Is Kp the Best Magnetic Activity Parameterization for Radial Diffusion Models in the Outer Radiation Belt?

Fira Fatmasiefa[1], Solène Lejosne[1]

[1]Space Sciences Laboratory, University of California, Berkeley, CA, USA.

Corresponding author: Fira Fatmasiefa (firafsiefa_@berkeley.edu)

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Key Points:

1. We have not found a single magnetic activity index or solar wind parameter that yield a better radial diffusion parameterization than Kp.

2. Hp60, Hp30 and the solar wind dynamic pressure yield radial diffusion parameterization of similar quality than Kp.

3. Hp60 and Hp30 can used in place of Kp in Kp-driven parameterizations of radiation belt radial diffusion to improve time resolution.

Abstract

Specifying radial diffusion magnitude is one of the main requirements for physics-based radiation belt models. Yet, radial diffusion quantification remains uncertain. The most commonly used parameterization for the logarithm of radial diffusion magnitude is a linear function of a magnetic index, Kp, with a coarse time resolution of three hours. This work presents alternate linear parameterizations of similar quality for the logarithm of radial diffusion magnitude, considering other magnetic indices and solar wind parameters. Using a time series for the logarithm of electromagnetic radial diffusion magnitude with a 1-minute time resolution, we investigate linear relationships with magnetic indices such as Kp, Hp60, Hp30, AE, SymH, and Dst and solar wind parameters such as solar wind dynamic pressure, solar wind speed and the north-south component of the interplanetary magnetic field. We find that Kp, Hp60, Hp30, and solar dynamical pressure yield the strongest linear correlation with the logarithm of radial diffusion magnitude. We also provide simple, linear models of the logarithm of radial diffusion magnitude that best fit the time series. This work contributes to improving the time resolution for radial diffusion parameterization and radiation belt models. In particular, it suggests that Hp60 and Hp30 could also be used in place of Kp in the most commonly used Kp-driven parameterization for radiation belt radial diffusion.

Plain Language Summary

There is an increasing need to better understand and predict the near-Earth environment, especially the Van Allen radiation belts as there is an increasing number of spacecraft operating within these regions filled with high-energy charged particles. One option is to use physics-based computer codes to simulate these environments. One of the key inputs for these codes is radial diffusion magnitude, which quantifies the efficiency of the radial diffusion process that takes place within the radiation belts. Radial diffusion is commonly parameterized by one index called the Kp index to quantify how the process varies with magnetic activity. We question this choice, with the objective of improving radial diffusion models. We explore the behavior of radial diffusion magnitude when other simple ways to quantify magnetic activity are used in place of Kp. We provide alternate radial diffusion parameterization of similar quality and higher time resolution. The objective of this work is to contribute to current efforts in increasing the time resolution of radiation belt models.

1 Introduction

Radial diffusion is a key factor in radiation belt modeling, not just at Earth but also at other strongly magnetized planets (Lejosne & Kollmann, 2020). The radial diffusion coefficient (D_{LL}) quantifies the efficiency of the process. Yet, determining radial diffusion magnitude remains challenging as radial diffusion coefficients cannot be measured directly. Many assumptions have to be made to determine the radial diffusion coefficient whether it is from models, field measurements and/or particle measurements (e.g., Lejosne, 2019, 2020, L.-F. Li et al., 2020, Olifer et al., 2019, Sandhu et al., 2021, Sarma et al., 2020).

The most common radial diffusion parameterization is the model by Brautigam and Albert (2000), in which the radial diffusion magnitude is parameterized by the equatorial radial distance normalized in units of Earth radii, L, and the magnetic activity index, Kp. Specifically:

$$\log_{10}\left(\frac{D_{\mathrm{LL}}}{L^{10}}\right) \quad = \ -9.325 + 0.506 \times Kp \quad [\log_{10}(\mathrm{da}y^{-1})] \ \#(1)$$

for equatorial radiation belt particles. The extension to off-equatorial particles of similar kinetic energy consists of multiplying the right hand side of the equation (1) by a factor that decreases with decreasing pitch angles, down to 0.1 for the most field aligned particles (Fälthammar, 1968).

