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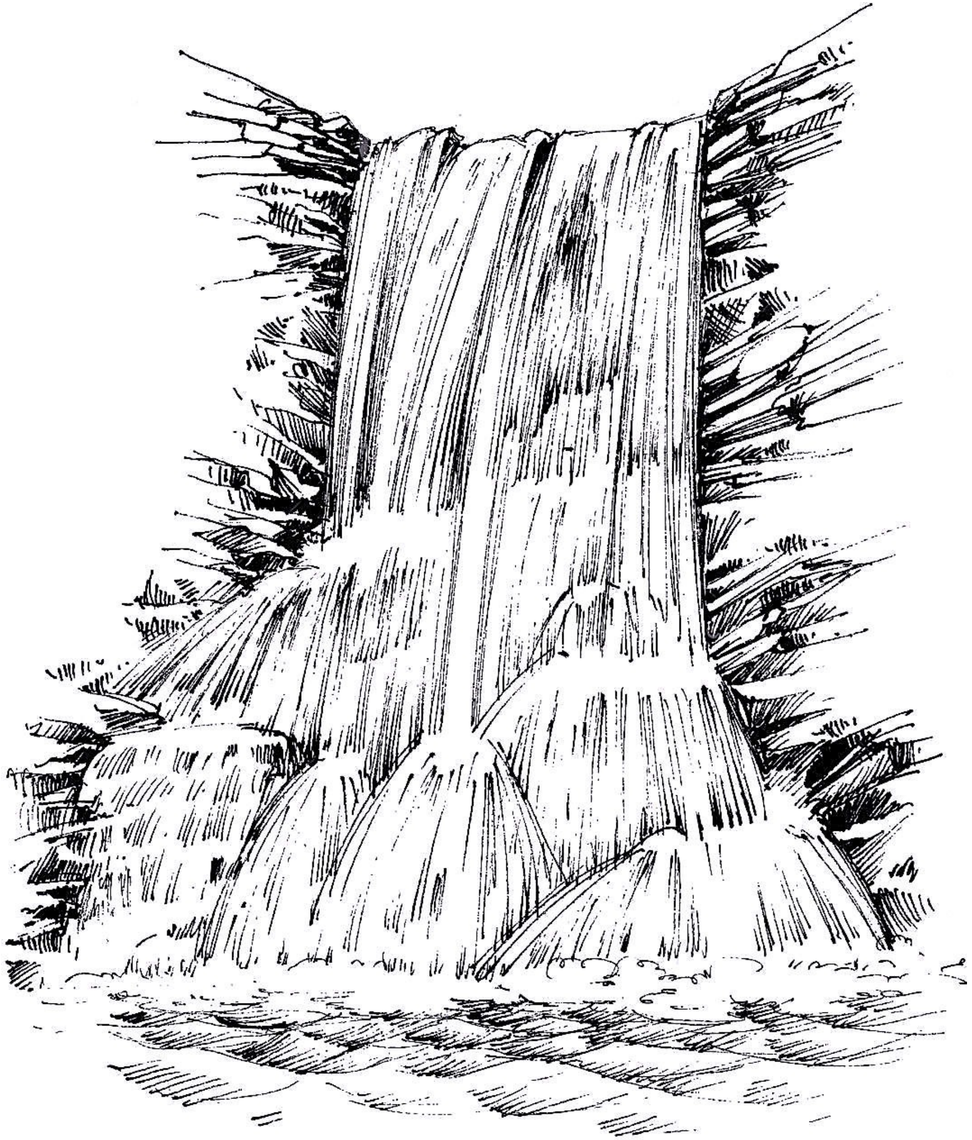
Reconnection is an Energy Cascade



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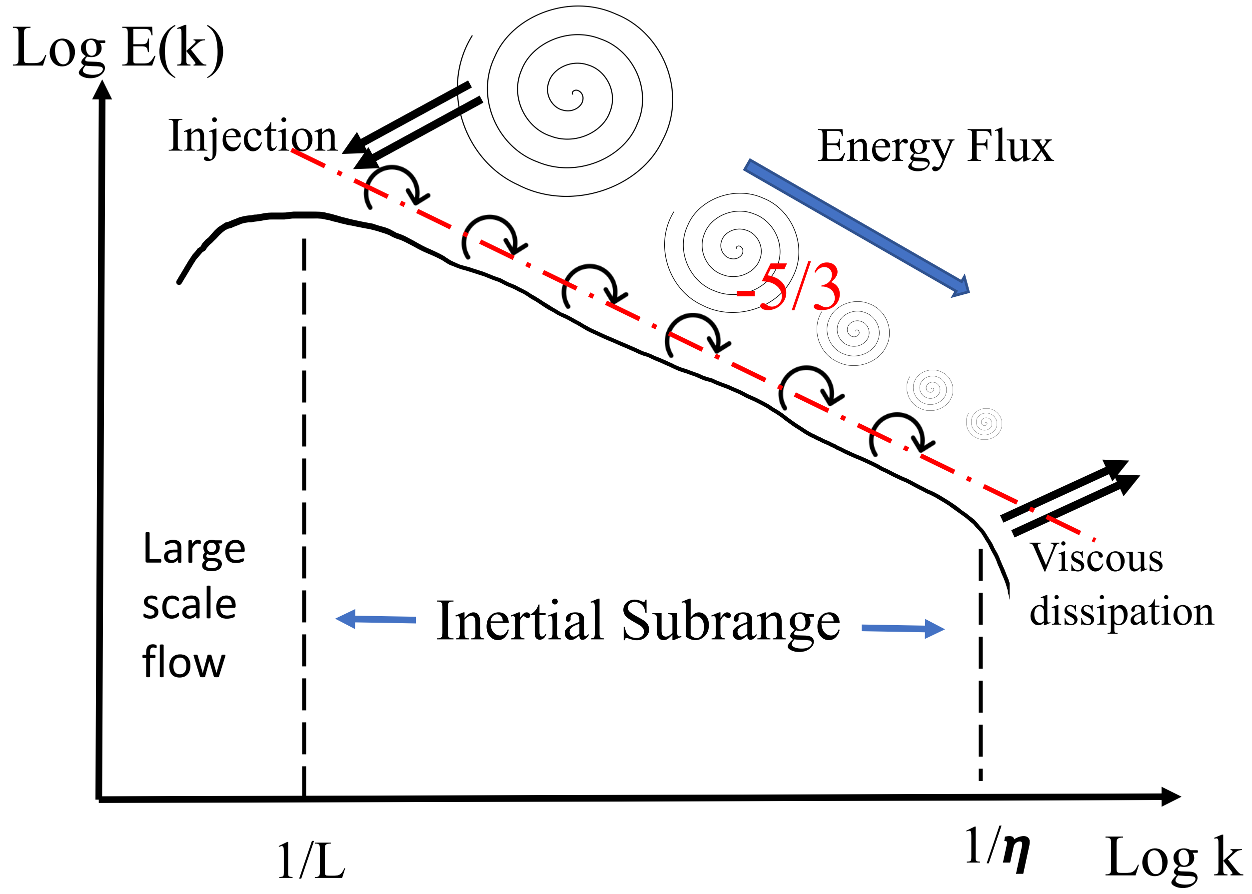
The rate of energy transfer is constant over the inertial subrange where the energy spectrum is Kolmogorov like, with spectral index -5/3.



