Ensemble Kalman Filter Outperforms Optimal Interpolation in Tsunami Waveform Assimilation

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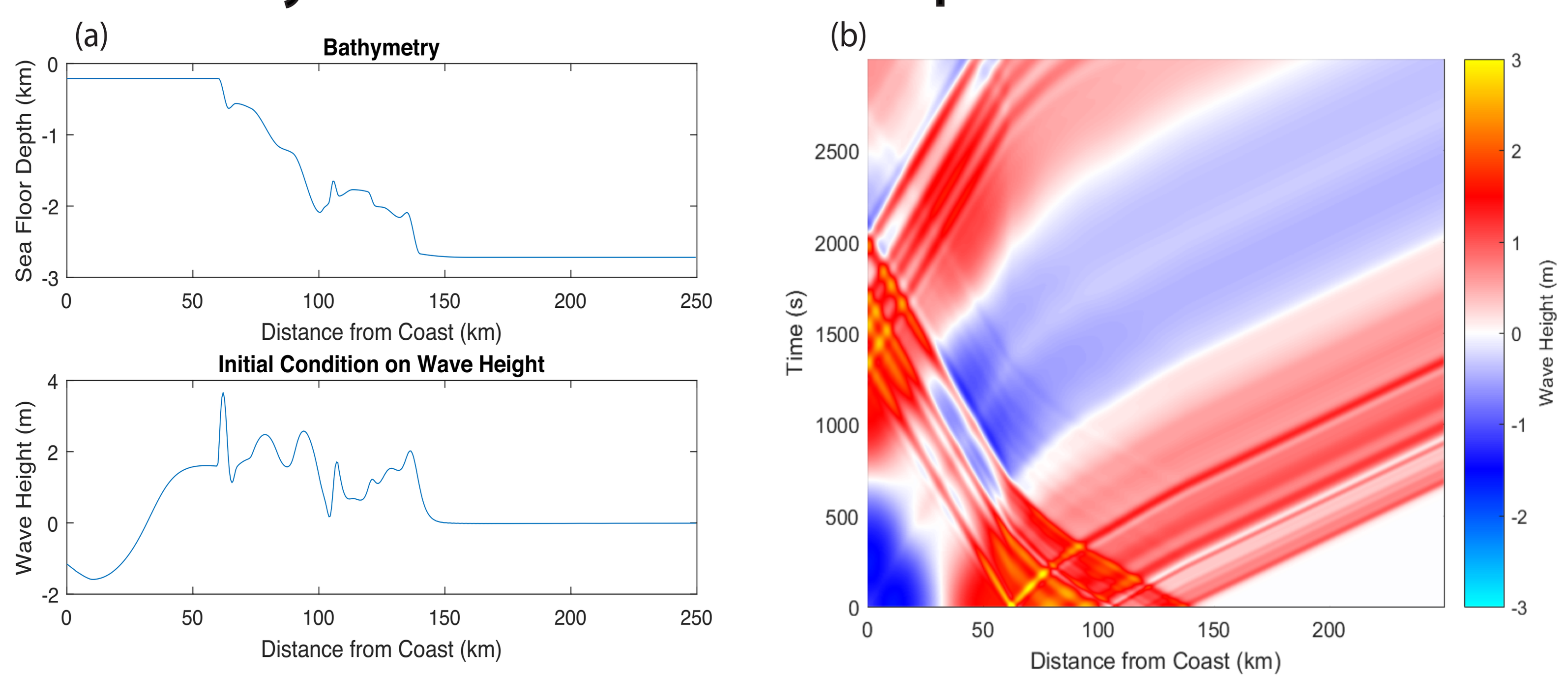
Background & Objectives

Ocean-bottom pressure sensor arrays, such as the S-NET and DONET in Japan, are revolutionary for local tsunami early warning. Maeda et al. (2015), Gusman et al. (2016), Wang et al. (2017, 2018) have used data assimilation via optimal interpolation to investigate how well we can use these data to perform tsunami forecasts. In this study, we introduce another data assimilation approach - the ensemble Kalman filter, and show that its improved performance could make it a viable candidate for real-time tsunami early warning.

Synthetic Tsunami Set-Up and Wavefield



Our simulation is based on a synthetic tsunami in the Cascadia subduction zone, with ocean bathymetry and initial condition on wave height shown on the left, and the space-time plot of true wavefield shown on the right. We used the linear long-wave model for wave propagation:

$$\partial\eta/\partial t = -\partial q/\partial x$$
$$\partial q/\partial t = -gH\partial\eta/\partial x$$

η is the wave height, q is the volumetric flux, g is gravitational acceleration, and H is the water depth.