The model presented in equation (1) results from a linear least-square fit of discrete values provided in the outer belt, at $L = 4$ (Lanzerotti and Morgan (1973)), and $L = 6.6$ (Lanzerotti et al., 1978), for different Kp values ($1 < Kp < 6$). The discrete values derived by Lanzerotti and Morgan (1973) and Lanzerotti et al. (1978), together with the L^{10} dependence assumed by Brautigam and Albert (2000), relied on the theoretical framework developed by Fälthammar (1965, 1966) for electromagnetic radial diffusion. This parameterization is favored by most radiation belt models because of its simplicity, and because it is able to render many features of the outer belt dynamics over long timescales (months to years) (e.g., Shprits et al., 2005).

Other parameterizations for radial diffusion have been proposed over the years (e.g., Ozeke et al., 2014; Ali et al., 2016; Liu et al., 2016). They rely on a new theoretical framework (Fei et al., 2006) well-suited for data analysis but whose theoretical validity has been challenged (e.g., Lejosne et al., 2013, Lejosne, 2019). These parameterizations also formulate magnetic activity dependence in terms of Kp, the main index used in radiation belt models (e.g., Drozdov et al., 2021).

A comparison between the outputs of the Versatile Electron Radiation Belt (VERB) code (Subbotin & Shprits, 2009) for long-term radiation belt modeling using different radial diffusion parameterizations and Van Allen Probes measurements shows that the parameterization by Brautigam and Albert (2000) for equatorial radiation belt particles still provides a slightly better agreement between simulations and observations (Drozdov et al., 2017, 2021). That said, taking into account the variation of radial diffusion magnitude with equatorial pitch angle has been shown to reduce the quality of the simulation (Drozdov et al., 2021).

When discrete radial diffusion estimates are computed, they can display significant deviations from values provided by the Kp-parameterization (e.g., Olifer et al., 2019, Sandhu et al., 2021). The existence of such discrepancies limit radiation belt modeling accuracy (e.g., Thompson et al., 2020) and they hamper further progress in major science questions, such as the relative contributions of radial transport and local acceleration in radiation belt energization (e.g., Drozdov et al., 2022). They also call for the development of event-specific radial diffusion coefficients (e.g., Tu et al., 2009), which usually require intensive work (e.g., Z. Li et al., 2017, George et al., 2022). Several works have circumvented the need for costly numerical simulations through simplifications or ad-hoc modeling. They have proposed ways to quantify radial diffusion directly from solar wind characteristics (e.g., X. Li et al., 2001, 2009, Lejosne, 2020, Xiang et al., 2021). In all cases, the level of uncertainty associated with the outputs remains unknown.

To our best knowledge, the time series provided by Lejosne (2020) is the only time series that yields a linear fit similar to Brautigam and Albert's (2000) formula when the logarithm of radial diffusion magnitude is binned as a function of the Kp index. The underlying theoretical framework is similar to Fälthammar (1966). It consists of connecting solar wind characteristics to the state of the outer belt, via Shue et al's (1998) magnetopause location model (a function of the solar wind dynamic pressure and the north-south component of the interplanetary magnetic field), and a time-varying geomagnetic field model parameterized by the magnetopause location. In the following, we further leverage the time series derived by Lejosne (2020) to test whether Kp is the best magnetic activity parameterization for radial diffusion in the outer radiation belt. This type of investigation had been precluded so far due to the lack of comprehensive database of radial diffusion time series. That said, statistical analyses of ULF waves suggest that Kp is the best single parameterization for ULF wave power, before Dst, solar wind speed, solar wind pressure, or the north-south component of the interplanetary magnetic field (e.g., Dimitrakoudis, 2015, 2022). Our results are consistent with these findings, as shown below.

The datasets and methods are described in **Section 2**, and the results are provided in **Section 3**. They are discussed in **Section 4**.

2 Data and Method

2.1. Data

The radial diffusion time series (Lejosne, 2020) provides the logarithm of electromagnetic radial diffusion magnitude, $\log_{10}(D_{\mathrm{LL}}/L^{10})$, for equatorial particles with angular drift frequency, $\Omega = 1mHz$, with a 1-minute time resolution for the years 1995–2019.

We first focus on a random year to investigate possible linear relationships between magnetic activity conditions and the logarithm of electromagnetic radial diffusion magnitude, $\log_{10}(D_{\mathrm{LL}}/L^{10})$: the year 2014. Three other years are also considered to determine the variability of the results with the solar cycle, namely, 1997, 2000, and 2017. While the year 2014 corresponds to the peak of the solar cycle 24, the year 1997 corresponds to the ascending phase of the solar cycle 23, 2000 is close to the peak of solar cycle 23, and 2017 is in the declining phase of solar cycle 24.

