Supporting Information for "On the relevance of aerosols to snow cover variability over High Mountain Asia"

Chayan Roychoudhury[1], Cenlin He[2], Rajesh Kumar[2], John M. McKinnon[1] and Avelino F. Arellano Jr.[1]

[1]Department of Hydrology and Atmospheric Sciences, University of Arizona, Tucson, AZ, USA

[2]Research Applications Laboratory, National Center for Atmospheric Research, Boulder, CO, USA

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Introduction Here, we outline the algorithm for relative importance analysis used in our study in Text S1. A summary of the sensitivity tests used for our study is in Table S1. Figure S1 consists of a graphical overview of various geophysical drivers that regulate snow cover over glacial regions.

Text S1. Relative Weight Analysis

We define a matrix $\mathbf{X}$ of dimensions $N \times P$ such that columns correspond to P predictors and the rows to N samples of space and time, with standardized values (centered by subtracting the mean from each column and dividing each column by its standard deviation). We also define the column vector $\mathbf{Y}$ of dimensions $N \times 1$ as the outcome/response/dependent variable containing standardized values of N samples of space and time.

Step 1: We then perform a singular value decomposition on $\mathbf{X}$ as follows,

$$\underset{N\times P}{\mathbf{X}} = \underset{N\times R}{\mathbf{U}}\,\underset{R\times R}{\mathbf{\Sigma}}\,\underset{R\times P}{\mathbf{V^T}} \tag{1}$$

where $R \leq \min\{N, P\}$, $\mathbf{U}$ is the eigenvector matrix associated with $\mathbf{XX^T}$, $\mathbf{V}$ is the eigenvector matrix associated with $\mathbf{X^TX}$ and $\mathbf{\Sigma}$ is a diagonal matrix with values equal to the square roots of the eigenvalues of $\mathbf{XX^T}$ and $\mathbf{X^TX}$.

Step 2: Then we find the orthogonal approximation of $\mathbf{X}$ by

$$\underset{N\times P}{\mathbf{Z}} = \underset{N\times R}{\mathbf{U}}\,\underset{R\times P}{\mathbf{V^T}} \tag{2}$$

where $\mathbf{Z}$ is related to the original $\mathbf{X}$ with a new set of P predictors that are uncorrelated with each other.

Step 3: We regress the response/outcome $\mathbf{Y}$ on the new set of predictors $\mathbf{Z}$ and estimate the regression coefficients β_p where $p = 1, 2, \ldots, P$

$$\underset{P\times 1}{\boldsymbol{\beta}} = \underset{P\times P}{(\mathbf{Z^TZ})^{-1}}\,\underset{P\times N}{\mathbf{Z^T}}\,\underset{N\times 1}{\mathbf{Y}} \tag{3}$$

$$\underset{N\times 1}{\mathbf{Y}} = \sum_{p=1}^{P} \beta_p \underset{N\times 1}{\mathbf{Z}_p} \tag{4}$$

Step 4: As $\mathbf{Z}$ is an approximation of $\mathbf{X}$, we also regress the original $\mathbf{X}$ on the orthogonal $\mathbf{Z}$ and estimate the regression coefficient $\lambda_{p'p}$ where p and $p' = 1, 2, \ldots, P$ as

$$\underset{P\times P}{\boldsymbol{\Lambda}} = \underset{P\times P}{(\mathbf{Z^T Z})^{-1}} \underset{P\times N}{\mathbf{Z^T}} \underset{N\times P}{\mathbf{X}} \tag{5}$$

$$\underset{N\times 1}{\mathbf{X}_p} = \sum_{p'=1}^{P} \lambda_{p'p} \underset{N\times 1}{\mathbf{Z}_p} \tag{6}$$

Step 5: We combine the regression coefficients β_p and $\lambda_{p'p}$ from both regressions to estimate the normalized relative importance of the original predictors $\mathbf{X}$ by,

$$\underset{P\times 1}{\mathbf{RW}_p} = \boldsymbol{\Lambda}^{[2]}\boldsymbol{\beta}^{[2]} = \frac{\sum\limits_{p'=1}^{P} \lambda_{p'p}^2 \beta_p^2}{\sum\limits_{p=1}^{P} \left(\sum\limits_{p'=1}^{P} \lambda_{p'p}^2 \beta_p^2 \right)} \tag{7}$$

where $\boldsymbol{\Lambda}^{[2]}$ and $\boldsymbol{\beta}^{[2]}$ refer to squared column elements of the regression coefficient matrices $\boldsymbol{\Lambda}$ and $\boldsymbol{\beta}$. The property of the normalized relative importance is such that $\sum\limits_{p=1}^{P} \mathbf{RW}_p = 1$

