The Perturbed Rainfall Uncertainty Unit Hydrograph Method

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Key Point:

- The PeRUUH Method derives a more reliable Unit Hydrograph when the special and temporary uncertainty in the rainfall cannot be neglected

The Perturbed Rainfall Uncertainty Unit Hydrograph Method

The Unit Hydrograph (UH) theory can be used to estimate the discharge hydrograph of a basin based on precipitation and runoff measurements. However, uncertainty in those measurements may unable a unique UH to be derived. It is well known that in comparison with runoff measurements, the spatial and temporal variability of rainfall is the main source of uncertainty. Hence, in this work we present a method to derive an improved unique UH assuming that the rainfall data is the main source of uncertainty. The Perturbed Rainfall Uncertainty Unit Hydrograph (PeRUUH) method is verified using a large scale rainfall simulator (LSRS). The method combines linear programming and reverse modelling to generate a perturbed rainfall intensity pattern, which accounts for the temporal and spatial rainfall uncertainty assuming discharge to be true. The method is validated in three different rainfall events using the LSRS. The PeRUUH method is more robust than the original UH, as it is shown to be able to estimate a unique UH for which the discharges have an error of (approx.) 10 times smaller than its counterpart.

Keywords: unit hydrograph; rainfall intensity; rainfall simulation; uncertainty control; runoff discharge hydrograph estimation

1. Introduction

A unit hydrograph (UH) is defined as a direct runoff hydrograph resulting from 1 mm of excess rainfall fallen uniformly over the drainage area for an effective duration (with unit intensity and duration). The UH theory (Dooge, 1959) has been widely implemented and verified, and can be obtained either by an explicit or an implicit procedure (Chow et al., 1988). This simple linear model can be used to estimate runoff hydrographs and discharge peaks resulting from a known amount of excess rainfall. The deconvolution equation is used to derive a UH for a specific watershed given precipitation and runoff discharge measurements. In order to apply the method correctly, five requirements should be met:

1. The excess rainfall has constant intensity within the effective duration and has a short duration. A single-peaked hydrograph of short time base will be generated.

2. Excess rainfall is uniformly distributed throughout the drainage area.
3. Constant base time for the duration of the direct runoff hydrograph.
4. The ordinates of all direct runoff hydrographs of a common base time are directly proportional to the total amount of direct runoff represented by each hydrograph
5. For a given watershed the UH is unique and constant at remaining channel conditions and without appreciable storage.

If one of the requisites is not met, the derivation of the UH will not be correct. In this case, solving the deconvolution equation for different rainfall events in the same catchment area may generate different UHs. In those cases the UH theory cannot be implemented to estimate the runoff discharge hydrographs. Some authors have proposed solutions to obtain a single solution for the UH using the method of successive approximations (Chow et al., 1988), or accounted for the occurrence of negative UH ordinates (Al-Gazali, 2015).

There are many sources of uncertainty in hydrological modelling, for example: randomness of natural processes, model input data (including assumed parameter values), data uncertainties (measurement errors, inconsistency and non-homogeneity of data), operation and construction deficiencies, amongst others (Zhao et al., 1997). One of the major sources of uncertainty is the rainfall intensity, which may compromise the derivation of the UH (Rew and McCuen, 2012). In order to improve the results obtained, Williams, Cameron, & Evans (1980) proposed the use of different UH for different rainfall events, and suggested the use of an average UH for the catchment, which is obtained from the different UHs used. The inaccuracy of the results was justified by the non-linearity of the catchment. In this case, the variation in the effective rainfall is not considered as a source of error – it is considered to be constant and proportional to the real total rainfall. Another approach to reduce uncertainties in the UH method, is the one from developed by Hromadka (1991), which reformulates the UH method into a stochastic integral equation in order to prepare probabilistic distributions of design criterion variable values.

Some methods to reduce the uncertainty in rainfall data and to improve the discharge estimation using UHs, include the quantification of uncertainties for multiple-storm UH ordinates, called storm resampling, which is based on the bootstrap resampling technique (Zhao et al., 1997) and reverse modelling (Kretzschmar et al., 2016). The former produces tighter confidence intervals around the peak than the no storm-scaling. The latter assumes that the streamflow is better estimated than the observing rainfall.

Another approach to analyse the rainfall-runoff relation is to use rainfall simulation prototypes. Nevertheless, these apparatus have some limitations in producing homogenous temporal and spatially rainfall. In the simulator built by Al Ali, Bonhomme, Dubois, & Chebbo (2017), an Uniformity Coefficient of 73% was obtained and the spatial distribution of rainfall showed a higher rainfall concentration at the centre, directly below the nozzle. In this work, despite the inhomogeneity, the spatial variability was assumed to be acceptable, and the rainfall intensity was assumed to be temporally constant. Deng, De Lima, & Singh (2005) developed a model for solute transport. In their experiments, they also assumed a constant rainfall. However, in the determination of overall uncertainty from all input data and coefficients, the assumed constant rainfall was considered to be the only significant source of uncertainty in the simulation.