In the inertial range,

Energy spectrum $E(k) \sim \epsilon^{\frac{2}{3}}\, k^{-\frac{5}{3}}$

Energy cascade rate (ϵ) = constant

$$E(k)\ \alpha\ \ k^{-\frac{5}{3}}$$

Mathematically, energy cascade rate is related to second and third-order structure functions through von Kármán Howarth equation.

For a Simple resistive MHD system:

Cascade rate $$\epsilon = -\frac{1}{4}\frac{\partial S}{\partial t} - \frac{1}{4}\nabla_l \cdot \mathbf{Y} \quad [1]$$

Increments → $\delta \boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}+\boldsymbol{l}) - \boldsymbol{u}(\boldsymbol{x})$

Assumptions: Incompressible, statistical homogeneity

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For a Simple resistive MHD system:

Cascade rate

$$\epsilon = -\frac{1}{4}\frac{\partial S}{\partial t} - \frac{1}{4}\nabla_l \cdot \mathbf{Y} \quad [1]$$

Second-order structure function

$$S_{\boldsymbol{b}} =< \delta\boldsymbol{b} \cdot \delta\boldsymbol{b} > = < |\delta\boldsymbol{b}|^2 >$$

$$S_{\boldsymbol{u}} =< \delta\boldsymbol{u} \cdot \delta\boldsymbol{u} > = < |\delta\boldsymbol{u}|^2 >$$

