Time-magnitude correlations and the time variation of the Gutenberg-Richter parameter in foreshock sequences

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Main point #1: (140 character limit including spaces) Time-magnitude correlations lead to a modified Gutenberg-Richter distribution

Main point #2: (140 character limit including spaces) Time distribution of the precursory magnitudes may give information about the main shock

Main point #3: (140 character limit including spaces) The above information is criticaly discussed

Abstract

The time dependence of the parameter of the Gutenberg-Richter (GR) magnitude distributions is computed for correlated foreshock sequences of earthquakes, by using the geometric-growth model of earthquake focus, the magnitude distribution of correlated earthquakes and the time-magnitude correlations, derived recently. It is shown that this parameter decreases in time in the foreshock sequence, from the background values down to the main shock. If correlations are present, this time dependence and the time-magnitude correlations provide a tool of monitoring the foreshock seismic activity. We discuss a possibility of estimating the moment of occurrence of a main shock by such an analysis of a foreshock sequence. The discussion is applied to two Vrancea main shocks. The appreciable limitations of such an estimation are discussed.

Plain language: Precursory seismic events may be correlated to the seismic main shock. If so, they may provide relevant information about the magnitude and occurrence time of the main shock, although with great limitations. We discuss the subject in this paper.

Key words: Gutenberg-Richter parameters; foreshock-aftershock sequences; correlated earthquakes

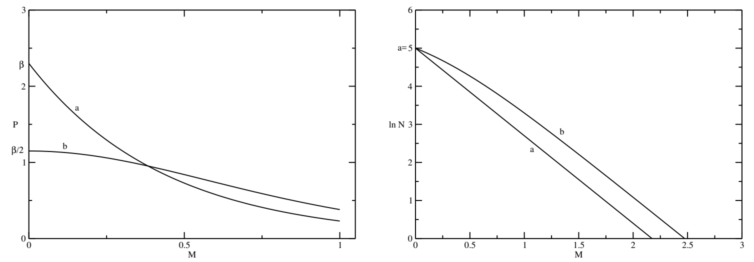
Recently, Gulia and Wiemer (2019) suggested that the difference between the parameters (β) of the Gutenberg-Richter (GR) magnitude distribution of the aftershocks and the foreshocks can be used to estimate the occurrence of main shocks. Acccompanying earthquake sequences have been analyzed by these authors for the Amatrice-Norcia earthquakes (24 August 2016, magnitude 6.2; 30 October 2016, magnitude 6.6) and the Kumamoto earthquakes (15 April 2016, magnitude 6.5 and 7.3). While the foreshock parameter β is lower than the background value (*e.g.*, by 10%), the value of the aftershock parameter is higher than the background value (*e.g.*, by 20%). A similar decrease in the parameter β has been reported for the foreshocks of the L'Aquila earthquake (6 April 2009, magnitude 6.3) by Gulia et al (2016) and the Colfiorito, Umbria-Marche, earthquake (26 September 1997, magnitude 6) by De Santis et al (2011).

The standard GR magnitude (M) distribution is $P(M) = \beta e^{-\beta M}$, where the parameter β varies in the range 1.15 to 3.45 (in decimal logarithms 0.5 to 1.5). The mean value $\beta = 2.3$ (in decimal logarithms $\beta = 1$) is usualy accepted as a reference value (Stein and Wysession, 2003; Udias, 1999; Lay and Wallace, 1995; Frohlich and Davis, 1993). It has been shown (Apostol, 2006) that $\beta = br$, where $b = 3.45$ (in decimal logarithms $3/2$) and r is a parameter characterizing the earthquake focus. This parameter varies beween $r = 1/3$ and $r = 1$, with a mean value $r = 2/3$ ($\beta = 2.3$). The standard cumulative (excedence) GR distribution (earthquakes with magnitude greater than M) is $P_{ex}(M) = e^{-\beta M}$; it is used in its logarithmic form $\ln N(M) = \ln N(0) - \beta M$, where $N(M)$ is the number of earthquakes with magnitude greater than M.