To quantify magnetic activity, we consider various indices with various time resolutions: Kp (3hr time resolution), Hp60 (60 min time resolution), Hp30 (30 min), AE (1hr), SymH (1 hr), and Dst (1 hr). We also consider the hourly solar wind dynamic pressure, as well as, the solar wind speed (1 min time resolution) and the north-south component of the interplanetary magnetic field (1 min). The logarithm of the solar wind pressure is further examined because that it is one of the inputs of the model quantifying $\log_{10}(D_{\mathrm{LL}}/L^{10})$ according to Lejosne (2020).

2.2. Method

The objective is to search for the magnetic index or solar parameter that can best parameterize radial diffusion magnitude in the simplest way possible. First, we search for linear correlations with various magnetic indices and solar wind parameters. Then, the parameters yielding the highest correlation coefficients are further analyzed to derive alternate radial diffusion models. The approach is similar to what has been done to parameterize plasmapause location (O'Brien and Moldwin, 2003). We focus on finding the magnetic index or the solar wind parameter with the strongest linear correlation with radial diffusion magnitude. We consider the value of each magnetic index or solar wind parameter, as well as the maximum within a preceding time interval of varying size. By doing so, we test whether the time history of the magnetic activity parameter influences radial diffusion magnitude. Then, we perform a linear interpolation between the logarithm of radial diffusion magnitude and the selected indices. Finally, we compare the linear models by calculating the root-mean-square error (RMSE) between the time series and the values provided by the model. The RMSE error is defined by:

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=1}^{N} R_j^2} \#(2)$$

Where $R_j = (x_j - \hat{x}_j)$ is the residual, defined as the difference between the reference value, x_j, and the value predicted by the model of best fit, $\hat{x}_j$.

3 Results

3.1. Correlation coefficients

$\log10(D_{\mathrm{LL}}/L^{10})$ vs	Kp	Hp60	Hp30	AE	$\log_{10}(Dp)$	-SymH	-Dst	Bz	Vsw
cc	0.6	0.6	0.6	0.4	0.6	0.1	0.2	0	0.2

Table 1. Pearson correlation coefficients for each magnetic index and solar wind parameter vs the logarithm of radial diffusion magnitude

Table 1 shows that the Kp, Hp60, and Hp30 indices as well as both the solar wind dynamic pressure yield the highest correlation coefficient values with the logarithm of radial diffusion magnitude. While the AE index provides correlation coefficient values of 0.4, Dst and SymH give low correlation coefficient values, indicating no strong linear correlation with the logarithm of radial diffusion magnitude. We did not find significant variations with the year considered (i.e., with the solar cycle phase). When considering the immediate time history of the magnetic activity parameters (maximum over the latest 3 hours, 6 hours, 12 hours or 24 hours), we did not find a higher correlation coefficient than when considering the real time magnetic activity parameters.

3.2. Linear Fits

We propose a parameterization following the linear form:

$$\log_{10}\left(\frac{D_{\mathrm{LL}}}{L^{10}}\right) = aQ + b \#(3)$$

where Q is an index representing magnetic activity, a is the slope of the linear fit and b is the intercept. The values of a and b are obtained by least squares fitting are provided in **Table 2** when the Pearson correlation coefficient between the magnetic activity index or solar wind parameter, Q, and the logarithm of radial diffusion magnitude, $\log_{10}\left(\frac{D_{\mathrm{LL}}}{L^{10}}\right)$, is greater than 0.3 (**Section 3.1**).

Index, Q	Linear Fit: $\log_{10}\left(\frac{D_{\mathrm{LL}}}{L^{10}}\right) = aQ + b$	RMSE	
	Slope, a	Intercept, b	
Kp	0.340	-9.049	0.66
Hp60	0.339	-9.047	

Hp30	0.340	-9.047	
$\log_{10}(Dp\ [nPa])$	1.838	-8.871	
AE [nT]	1.713 $\times 10^{-3}$	-8.729	0.73

Table 1. Linear models of best fit of radial diffusion magnitude given by each magnetic index and solar wind parameter and their respective root mean square error (RMSE) values.

