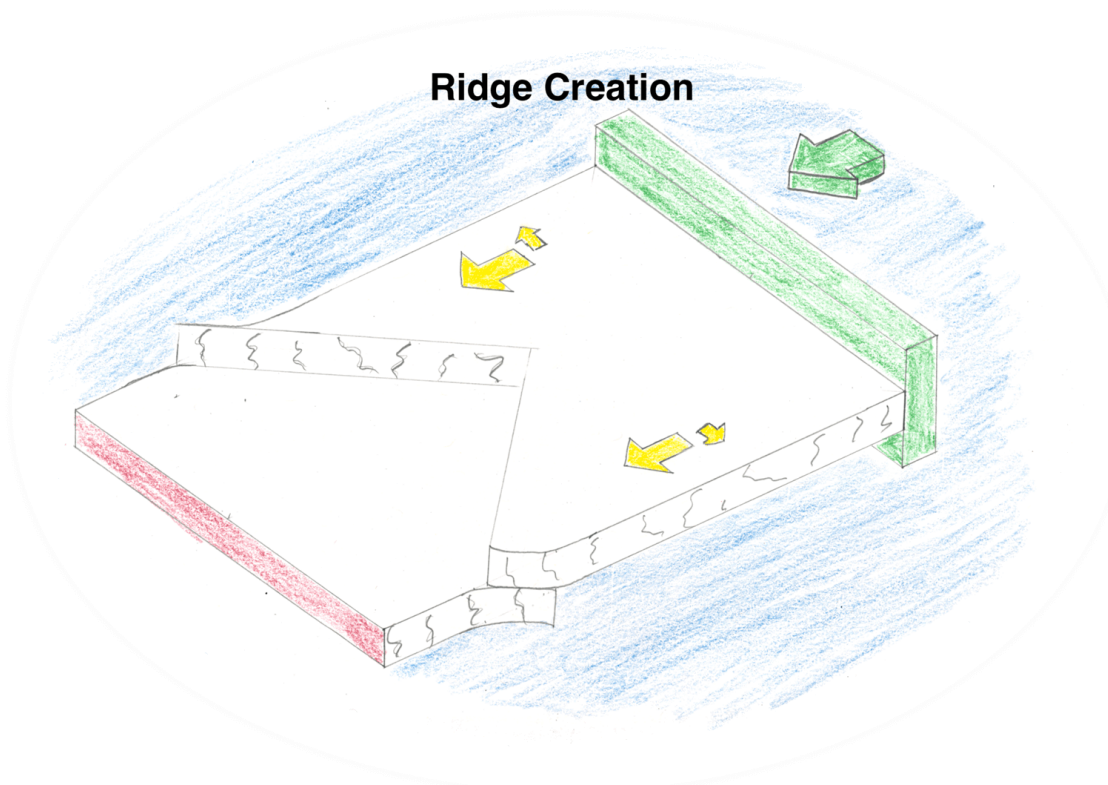
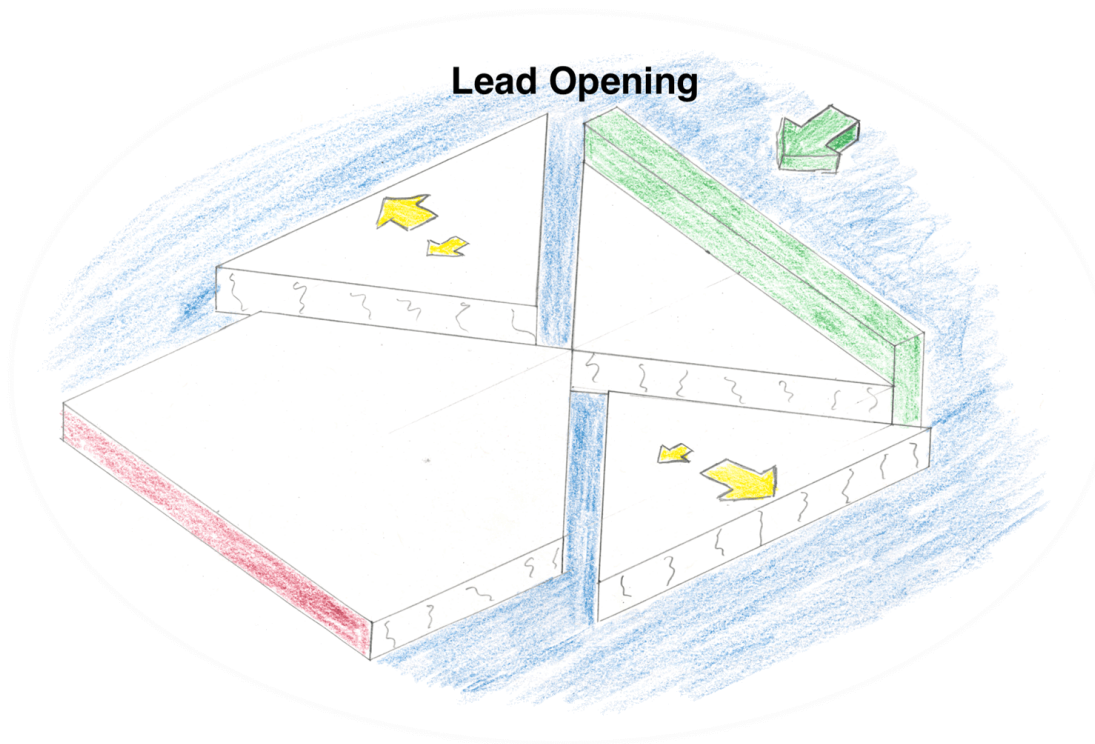
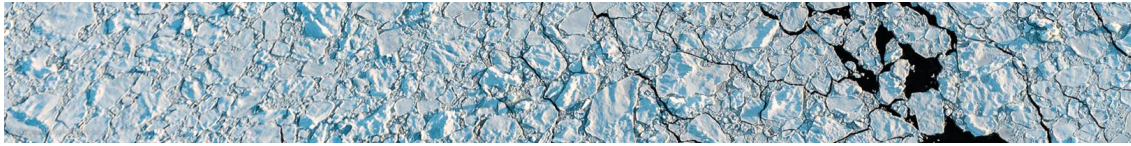
To study the flow rule's effect on the fracture angles, we created a new rheology with an elliptical yield curve and a non-normal flow rule by defining a different elliptical plastic potential.