$$S = S_{\boldsymbol{u}} + S_{\boldsymbol{b}}$$

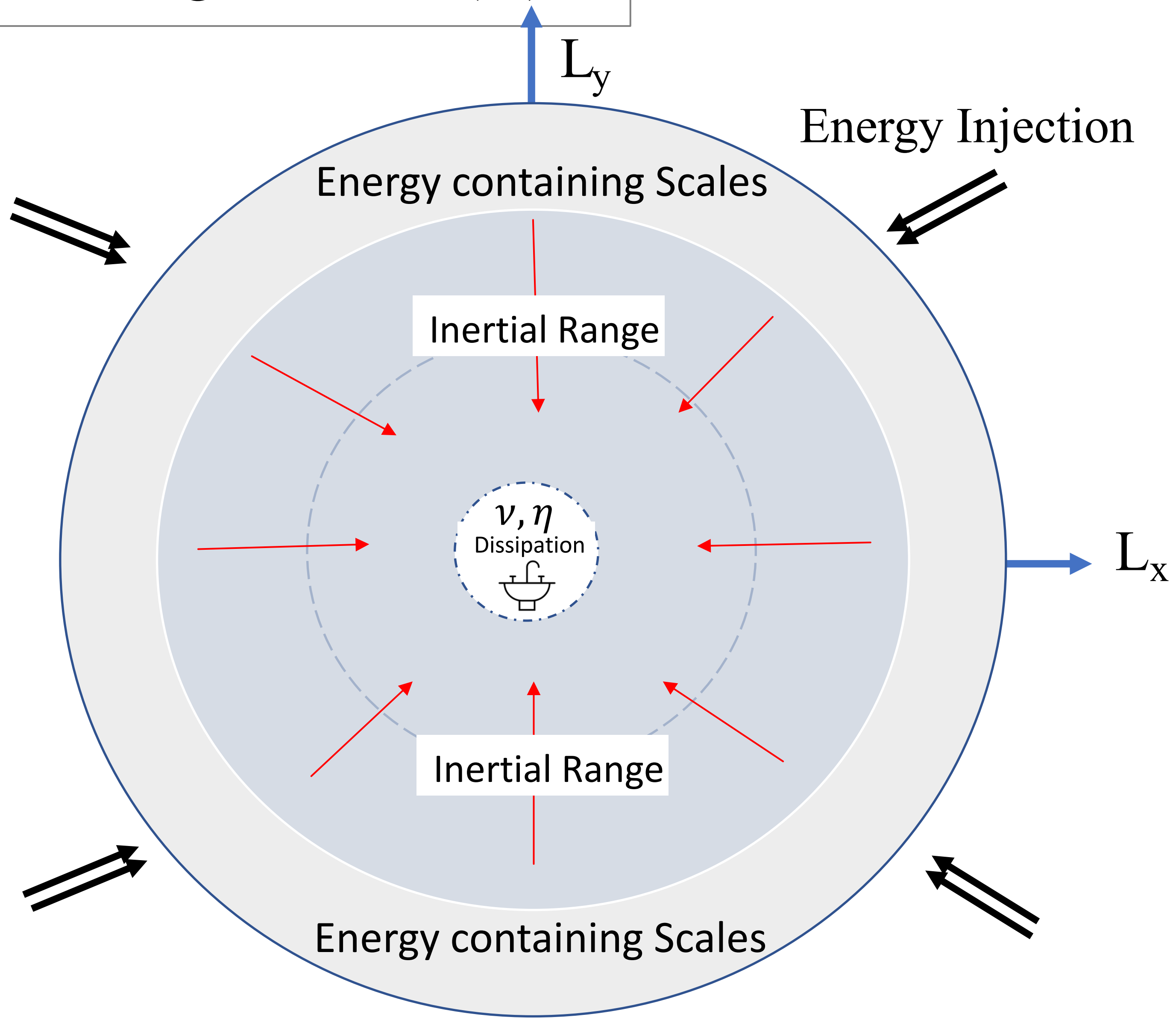
MHD/Yaglom flux

$$\boldsymbol{Y} =< \delta\boldsymbol{u}(|\delta\boldsymbol{u}|^2 + |\delta\boldsymbol{b}|^2) - 2\delta\boldsymbol{b}(\delta\boldsymbol{u} \cdot \delta\boldsymbol{b}) >$$

For a steady state turbulence, the energy cascade in the inertial range is solely dependent on the Yaglom flux (Y).

Cascade rate $\epsilon = -\frac{1}{4}\nabla_l \cdot \mathbf{Y}$

The red arrows represent the direction of energy flux.



If one includes the Hall term, then a Hall correction flux (H) becomes significant for smaller lags.

Cascade rate $$\epsilon = -\frac{1}{4}\frac{\partial S}{\partial t} - \frac{1}{4}\nabla_l \cdot \mathbf{Y} - \frac{1}{4}\nabla_l \cdot \boldsymbol{H}/2$$ [2]

Ferrand et al. ApJ (2019)

Hellinger et al. ApJL (2018)

Second-order structure function

$$S_{\boldsymbol{b}} = <\delta\boldsymbol{b}\cdot\delta\boldsymbol{b}> = <|\delta\boldsymbol{b}|^2>$$

$$S_{\boldsymbol{u}} = <\delta\boldsymbol{u}\cdot\delta\boldsymbol{u}> = <|\delta\boldsymbol{u}|^2>$$

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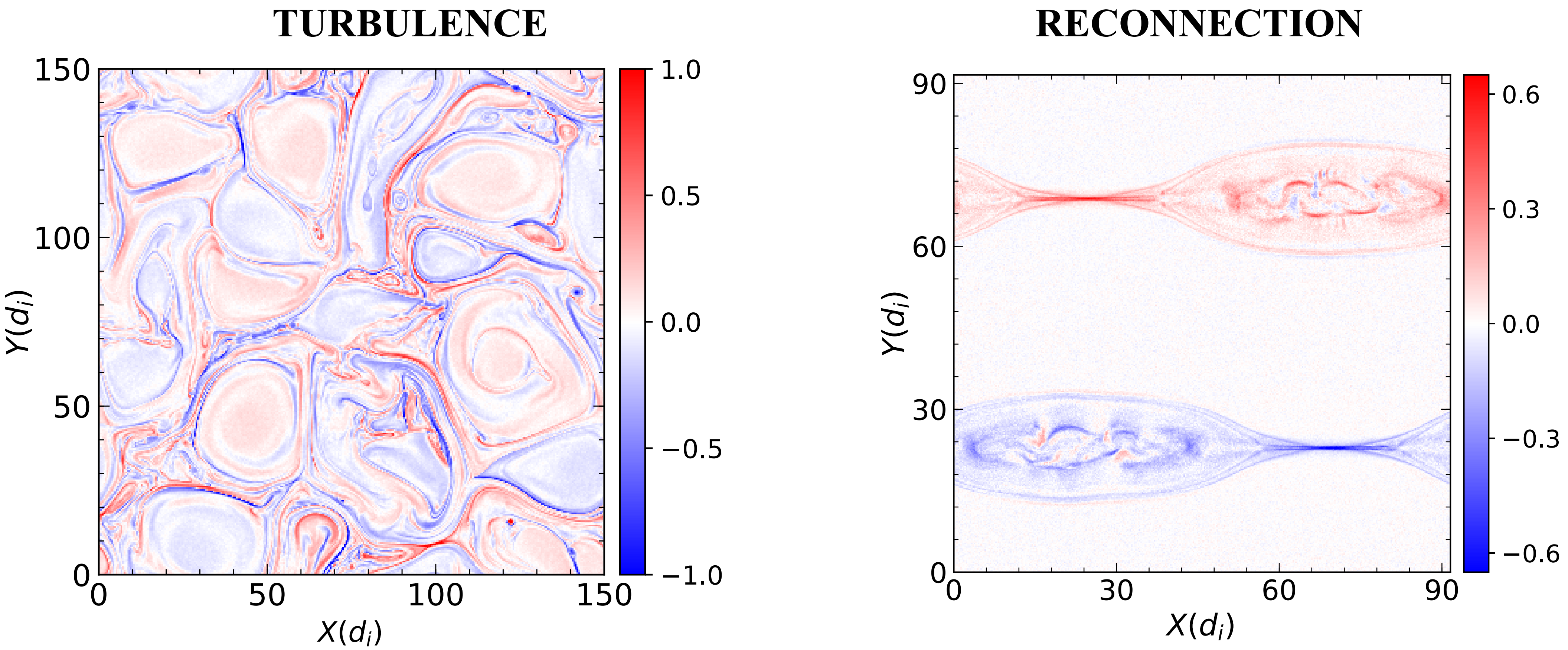
MHD/Yaglom flux

$$\boldsymbol{Y} = <\delta\boldsymbol{u}(|\delta\boldsymbol{u}|^2 + |\delta\boldsymbol{b}|^2) - 2\delta\boldsymbol{b}(\delta\boldsymbol{u}\cdot\delta\boldsymbol{b})>$$

Hall correction term (Hall flux)

$$\mathbf{H} = <2\delta\boldsymbol{b}(\delta\boldsymbol{j}\cdot\delta\boldsymbol{b}) - \delta\boldsymbol{j}(|\delta\boldsymbol{b}|^2)>$$

Testing the validity of Eqn. [2] for fully kinetic (PIC) simulations of turbulence and reconnection, as they look very different.

