

An analytical approach to understanding the morphologies of glaciovolcanic caves and chimneys

SFU An analytical approach to understanding the morphologies of glaciovolcanic caves and chimneys
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Introduction
0:00 / 1:04

Analytical models of caves and chimneys
0:00 / 2:07

The radial geometry of a snow-capped cave is determined by balancing the creep closure of the radial rock rate, under the assumption that $P \ll \rho g H$.

$$r = \frac{r_0 \Delta T}{\rho g L \Delta T} \left(\frac{r_0}{r} \right)^n$$

However, if the length of the cave is not much smaller than the glacier thickness, the balance has to be adjusted to supply in the direction parallel to its surface. Assuming that the creep closure has an isotropic geometry, the radial geometry can be described as a function of r , the radial distance, and θ , the polar angle. The rock opening and creep closure balance at the cave's surface is hence:

$$r N \left(1 - \cos \theta \frac{r}{N} \right)^n = \frac{r_0 \Delta T}{\rho g L \Delta T} \left(\frac{r_0}{r} \right)^n$$

where $r N$ is the radial component of the radial distance. This allows us to numerically solve for the geometry of the void for a given time t .

One way to simplify a geometry in steady state is to model it as a radially symmetric vertical column. If the radial profile $r(r)$ of the steady state is known, then the total growth rate is given by:

$$\dot{Q}_{\text{cave}} = 2\pi \rho L \Delta T \int_0^R r(r) \left(\frac{r_0 \Delta T}{\rho g L \Delta T} \right)^{1/n} dr$$

This is an underestimate as it ignores the effect of the lateral flow of ice and the influence of the ice on the cave's geometry.

Evolution
0:00 / 0:49

Future work and Acknowledgements
0:00 / 0:25

Future work
Using the insight gained from these analytical investigations, we can use further investigate these glaciovolcanic phenomena by employing a numerical full Stokes ice-flow model. The influence of these glaciovolcanic features on the future evolution of Jökull Guller in the Mt. Meager Volcanic Complex will also be tested with respect to changing climate.

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PRESENTED AT:



INTRODUCTION

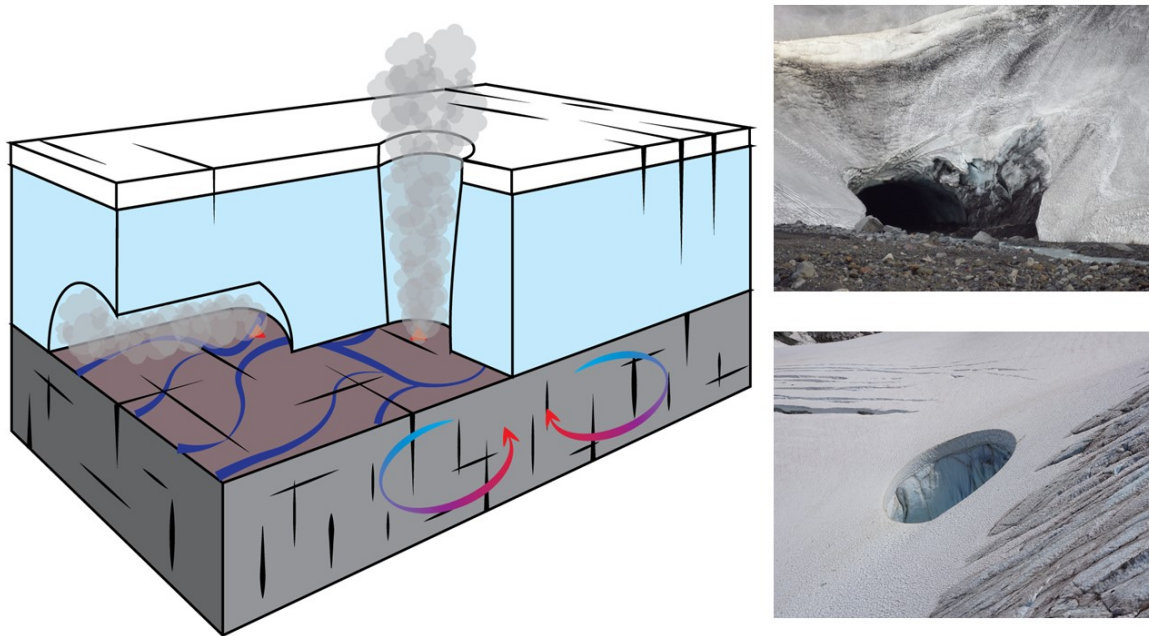


Figure 1: **Left:** A schematic of a glaciovolcanic cave and chimney above subglacial geothermal sources. **Top right:** A glaciovolcanic cave in Kverkfjöll, Iceland. **Bottom right:** A glaciovolcanic chimney in Job Glacier, Mt. Meager, British Columbia, Canada.

Gas and vapour emissions from subglacial or subnivean volcanic and geothermal sources are capable of melting voids and passageways into the overlying ice/snow. We identify two distinct void morphologies, glaciovolcanic caves: sub-horizontal voids that form between the bedrock and the overlying ice over geothermal sources that do not have sufficient energy flux to fully melt through the ice; and chimneys: vertical shafts formed where subglacial geothermal sources completely melt through the glacier. Here we seek to describe the morphology of these glaciovolcanic features, with respect to glaciological conditions and geothermal heat fluxes, using existing analytical models.

This research project was sparked by the 2016 discovery of degassing glaciovolcanic chimneys within Job Glacier in the Mt. Meager Volcanic Complex, British Columbia, Canada. Their formation has been a source for speculation, as it could have been driven by either changes in geothermal or glaciological conditions. Here we present analytical models to test these hypotheses and shed a broad light on the relationships between geothermal heat fluxes and water-drained glaciological conditions.

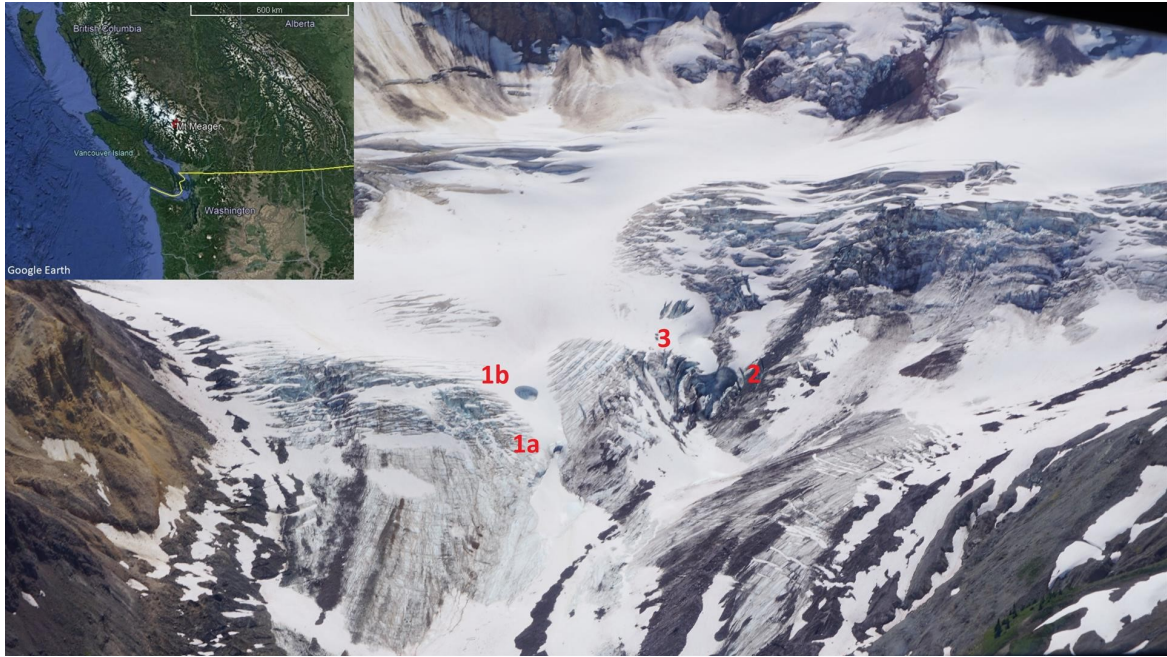


Figure 2: Job Glacier in the Mt. Meager Volcanic Complex with its glaciovolcanic features labelled. The photo was taken in August 2020 and is looking south. **1a:** A, now, non-active chimney discovered in 2016. **1b:** A new chimney, discovered in 2020. Likely formed above the source of 1a. **2:** A chimney that was discovered in 2016. It has two active fumaroles that have caused significant surface lowering around them. **3:** A surface depression that was first observed in 2016. As of November 2020 it has opened up, revealing another chimney.