Iserloh et al. (2013) analysed different rainfall simulators, showing a wide range of generated intensities, drop size and kinetic energy, as well as a spatial variability among 1,2 and 13,2 %. Moreover, the rainfall simulators of Abudi, Carmi, & Berliner (2012) and Aksoy et al. (2012), show that the apparatuses require strenuous adjustments to be calibrated, and yet the rainfall intensity cannot be adjusted to an exact expected value. Hence, it can be understood that, when operating rainfall simulators, not all variables can be controlled completely, and therefore, some assumptions have to be met for simplification of the system; the most common assumption suggest that the spatial and temporal variability of the rainfall can be ignored.

Similarly to other experimental facilities (Isidoro et al., 2012a)(Isidoro et al., 2012b), the analysis of the rainfall variability made by Reis (2015) showed that the Large Scale Rainfall Simulator (LSRS) at the Federal University of Alfenas (UNIFAL-MG) has a higher rainfall intensity at the nozzle and a radial intensity decline, as it can be seen in Figure 1. Hence, it is clear that the application of UH derivation methods with the measurement on this LSRS, assuming a constant temporal and spatial rainfall intensity, will likely lead to incorrect results.

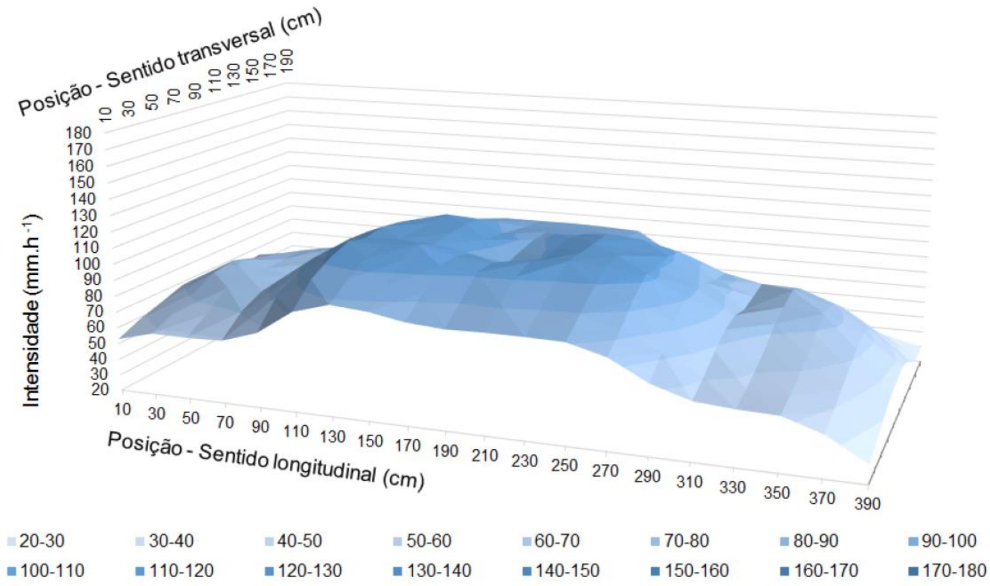

Figure 1: Spatial rainfall intensity distribution in the LSRS (Reis, 2015)

In order to reduce the errors when applying UH derivation methods, an alternative is to focus on more reliable input data, as for example the runoff discharge measurements. The work of Kretzschmar et al. (2016) is based on the fact that rainfall input data has inherent uncertainty because it is variable in time and space, which causes significant errors in hydrological models. To reduce the uncertainty, their work proposes the implementation of reverse hydrology methods, using the streamflow information of the catchment area to infer the rainfall over the whole catchment, rather than the amount measured at an individual rain gauge. The authors highlight that the "streamflow is better estimated using inferred rainfall than observed rainfall".

As such, we derive in this study the Perturbed Rainfall Uncertainty Unit Hydrograph (PeRUUH) method under the assumption that the runoff discharge measurements are more reliable (and less uncertain) than the rainfall input data. The PeRUUH method applies linear programming and inverse modelling to the UH theory, to obtain a perturbed rainfall intensity pattern that accounts for the temporal and spatial uncertainty in rainfall, and obtain a unique UH for a given watershed.

2. Large Scale Rainfall Simulator (LSRS)

2.1. Description of the Apparatus

The work was realized under laboratory conditions, simulating rainfall over an impermeable concrete surface of 4.3 m² and with an inclination of 8.1%. The rainfall was produced by a pressurized rainfall simulator created by Reis (2015), which consists of three conical sprinklers (FullJett® HH- W ¼) controlled by a solenoid valve and a digital manometer. The vertical distance from the nozzle to the concrete surface is 2,5 m. The digital manometer monitors the pressure to maintain the same experiment conditions. The solenoid valves control the start and stop of the flow and are connected to the sprinklers via a PVC tubes with a diameter of 0.0125 m, which are separated by 1.3 m. To feed the system, a motor-pump (power 1 CV) is connected through a PVC tube with a diameter of 0.025 m to a tank with a constant water level. All experiments where realized at a pressure of 0.11 MPa and a water temperature between 22.5 and 23.5 °C (Pessoti et al., 2020). The experimental apparatus can be observed in Figure 2.

