

An Integrated Evaluation Framework based on Generalized Likelihood Uncertainty Estimation for Quantifying Uncertainty in Flood Modeling

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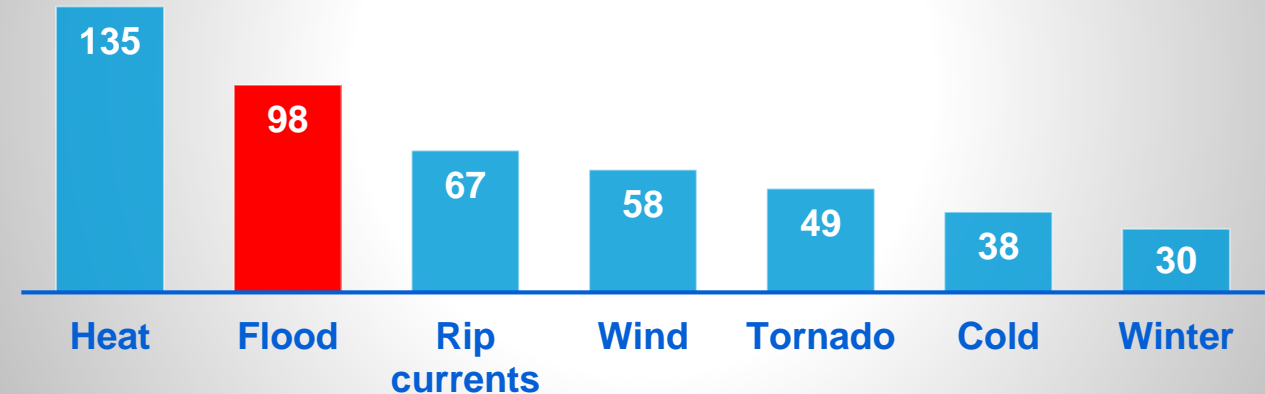
Introduction

➤ Flood & Flood model

- Flooding is one of the most devastating natural disasters in the world
- Evaluation of reliability and accuracy of model predictions is a critical issue
- How to demonstrate and quantify the uncertainty in evaluation metrics?

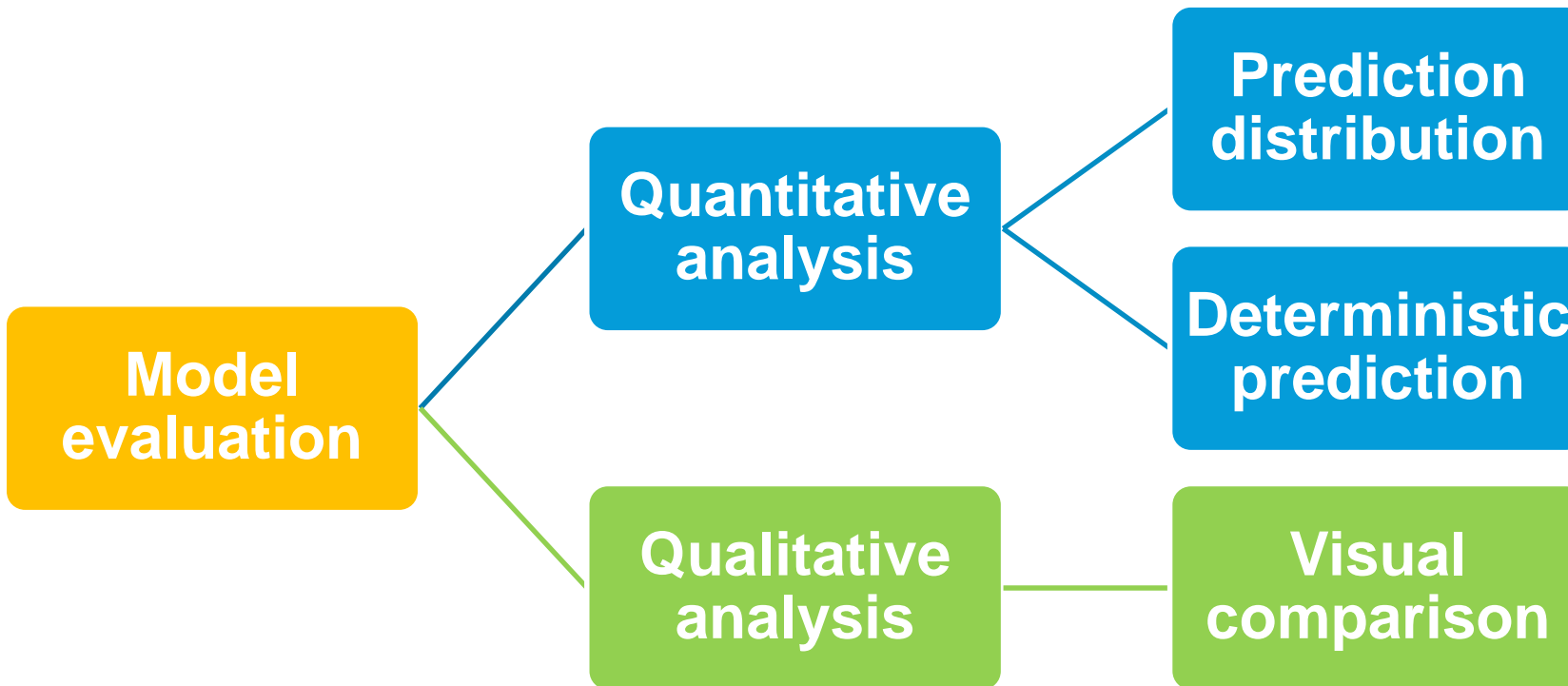


Weather Fatalities in USA 10-year Average (2012-2021)



