Estimating Bayesian Model Averaging Weights and Variances of Ensemble Flood Modeling Using Multiple Markov Chains Monte Carlo

Tao Huang and Venkatesh Merwade

Lyles School of Civil Engineering, Purdue University, USA

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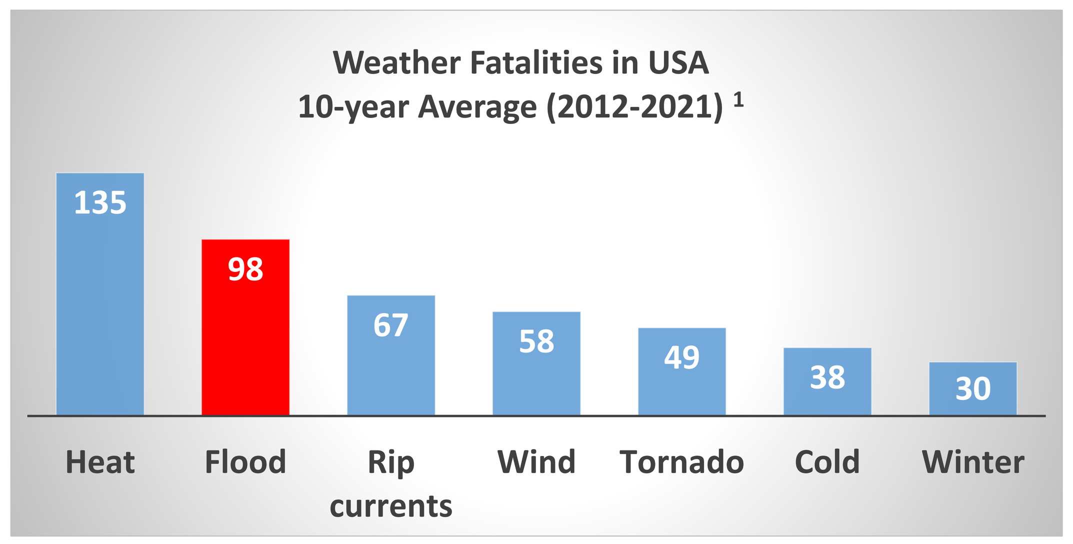
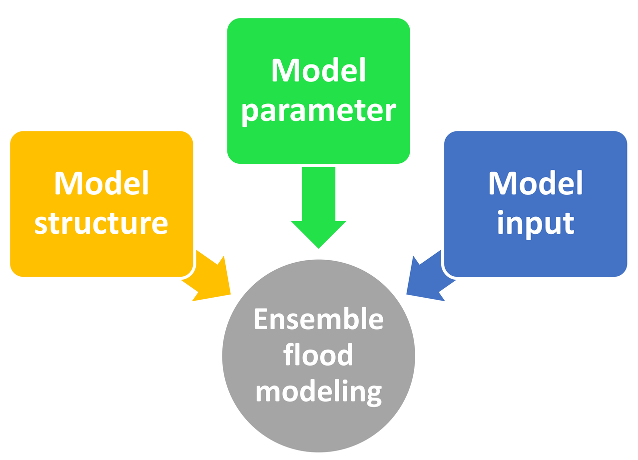
Introduction





Multi-model method

- physics-based and data-driven models
- No "perfect" single model
- "Equifinality" in hydrologic & hydraulic modeling
- Fixed estimates cannot fit all kinds of scenarios