Kp, Hp60, and Hp30 yield similar linear fits for the logarithm of radial diffusion magnitude. The Kp-parameterization of radial diffusion is similar to the one proposed by Brautigam and Albert (2000) (**equation (1)**). The intercepts differ by 3% while the slopes differ by 33%, with the highest discrepancies for the highest Kp values (**Figure 1**). The similarity between the linear fits as a function of Kp, Hp60, and Hp30 suggests that Kp-parameterizations of radial diffusion are still valid when formulated in terms of Hp60, or Hp30, allowing for a simple parameterization for radiation belt radial diffusion with a higher time resolution.

A parameterization of similar quality (i.e., similar RMSE) is obtained when formulating radial diffusion magnitude in terms of solar wind dynamics pressure (**Figure 2**). On the other hand, while radial diffusion magnitude tends to increase with the AE index, it can reach maximum values regardless of the value of AE (**Figure 3**). This "triangle shape" distribution (e.g., Reeves et al., 2011), results in a smaller correlation coefficient (**Table 1**) and a linear fit of lesser quality (**Table 2**).

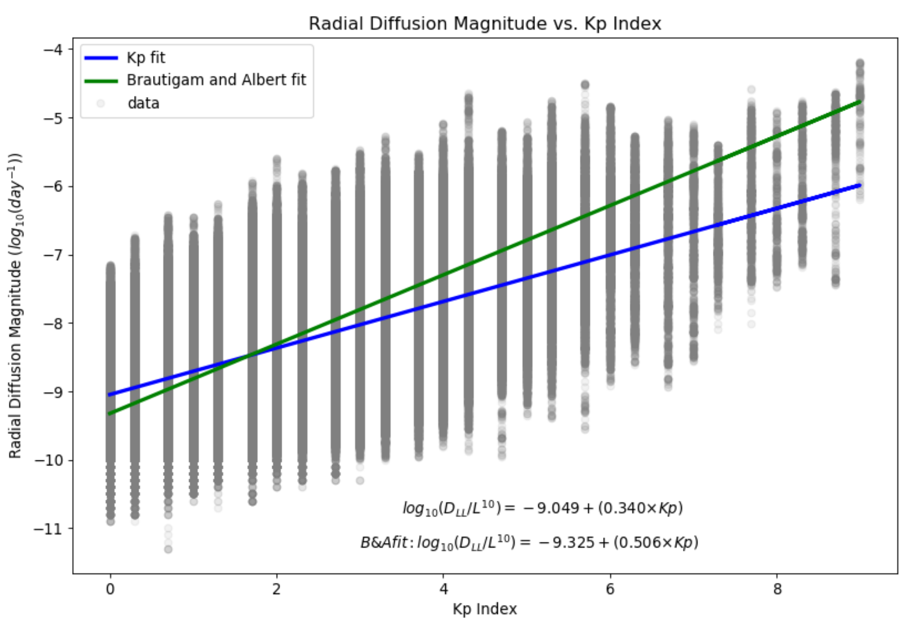

Figure 1. Model of best fit for Kp index vs. radial diffusion magnitude. The blue line represents our proposed parameterization, while the green line represents Brautigam and Albert's model for Kp vs. radial diffusion magnitude (2000).

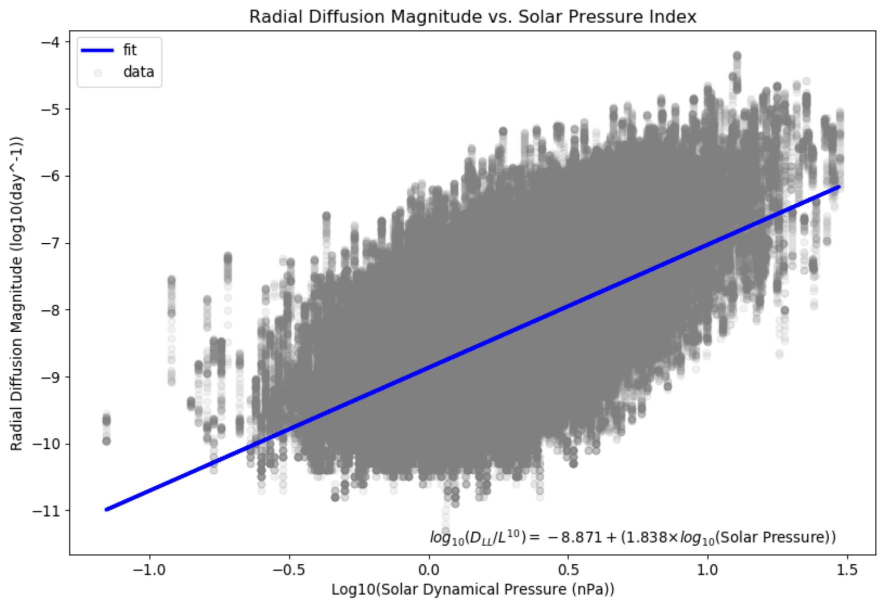

Figure 2. Model of best fit for the logarithm of solar dynamic pressure vs. radial diffusion magnitude