Introduction

- **Uncertainty in evaluation criteria for flood models**



- **Statistical metrics**

- ❑ Reliability of prediction distribution
- ❑ *MSE/RMSE, NSE, KGE, R^2 , etc.*

- **Limitations**

- ❑ No “perfect” single metric
- ❑ No broad consensus on uncertainty quantification for flood models

Introduction

- **Objectives:** quantify uncertainty in evaluation metrics
- ✓ revisit the statistical meanings of existing metrics and propose an integrated evaluation framework
- ✓ investigate the effect of different prior distributions in GLUE analysis on the uncertainty metrics
- ✓ evaluate the effect of different high-flow scenarios on the uncertainty metrics
- ✓ explore the impact of different types of measurement errors on the uncertainty metrics

Integrated Evaluation Framework

➤ Uncertainty coefficients

$$UC1 = \frac{N_{obs-90\%}}{n} \cdot 100\%$$

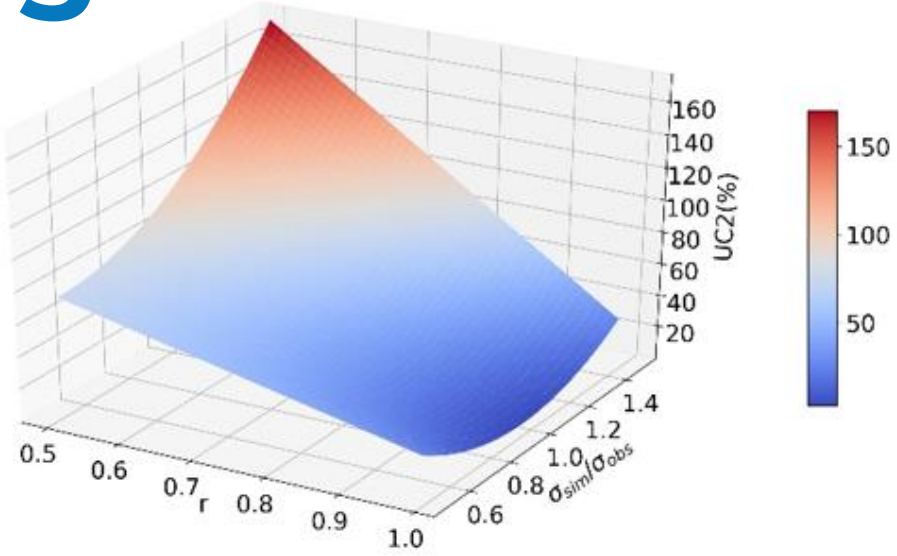
$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_{obs,i} - y_{sim,i})^2}{n}}$$

$$UC2 = 1 - NSE = \frac{\sum_{i=1}^n (y_{obs,i} - y_{sim,i})^2}{\sum_{i=1}^n (y_{obs,i} - \bar{y}_{obs,i})^2} = \frac{\sum_{i=1}^n (y_{obs,i} - y_{sim,i})^2 / n}{\sum_{i=1}^n (y_{obs,i} - \bar{y}_{obs,i})^2 / n} = \frac{RMSE^2}{\sigma_{obs}^2} \cdot 100\%$$

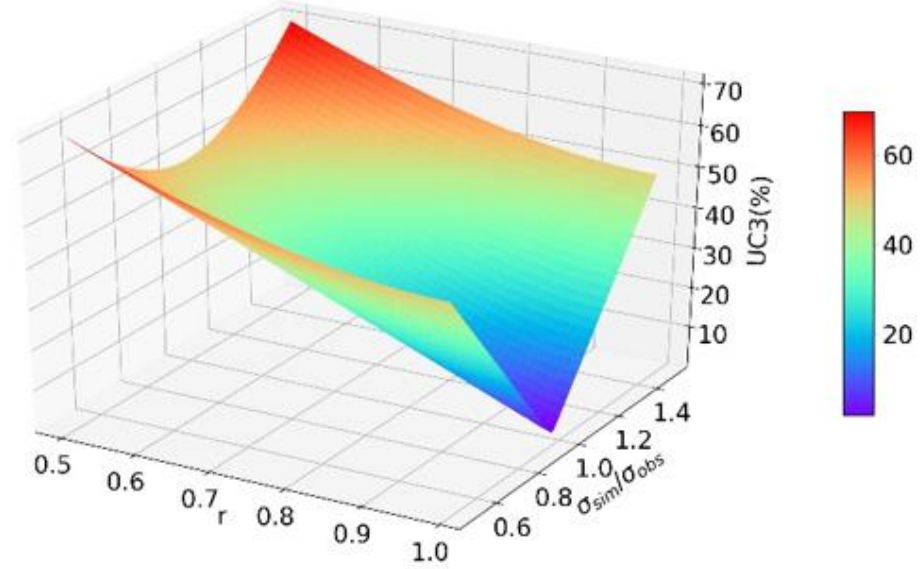
$$UC3 = 1 - KGE = \sqrt{(r - 1)^2 + \left(\frac{\sigma_{sim}}{\sigma_{obs}} - 1\right)^2 + \left(\frac{\bar{y}_{sim}}{\bar{y}_{obs}} - 1\right)^2} \cdot 100\%$$

$$UC4 = 1 - R^2 = \frac{\sum_{i=1}^n (y_{obs,i} - \hat{y}_i)^2}{\sum_{i=1}^n (y_{obs,i} - \bar{y}_{obs,i})^2} \cdot 100\% = \frac{\sum_{i=1}^n (y_{obs,i} - Slope \cdot y_{sim,i})^2}{\sum_{i=1}^n (y_{obs,i} - \bar{y}_{obs,i})^2} \cdot 100\%$$

Integrated Evaluation Framework



(a) UC2.