How Kalman Filter Works

Kalman filter first produces a prior prediction of the state vector x = [q, η], based on the physical model (with error covariance P) from time k - 1 to k:

$$\hat{x}_k^- = F_k\hat{x}_{k-1}, \quad P_k^- = F_kP_{k-1}F_k^T$$

To assimilate observations z at select locations, we update state variable x, and obtain a posterior prediction, assuming the observations have an error covariance matrix R:

$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$, where $K_k = P_k^-H^T(HP_k^-H^T + R_k)^{-1}$

We update the error covariance matrix of the physical model before the next time step: $P_k = (I - K_kH)P_k^-$

The **ensemble Kalman filter** is a Monte Carlo approximation of the Kalman filter: it runs an ensemble of N realizations of the prior prediction. The error covariance matrix P is approximated by the covariance of the ensemble: a form of dimensionality reduction, lessening the computational load compared to the full Kalman filter. Optimal interpolation keeps both P and K constant - an even greater form of simplification.

Results from Ensemble Kalman Filter and Optimal Interpolation for Different Station Spacings

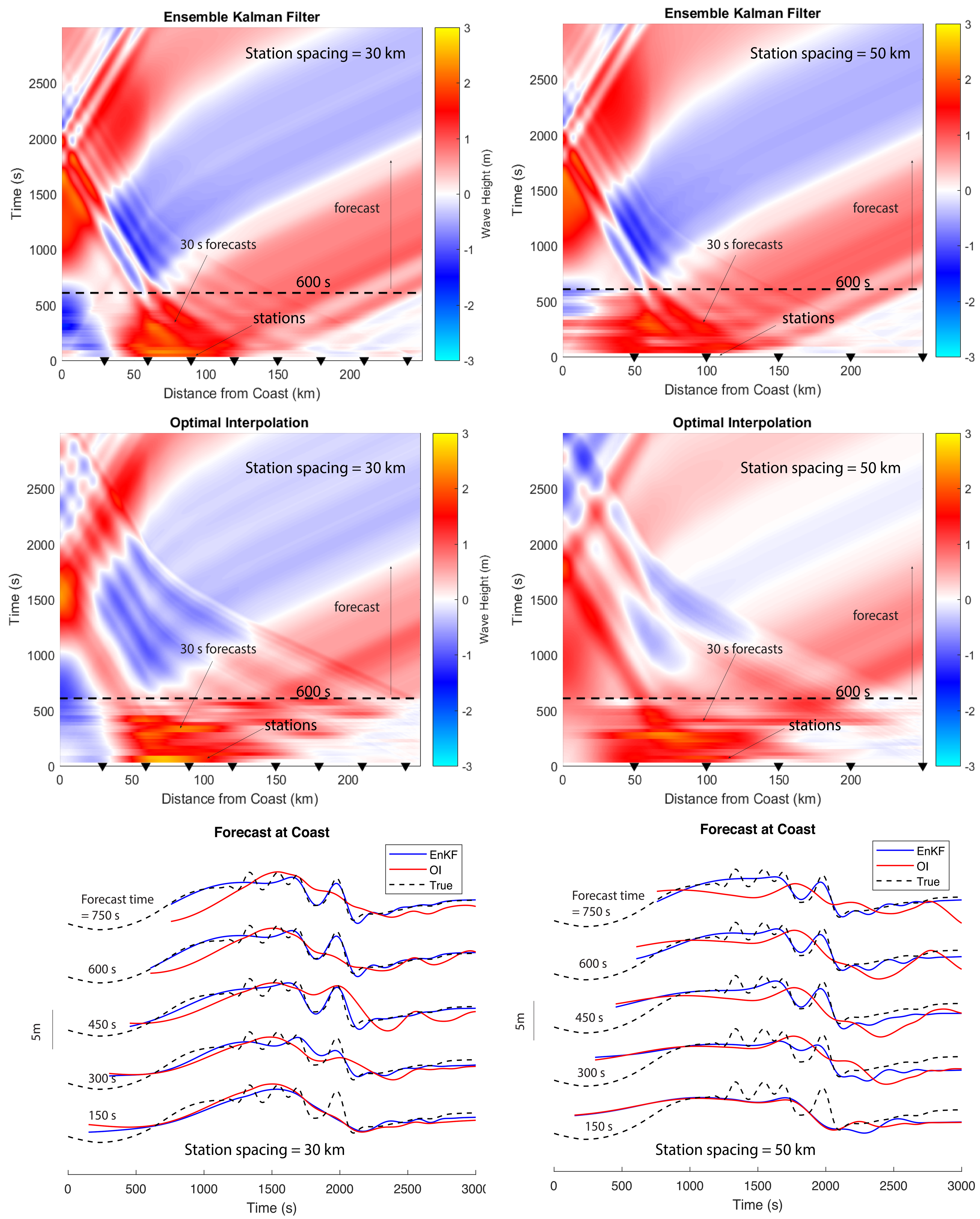
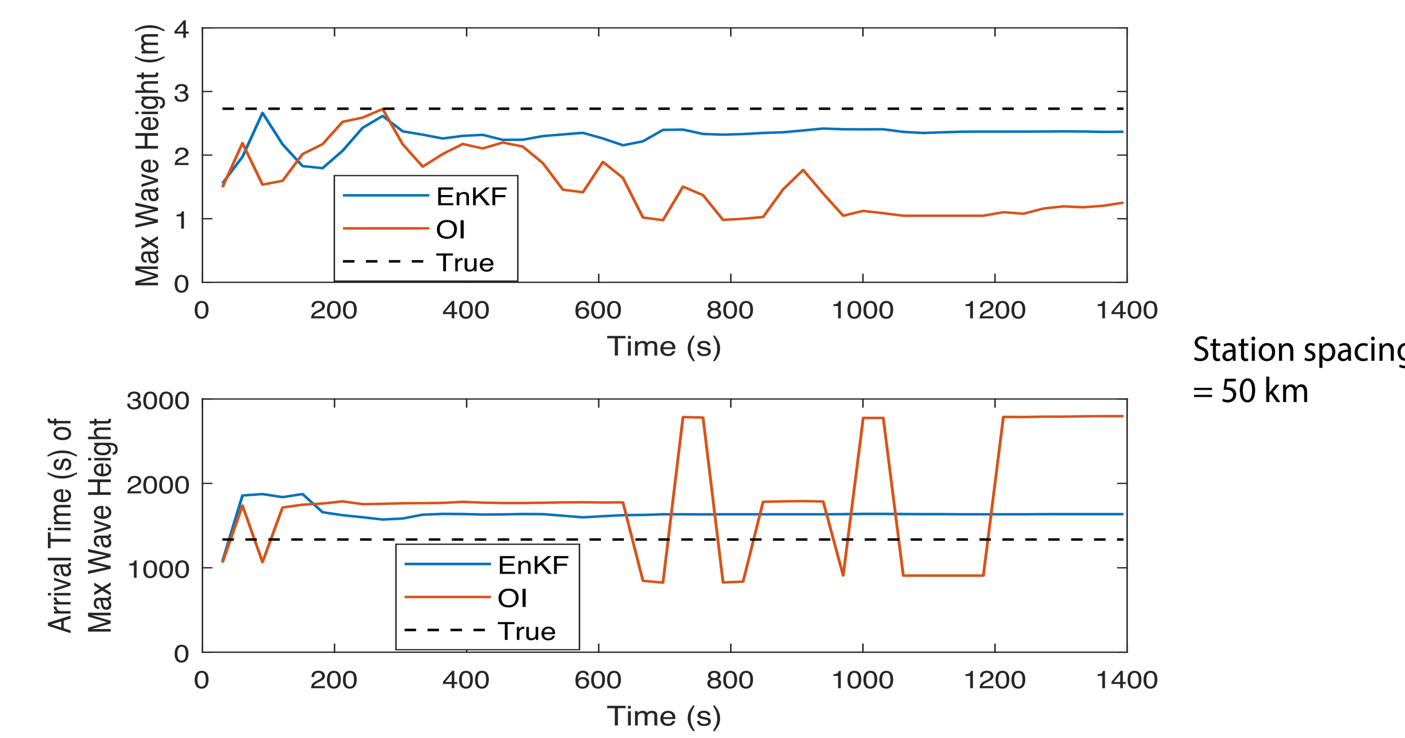