According to these standard formulae, an increase in β indicates the occurrence of more small-magnitude earthquakes, which may appear in the aftershock region, while a decrease in β indicates, comparatively, more greater-magnitude earthquakes. A decrease in β in the foreshock region has been reported in many instances (see, *e.g.*, Gulia et al, (2016) and References therein), as well as an increase in the aftershock region (Gulia et al, 2018). In principle, a statistical description of the accompanying seismic activity implies a symmetric distribution in the foreshock-aftershock regions. However, after a main shock the condition of the seismic region may change appreciably, such that it is difficult to view the foreshocks and the aftershocks as members of the same statistical ensemble.

Earthquakes associated in time and space, like the earthquake sequence accompanying a main shock, may exhibit correlations. The magnitude distribution of the correlated earthquakes differs from the standard Gutenberg-Richter distribution discussed above (Apostol, 2021). Judged by their time-dependence shape, the first part of the foreshock distribution indicated by Gulia and Wiemer (2019) may exhibit correlations, but the existence of correlations cannot be definitely assessed in the aftershocks distribution, where a change in the seismicity conditions may be present. We discuss below a possible relevance of a correlated foreshock sequence for the occurrence of a main shock.

The correlation-modified magnitude distribution (modified GR distribution,



(a) (b)

Figure 1: The standard GR distribution $\beta e^{-\beta M}$ (panel (a), curve a) compared to the correlation-modified GR distribution, equation (1) (panel (a), curve b) and the standard cumulative GR distribution $\ln N = a - \beta M$ (panel (b), curve a) compared to the correlation-modified cumulative GR distribution, equation (3) (panel (b), curve b) for $\beta = 2.3$ and an arbitrary value $a = 5$.

Apostol, 2021) is

$$P^c(M) = \beta e^{-\beta M} \frac{2}{(1+e^{-\beta M})^2} \ ; \tag{1}$$

without other specifications, this distribution includes the so-called dynamical correlations, which affect mainly the small-magnitude earthquakes. We expect such correlations to be present in foreshock sequences. From equation (1) we get the correlation-modified cumulative distribution

$$P^c_{ex}(M) = e^{-\beta M} \frac{2}{1+e^{-\beta M}} \ . \tag{2}$$

The logarithmic form of this distribution

$$\ln N^c(M) = \ln N(0) + \ln 2 - \ln\left(1 + e^{\beta M}\right) \tag{3}$$

should be compared to the standard logarithmic form

$$\ln N(M) = \ln N(0) - \beta M \ . \tag{4}$$

We can see that the modified GR distributions (equations (1) and (2)) differ from the standard GR distributions, as shown in Fig. 1. It seems that such a qualitative difference has been found for southern California earthquakes recorded between 1945-1985 and 1986-1992 (Jones, 1994). The difference arises mainly in the small-magnitude region $M \lesssim 1$, where the distributions are flattened. For instance, in this region the parameter β of the cumulative distribution tends to $\beta/2$, according to equation (3). This deviation, known as the roll-off effect (Pelletier, 2000; Bhattacharya et al, 2009), is assigned usually to an insufficient determination of the small-magnitude data. We can see that it may be due, at least partially, to correlations. For large magnitudes the logarithmic cumulative distribution is shifted upwards by $\ln 2$ (equation (3)), while its slope is very close to the slope of the standard cumulative GR distribution (β).