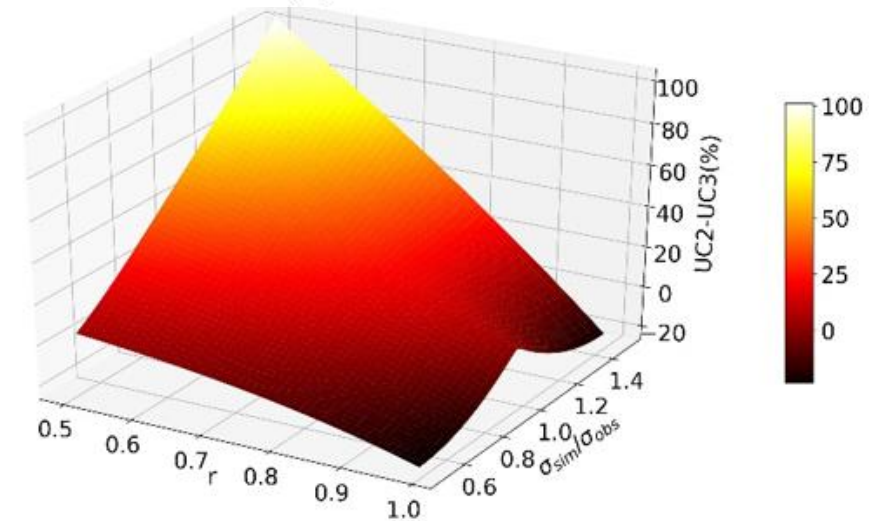
(b) UC3.

Assuming

$$\bar{y}_{sim} = \bar{y}_{obs}$$

$$UC2 = 1 - NSE = \left[1 - r^2 + \left(r - \frac{\sigma_{sim}}{\sigma_{obs}} \right)^2 + \left(\frac{\bar{y}_{sim} - \bar{y}_{obs}}{\sigma_{obs}} \right)^2 \right] \cdot 100\%$$

$$UC3 = 1 - KGE = \sqrt{(r - 1)^2 + \left(\frac{\sigma_{sim}}{\sigma_{obs}} - 1 \right)^2 + \left(\frac{\bar{y}_{sim}}{\bar{y}_{obs}} - 1 \right)^2} \cdot 100\%$$



(c) UC2-UC3.

Integrated Evaluation Framework

Integrated criterion for
uncertainty quantification

$$IUC = \alpha \cdot UC1 + (1 - \alpha) \cdot \frac{UC2 + UC3 + UC4_{adj}}{3}$$

Reliability of distributions

Accuracy of deterministic predictions

$UC1$

$UC2$
(1-NSE)

$UC3$
(1-KGE)

$UC4$
(1- R^2)