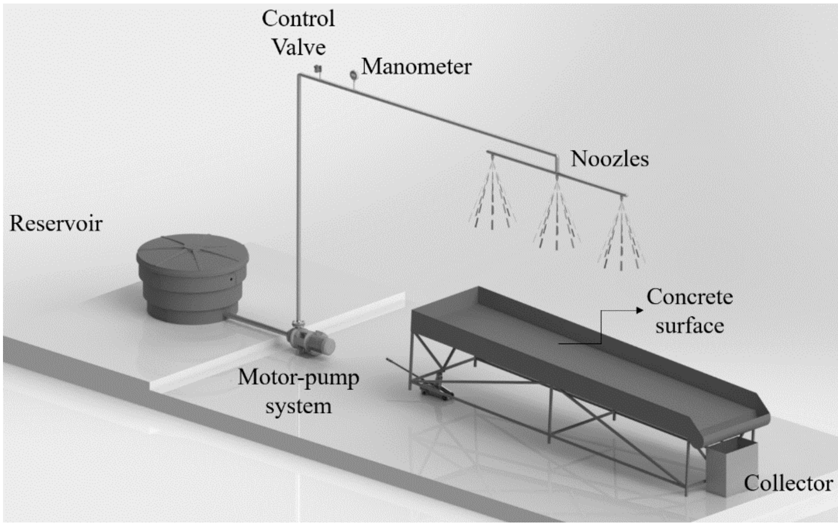

Figure 2: Experimental Apparatus for rainfall simulation and calculation of runoff hydrographs in a reduced impermeable surface (Pessoti et al., 2020) (Reis, 2015).

The runoff is collected periodically in a total of ten receptacles, and the discharge hydrograph is determined using the volumetric method. The measurement of the discharge hydrograph are measured straight forward by the volume of water leaving the LSRS. Hence, this measure can be considered highly accurate.

Even though the spatial variation of the apparatus has been analysed and quantified (as seen in Figure 1), for former experiments, the rainfall uncertainty was assumed to be acceptable and ignored.

2.2. 1.2 Description of the set of tests performed at the LSRS

Three different experiments with similar rainfall intensities were performed at the LSRS under the same conditions, which are presented in Table 1.

Experiment	A	B	C
Intensity [mm/s]	0.0198	0.0196	0.0200

Table 1: Average rainfall intensity of the experiments performed at the LSRS.

Nevertheless, the exact timely variation of the rainfall intensity has not been measured. Therefore, the values presented in Table 1 are an estimation of the average rainfall intensity of each experiment. These average rainfall intensity values are given by the total runoff discharge volume, which is measured accurately.

The runoff coefficient of the concrete surface of the LSRS is calculated to be between 95 and 98 %.

3. Methods

The Perturbed Rainfall Uncertainty Unit Hydrograph (PeRUUH) method is developed and tested with the data from the LSRS (Pessoti et al., 2020). The results of the PeRUUH method are compared with the UH theory as described in (Chow et al., 1988).

3.1. Description of conventional UH derivation

The Unit Hydrograph (UH) can be derived with two different methods, explicit and implicit:

Explicit method. Conventional method using the deconvolution equation, to calculate an UH having M-number of excess rainfall pulses (volume fallen in a given time span, including a specific rainfall intensity) and N-number of direct runoff pulses (runoff volume measured in a given time span). This is the reverse process from the discrete convolution equation, which is presented in EQ (1).

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1} \qquad \text{EQ (1)}$$

Where in Q_n is the computed value of the direct runoff, P_m is the excess rainfall and U_{n-m+1} is the unit hydrograph (Chow et al., 1988). All processes and results calculated with the explicit method will be notated with an "**e**" for identification, for example, the UH calculated with this methods is noted as "**UHe**".

Implicit method. This method derives the UH by matrix multiplication, using the formula:

$$[Q] = [P] * [U]$$

Where [Q] is the runoff discharge matrix, [U] is the UH matrix, and [P] is the rectangular matrix for precipitation. Complete details of the method can be found in (Chow et al., 1988). All processes and results calculated with this method are marked with "**i**" for identification.

3.2. Description of PeRUUH method

As mentioned in the introduction, the PeRUUH method applies linear programming and inverse modelling to the UH theory to generate a perturbed rainfall intensity (time) pattern. The volume of the rain, as well as the start and end time are easily measured, obtaining an average rainfall intensity. This homogenous pattern is the initial value for iterative calculation, which generates an optimized rainfall intensity pattern. This is done automatically for a given number of times "**j**". In the proposed method, each loop consists of four steps:

(I) The excess rainfall volume is randomly redistributed between the number of rainfall pulses, assigning a specific volume (representing a given intensity) to each rainfall pulse and keeping the sum of the total volume constant. A perturbed rainfall intensity pattern is generated in every loop. The new pattern is named "$\mathbf{Rw_j}$".

(II) For optimization of the calculation process, the UH is generated implementing the implicit method (section 2.1), using the original discharge runoff "$\mathbf{Q_{orig}}$" and the new perturbed rainfall intensity pattern (Rw). The array generated at this step is named "$\mathbf{UHw_j}$".

(III) A new runoff discharge hydrograph “$\mathbf{Qw_j}$” is calculated in each loop, using the given $Rw_{\mathbf{j}}$ and $UHw_{\mathbf{j}}$ values.

(IV) The calculated discharge hydrograph in each loop (Qw_j) is compared with the original discharge hydrograph (Q_{orig}). The Objective Function “$\mathbf{DifQ_j}$” is the sum of the differences of the calculated and the original discharge hydrograph at each point using the Root-mean-square deviation (RMSE), as seen in EQ (2).

$$DifQ_j = sum(abs(Qw_j - Q_{orig})) \qquad \text{EQ (2)}$$

This value represents the difference between the calculated runoff discharge hydrograph and the original one. A value of zero means that both hydrographs are the same. A value greater means that the hydrographs are different. This serves as an indicator of the improvement obtained by the PeRUUH method.