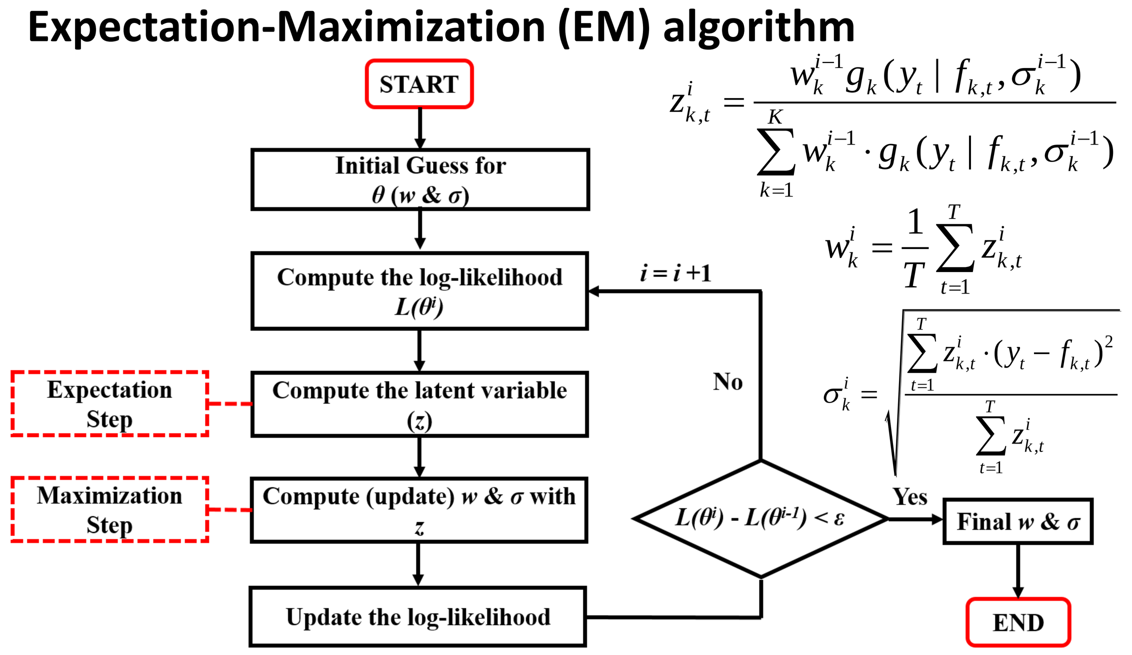
Bayesian Model Averaging (BMA)[2]

- **Law of total probability**

$$p(y \mid D) = \sum_{k=1}^{K} p(f_k \mid D) \cdot p_k(y \mid f_k, D) = \sum_{k=1}^{K} w_k \cdot p_k(y \mid f_k, D) \qquad \sum_{k=1}^{K} w_k = 1$$

$$L(\theta) = \sum_{t=1}^{T} \log\left(\sum_{k=1}^{K} w_k \cdot p_k(y_t \mid f_{k,t}, \sigma_k)\right) = \sum_{t=1}^{T} \log\left(\sum_{k=1}^{K} w_k \cdot \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_t - f_{k,t}}{\sigma_k}\right)}\right)$$

- **Expectation-Maximization (EM) algorithm**



$$z_{k,t}^{i} = \frac{w_k^{i-1} g_k(y_t \mid f_{k,t}, \sigma_k^{i-1})}{\sum_{k=1}^{K} w_k^{i-1} \cdot g_k(y_t \mid f_{k,t}, \sigma_k^{i-1})}$$

$$w_k^{i} = \frac{1}{T} \sum_{t=1}^{T} z_{k,t}^{i}$$

$$\sigma_k^{i} = \sqrt{\frac{\sum_{t=1}^{T} z_{k,t}^{i} \cdot (y_t - f_{k,t})^2}{\sum_{t=1}^{T} z_{k,t}^{i}}}$$