Schematic of the new rheology with an elliptical yield curve (blue) and a plastic potential (red) of different ellipse ratios, respectively e_F and e_G . The resulting flow rule is shown with orange arrows, while the normal flow rule is shown in gray.

With this new rheology, we can isolate the effect of the flow rule from the yield curve. The stress state at fracture can be kept the same while changing the post-fracture deformation.





6 - CONCLUSION

- The flow rule orientation sets the fracture angles in agreement with the predictions of the Roscoe Angle.
- By introducing an independent plastic potential, we decouple the fracture angle and the stress parameterization.
- The new rheology can also create fracture angles smaller than 30° .
- The flow rule has the advantages of being observable with remote sensing methods, unlike the stresses.
Observations could be used to design new rheologies.

Joint high-resolution observations of sea ice deformation and sea ice stresses could help to design new VP rheologies, for example, from the MOSAiC expedition.

DISCLOSURES

The content of this poster is included in the manuscript

Ringeisen, D., Tremblay, L. B., and Losch, M.: Non-normal flow rules affect fracture angles in sea ice viscous-plastic rheologies, *The Cryosphere Discuss.*, <https://doi.org/10.5194/tc-2020-153>, in review, 2020.

currently in review for the journal *The Cryosphere*.

The paper can be found at the address: <https://doi.org/10.5194/tc-2020-153> (<https://doi.org/10.5194/tc-2020-153>)

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ABSTRACT

High-resolution viscous-plastic (VP) sea ice models reproduce the narrow deformation lines observed in the Arctic sea ice, called the Linear Kinematic Features (LKF). Recent studies showed that standard VP models overestimate the intersection angles between the LKFs when compared to observations. We investigate fracture angles in a uniaxial compression test and two different VP rheology. The first one uses an elliptical yield curve and a normal flow rule. In contrast, the second rheology uses a different elliptical plastic potential that creates a non-normal flow rule. Results show that the non-normality of the flow rule changes the angles of fracture. This new rheology can create fracture angles as low as 22° when the rheology with normal flow rule is limited to angles above 30° . A newly adapted theory – based on one developed from granular material observations – predicts the modeled fracture angles accurately. Using a non-normal flow rule takes longer to solve numerically, but allow reductions of the fracture angle to values within the range of satellite observations and decouples the angle of fracture from the shape of the yield curve.

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