Table S1. Summary of the sensitivity tests. Case 0 refers to the original MLR model ('control') used in our study to estimate monthly relevance. Aerosol interactions are defined for each sensitivity test in reference to the groups of interaction terms used in our MLR model.

Cases	**Comment**	**Aerosol Interactions**
Case 0	Control Case	AER-MET + AER-AER + AER
Case 1	No elevation and associated interactions	AER-MET + AER-AER + AER
Case 2	Aggregating species AOD and SMXR to total AOD and SMXR with their interactions	AER-MET + AER-AER + AER
Case 3	No aerosol – meteorology interactions	AER-AER + AER
Case 4	Only individual aerosol variables, no aerosol related interactions	AER

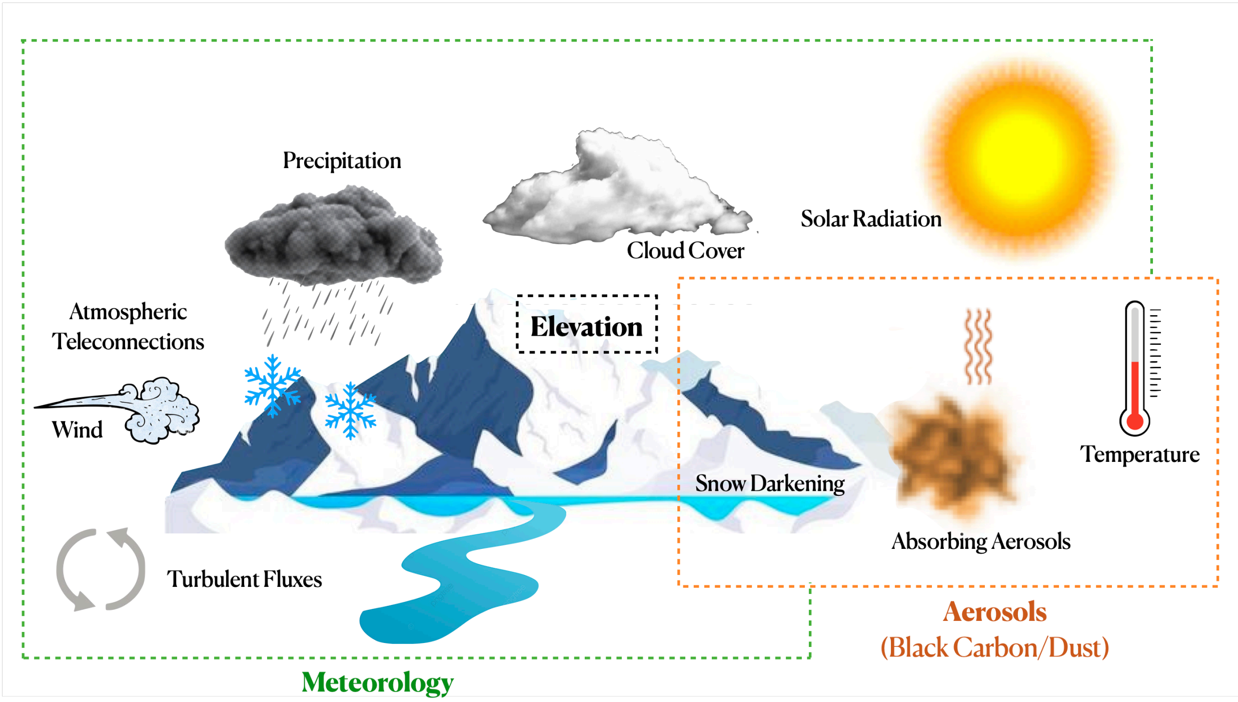

Figure S1. Graphical summary of various drivers behind snow cover in High Mountain Asia.