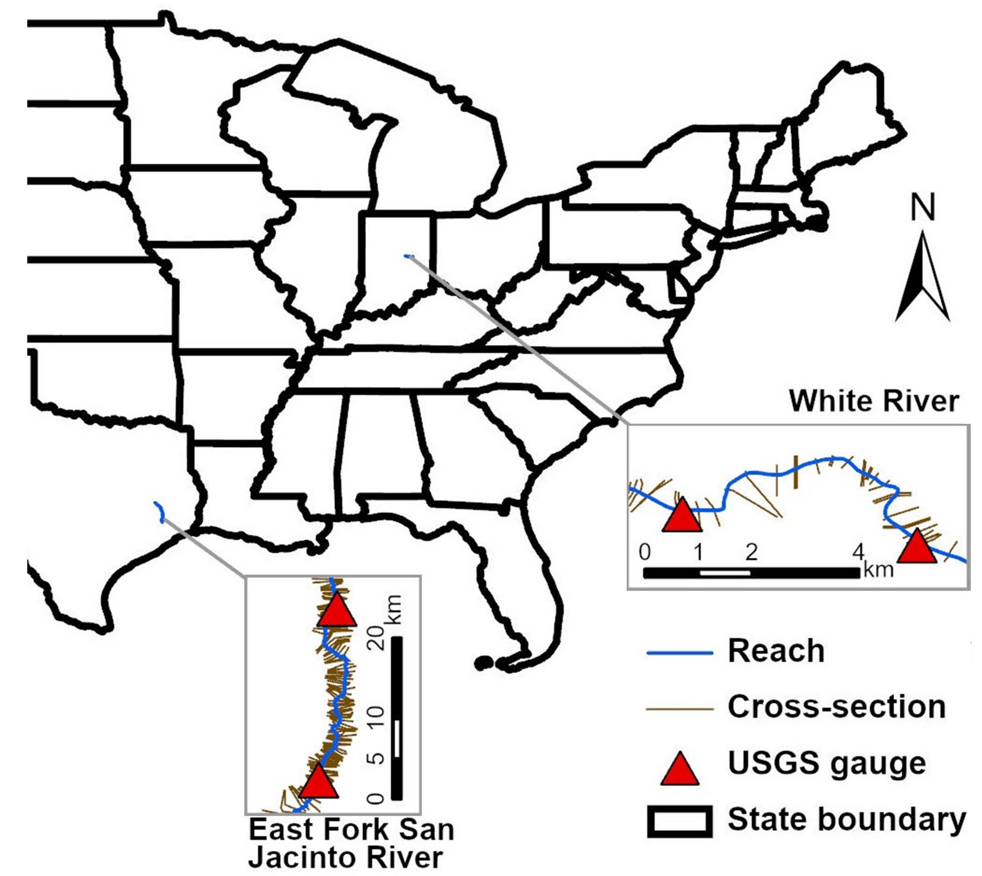
✓	✗
• combine estimations from multiple competing models • BMA weights are interpretable • provide a prediction distribution of variables of interest	• EM algorithm yielded fixed BMA weights and variances • Conditional PDF is limited by Gaussian assumption • Markov Chain Monte Carlo (MCMC) is rarely examined in BMA analysis

Research Objectives

Quantify the uncertainty in BMA parameters (weight & variance):

- estimate the BMA parameters using different numbers of samples in each Markov chain with the Metropolis-Hastings (M-H) algorithm
- compare the performance of EM and M-H MCMC algorithms for estimating the BMA parameters
- estimate the BMA weights using different proposal distributions in the M-H MCMC algorithm
- investigate the impact of different conditional PDFs of the predictor variable on the BMA parameters

Study Area and Data



Study stream	Channel length (km)	Average channel width (m)	Channel slope (%)
White	6.76	64	0.0631
East Fork San Jacinto	50.11	76	0.0438

Study stream	Upstream USGS gauge	Downstream USGS gauge	Simulation Period (100 days)
White	03348000	03348130	2021-3-15 to 2021-6-22
East Fork San Jacinto	08070000	08070200	2021-4-15 to 2021-7-23

Methodology

- **Numerical experiment (ensemble of 10 members)**

D = 100 days of daily water stage data (in meters)	
$f_1 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.06^2)$	$f_2 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.06^2)$
$f_3 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.12^2)$	$f_4 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.12^2)$
$f_5 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.18^2)$	$f_6 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.18^2)$
$f_7 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.24^2)$	$f_8 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.24^2)$
$f_9 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.30^2)$	$f_9 = D + \varepsilon$, where $\varepsilon \sim N(0, 0.30^2)$