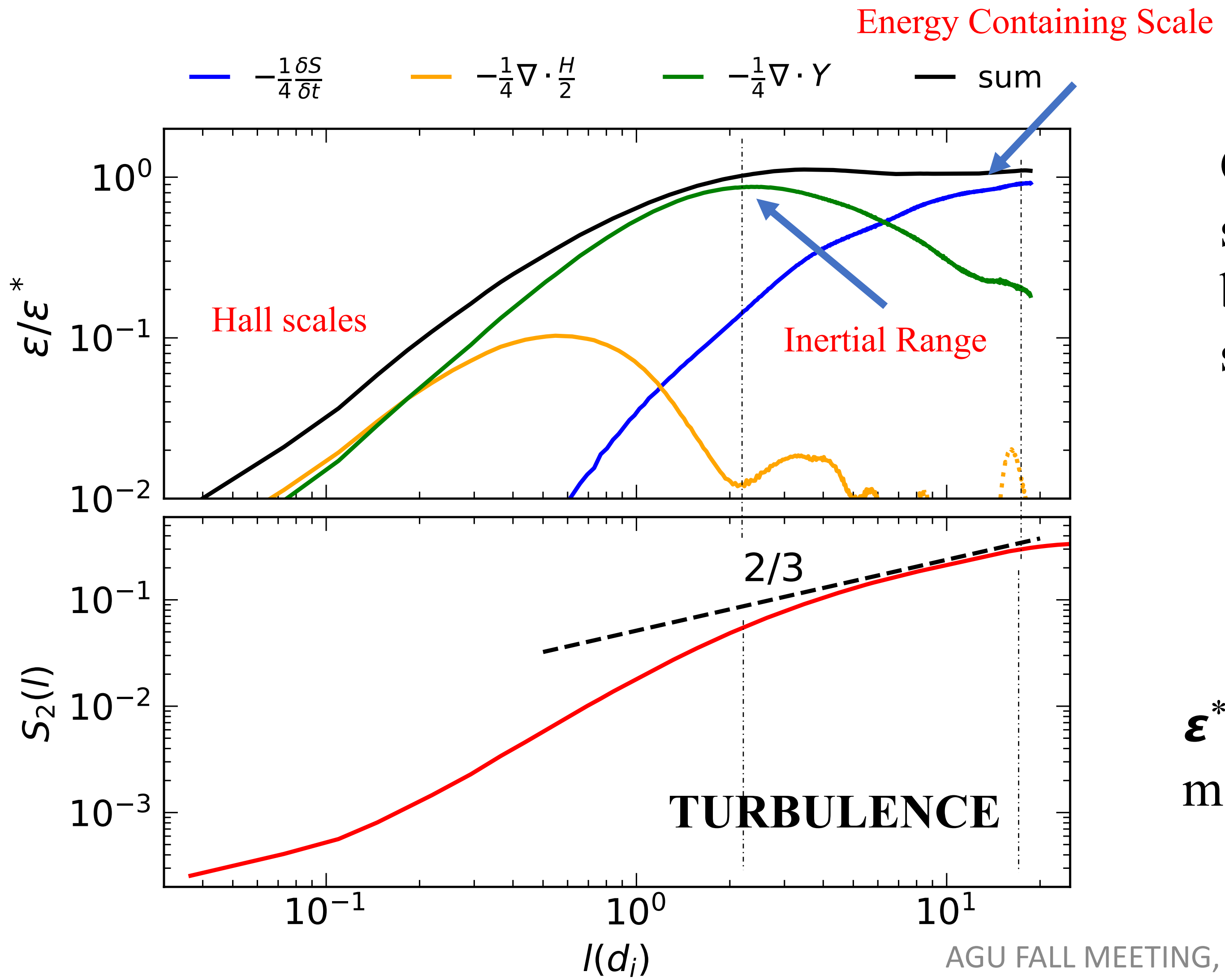


Out of plane current density for turbulence (left) when the mean square current is maximum, and for reconnection(right) when the reconnection is quasi-steady.

However, both the simulations (turbulence and reconnection) have same initial turbulence amplitude and background density.

S.N.	L_{box} [d_i]	Grids	B_g	n_b	T_e/T_i	Δx	δb_{rms}	δu_{rms}	Type	Initial Conditions	Boundary conditions
1	149.6	4096^2	$B_z=1$	1.0	0.3/0.3	0.0365	$1/\sqrt{10}$	$1/\sqrt{10}$	Turbulence	Fourier modes ($2 \leq \|k\| \leq 4$)	Periodic
2	91.6	4096^2	$B_z=0$	1.0	0.01/0.05	0.0223	$1/\sqrt{5}$	0	Reconnection	Double Harris current sheet	Periodic