average 90% interval width

$RMSE$

σ_{obs}

$Mean_{obs}$

σ_{sim}

$Mean_{sim}$

r

$Slope$

Integrated evaluation framework for flood models

$$UC4_{adj} = 1 - R^2 + |1 - Slope|$$

Integrated Evaluation Framework

- Values of the empirical factor (α) in *IUC*

$$IUC = \alpha \cdot UC1 + (1 - \alpha) \cdot \frac{UC2 + UC3 + UC4_{adj}}{3}$$

90% prediction interval of water stage distribution

α

average interval width ≤ 0.3 m

0.1

$0.3 \text{ m} < \text{average interval width} \leq 0.9 \text{ m}$

0.25

$0.9 \text{ m} < \text{average interval width} \leq 1.2 \text{ m}$

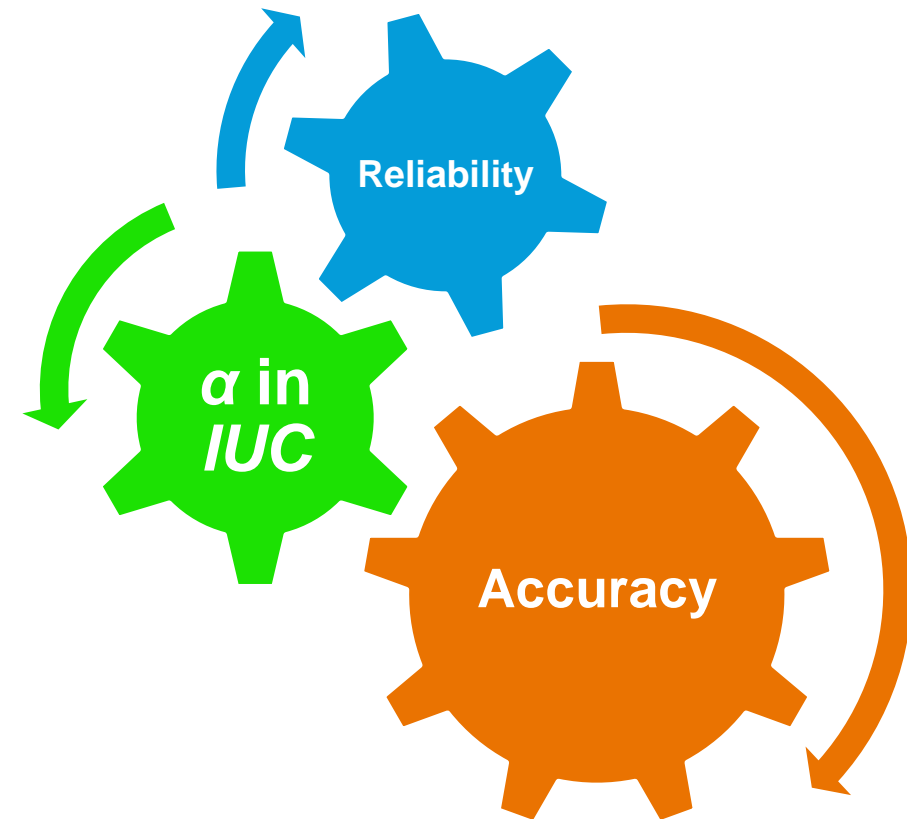
0.5

$1.2 \text{ m} < \text{average interval width} \leq 1.8 \text{ m}$

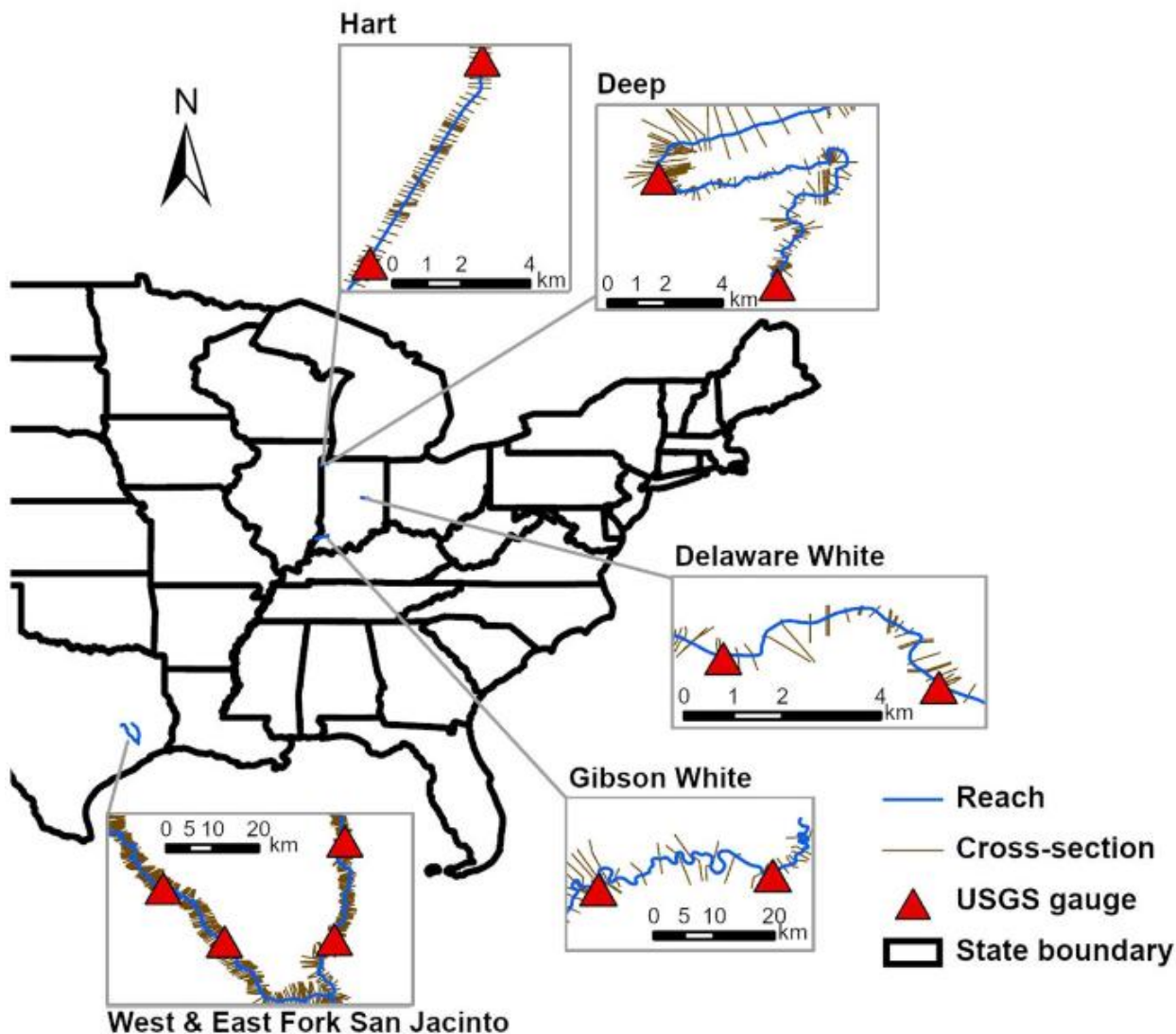
0.25

average interval width > 1.8 m

0.1



Study Area and Data



Study Stream (State-No.)	Channel length (km)	Average channel width (m)	Channel slope (%)
Hart (IN-1)	8.45	16	0.1037
Deep (IN-2)	19.55	48	0.0095
Delaware White (IN-3)	6.76	64	0.0631
Gibson White (IN-4)	70.38	182	0.0087
West Fork San Jacinto (TX-1)	56.31	227	0.1624
East Fork San Jacinto (TX-2)	50.11	76	0.0438

FEMA model source: <https://dnrmmaps.dnr.in.gov/appsphp/model/index.php>
<https://webapps.usgs.gov/infrm/estbfe/>

Simulation period: 200 days (summer & fall in 2021)

Methodology

- **Uncertainty quantification based on GLUE for FEMA models (1D HEC-RAS)**
 - **Generalized likelihood uncertainty estimation (GLUE)** incorporates both Monte Carlo sampling and the Bayesian analysis (Beven and Binley, 1992).