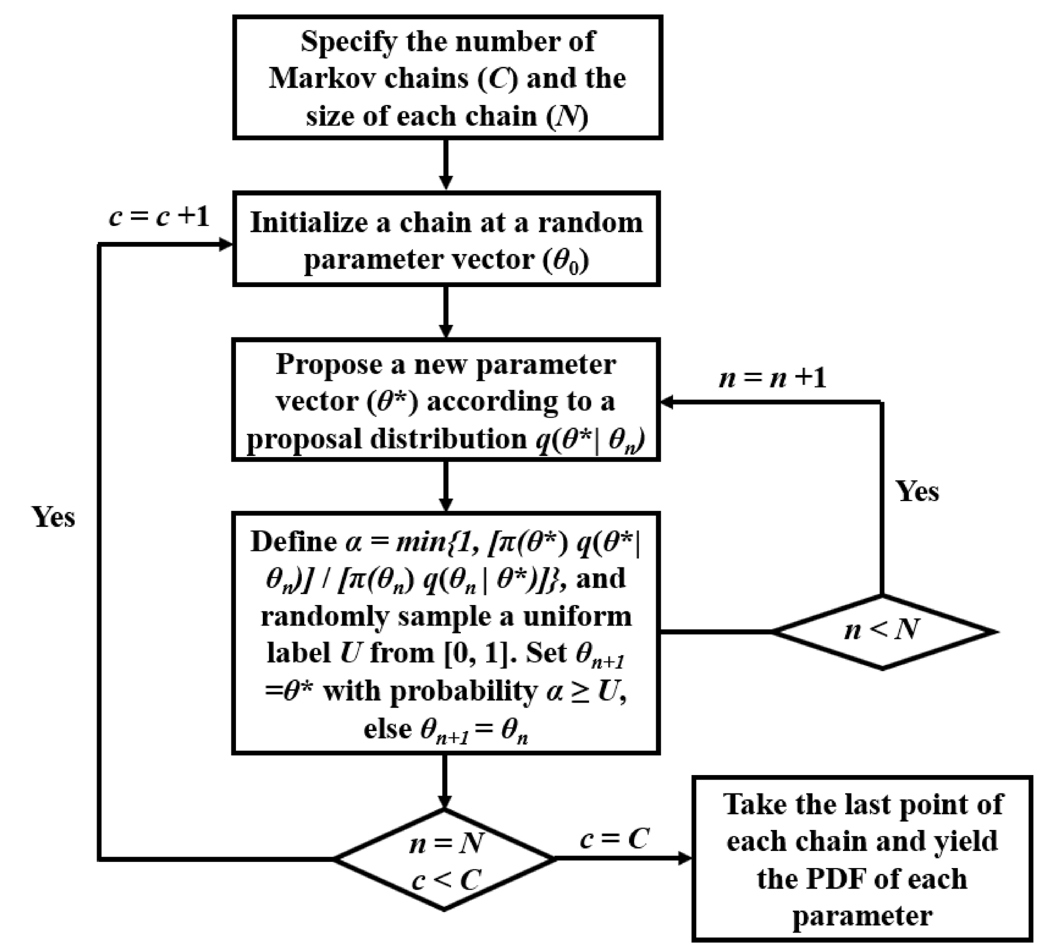
- **Ensemble flood modeling in 1D HEC-RAS**

No.	Channel Roughness	Upstream Flow Input	HEC-RAS Plan Files
1	0.8n	0.8Q	g01 & u01
2	0.8n	Q	g01 & u02
3	0.8n	1.2Q	g01 & u03
4	n	0.8Q	g02 & u01
5	n	Q	g02 & u02
6	n	1.2Q	g02 & u03
7	1.2n	0.8Q	g03 & u01
8	1.2n	Q	g03 & u02
9	1.2n	1.2Q	g03 & u03
10	Average of simulations from No.1-No.9		

Note: n is the Manning's n value for the main channel in the original HEC-RAS models, Q is the streamflow from USGS gauge stations, g** represents a geometry file of a HEC-RAS project, and u** represents a flow data file of a HEC-RAS project.

