Supporting Information for "Baroclinic control of Southern Ocean eddy upwelling near topography"

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This supporting information provides details of the energy equations in a two-layer isopycnal framework.

Energy budget in a two-layer isopycnal framework

1. Definitions and decompositons

In this study, we use a thickness-weighted energy framework similar to that used by Barthel et al. (2017). The two-layer system has four main energy reservoirs, defined as follows. APE_{bt} is the available potential energy due to the free surface elevation η_0 (or 'barotropic' potential energy)

$$APE_{bt} = \frac{\rho_0}{2} g {\eta_0}^2, \tag{1}$$

APE_{bc} is the available potential energy due to the motions of the interface separating the upper and lower layers η_1 (or 'baroclinic' potential energy)

$$APE_{bc} = \frac{\rho_0}{2} g' {\eta_1}^2, \tag{2}$$

and KE_i is the kinetic energy in each layer ($i = 1, 2$),

$$KE_i = \frac{\rho_0}{2} h_i |\mathbf{u}_i|^2, \quad \text{(for } i = 1, 2\text{)}. \tag{3}$$

Here, ρ_0 is the reference density of the Boussinesq approximation, g is the acceleration due to gravity, $g' = \frac{g\Delta\rho}{\rho_0}$ is the reduced gravity of the interface between the two layers, h_i is the i-th layer thickness, and $\mathbf{u}_i = [u_i, v_i]$ is the horizontal velocity in layer i.

To separate the mean and eddy terms, we define the traditional Reynolds decomposition for most variables in our model. For example, the layer thickness becomes:

$$h_i \equiv \overline{h_i} + h_i', \tag{4}$$

where the overbar and prime symbols denote a three-year time mean and the associated deviation respectively. Following the methodology used in Aiki & Richards (2008), the velocity variable is decomposed into a thickness-weighted mean (TWM) velocity $\widehat{\mathbf{u}}$ and deviation from the TWM mean $\mathbf{u}_i''$,

$$\mathbf{u}_i \equiv \widehat{\mathbf{u}}_i + \mathbf{u}_i'', \text{ with } \widehat{\mathbf{u}}_i \equiv \frac{\overline{h_i \mathbf{u}_i}}{\overline{h_i}}. \tag{5}$$

In a thickness-weighted framework, each energy reservoir can be decomposed into contributions from the mean and eddy, as proposed by **?**,

$$\overline{APE_{bt}} = \overline{\frac{\rho_0}{2} g \eta_0{}^2} = \underbrace{\frac{\rho_0}{2} g \overline{\eta_0}^2}_{MPE_{bt}} + \underbrace{\frac{\rho_0}{2} g \overline{\eta_0'^2}}_{EPE_{bt}}, \tag{6}$$

$$\overline{APE_{bc}} = \overline{\frac{\rho_0}{2} g' (\eta_1)^2} = \underbrace{\frac{\rho_0}{2} g' (\overline{\eta_1})^2}_{MPE_{bc}} + \underbrace{\frac{\rho_0}{2} g' \overline{\eta_1'^2}}_{EPE_{bc}}, \tag{7}$$

$$\overline{KE_i} = \overline{\frac{\rho_0}{2} h_i |\mathbf{u}_i|^2} = \underbrace{\frac{\rho_0}{2} \overline{h_i} |\widehat{\mathbf{u}}_i|^2}_{MKE_i} + \underbrace{\frac{\rho_0}{2} \overline{h_i |\mathbf{u}_i''^2}}_{EKE_i}, \quad \text{(for } i = 1,2). \tag{8}$$

Note that the kinetic energy is decomposed using the TWM decomposition of velocity. Note also that this eddy-mean decomposition is based on the separation between stationary (i.e. mean) and transient (i.e. eddy) features. Thus, the contribution of stationary meanders, or stationary eddies, are included in the contribution of the time-mean flow.