Sampling
from prior
distributions
of
uncertainty
sources



Model
ensemble
and multiple
model runs



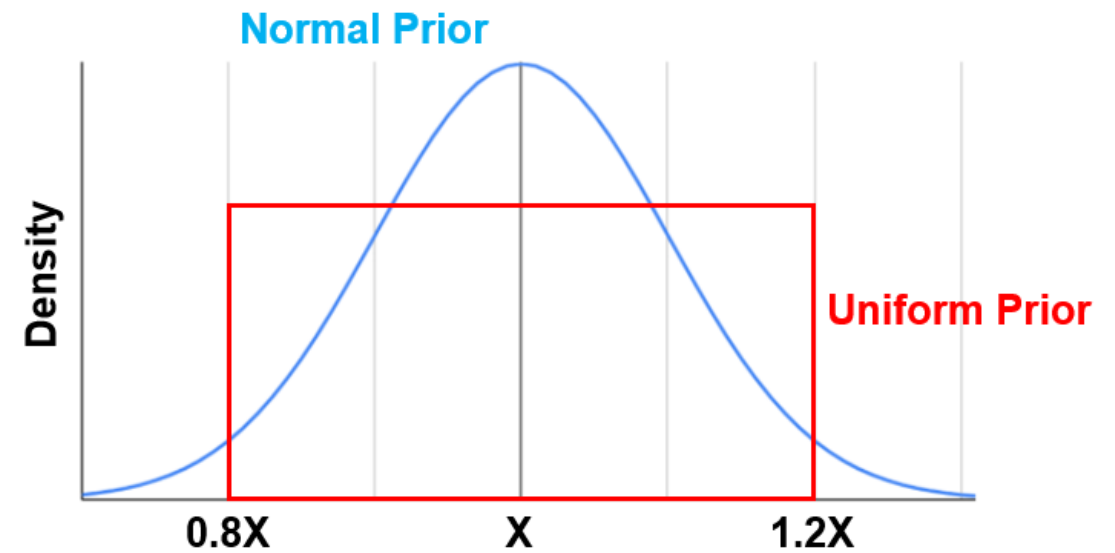
Multiple
results and
compare to
observations
based on
Likelihood



Estimate
uncertainty
based on
behavioral
outputs

Methodology

- **Uncertainty quantification based on GLUE for FEMA models (1D HEC-RAS)**



Uncertainty Type	Uncertainty Source	Prior Distribution-1 (Uniform)	Prior Distribution-2 (Normal)
Model Parameter	Channel roughness (n)	$U(0.8n, 1.2n)$	$N(n, 0.1n)$
Input Data	Upstream flow input (Q)	$U(0.8Q, 1.2Q)$	$N(Q, 0.1Q)$

400 (=20 × 20) model configurations (plan files) in HEC-RAS

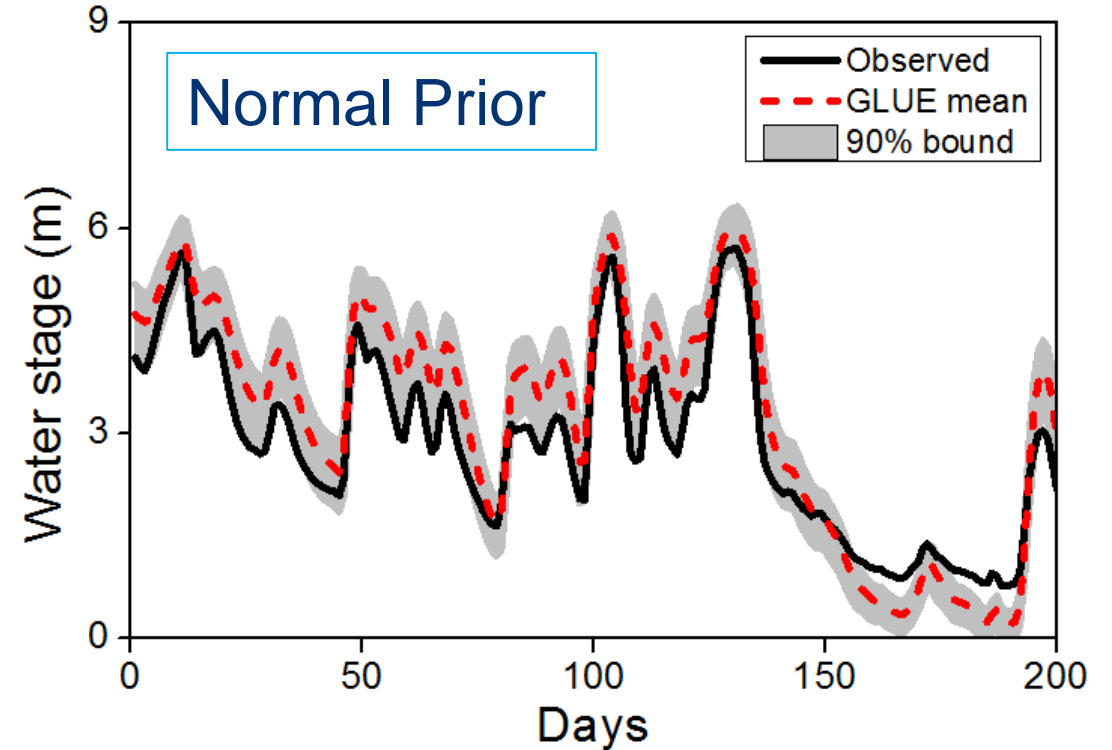
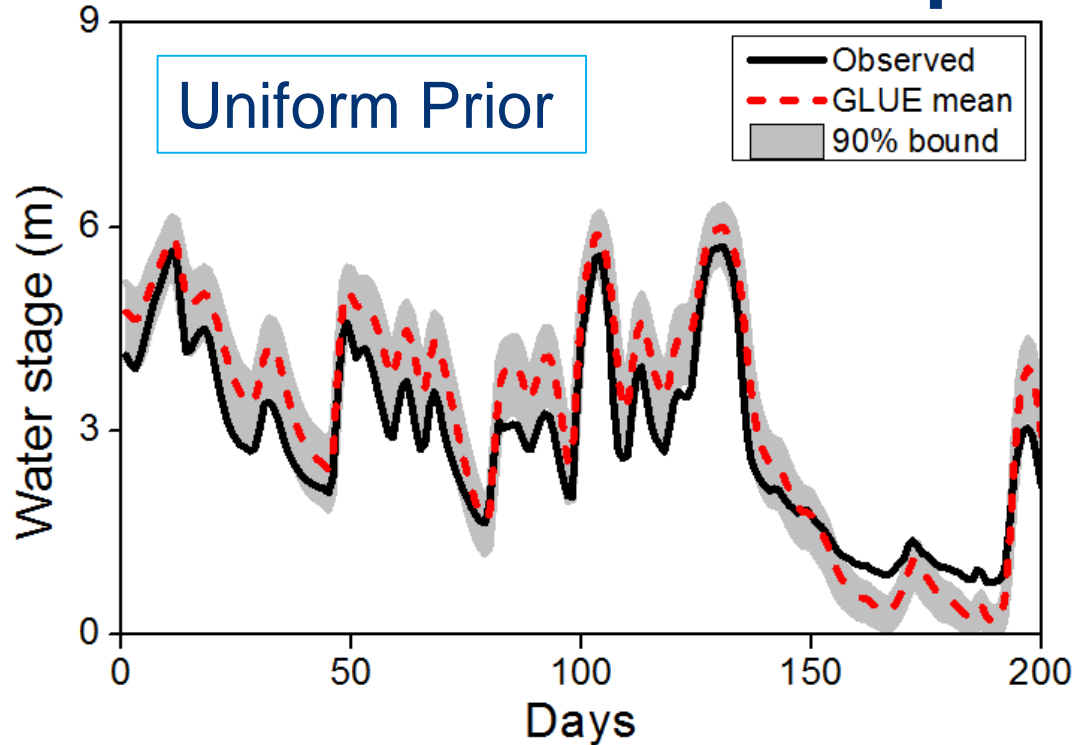
- **Likelihood function:**