The correlation-modified cumulative distribution given by equation (2) can be used to identify a correlated sequence of foreshocks. We consider a seismic region with a background of (regular) earthquakes extended over a long period of time T, interrupted from time to time by (rare) big seismic events. We may assume that some of these large earthquakes are main shocks in associated sequences of correlated foreshocks (and aftershocks). For moderate and large magnitudes we may fit the seismic activity by the standard cumulative GR distribution given by equation $\ln N(M) = \ln N(0) - \beta M$. Usually, such fits are done by using a small-magnitude cutoff, such that the slope of the distribution (β) is not affected by correlated small-magnitude earthquakes. It is convenient to introduce the seismicity-rate parameter $t_0 = T/N(0)$, which, due to the small-magnitude cutoff, becomes a fitting parameter. The standard GR cumulative distribution reads

$$\ln\left[N(M)/T\right] = -\ln t_0 - \beta M \ . \tag{5}$$

By fitting this law to the empirical data we get the parameters β (and r) and t_0. For instance, such a fit, done for a set of 3640 earthquakes with magnitude $M \geq 3$ which occurred in Vrancea during $1981-2018$, leads to $-\ln t_0 = 11.32$ (t_0 measured in years) and $\beta = 2.26$ ($r = 0.65$), with an estimated 15% error. We note that the value $\beta = 2.26$ is close to the reference value given above (2.3). (The data for Vrancea have been taken from the Romanian Earthquake Catalog, 2018; a completenesss magnitude $M = 2$ is estimated and the magnitude average error is $\Delta M = 0.1$). A similar fit, with slightly modified parameters, holds for 8455 Vrancea earthquakes with magnitude $M \geq 2$ (period $1980-2019$). This way we get the parameters of the background sesimic activity (β, r, t_0).

Let us assume now that we are in the proximity of a main shock with magnitude M_0, at time τ until its occurence, and we monitor the sequence of correlated foreshocks. It was shown (Apostol, 2021) that the magnitudes of the (correlated) foreshocks M ($< M_0$) are related to the time τ by

$$M = \frac{1}{b}\ln(\tau/\tau_0) \ , \tag{6}$$

where

$$\tau_0 = rt_0 e^{-b(1-r)M_0} \tag{7}$$

is a cutoff time, which depends on the magnitude of the main shock, the seismicity-rate parameter t_0 and the parameter $r = \beta/b$. All these parameters are provided by the analysis of the background seismic activity. The small threshold time τ_0 corresponds to a very short quiescence time (Ogata and Tsuruoka, 2016) before the occurrence of the main shock. In addition, the time τ should be cut off by an upper threshold, corresponding to the magnitude of the main shock ($\tau < \tau_0 e^{bM_0}$). We limit ourselves to small and moderate magnitudes M in the accompanying seismic activity, such that the magnitude of the main shock may be viewed as being sufficiently large (in this respect, the so-called statistical correlations discussed by Apostol, (2021), are not included). Equation (6) is derived by analyzing the time-magnitude correlations predicted by the geometric-growth model of earthquake focus

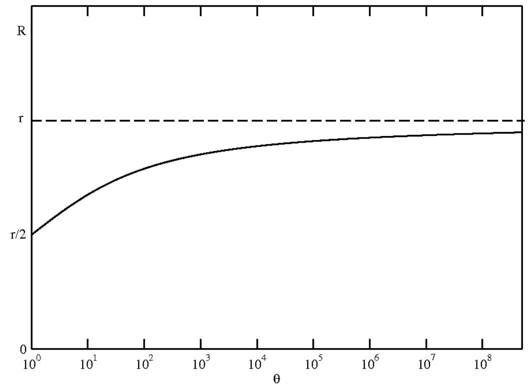


Figure 2: Function $R(\theta)$ vs θ for $r = 2/3$ (equation (9)).

(Apostol, 2006). According to this model the accumulation time of an earthquake with energy E is $t = t_0(E/E_0)^r = t_0e^{\beta M}$, where E_0 is a cutoff energy. By means of this model, Bath's law is derived and the occurrence time of the Bath partner is calculated, as well as the cumulative magnitude distribution of the accompanying seismic activity.