THERMODYNAMICS

Similar to R  thlisberger channels (R  thlisberger, 1972), in order for glaciovolcanic caves and chimneys to exist in a steady state, the geothermally induced melt has to be in balance with the ice creep closure. The creep closure for a radially symmetric tunnel is described by:

$$\frac{1}{r} \frac{dr}{dt} = -A \left(\frac{p_i}{n} \right)^n,$$

where A and n are the ice-flow coefficient and exponent, respectively, $p_i = \rho_i g H$ is the ice overburden pressure, ρ_i is the ice density, H is the glacier thickness and r is the tunnel radius (Nye, 1953).

The radial melt rate is:

$$\frac{dr}{dt} = \frac{\dot{q}_M}{\rho_i L_f},$$

where L_f is the latent heat of fusion and \dot{q}_M is the specific energy flux supplied to the ice for melt. It is outside the scope of this research to model the gas flow within the voids in detail and hence we define the heat flux in the form of a simple temperature-index model:

$$\dot{q}_M = \mathcal{F}_M \Delta T,$$

where ΔT is the temperature contrast between the gases and the ice and \mathcal{F}_M is a melt factor that depends on the water-vapour concentration and the mode of convection. It is, however, possible to draw some basic conclusions regarding glaciovolcanic void formation and evolution, from the Boussinesq approximation of the momentum conservation for fluids, which describes forced and free convection:

$$\rho \frac{D\vec{v}}{Dt} = (-\vec{\nabla} p + \rho \vec{g}) - \vec{\nabla} \cdot \vec{\tau} - \rho \vec{g} \beta \Delta T,$$

where ρ is the gas density, \vec{v} the gas velocity, $\vec{\nabla} \cdot \vec{\tau}$ is a viscosity term and β is the coefficient of thermal expansion (Bird et al., 2006).

References:

- Bird, R. B., Stewart, W. E. & Lightfoot, E. N. (2006). *Transport phenomena* (2nd). John Wiley & Sons.
- Nye, J. F. (1953). The flow law of ice from measurements in glacier tunnels, laboratory experiments and the Jungfraufirn borehole experiment. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 219(1139), 477-489. doi: 10.1098/rspa.1953.0161
- R  thlisberger, H. (1972). Water pressure in intra- and subglacial channels. *Journal of Glaciology*, 11(62), 177-203. doi: 10.3189/S0022143000022188

ANALYTICAL MODELS OF CAVES AND CHIMNEYS

The radial geometry of a semi-cylindrical cave is obtained by balancing the creep closure and the radial melt rate, under the assumption that $r \ll H$:

$$r = \frac{\mathcal{F}_M \Delta T}{\rho_i L_f A} \left(\frac{n}{\rho_i g H} \right)^n.$$

However, if the height of the cave is not much smaller than the glacier thickness, the balance has to be adjusted to only apply in the direction perpendicular to its surface. Assuming that the morphology has an azimuth symmetry, the void geometry can be described as a function of r , the radial distance, and θ , the zenith angle. The melt-opening and creep-closure balance at the cave's surface is hence:

$$r_N \left(1 - \cos \theta \frac{r}{H} \right)^n = \frac{\mathcal{F}_M \Delta T}{\rho_i L_f A} \left(\frac{n}{\rho_i g H} \right)^n,$$

where r_N is the normal component of the radial distance. This allows us to numerically solve for the geometry of the void for a given heat flux.