$$L(f_k | D) = \frac{1}{\sum_{t=1}^T (f_{k,t} - y_t^{obs})^2}$$

- **Cut-off threshold: top 75%**

Results and Discussion

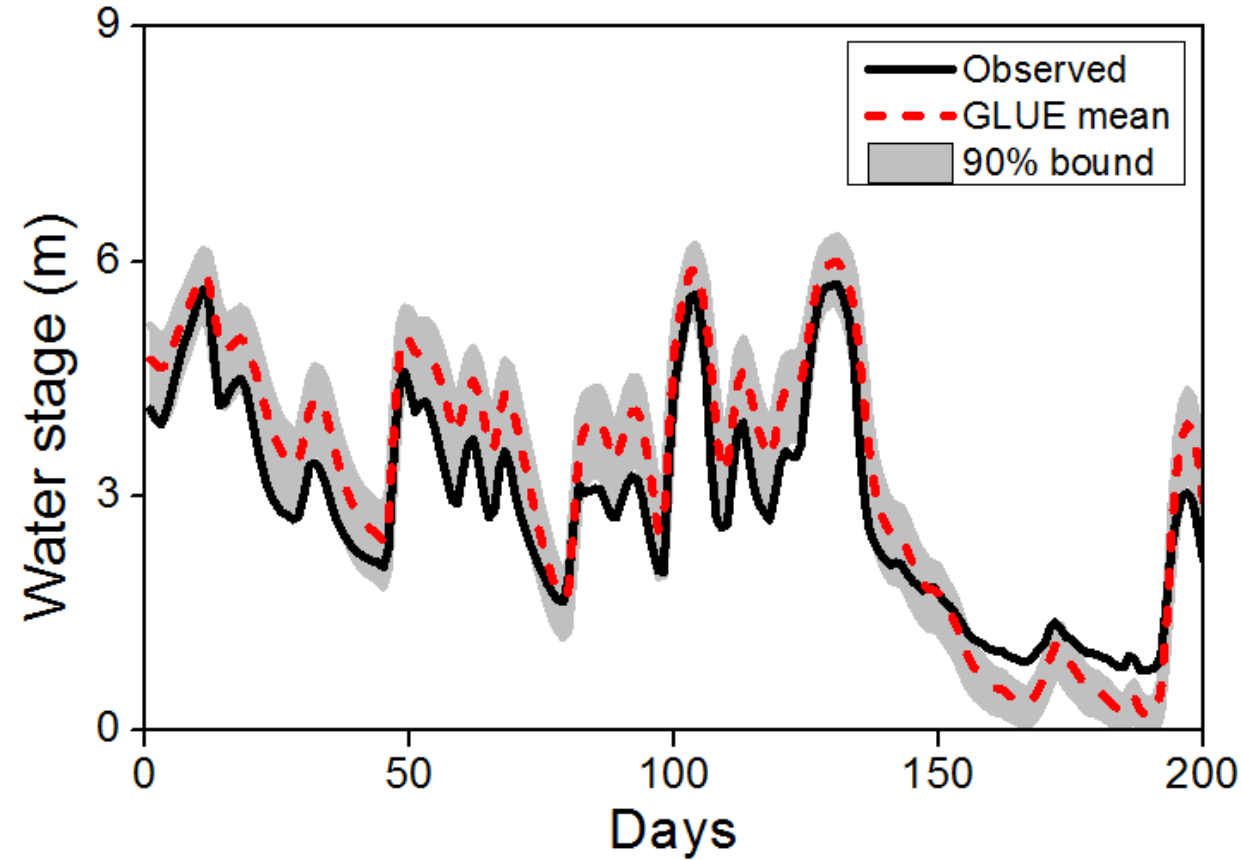
➤ Effect of different prior distributions in GLUE