Methodology (cont.)

- **BMA analysis & Metropolis-Hastings (M-H) algorithm**



$$L(\theta) = \sum_{t=1}^{T} \log\left(\sum_{k=1}^{K} w_k \cdot p_k(y_t \mid f_{k,t}, \sigma_k)\right) = \sum_{t=1}^{T} \log\left(\sum_{k=1}^{K} w_k \cdot \frac{1}{\Gamma(\alpha)\beta^{\alpha}} y_t^{\alpha-1} e^{-\frac{y_t}{\beta}}\right)$$

$$\alpha = f_{k,t}^{\ 2} / \sigma_k^{\ 2} \qquad \beta = \sigma_k^{\ 2} / f_{k,t}$$

- **Evaluation metrics for model performance[3]**

$$UC1 = \frac{N_{obs-90\%}}{n} \cdot 100\%$$

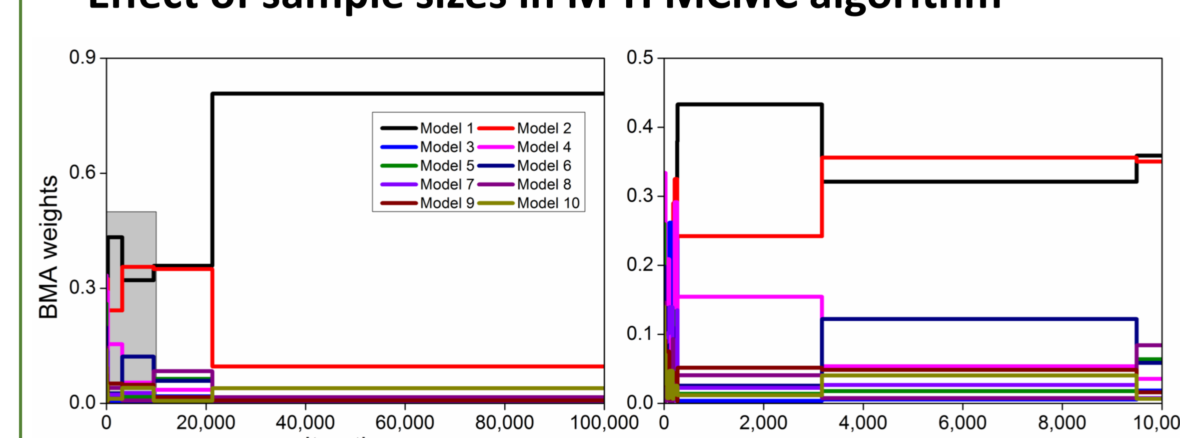
$$UC2 = 1 - NSE = \frac{RMSE^2}{\sigma_{obs}^{\ 2}} \cdot 100\% \qquad RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_{obs,i} - y_{BMA,i})^2}{n}}$$

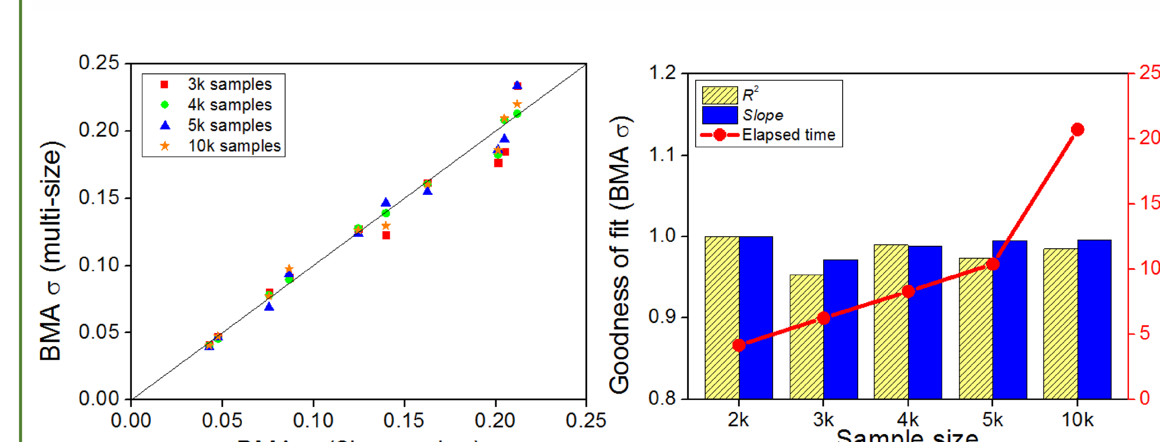
$$UC3 = 1 - KGE = \sqrt{(r-1)^2 + (\frac{\sigma_{BMA}}{\sigma_{obs}} - 1)^2 + (\frac{\bar{y}_{BMA}}{\bar{y}_{obs}} - 1)^2} \cdot 100\%$$