The distribution given by equation (2) indicates a change in the parameter β of the standard GR distribution. We denote by B the modified parameter β; it is given by

$$e^{-\beta M}\frac{2}{1+e^{-\beta M}} = e^{-BM} \quad , \tag{8}$$

where B is a function of M ($B(M)$). It is convenient to introduce the ratio $R = B/b$ (similar to $r = \beta/b$ given above), such that equation (8) becomes

$$R = \frac{1}{\ln\theta}\ln\left[\frac{1}{2}\left(1+\theta^r\right)\right] \quad , \tag{9}$$

where $\theta = \tau/\tau_0$. The parameter R varies from $R = r$ for large values of the variable θ to $R = r/2$ for $\theta \to 1$ ($\tau \to \tau_0$). The function $R(\theta)$ is plotted in Fig. 2 *vs* θ for $r = 2/3$. The decrease of the function $R(\theta)$ for $\theta \longrightarrow 1$ indicates the presence of the correlations.

According to equation (8), the modified GR parameter B is given approximately by

$$B(M) \simeq \beta - \frac{\ln 2}{M} \ , \tag{10}$$

or

$$R(\tau) \simeq r - \frac{\ln 2}{\ln(\tau/\tau_0)} \tag{11}$$

for a reasonable range of foreshock magnitudes $M > 1$. Equations (9)-(11) show the decrease of the GR parameter in a foreshock sequence. For instance, a 10% decrease is achieved for $M = 3$, or $\tau/\tau_0 \simeq 3.6 \times 10^4$ ($\beta = 2.3$, $r = 2/3$). It is worth noting that smaller magnitudes occur in the sequence of correlated foreshocks for shorter times, measured from the occurrence of the main shock (the nearer main shock, the smaller correlated foreshocks).

On the other hand, the time-magnitude correlations expressed by equation (6) lead to $\tau = \tau_0 e^{bM}$ for the accumulation time elapsed from the main shock to an aftershock. This relation shows a change in the seismicity conditions, where t_0 is replaced by τ_0 and β is replaced by b in the regular accumulation time $t = t_0 e^{\beta M}$. The magnitude distribution $(t_0/t^2)\, dt = \beta e^{-\beta M} dM$, which follows from this accumulation time (Apostol, 2006), is changed in this case to $be^{-bM} dM$, which indicates an increase in the GR parameter ($b = 3.45$) with respect to its background value β. Such a deviation holds up to a cutoff magnitude M_c where the two distributions become equal, such that we may estimate an average increase in the parameter β as $(b - \beta)/2\beta = 25\%$ for $\beta = 2.3$. The cutoff magnitude is given by $be^{-bM_c} = \beta e^{-\beta M_c}$, whence $M_c = 0.36$ for $r = 2/3$, $b = 3.45$ ($\beta = 2.3$). Both these estimated deviations of the GR parameter for foreshocks and aftershocks are in agreement with data reported by Gulia et al (2016, 2018) and Gulia and Wiemer (2019).

The logarithmic law expressed by equation (6) for the time-magnitude correlated foreshocks provides a means of estimating the occurrence time of the main shock. Indeed, if we update the slope B of the cumulative distribution $\ln[N(M)/N(0)] = -BM$ at various successive times t (equation (8)), and if this B fits equation (10), then we may say that we are in the presence of a

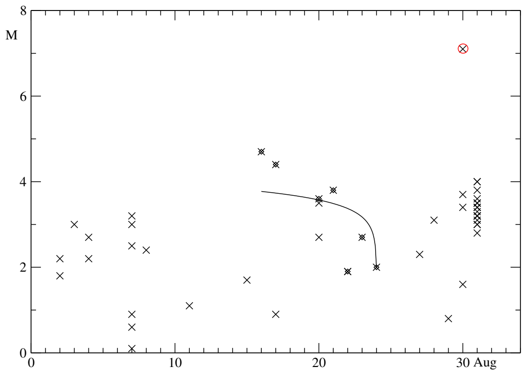


Figure 3: Vrancea seismic activity in the period 1 August - 31 August 1986 (Romanian Earthquake Catalog, 2018). The curve is the fit of equation (12) to data from 16 August to 24 August (fitting parameters $t_{ms} = 24$ August and $\tau_0 = 10^{-4.76}$ days; see the text).