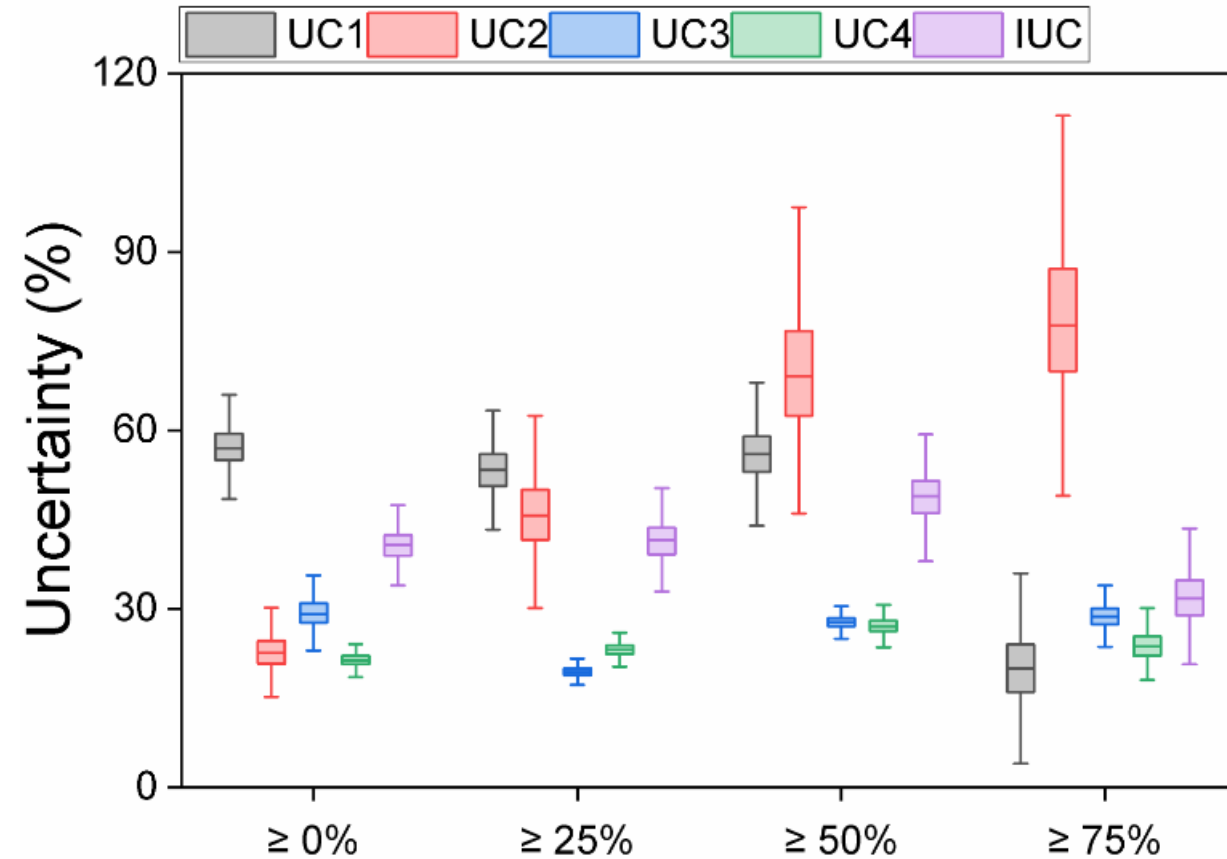
Model	Type	Mean (U)	90% CI (U)	Mean (N)	90% CI (N)
IN-4	$UC1$ (%)	57.36	[52.00, 63.50]	59.67	[54.00, 65.50]
	$UC2$ (%)	22.88	[18.72, 27.86]	23.06	[18.76, 28.06]
	$UC3$ (%)	29.36	[25.34, 33.41]	29.22	[25.56, 33.47]
	$UC4$ (%)	21.42	[19.71, 23.15]	21.58	[19.88, 23.35]
	IUC (%)	40.55	[35.20, 45.01]	39.62	[30.95, 45.91]