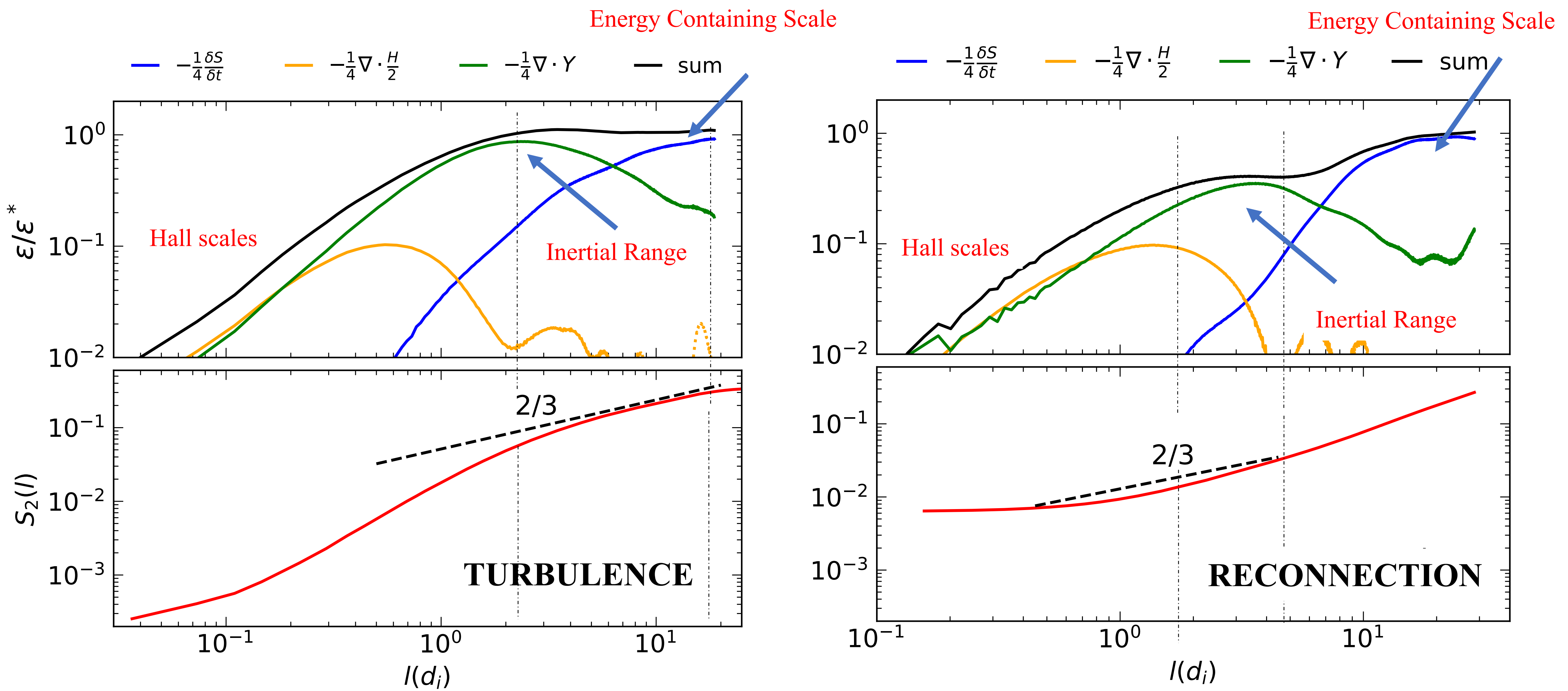
The PIC simulation of turbulence displays an energy cascade in the inertial range as seen by the constant energy transfer rate.



Cascade rate estimation for the turbulence simulation when current is maximum. The bottom plot represents the total second order structure function.

$\boldsymbol{\varepsilon}^*$ = energy decay rate obtained from the magnetic and Ion flow energy.

Similarly, reconnection simulaton also displays a constant cascade rate in the inertial range. Qualitative behavior of S, Y and H are identical.



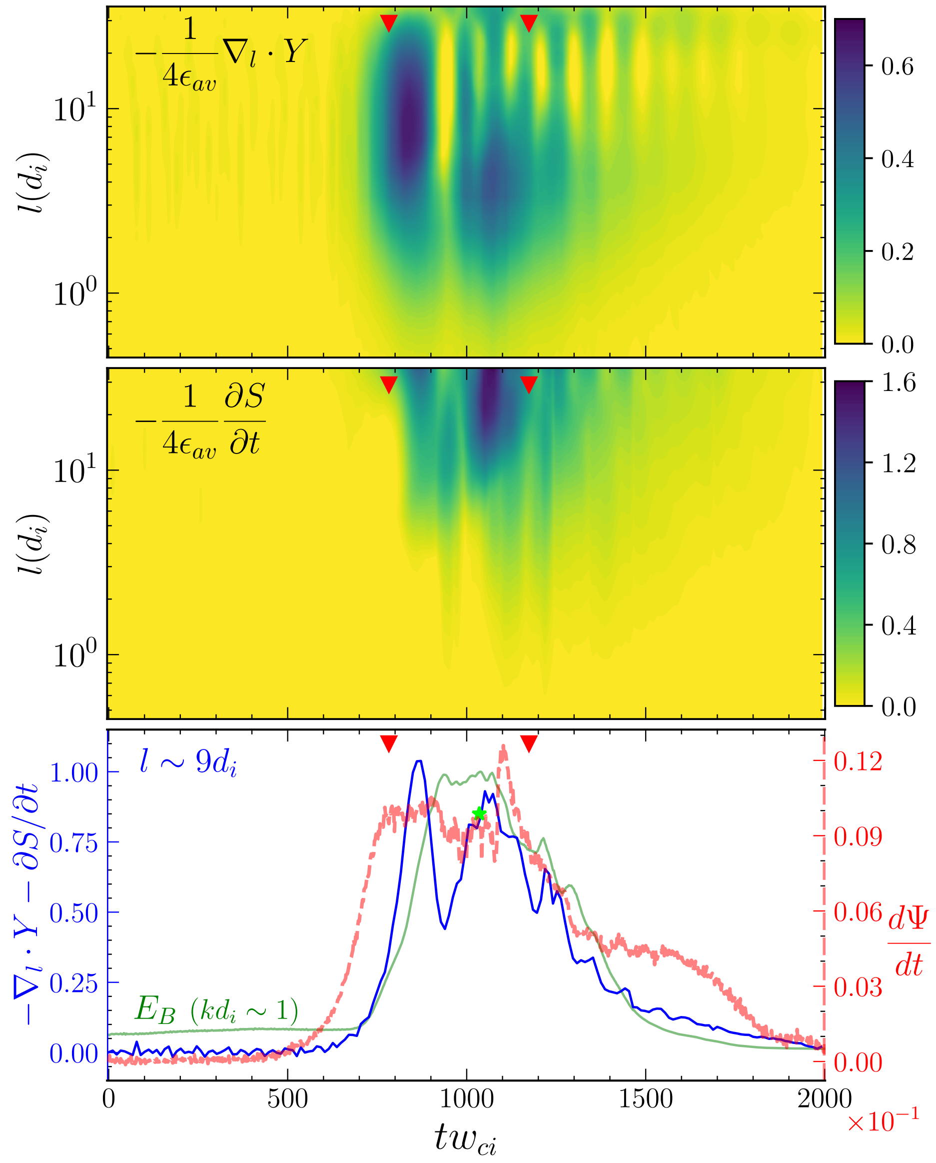
Estimation of von Kármán constant (C_{vk}) shows that for reconnection it is about 33% smaller than that of turbulence, something associated with the maximum size of energy containing scales.