correlated sequence of foreshocks which may announce a main shock at the moment $t_{ms} = t + \tau$. (In particular, the probability of occurrence of a main shock with magnitude M_0 increases in this case by a factor $\frac{\overline{B}}{\beta} e^{(\beta-\overline{B})M_0}$, where $\overline{B}$ is the average value of the parameter B). For practical use it is more convenient to use directly equation (6), which leads to the time dependence

$$M(t) = \frac{1}{b} \ln \frac{t_{ms} - t}{\tau_0} \tag{12}$$

of the foreshock magnitudes, for $(1-r)t_{ms} < t < t_{ms} - \tau_0$ $(0 < M < M_0)$. This formula provides an estimate of the occurrence moment of the main shock t_{ms} from the correlated-foreshock magnitudes $M(t)$ and the background seismicity parameter τ_0; the occurrence time is given by

$$t_{ms} = t + \tau_0 e^{bM(t)} \ . \tag{13}$$

It is worth noting that the time t_{ms} depends on the magnitude of the main shock, as expected (M_0, which enters τ_0, equation (7)). For instance, a magnitude M indicates a time $\tau = \tau_0 e^{bM}$ up to the main shock (equation

(6)). Let us assume that we are interested in a main shock with magnitude $M_0 = 7$; then by using $t_0 = e^{-11.32}$ (years, for Vrancea) and $r = 2/3$ given above, we get $\tau_0 = \frac{2}{3}10^{-8.42}$ (years); a foreshock with magnitude $M = 5$ would indicate that we are at $\tau = \frac{2}{3}10^{-8.42}10^{7.5} = 0.079$ years, *i.e.* $\simeq 29$ days, from that main shock. The time t_{ms} of the occurrence of the main shock is obtained from equation (12) as a fitting parameter of the correlated-foreshock magnitudes $M(t)$. In practice, it is also convenient to view τ_0 as a fitting parameter. Since, for moderate magnitudes, the variation of the parameter R is small (equations (10) and (11)), we may use the background value for r in the expression of τ_0 (*e.g.*, $r = 2/3$), which leads to an estimate of the expected main-shock magnitude M_0 from the fitting parameter τ_0. However, a reliable prediction of the time t_{ms} provided by equation (13) requires a very high slope of the decreasing magnitudes $M(t)$ in the neighbourhood of t_{ms}, which can only be attained by a special data set, including many small-magnitude foreshocks whose magnitudes fall rapidly to zero.

The application of equations (12) and (13) to fitting the correlated foreshocks, in order to forecast an occurrence time t_{ms} for a main shock with magnitude M_0 given by the other fitting parameter τ_0, involves certain particularities. First, we should note that not all the precursory seismic events are foreshocks correlated with the main shock. Second, small clusters of precursory events may exist, which may include second-order correlated earthquakes, *i.e.* events which accompany foreshocks, according to the epidemic-type model (see, for instance, Ogata, 1988, 1998). These secondary events have little relevance upon the main shock, such that they may be left aside. We limit ourselves to the highest foreshocks occurring in short periods of time (though an average magnitude for each small cluster may also be used). Third, the relevant part of the logarithmic curve given by equation (12) (or the exponential in equation (13)) is its abrupt decrease in the immediate proximity of t_{ms} (of the order of days for Vrancea), such that we should limit ourselves to foreshocks which occur in the last few days. In this regard, a reliable estimation of forecasting parameters is conditioned by a rich seismic activity in the immediate vicinity of the occurrence moment of a main shock (this would be a very short-time forecasting, of the order of days). This limitation is related to the very small values of the parameter t_0 and the large magnitude M_0, of interest for the main shock (small values of the parameter τ_0).

