Supporting Information for "A physics-based universal indicator for vertical decoupling and mixing across canopies architectures and dynamic stabilities"

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Text S1: Derivation of $w_{e,crit}$

Derivation of $w_{e,crit}$ relies on the assumption that in order for a downward moving air parcel to reach the ground its kinetic energy must match the work needed to counterbalance the forces hindering the downward movement. Under stable stratification downward movement is hindered by buoyancy force F_B:

$$F_B = g\frac{\rho - \rho_e}{\rho_e}, \tag{1}$$

where g is acceleration due to gravity (m s^{-2}), ρ_e is density of the downward moving air parcel (kg m^{-3}) and ρ is the air density of air surrounding the air parcel. Note that F_B is relative to unit mass and both ρ and F_B depend on height z. Also canopy drag hinders air movement through the canopy. The drag force (F_D) per unit mass can be approximated with (e.g. Poggi, Katul, & Albertson, 2004; Cescatti & Marcolla, 2004; Watanabe, 2004):

$$F_D = -c_d a U w_e, \tag{2}$$

where c_d is drag coefficient (unitless), a is leaf area density (m^2 m^{-3}), U is horizontal wind speed (m s^{-1}) and w_e is the speed of the air parcel (m s^{-1}). All these four variables vary with height z. The work (W) needed to offset these two forces can be calculated as line integral from height h to the surface ($z = 0$ m):

$$\begin{aligned} W &= -\int_h^0 (F_B + F_D)\, dz && (3)\\ &= -\int_h^0 \left(g\frac{(\rho - \rho_e)}{\rho_e} - c_d a U w_e\right) dz && (4)\\ &= gh\frac{\hat{\rho} - \rho_e}{\rho_e} + \int_h^0 c_d a U w_e dz && (5)\end{aligned}$$

where $\hat{\rho}$ is the average air density in the air column below h. Following prior studies (Inoue, 1963; Amiro, 1990; Poggi, Porporato, et al., 2004; Yi, 2008) U and w_e profiles below canopy height were parameterized as $U(z) = U(h)e^{\beta(z/h-1)}$ and $w_e(z) = w_e(h)e^{\alpha(z/h-1)}$. The coefficients α and β were obtained by fitting to observations ($\beta = 2.0, R^2 = 0.98$ and $\alpha = 1.5, R^2 = 0.96$). σ_w profiles measured at the same site in a prior study were used (Launiainen et al., 2007) for determining α. This approach assumes that σ_w below canopy is governed by downward penetrating sweeps. Now if we assume that c_d and a are constant with height ($\hat{c_d}$ and $\hat{a}$, respectively), after integration we find

$$W \approx gh\frac{\hat{\rho}-\rho_e}{\rho_e} + \hat{c_d}\hat{a}U(h)w_e(h)\frac{h}{\beta+\alpha}\left(e^{-\beta-\alpha}-1\right), \qquad (6)$$

which can be further reduced to

$$W = gh\frac{\hat{\rho}-\rho_e}{\rho_e} - \gamma\hat{c_d}\mathrm{LAI}U_h w_e(h), \qquad (7)$$

where LAI is leaf area index (LAI $= h\hat{a}$), $U_h = U(h)$ and γ is a constant depending on the horizontal wind and downward penetrating air parcel speed profiles below h ($\gamma = \frac{1-e^{-\beta-\alpha}}{\beta+\alpha}$). Note that since $\alpha > 0$ and $\beta > 0$, therefore also $\gamma > 0$.

Now since kinetic energy of downward moving air parcel ($\frac{1}{2}w_e(h)^2$) must match the work, we can equate

$$\frac{1}{2}w_e(h)^2 = gh\frac{\hat{\rho}-\rho_e}{\rho_e} - \gamma\hat{c_d}\mathrm{LAI}U_h w_e(h), \qquad (8)$$

which can be solved for $w_e(h)$ to get $w_{e,crit}$:

$$w_{e,crit} = -\gamma\hat{c_d}\mathrm{LAI}U_h - \sqrt{\gamma^2\hat{c_d}^2\mathrm{LAI}^2U_h^2 + 2gh\frac{\hat{\rho}-\rho_e}{\rho_e}}. \qquad (9)$$

Here only the negative root was selected as physically meaningful. Assuming that air density changes only due to temperature and that the air parcel heats up adiabatically during its descent, then $w_{e,crit}$ can be written using potential temperature (θ)

$$w_{e,crit} = -\gamma\hat{c_d}\mathrm{LAI}U_h - \sqrt{\gamma^2\hat{c_d}^2\mathrm{LAI}^2U_h^2 + 2gh\frac{\theta_e-\hat{\theta}}{\hat{\theta}}}, \qquad (10)$$

which equals Eq. (1) in the main text.

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