For each loop, the generated Rw_j and UHw_j are stored together with the respective $DifQ_j$, only if both following criteria are met: (i) $DifQ_j$ is smaller than the given limit value “$value_{limit}$”, and (ii) all values in the UH array are greater than zero.

After “j”-number of loops, all the stored $DifQ_j$ values are compared and the smallest (corresponding to the best fitting regenerated discharge hydrograph) is selected, together with the corresponding Rw_j values. With these values, the UH is derived using both methods (explicit and implicit; section 2.1). These UHs are named “$\mathbf{UHw_e}$” and “$\mathbf{UHw_i}$” (for the explicit and the implicit method respectively). If these UHs are equal, then an improved UH for the catchment area is found. If these UHs are quite similar but not exactly equal, an average of both arrays will be generated “**UHx**”.

If (for each value in the array): $UHw_e = UHw_i$

Then: $UHx = UHw_e = UHw_i$

Else: $UHx = (UHw_e + UHw_i)/2$

Using UHx (the improved UH for the catchment area) and Rw (the improved rainfall intensity pattern), the calculation of the runoff discharge hydrograph with the explicit and the implicit method (Qx_e and Qx_i) should be equal (or very similar) to the original discharge hydrograph. The whole calculation process can be seen in Figure 3.

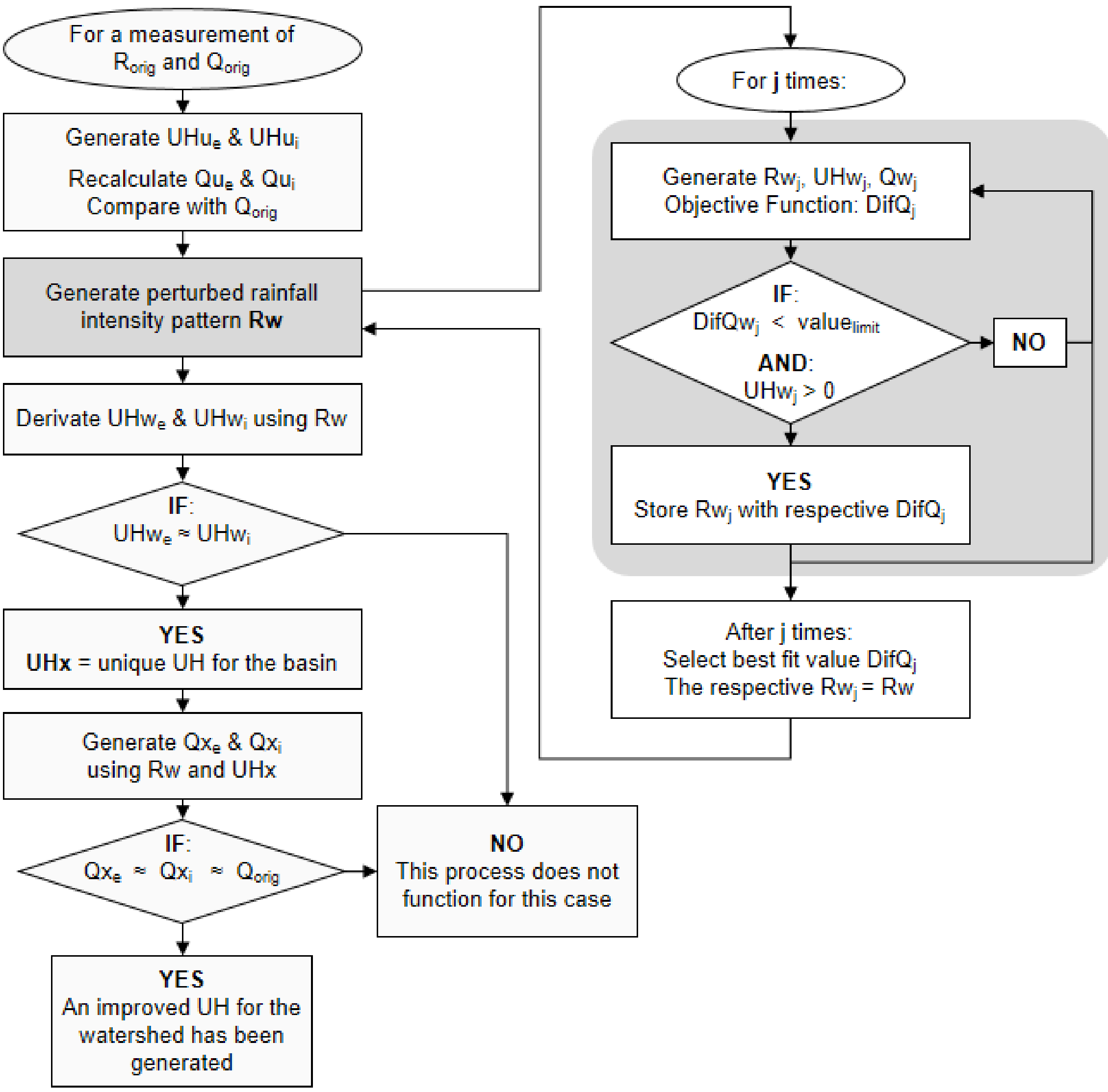

Figure 3: Process of the PeRUUH method

3.3. Validation of the method

After obtaining an improved UH with the PeRUUH method for one experiment (named A for identification), the same process is repeated independently for other experiments (named B and C for identification) under the same conditions but with different rainfall intensities. All perturbed rainfall intensity patterns (Rw_A, Rw_B, Rw_C) and generated UHs (UHw_{eA}, UHw_{iA}, UHw_{eB}, UHw_{iB}, UHw_{eC}, UHw_{iC}) for both derivation methods are then compared. If these are approximately the same (e.g. threshold of 3%), then it is shown that the method is able to derive a unique UH for the catchment area which is independent of the rainfall intensity, under the assumptions that rainfall input data is likely the main source of error, and discharges can be considered accurate