Run	$k_{av} = \frac{2\pi}{L_{box}}\bar{k}$	$\lambda = \frac{1}{k_{av}}$	ϵ (x 10^{-4})	ϵ^* (x 10^{-4})	$\frac{\delta Z^3}{\lambda}$	$\frac{\epsilon}{\epsilon^*}$	$C_{vk} = \frac{\boldsymbol{\epsilon}^*}{\frac{\delta Z^3}{\lambda}}$
Turbulence	0.125	8	1.51	1.71	0.011	0.88	0.0155
Reconnection	0.068	14.7	0.24	0.636	0.006	0.38	0.0106

Time evolution of the Yaglom flux (Y), and second-order structure function (S) term shows their correlation with the reconnection rate of the system.

Shutdown of the cascade preceeds the shutdown of reconnection.

Energy at ion inertial scales (E_B) is correlated with the reconnection rate.

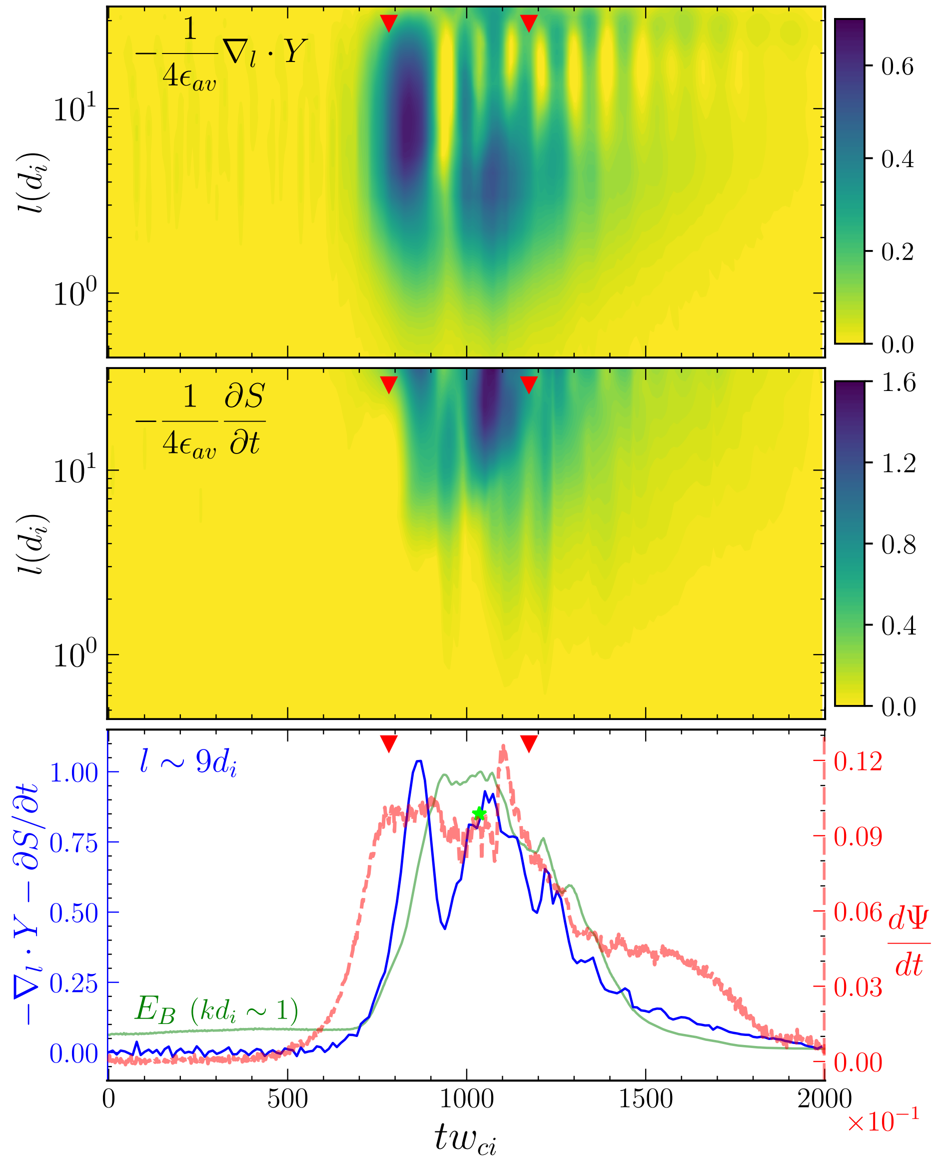


Conclusion:

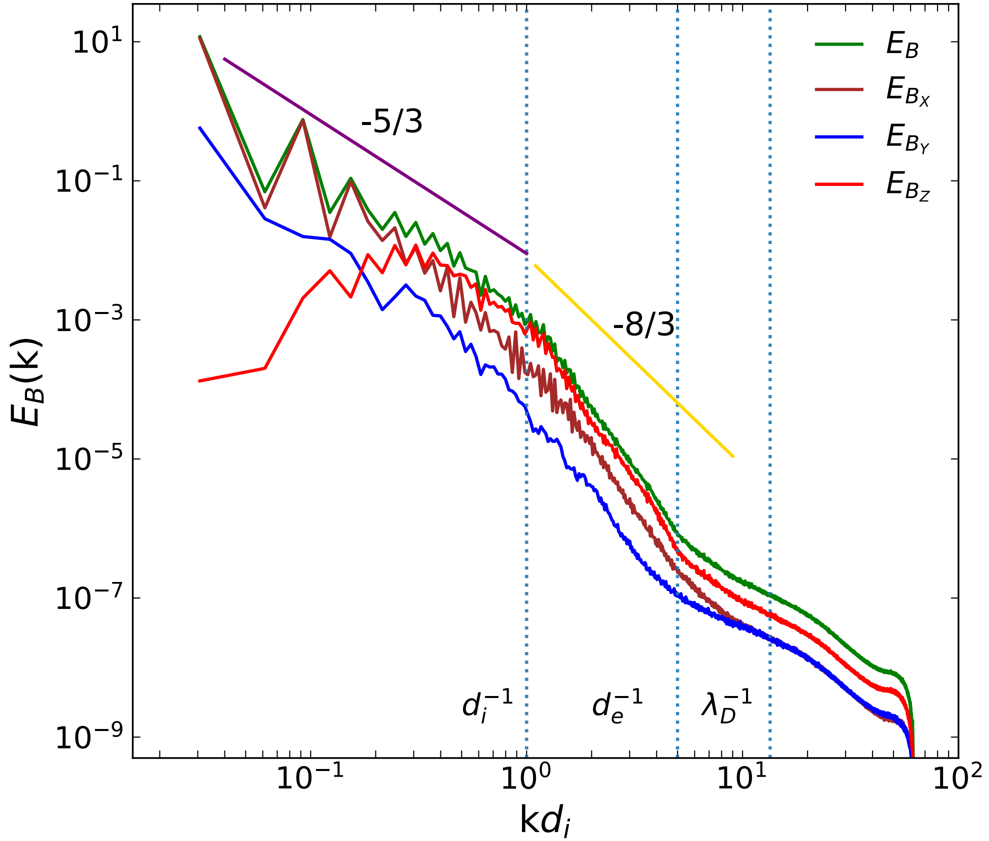
Reconnection is a standard turbulence cascade.

Shutdown of the cascade preceeds the shutdown of reconnection.

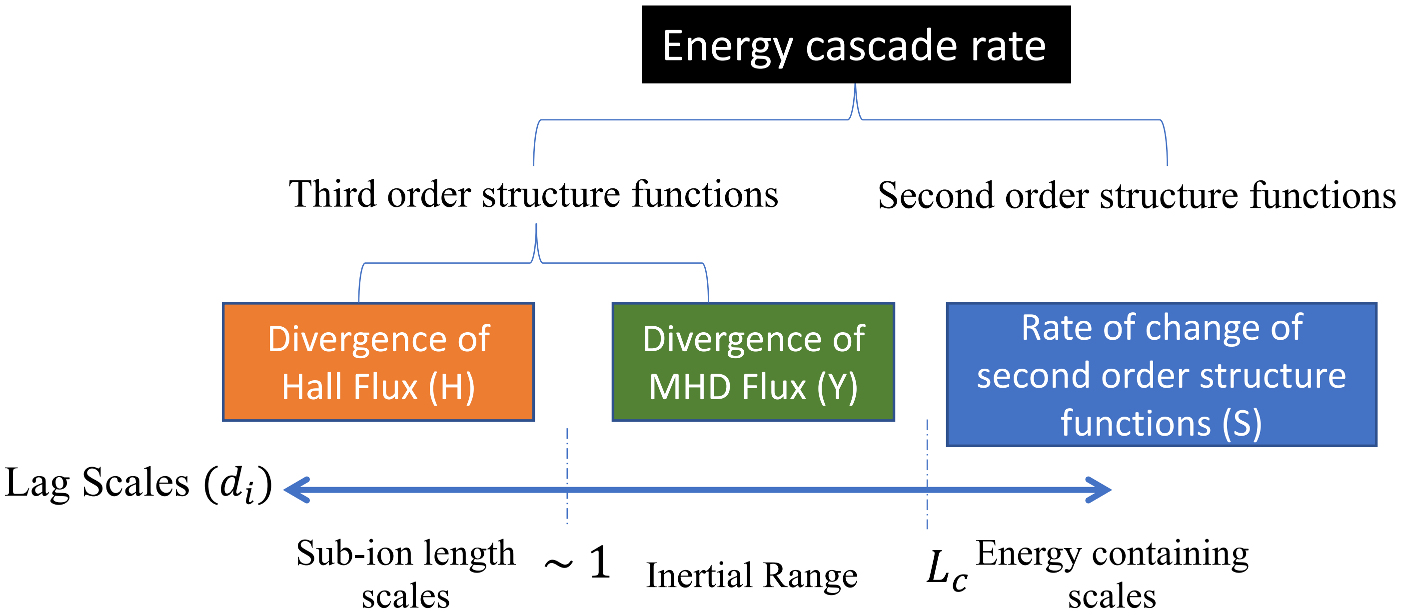
Energy at ion inertial scales (E_B) is correlated with the reconnection rate.



Motivation: Laminar reconnection exhibits a Kolmogorov like energy spectrum[1] (spectral index -5/3). How about the energy trasnfer? Is reconnection a cascade process?



Omni-directional magnetic energy spectrum for quasi-steady reconnection ([1]Adhikari et al., Physics of Plasmas **27**, 042305 (2020).



Energy cascade rate related to the structure functions with the lag scale of their dominance.