One way to analyse a chimney in steady state is to model it as a radially symmetric vertical conduit. If the radial profile $r(z)$ of the chimney is known, then the total geothermal energy needed to sustain the given geometry is:

$$\dot{Q}_{TOT} = 2\pi \rho_i L_f A \int_0^H r(z)^2 \left(\frac{\rho_i g (H-z)}{n} \right)^n dz.$$

This is an underestimate as energy losses due to turbulence, heat transfer efficiency and heat loss to the atmosphere are not considered.

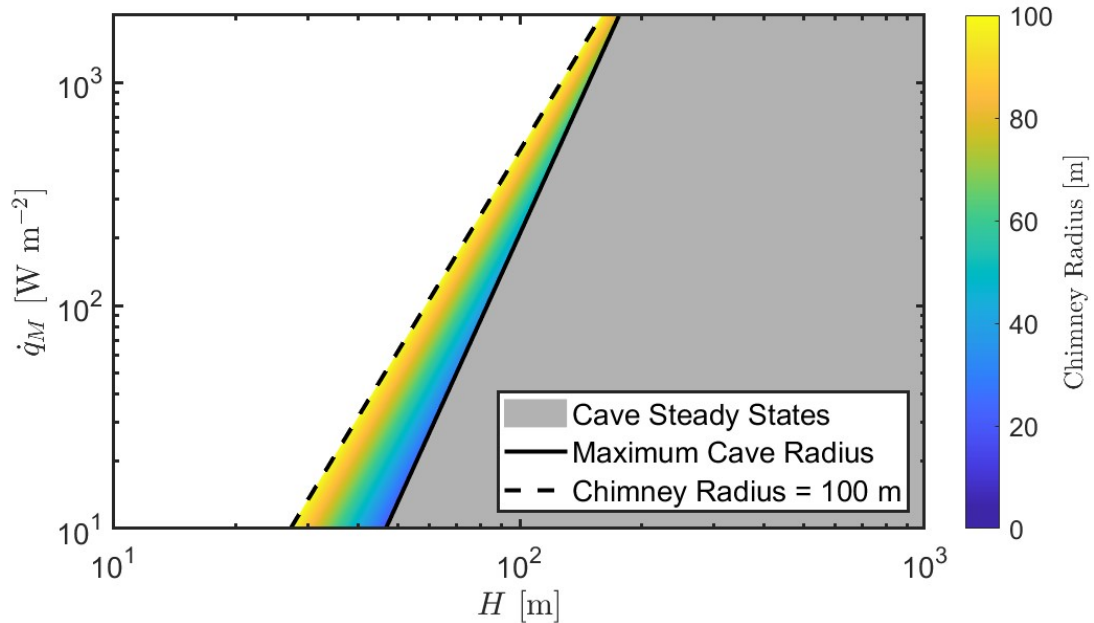


Figure 3: The shaded area shows the theoretical parameter space where steady state cave geometries exist, with the solid black line being the maximum cave radius. Left of the solid black line, the cave solution becomes unstable, indicating that a cave can either not exist or has to melt through the ice and form a chimney, with the associated chimney radius shown in colour.

It is possible to further increase the model complexity without modelling the gas flow by making several assumptions: for a finite volume of the conduit, creep closure only works in the radial direction; heat flux

through a cross-section of the chimney is $\dot{Q}(z) = \pi r(z)^2 c \dot{m}(z) \Delta T$ where c is the specific heat capacity of the gas and $\dot{m}(z)$ is the gas mass flux; and loss in gravitational potential energy is the only energy loss within the turbulent flow. Balancing all energy terms results in a lapse rate within the chimney:

$$-\frac{dT}{dz} = \frac{g}{c} + \frac{2\pi\mathcal{F}_M(\Delta T)^2}{\rho_i L A \dot{m}_0 c} \left(\frac{n}{\rho_i g(H-z)} \right)^n,$$

which can be solved numerically to obtain a temperature profile. The radial geometry is then:

$$r(z) = \frac{\mathcal{F}_M \Delta T(z)}{\rho_i L A} \left(\frac{n}{\rho_i g(H-z)} \right)^n.$$

Table of properties.