4. Results

4.1. Results of Conventional UH derivation

The UHs are derived using the original measured rainfall intensity and the original discharge hydrograph (Q_{orig}) for the explicit and implicit methods described in Description of conventional UH derivation. The resulting UH are compared in Figure 4.

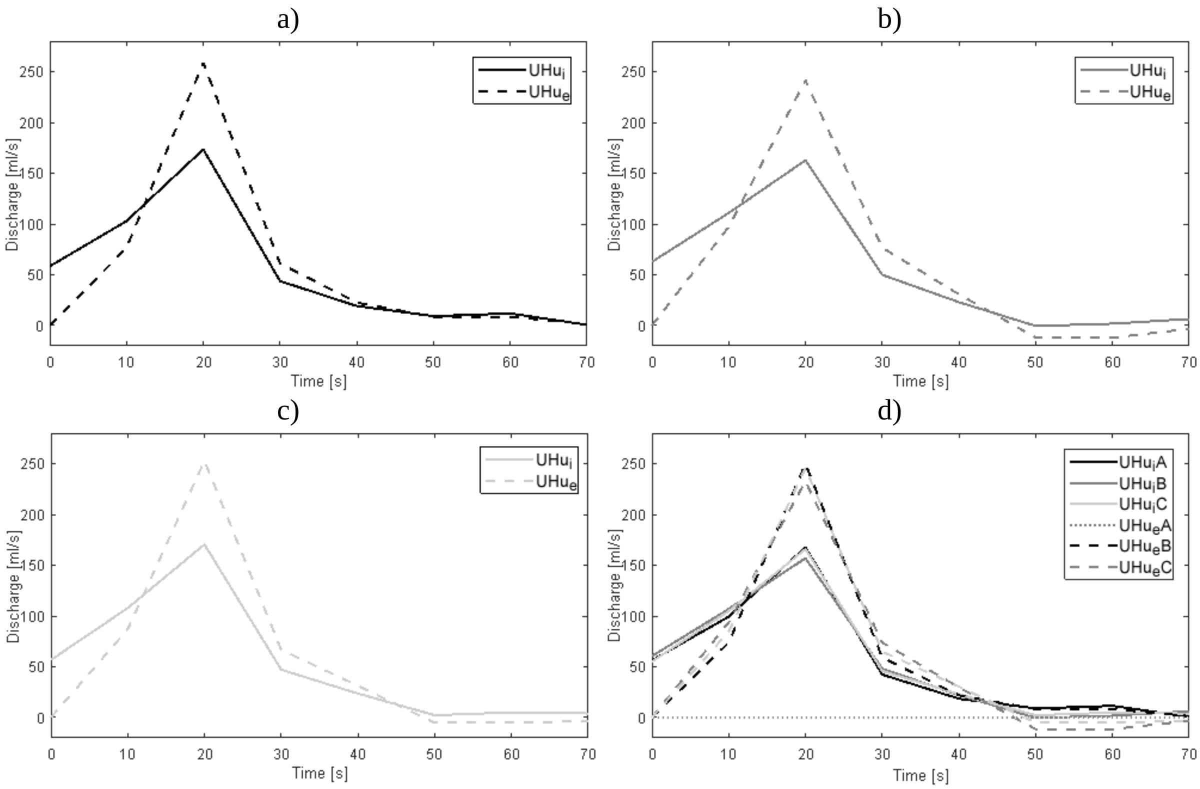

Figure 4: Resulting UHs using the conventional derivation methods. a-c) for experiment A, B and C respectively; d) all resulting UHs together.

The calculation of the Objective Function (difference between the original and the recalculated runoff hydrograph) for each experiment (A, B and C) and for each UH derivation method can be seen in Table 2 and in Figure 5.

Experiment	A		B		C	
	Qx_i	Qx_e	Qx_i	Qx_e	Qx_i	Qx_e
Total Volume [ml]	9 890		9 878		9 960	
Objective Function: DifQ	1044	917	1074	1045	1043	947
% Volume	10.6	9.3	10.9	10.6	10.5	9.5

% Average	9.9	10.7	10.0

Table 2: Comparison of the generated runoff discharge values (conventional method) and the original values. The Total Volume represents the runoff discharge. DifQ: root-mean-square deviation of the differences between the original and the recalculated discharge hydrograph. % Volume: percentage of the volume which is erroneous, considering the total volume as 100%. % Average: average of the error volume percentage for the conventional UH derivation methods.