$$UC4 = (1 - R^2 + |1 - Slope|) \cdot 100\%$$

Results and Discussion

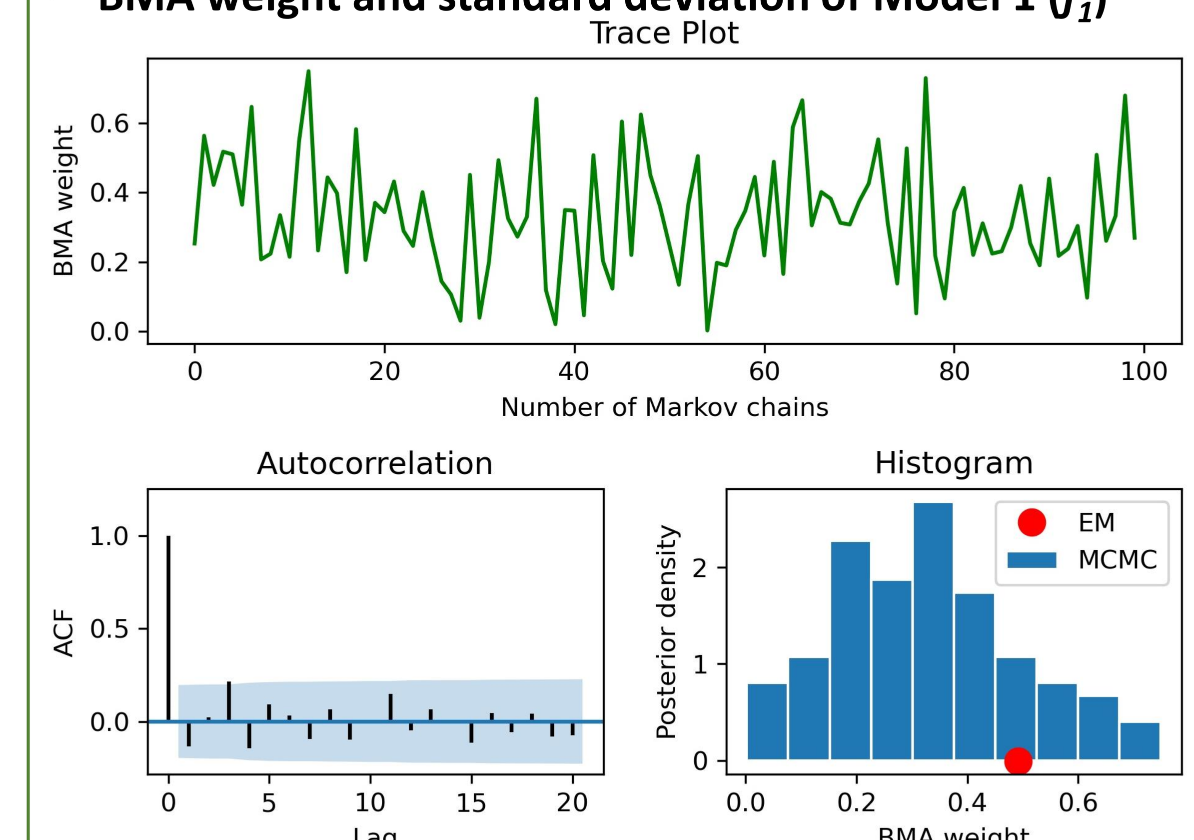
- **Effect of sample sizes in M-H MCMC algorithm**





Results and Discussion (cont.)