Unfortunately, these conditions cannot be met easily for Vrancea earthquakes, where the correlation statistics seems to be poor. Vrancea is the main seismic region of Romania. Reliable recordings of earthquakes started in Romania around 1980. Since then, three major earthquakes occurred in Vrancea: 30 August 1986, magnitude $M = 7.1$; 30 May 1990, magnitude $M = 6.9$; 31 May 1990, magnitude $M = 6.4$ (Romanian Earthquake Catalog, 2018). The 7.1-earthquake (depth $131km$) is shown in Fig. 3, together with all its precursory seismic events from 1 August to 31 August. All these earthquakes occurred in an area with dimensions $\simeq 100km \times 80km$ ($45° - 46°$ latitude, $26° - 27°$ longitude), at various depths in the range $30km - 170km$, except for the events of 7-8 August and the 1.6-event of 30 August, whose depth was $5km - 20km$. Almost no subset of these earhquakes can be fitted by equation (12) with a reasonable accuracy. For example, the sub-set of earthquakes from 16 August to 24 August is fitted by equation (12) with the fitting parameters $t_{ms} = 24$ August, $\tau_0 = 10^{-4.76}$ days and a large rms relative error 0.32. The maximum magnitude has been used for the earthquakes which occurred in the same day, because, very likely, those with smaller magnitude are secondary accompanying events of the greatest-magnitude shock. If we assume that this is a correlated-foreshock sub-set, it would indicate the occurence of a main shock with magnitude 4.4 on 24 August. The three earthquakes from 27 August to 29 August may belong to a similar sub-set, which, however, is too poor to be relevant. A main shock with magnitude 7.1 ($\tau_0 = 10^{-6.06}$ days) and an average magnitude for the days with multiple events leads to a fit with a larger rms relative error 0.6. The quality of all these fits is poor. Moreover, we cannot identify a correlated sub-set of foreshocks for the earthquake pair of 30-31 May 1990 (depth $87-91km$), *i.e.* a sequence of precursory events with an average magnitude, or a maximum-magnitude envelope, decreasing monotonously in a reasonable time range. Another particularity in this case, in comparison with the earthquake of 1986, is the quick succession (30-31 May) of two comparable earthquakes (magnitude 6.9-6.4). Although of a very limited relevance for Vrancea, the method described above may be of interest for other seismic regions, where the correlated-foreshock statistics is richer (in the sense discussed above).

In conclusion, the GR distributions modified by correlations in the foreshock region and the time dependence of the foreshock magnitudes (Apostol, (2021)) can be used, in principle, to estimate the moment of occurrence of the main shock and its magnitude, although with appreciable limitations. The main source of errors arises from the quality of the fit $B(t)$ *vs* $M(t)$ (equation (10)), or, equivalently, the fit of the function $R(\theta)$ given by equation (9), or the fit given by equations (12) and (13). In order to improve the quality of these fits we need a rich correlated-foreshock activity in the immediate proximity of the main shock. Another source of errors arises from the background parameters t_0 and r (β), which may affect considerably the exponentials in the formula of the time cutoff τ_0 (equation (7)). Also, a limitation of the procedure described above arises from the fact that a relevant decrease of the parameters $B(M)$, $R(\tau)$ or $M(t)$ occurs for small values of the variables M and τ, *i.e.* very near to the occurrence moment of the main shock. The procedure described above is based on the assumption that the foreshock magnitudes are ordered in time according to the law given by equation (6). However, according to the epidemic-type model (see, for instance, Ogata, 1988, 1998), the time-ordered magnitudes may be accompanied by smaller-magnitudes earthquakes, such that the law given by equation (6) may exhibit lower-side oscillations, and the slope given by equation (11) may exhibit upper-side oscillations. Several sub-sets of correlated foreshocks may be identified (in accordance with the epidemic-type model), as well as the absence of correlations. However, the decrease of the GR parameter in the correlated foreshock sequences and its increase in aftershock sequences remain a valuable piece of information.

Acknowledgements

The authors are indebted to the colleagues in the Institute of Earth's Physics, Magurele-Bucharest, for many enlightening discussions. This work was partially carried out within the Program Nucleu 2019, funded by Romanian Ministry of Research and Innovation, Research Grant #PN19-08-01-02/2019. Data used for the Vrancea region have been extracted from the Romanian Earthquake Catalog, 2018.

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