Figure above shows the forecast results for two different station spacings: 30 km (left column), and 50 km (right column). Top row shows forecast at 600 s from ensemble Kalman filter. Second row shows forecast at 600 s from optimal interpolation. The last row shows the coastal forecasts at different times for both methods, compared to the true solution.

Ensemble Kalman Filter is More Accurate and Stable than Optimal Interpolation

- At 600 s, ensemble Kalman filter achieves a forecast that is much closer to the true waveform for both station spacings
- Optimal interpolation's disadvantage is clearer when the station spacing is increased to 50 km
- For station spacing of 30 km, 30 s forecasts of ensemble Kalman filter before 600 s are more consistent
- The last two graphs shows how coastal forecasts of ensemble Kalman filter improves with time due to the continual update of the covariance matrix and Kalman gain matrix, butoptimal interpolation shows less improvement
- Ensemble Kalman filter can resolve the high-frequency components of the waveform much better

Real-Time Coastal Forecasts for Sparse Station Spacing



Conclusion & Future Prospects

Improved stability and accuracy of the ensemble Kalman filter make it attractive, especially as most offshore networks suffer from sparse station spacing. Although optimal interpolation is much more computationally efficient, developments in high-performance computing and parallel implementation of ensemble Kalman filter make it a promising candidate for real-time local tsunami early warning.

Selected References

Maeda, T., K. Obara, M. Shinohara,T. Kanazawa, and K. Uehira (2015), Successive estimation of a tsunami wavefield without earthquake source data: A data assimilation approach toward real-time tsunami forecasting, Geophys. Res. Lett., 42, 7923–7932.

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