Parametric Decay: A Comparison of Analytic and Numerical Results

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INTRODUCTION

Alfvénic fluctuations are important in a range of astronomical scenarios. In the solar corona and solar wind, Alfvén waves transport fluctuating magnetic energy. Furthermore, circularly polarized Alfvén waves of any amplitude have long been recognized to be exact solutions of the nonlinear ideal magnetohydrodynamic (MHD) equation (Ferraro 1955) . Alfvén waves in solar wind have large amplitudes so that nonlinear wave–wave and wave–particle interactions are expected to be important. E. g., in compressible plasmas, large amplitude Alfvén waves are subject to a parametric decay instability (PDI), where a forward-propagating Alfvén wave (pump wave) decays into a backward-propagating Alfvén wave and a forward-propagating ion acoustic wave or slow wave. Therefore, extensive theoretical research has been conducted on parametric decay instabilities of Alfvén waves in quiescent plasmas.

Sagdeev & Galeev (1969) were the first to study the evolution of monochromatic Alfvén waves, demonstrating that a coherent circularly polarized Alfvén wave was unstable and decayed into a backward-propagating Alfvén wave and a sound wave in the low-beta limit. Following that, various writers relaxed several limiting assumptions to study the stability properties of finite-amplitude Alfvén waves.

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METHOD

The low-frequency behavior of a magnetized plasma is represented by the set of ideal MHD equations for which a large-amplitude, circularly polarized wave is an exact solution. Applying a linear perturbation analysis, one can find the dispersion relation governing instabilities, derived by Derby (1978) :

$$(\omega^2 - \beta k^2)(\omega - k)[(\omega + k)^2 - 4] = \eta^2 k^2 (\omega^3 - k\omega^2 - 3\omega + k) \quad (1)$$

where ω and k are normalized by the frequency and wavenumber of the pump Alfvén wave and $\eta = \delta B / B_0$. Note that β in Equation (1) is defined as C_s^2 / v_A^2.

To find instabilities one looks for a range of real values of k for which the dispersion relation yields complex roots for ω. The dispersion relation (1) is a fifth-order polynomial and can be solved numerically for the complex values of $\omega = \omega_r + i\gamma$.

A large-amplitude, circularly polarized Alfvén wave subject to the parametric decay instability(PDI) provides an unstable eigenmode consisting three waves: a longitudinal (sound-like) wave involving density fluctuations, and two transverse (Alfvén-like) waves involving magnetic field fluctuations.

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RESULT

Simulations are performed using the ideal MHD module of MRC code. The simulation domain is a 1-D box elongated along the background magnetic field: $\boldsymbol{B_0} = B_0\hat{z}$. Simulation grid size is (1, 1, 16384) in x, y, z direction, respectively and periodic boundary conditions are applied.

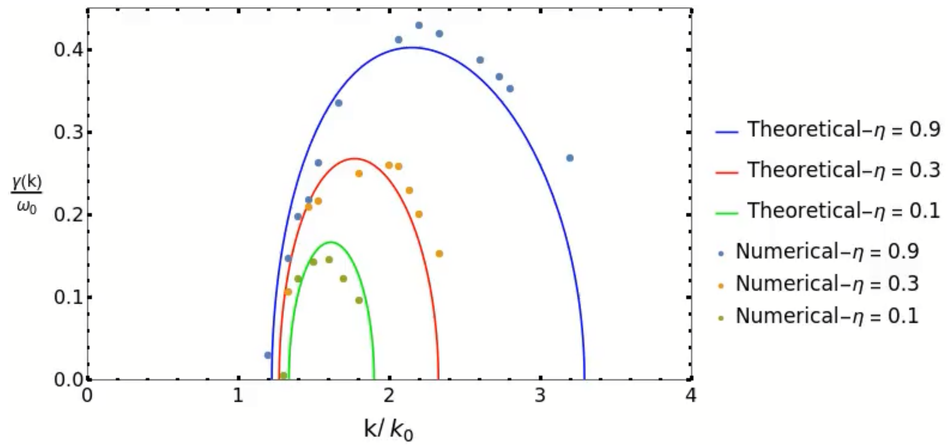

Figure 2. The growth rate of the daughter waves, γ(k), versus wavenumber for β, and for η= 0.1, 0.3 and 0.9

In Figure 2, graphs of theoretical and numerical values of γ(k) versus k are shown for β= 0.1, and for three different values of η. The linear theory predicts that equation (1) can have complex solutions for ω only when $k_L < k < k_U$, where the values of k_L and k_U depend on β and η. The range of k for PDI to occur, calculated for β = 0.1, and η = 0.1, 0.3, and 0.9 are (1.33, 1.89), (1.27,2.32) and (1.22, 3.29), respectively. For our simulations, the k values for the daughter waves were selected from the

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CONCLUSIONS

We have performed 1D ideal MHD simulations to study the parametric decay instability of a circularly polarized Alfvén wave. We inject a circularly polarized Alfvén wave of amplitude δB as the pump wave and four Alfvén waves of amplitude 10^{4} δB to produce a background in a simulation domain with a uniform background in density, pressure and magnetic field.

We employ the spatial FFT method to analyze the magnetic growth rates from the simulations with the theoretical linear MHD growth rates. We find that the growth rates of daughter waves and the range of unstable wavenumbers are consistent with those predicted by the linear theory. As one may expect, the maximum growth rate increases as $\eta = \delta B / B_0$ gets bigger.

Future Work:

- The model made use of one-dimensional geometry simplification. However, since natural systems are not limited to a one-dimensional geometry only, it is crucial to understand the consequences of higher dimentionality.
- PDI provides not only a robust mechanism for generating backward-

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