2. Time evolution of the mean and eddy energy

The equations governing the two-layer system can be derived from the incompressible hydrostatic equations of motion in isopycnal coordinates (see Barthel et al. (2017) for the full derivation). In particular, the time-mean energy reservoirs are governed by:

$$\partial_t MPE_{bt} = (\overline{h_1}\widehat{\mathbf{u}}_1 + \overline{h_2}\widehat{\mathbf{u}}_2) \cdot \nabla \overline{\phi_1} - \nabla \cdot (\overline{\phi_1}(\overline{h_1}\widehat{\mathbf{u}}_1 + \overline{h_2}\widehat{\mathbf{u}}_2)) \, , \tag{9}$$

$$\partial_t MPE_{bc} = \overline{h_2}\widehat{\mathbf{u}}_2 \cdot \nabla(\overline{\phi_2} - \overline{\phi_1}) - \nabla \cdot ((\overline{\phi_2} - \overline{\phi_1})\overline{h_2}\widehat{\mathbf{u}}_2)) \, , \tag{10}$$

$$\begin{aligned} \partial_t MKE_i = & -\nabla \cdot (\widehat{\mathbf{u}}_i MKE_i) - \overline{h_i}\widehat{\mathbf{u}}_i \cdot \nabla \overline{\phi_i} - \widehat{\mathbf{u}}_i \cdot \overline{h_i' \nabla \phi_i'} \\ & -\rho_0 (\widehat{\mathbf{u}}_i \cdot \nabla) \cdot \overline{(h_i \mathbf{u}_i'' \mathbf{u}_i'')} + \rho_0 \overline{h_i \mathbf{F}_{\tau \mathbf{i}}} \cdot \widehat{\mathbf{u}}_i, \quad \text{(for } i = 1,2). \end{aligned} \tag{11}$$

The equation governing the mean component of the layer MP flux divergence is

$$\nabla \cdot (\overline{\phi_i}\, \overline{h_i}\widehat{\mathbf{u}}_i) = -\overline{\phi_i}\, \overline{\partial_t h_i} + \overline{h_i}\widehat{\mathbf{u}}_i \cdot \nabla \overline{\phi_i}, \quad \text{(for } i = 1,2). \tag{12}$$

Likewise, the eddy energy reservoirs are governed by the following equations:

$$\partial_t EPE_{bt} = \overline{\phi_1' \partial_t h_2'} + \overline{\phi_1' \partial_t h_1'} \tag{13}$$

$$\partial_t EPE_{bc} = \overline{\phi_2' \partial_t h_2'} - \overline{\phi_1' \partial_t h_2'} \tag{14}$$

$$\begin{aligned} \partial_t EKE_i = & -\nabla \cdot (\widehat{\mathbf{u}}_i \; EKE_i) - \nabla \cdot (\overline{\mathbf{u}_i'' EKE_i}) - \overline{\mathbf{u}_i'' \cdot h_i \nabla \phi_i'} \\ & + \rho_0 \widehat{\mathbf{u}}_i \cdot \nabla \cdot \overline{(h_i \mathbf{u}_i'' \otimes \mathbf{u}_i'')} + \rho_0 \overline{h_i \boldsymbol{F_{\tau i}} \cdot \mathbf{u}_i''}, \quad (\text{for } i = 1, 2), \end{aligned} \tag{15}$$

where $\otimes$ denotes the outer product of two vectors. The associated eddy MP flux divergence equation is:

$$\nabla \cdot (\overline{\phi_i' h_i'} \widehat{\mathbf{u}}_i + \overline{\phi_i' h_i \mathbf{u}_i''}) = -\overline{\phi_i' \partial_t h_i'} + \widehat{\mathbf{u}}_i \cdot \overline{h_i' \nabla \phi_i'} + \overline{h_i \mathbf{u}_i'' \nabla \phi_i'} \tag{16}$$

These equations include advective terms, expressed as flux divergences, and local conversion terms. Only two terms locally convert energy between the time-mean and the eddy components of the system in layer i:

1. the work of interfacial form stress, $-\widehat{\mathbf{u}}_i \cdot \overline{h_i' \nabla \phi_i'}$, responsible for the generation of eddy energy in baroclinic instability, and
2. the work of Reynolds stress due to the horizontal convergence of momentum, $\rho_0 \widehat{\mathbf{u}}_i \cdot \nabla \cdot \overline{(h_i \mathbf{u}_i'' \otimes \mathbf{u}_i'')}$, which is associated with barotropic instability.

These terms can be bi-directional but are here defined as positive when converting energy from the mean to the eddy field.

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