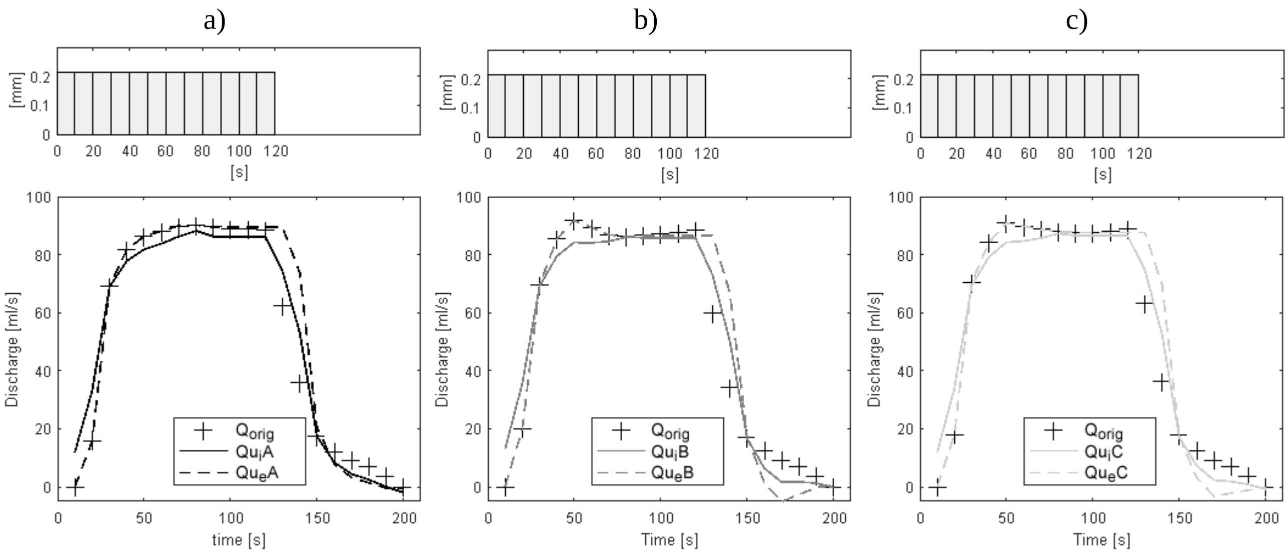

Figure 5: Recalculation of runoff discharge hydrograph using the assumed constant precipitation and comparison with the original. a-c) for case A, B and C respectively.

4.2. Results of PeRUHH method

Running the PeRUUH code (implemented in MATLAB), three different perturbed rainfall intensity patterns where generated from the measured total discharge volume. Comparing the values of the Objective Function, the best fitting rainfall intensity parameter is selected for each experiment independent from each other (Rw_A, Rw_B, Rw_C). These results can be seen in Figure 6.

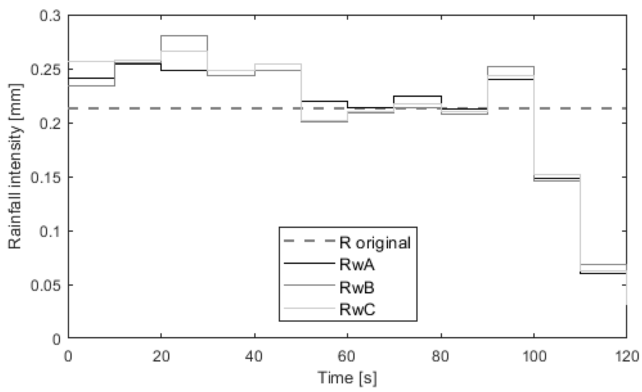

Figure 6: Perturbed rain intensity pattern for experiments A, B and C

For each experiment independently, using the optimized rainfall intensity pattern (Rw_A, Rw_B, Rw_C) and the original runoff discharge hydrographs, six new UHs are derived (UHw_{eA}, UHw_{iA}, UHw_{eB}, UHw_{iB}, UHw_{eC}, UHw_{iC}). The average of these UHs is UHx. These values can be seen in and Figure 7.

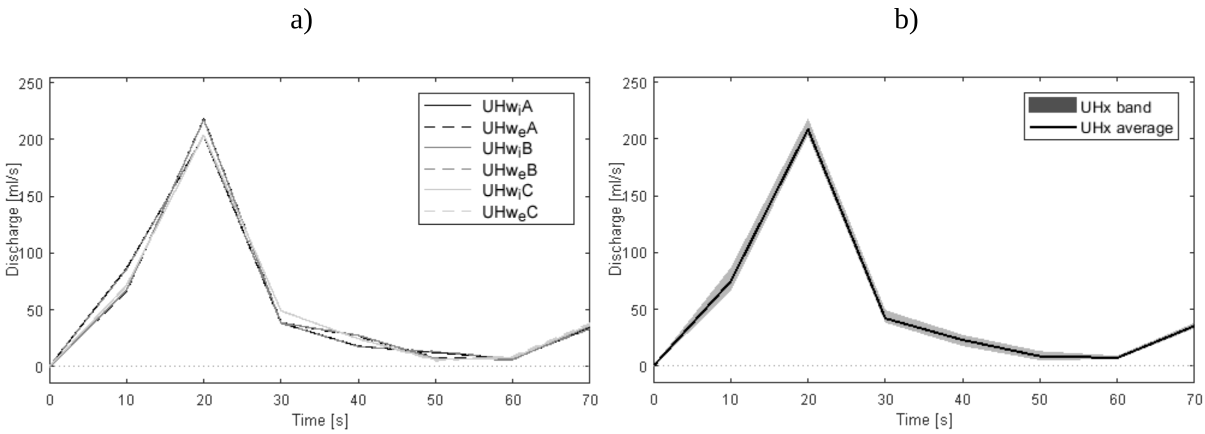

Figure 7: Comparison of the generated UH after applying the PeRUUH method, using the rainfall intensity pattern optimized for each experiment (Rw). a) six UHs: one for each of the three experiments and with each of the two UH derivation methods (UHw); b) average of the six UHw, the average optimized UH of the “catchment area” (UHx) and Band.