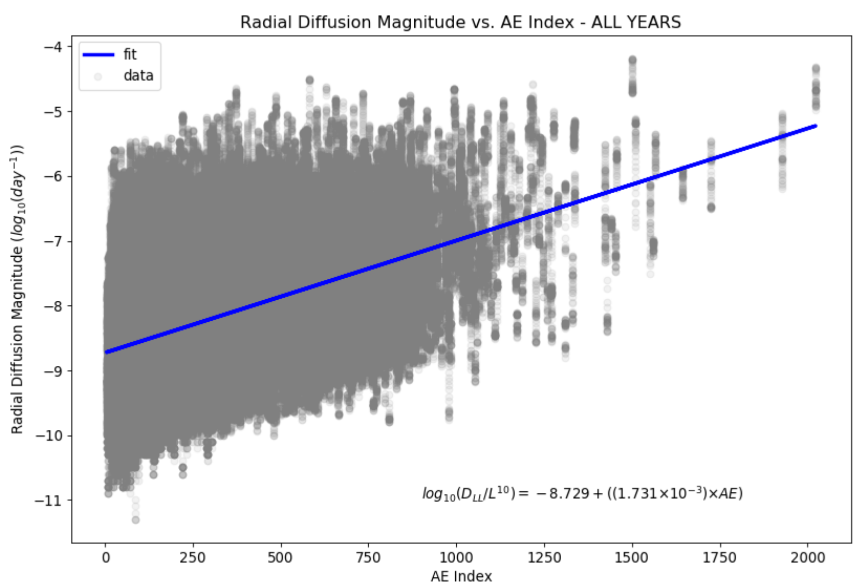

Figure 3. Model of best fit for the AE index vs. radial diffusion magnitude

4 Discussion

This work investigates how radial diffusion magnitude varies with different magnetic activity indices and solar wind parameters during different phases of the solar cycle. Out of all the magnetic indices and solar parameters considered, Kp, Hp60, and Hp30 along with the hourly solar dynamic dynamics pressure yield the highest correlation coefficient values with the logarithm of radial diffusion magnitude $\log_{10}(D_{\mathrm{LL}}/L^{10})$. The resulting linear fits are of similar quality, providing alternative models of higher time resolution to the well accepted Kp-driven parameterization for radiation belt radial diffusion. We found no significant variation in the results when considering different years at different phases of the solar cycle.

That said, radial diffusion magnitude cannot be measured accurately and there is no consensus on the value of radial diffusion at a given time. Therefore, a similar analysis applied to a different time series for radial diffusion magnitude could yield different parameterizations.

This work relies the theoretical model derived by Lejosne (2020), based on Mead's (1964) geomagnetic field model. Thus, it faces similar limitations, and errors are expected at low L values as well as high L values (see also Lejosne, 2020). In particular, since this Mead's (1964) geomagnetic field model is curl-free, it cannot render the effects of currents within the magnetosphere. This may explain the absence of correlation with SymH and Dst. In addition, Lejosne's model (2020) leverages the magnetopause location model developed by Shue et al (1998), which also faces limitations (Staples et al., 2020).

In the absence of error bars for radial diffusion coefficients, one possible way forward could be to compare different radial diffusion time series. Such comparison could bring forward times of similarities, and times of high discrepancies, signaling a need for further investigation. Implementing the proposed parameterizations in radiation belt codes solving the Fokker-Planck equation could also provide insight on their performance.

Data Availability Statement

The time series for radial diffusion magnitude is publicly accessible and can be downloaded at: https://doi.org/10.5281/zenodo.3731708

The time series for the Kp, AE, SymH and Dst magnetic indices, as well as the solar wind parameters, are obtained from OMNI WEB: https://omniweb.gsfc.nasa.gov

The time series for Hp60 and Hp30, were obtained from the GFZ Helmholtz Centre Postdam website: https://www.gfz-potsdam.de/en/hpo-index/

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