Symbol	Description	Value	Units
r	Radial distance		m
θ	Zenith angle		rad
z	Elevation above bedrock		m
t	Time		s
A	Ice flow-law coefficient	2.4×10^{-24}	$\text{Pa}^{-3} \text{ s}^{-1}$
n	Ice flow-law exponent	3	
p	Pressure		Pa
g	Gravitational acceleration	9.81	m s^{-2}
ρ	Density		kg m^{-3}
H	Glacier thickness		m
\dot{q}	Specific energy flux		W m^{-2}
\dot{Q}	Energy flux		W
T	Temperature		K
\dot{m}	Mass flux		kg s^{-1}
L_f	Latent heat of fusion	3.34×10^5	J kg^{-1}
\mathcal{F}_M	Heat transfer coefficient		$\text{W m}^{-2} \text{ K}^{-1}$
β	Thermal expansion coefficient		K^{-1}
c	Specific heat capacity		$\text{J kg}^{-1} \text{ K}^{-1}$

EVOLUTION

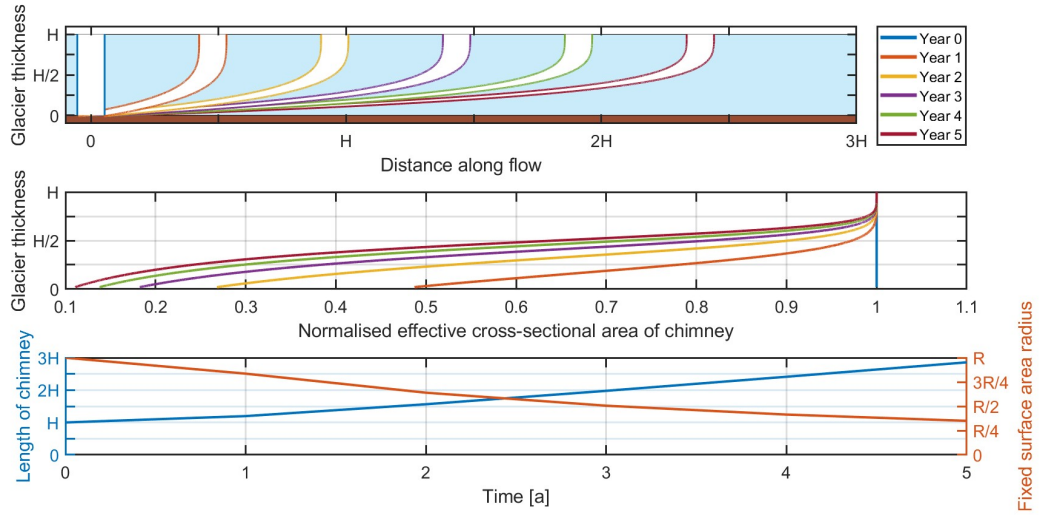


Figure 4: **Top:** The elongation of a glaciovolcanic chimney subject to a horizontal velocity field. **Middle:** The effective cross sectional area, i.e. perpendicular to the gas flow, at different time steps. **Bottom:** The total length of a chimney over time and the fraction of the initial radius required to conserve the initial chimney surface area.

As has been observed at Job Glacier in the Mt. Meager Volcanic Complex, chimney openings can be advected downstream with glacier flow. At a certain point the chimney becomes detached from the geothermal source and a new chimney forms above the original geothermal source. A glaciovolcanic chimney that is subjected to a velocity field but does not otherwise deform, becomes elongated. The surface area of the elongated chimney is directly proportional to its length. It therefore follows that a chimney, originally in a thermal equilibrium, can be advected into an unstable thermal state, where the geothermal input can no longer keep up with the increase in surface area. Secondly, as the chimney is advected, the effective cross-sectional area significantly decreases, creating conditions where wall debris or snow can more easily block the gas flow. Once the chimney has reached the aforementioned states, the pressure gradient within the chimney is reduced and the buoyancy term becomes dominant for the gas flow, initiating the vertical growth of a new chimney.

FUTURE WORK AND ACKNOWLEDGEMENTS

Future work

Using the insight gained from these analytical formulations, we aim to further investigate these glaciovolcanic phenomena by employing a numerical full-Stokes ice-flow model. The influence of these glaciovolcanic features on the future evolution of Job Glacier in the Mt. Meager Volcanic Complex will also be tested with respect to changing climate.

Acknowledgements

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