The recalculation of the runoff discharge hydrographs (using UHx and Rw) and the comparison of these hydrographs with the originals for each experiment independently, can be seen in Table 3 and Figure 8.

Experiment	A		B		C	
	Qx_i	Qx_e	Qx_i	Qx_e	Qx_i	Qx_e
Total Volume [ml]	9890		9878		9960	
Objective Function: DifQx	228	167	161	128	91	97
% Volume	2.3	1.7	1.6	1.3	0.9	1.0
% Average	2.0		1.5		0.9	

Table 3: Comparison of the generated runoff discharge values after applying the PeRUUH method and the original values. The Total Volume represents the runoff discharge. DifQ: root-mean-square deviation of the differences between the original and the recalculated discharge hydrograph. % Volume: percentage of the volume which is erroneous, considering the total volume as 100%. % Average: average of the error volume percentage for the conventional UH derivation methods.

a) b) c)

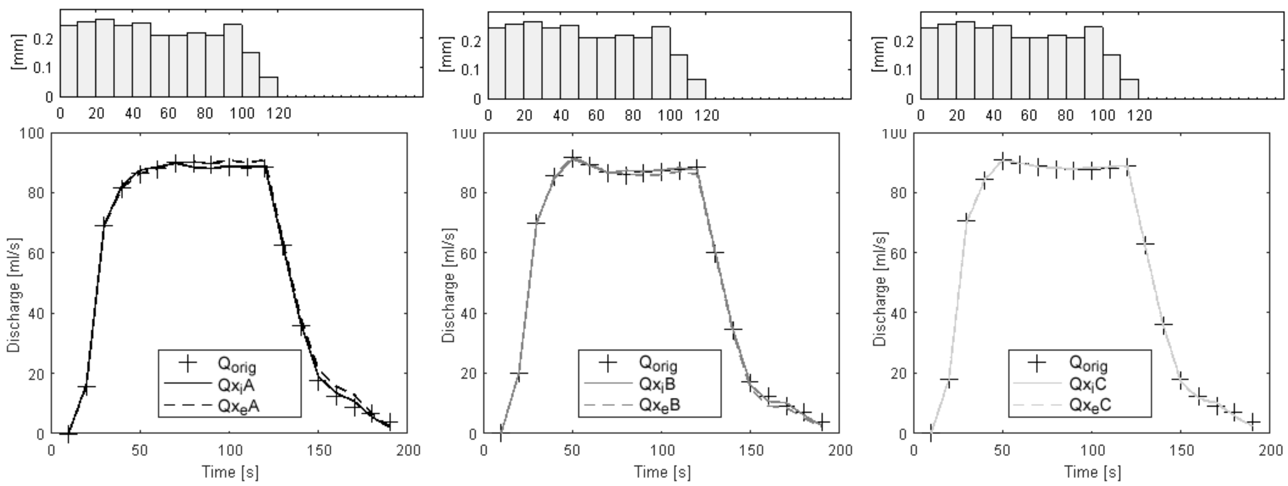

Figure 8: Recalculation of runoff discharge after derivation of the UH applying the PeRUUH method, using the improved rainfall intensity pattern (Rw), and comparison with the original; a-c) for experiment A, B and C respectively.

5. Discussion

When deriving an UH, if rainfall and discharge values are inconsistent, the resulting UH will be not be consistent when using the two derivation methods (see Figure 5). Assuming that the runoff discharge hydrograph is the most reliable input parameter, it is possible to estimate a perturbed rainfall intensity pattern, which accounts for the temporal and spatial uncertainties in the rainfall measurements, for which a unique UH for the catchment area can be generated (see Figure 7).

5.1. Conventional UH derivation

As it can be seen in 3.1, the two methods for UH derivation generate different UHs, thus, none of them is appropriate for being applied to the catchment area. Considering the three experiments (A, B and C), all their respective UHs should be very similar, but, as shown in Figure 4e, the uncertainty band for the six UHs is very large. Additionally, the re-generated runoff discharge hydrographs do not match the original one, as observed data Figure 5. Since the discharge is accurately measured using the volume method, it is possible to concluded that the rainfall input data is the main source of error.

The works of Rew & McCuen (2012), Williams et al. (1980), Hromadka (1991) and Isidoro, de Lima, & Leandro (2012) state that production of artificial rainfall cannot be temporal and spatially homogenous. Nevertheless, in order to simplify the system, Al Ali et al. (2017), Deng et al. (2005), Abudi et al. (2012) and Aksoy et al. (2012) assumed that the rainfall simulators generate a constant rainfall (spatially and temporally); the authors consider the impact of this assumption to be negligible, even though the impacts have not been quantified.