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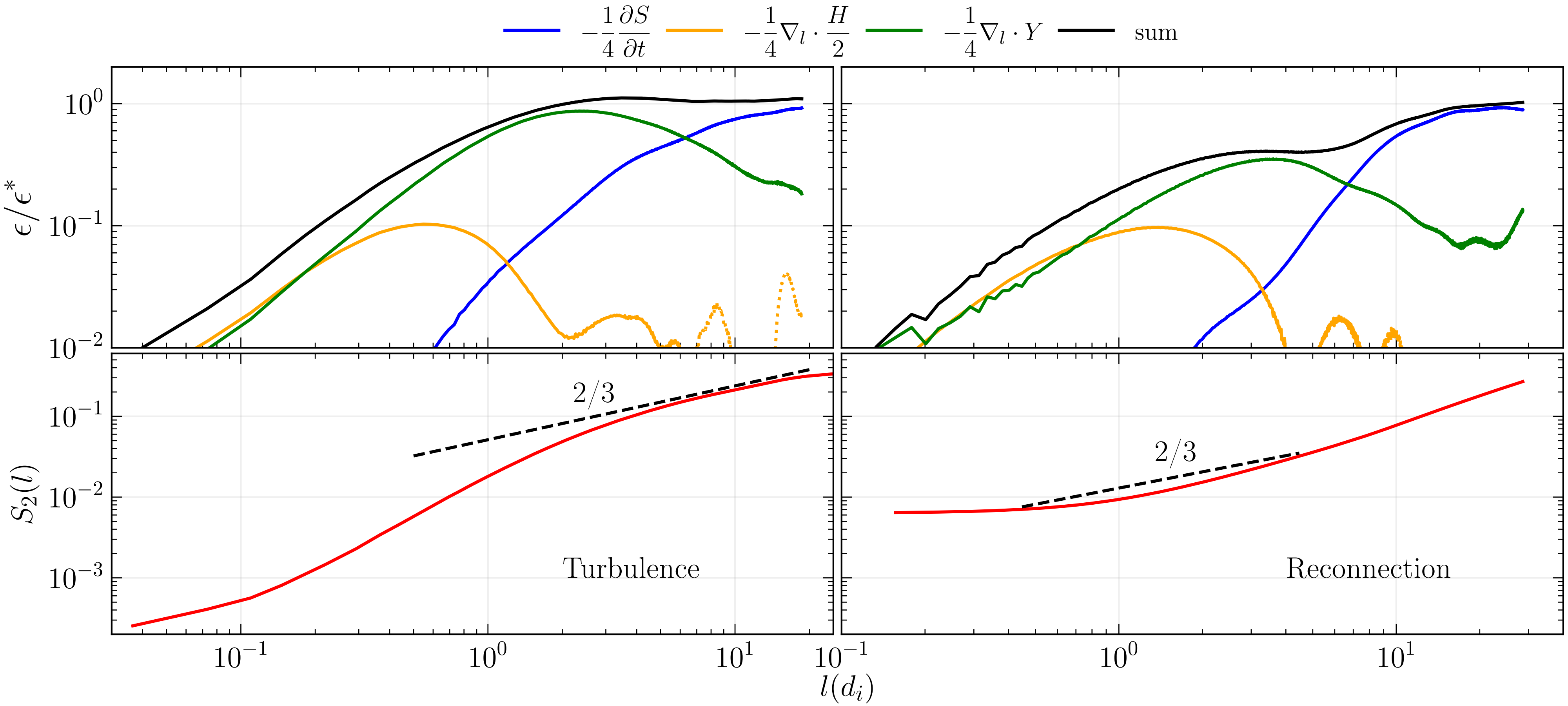
Results: Strong similarities of the energy cascade process in the turbulence & reconnection case. A constant cascade rate is observed in the inertial range.



Cascade rate estimation (top) for PIC simulations of turbulence (left), reconnection (right) and total second order structure function (bottom) versus lag.

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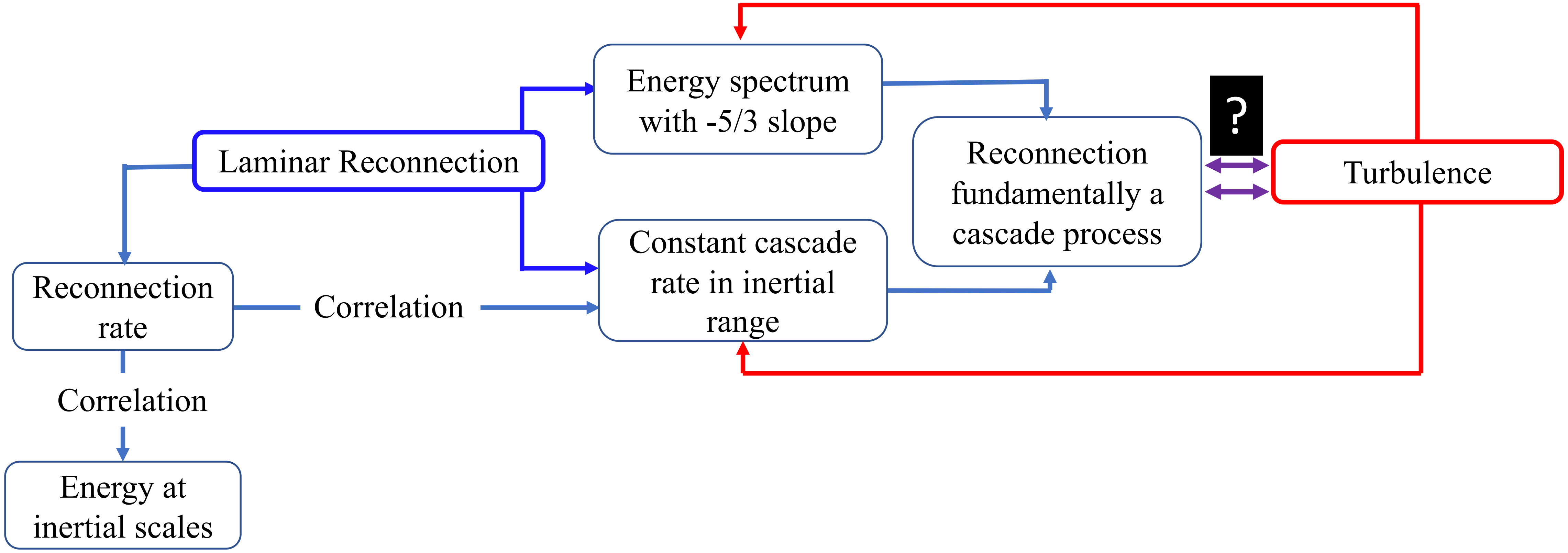
Significance: Is there a universal law governing reconnection and turbulence?



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