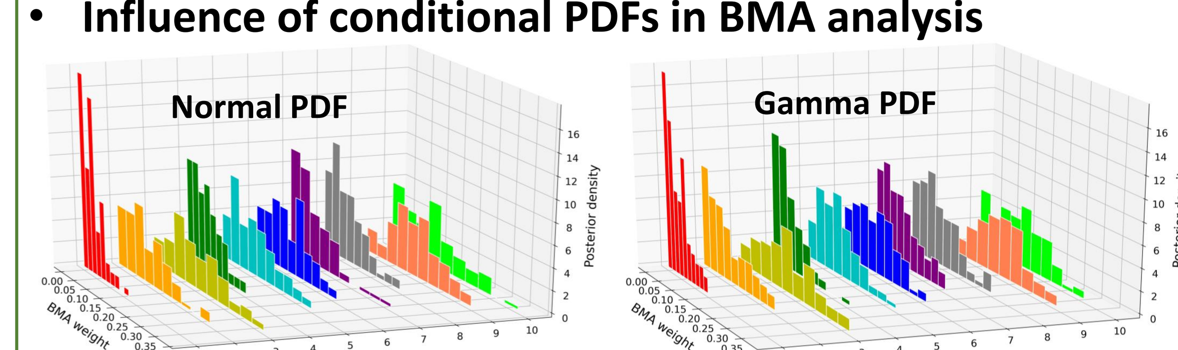
- **BMA weight and standard deviation of Model 1 (f_1)**



No.	EM weight	EM σ (m)	MCMC weight (Uniform & Normal)	MCMC σ (m)	Given σ (m)
1	0.492	0.04	0.332 & 0.354	0.05	0.06
2	0.497	0.04	0.320 & 0.35	0.04	0.06
3	0.0002	0.02	0.068 & 0.062	0.08	0.12
4	0	0.05	0.059 & 0.053	0.09	0.12
5	0	0.08	0.040 & 0.034	0.14	0.18
6	0	0.02	0.044 & 0.033	0.12	0.18
7	0	0.11	0.035 & 0.029	0.16	0.24
8	0	0.05	0.032 & 0.028	0.20	0.24
9	0.01	0.001	0.037 & 0.028	0.20	0.30
10	0	0.09	0.033 & 0.028	0.21	0.30

Algorithm	RMSE (m)	Average 90% prediction interval (m)	UC1 (%)	UC2 (%)	UC3 (%)	UC4 (%)
EM	0.041	0.19	5.00	1.61	1.06	2.00
MCMC (Uniform)	0.037	0.47	0.00	1.33	1.04	1.66
MCMC (Normal)	0.037	0.41	0.00	1.33	0.96	1.64

- **Influence of conditional PDFs in BMA analysis**



Algorithm	RMSE (m)	Average 90% prediction interval (m)	UC1 (%)	UC2 (%)	UC3 (%)	UC4 (%)
Normal PDF	0.421	1.32	13.00	7.54	5.69	7.76
Gamma PDF	0.419	1.34	11.00	7.48	5.67	7.71

Conclusions

- MCMC method with multiple independent chains is applicable in the BMA analysis and can demonstrate a full view of the uncertainty of BMA weights and variances.

References

1. Data source: https://www.weather.gov/hazstat/
2. Raftery, A. E., Gneiting, T., Balabdaoui, F., Polakowski, M. (2005). Using Bayesian model averaging to calibrate forecast ensembles. Monthly weather review, 133(5), 1155-1174.
3. Huang, T., & Merwade, V. (2023). Uncertainty Analysis and Quantification in Flood Insurance Rate Maps Using Bayesian Model Averaging and Hierarchical BMA. Journal of Hydrologic Engineering, 28(2), 04022038.