5.2. PeRUUH method

After applying the PeRUUH method, the perturbed rainfall intensity pattern (Rw) obtained for the three experiments are quite similar, as can be seen in Figure 6. Considering the known differences in the spatial intensity of the LSRS (Reis, 2015), this pattern can be explained by the spatial and the temporal variability of the rainfall intensity. Likewise, all the six newly derived UHs (for each of the three experiments and with both methods) are similar, as can be seen in and Figure 7. This proves that the PeRUUH method is able to derive a unique UH for the catchment area (UHx).

In order to corroborate the results, the runoff discharge hydrographs of every experiment were recalculated using UHx and Rw, and then compared to the original hydrograph of each experiment. This showed that all recalculated hydrographs are quite similar to the original one, as can be seen in Figure 8. The Objective Function indicates a big improvement when comparing the results of the conventional and the PeRUUH method (see Table 4).

DifQ	A	B	C
Conventional method	9.9 %	10.7 %	10.0 %
PeRUUH method	2.0 %	1.5 %	0.9 %

Table 4: comparison of the Objective Function for the calculations using the conventional and the PeRUUH method.

A remarkable phenomena can be observed in Figure 7: the last ordinates of all the UHs increase. This may be an effect of the characteristics of the catchment area or of the rainfall-runoff process. In all discharge runoff hydrographs of Figure 8, a change in the tendency at 150 s can be seen. This behaviour has also been observed by other authors, for example in the work of Isidoro et al. (2012), where a slight increase of runoff was seen in all experiments after the rainfall as stopped.

Furthermore, even though the average rainfall intensity of all experiments is quite similar, the form of the runoff discharge hydrograph of experiment A is slightly different from the one from experiments

B and C. This indicates that, in this LSRS, the average intensity is likely the same, but the rainfall distribution varies. Considering that the improved UH derived is the same for all three experiments, this result strongly supports the assertion that the differences in the runoff discharge hydrograph are caused by the differences in the rainfall spatial distribution.

5.3. From Lab to real catchments

The method developed in this study resulted in an improvement of the UH derivation in a rainfall simulator. The implementation of inverse modelling in hydrology could be an important tool to develop and understand the rainfall distribution in a real catchment, and, more importantly the catchment response i.e. the transformation from precipitation into discharge. Inverse modelling methods can also lead to reduced uncertainty and results improvements, as discussed also by Kretzschmar et al. (2016). Nevertheless, a perfect match can never be achieved with this method due to uncertainties steaming from other sources and non-linearity in the rainfall-runoff process. In real catchment applications, Zhao, Tung, Yeh, & Yang (1997) reported uncertainty steaming from non-linearity of the catchment and from the limitations of hydrological models. In any case, as some authors as Rew & McCuen (2012) and Hromadka (1991) indicate, it is well-known that rainfall is the major contributor to the overall uncertainty, particularly because of the rainfall strong spatial and temporal variability. Hence, the PERUHH method can become a very useful tool for real catchments.

6. Conclusion

The UH Theory can be used to estimate the discharge hydrograph of a basin, assuming that each catchment area with unchanging characteristics has a unique UH. However, when the spatial and temporal rainfall variations cannot be neglected, the UH obtained with the conventional method will not be correct. In this case, the application of the PeRUUH method allows to derivate a more reliable UH assuming that the rainfall measurements are uncertain.

In this work, the PeRUUH method is derived, and applied in a large scale rainfall simulator with an impermeable surface. It was shown that a valid unique UH using the conventional method could not be obtained with the original rainfall intensity pattern. The PeRUHH method, on the other hand, was able to estimate a unique UH for which the discharges have an error of (approx.) 10 times smaller than the UH derived with the conventional method. All the six resulting UHs for three independent experiments with different rainfall intensities, using two derivation methods, converge to a single UH. In addition, the calculated runoff discharge matchs the original one in all three independent experiments. This validates the application of the PeRUUH method and proves that the method is able to derive a unique UH for the large scale rainfall simulator, independently from the given rainfall intensity.

In large catchment areas, it is often difficult to measure a rainfall event exactly because of the large spatial and temporal rainfall variability. Therefore, it is likely that similar improvements will be found in real live applications. Hence, future work will see the implementation of the PeRUUH method in a real catchment area. Regardless of recent computational advances, research on UH methods is still relevant today. UH methods provide us a relative simple and quick way to quickly estimate high discharges in face of uncertain rainfall measurements, and thereby improve how we understand and manage floods.

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Notation:

"$-_{e}$"	applying the explicit method (linear equations)
"$-_{i}$"	applying the implicit method (matrix)
DifQ	Objective Function
j	number of loops to be run with the PeRUUH method
Q_{orig}	Original runoff discharge hydrograph
Q_u	recalculated runoff discharge with R_{orig} (conventional method)
Q_w	recalculated runoff discharge with Rw (PeRUUH method)
Q_x	recalculated runoff discharge hydrograph with Rw, UHx (PeRUUH method)
R_{orig}	original rainfall intensity pattern
R_w	Perturbed rainfall intensity pattern, created with PeRUUH
UH_u	UH calculated with R_{orig} (conventional method)
UH_w	UH calculated with Rw (PeRUUH method)

UH_x	unique UH for the basin, average of UHwe and UHwi
$value_{limit}$	limit value accepted for the Objective Function at each loop

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