Results and Discussion

➤ Evaluation under different high-flow scenarios



Water stage predictions of IN-4 model

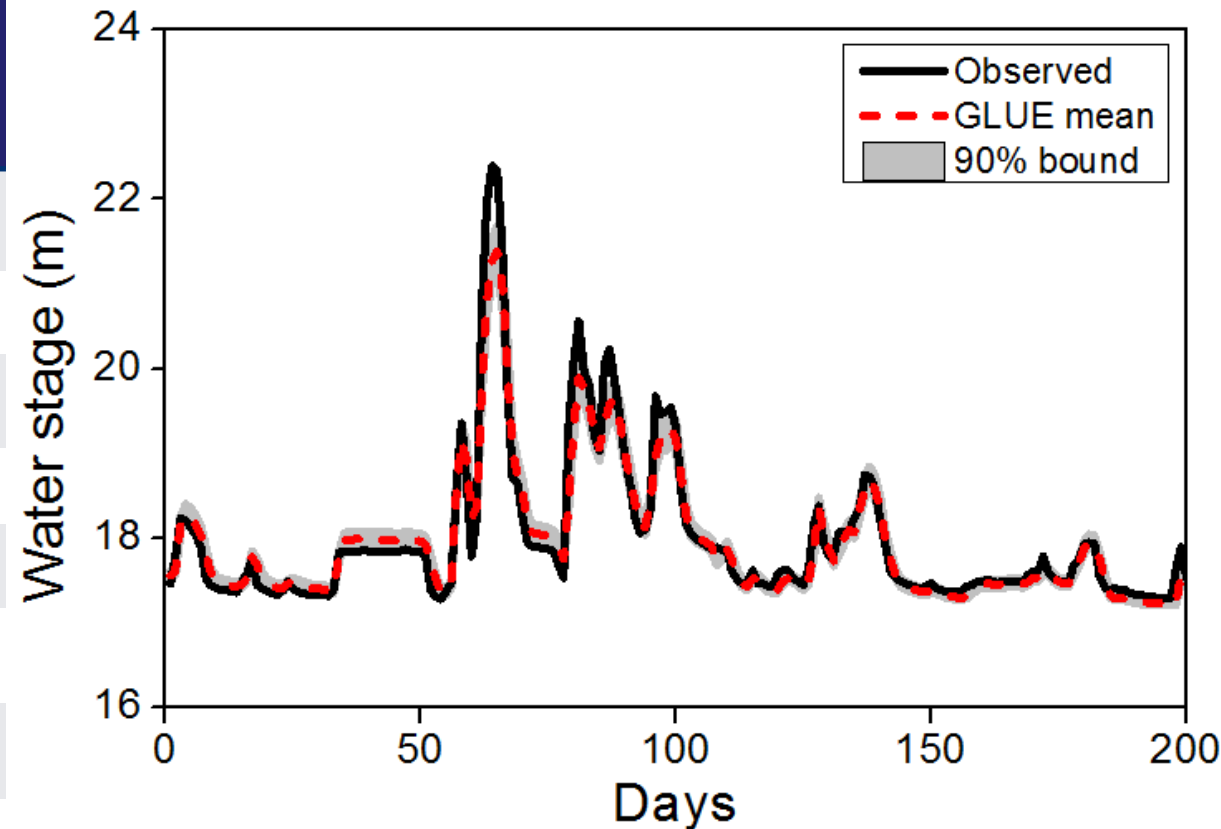


UC distributions of IN-4 model

Results and Discussion

➤ Integrated evaluation framework for flood models

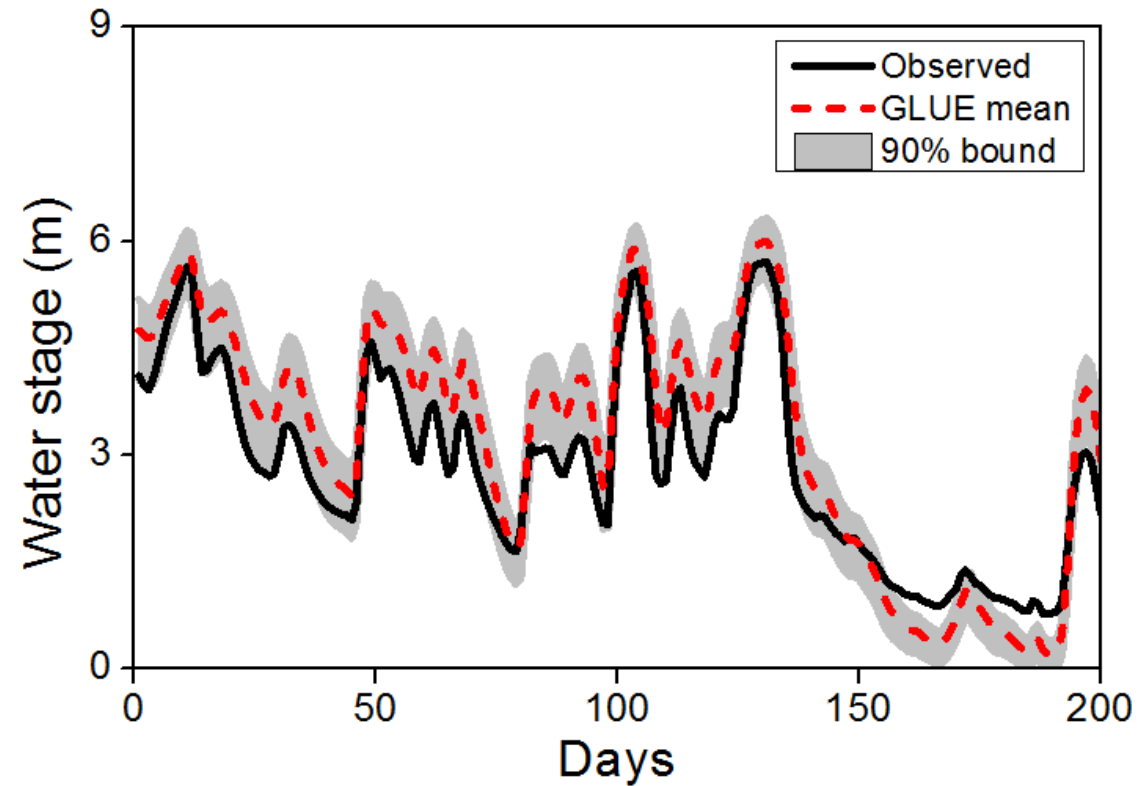
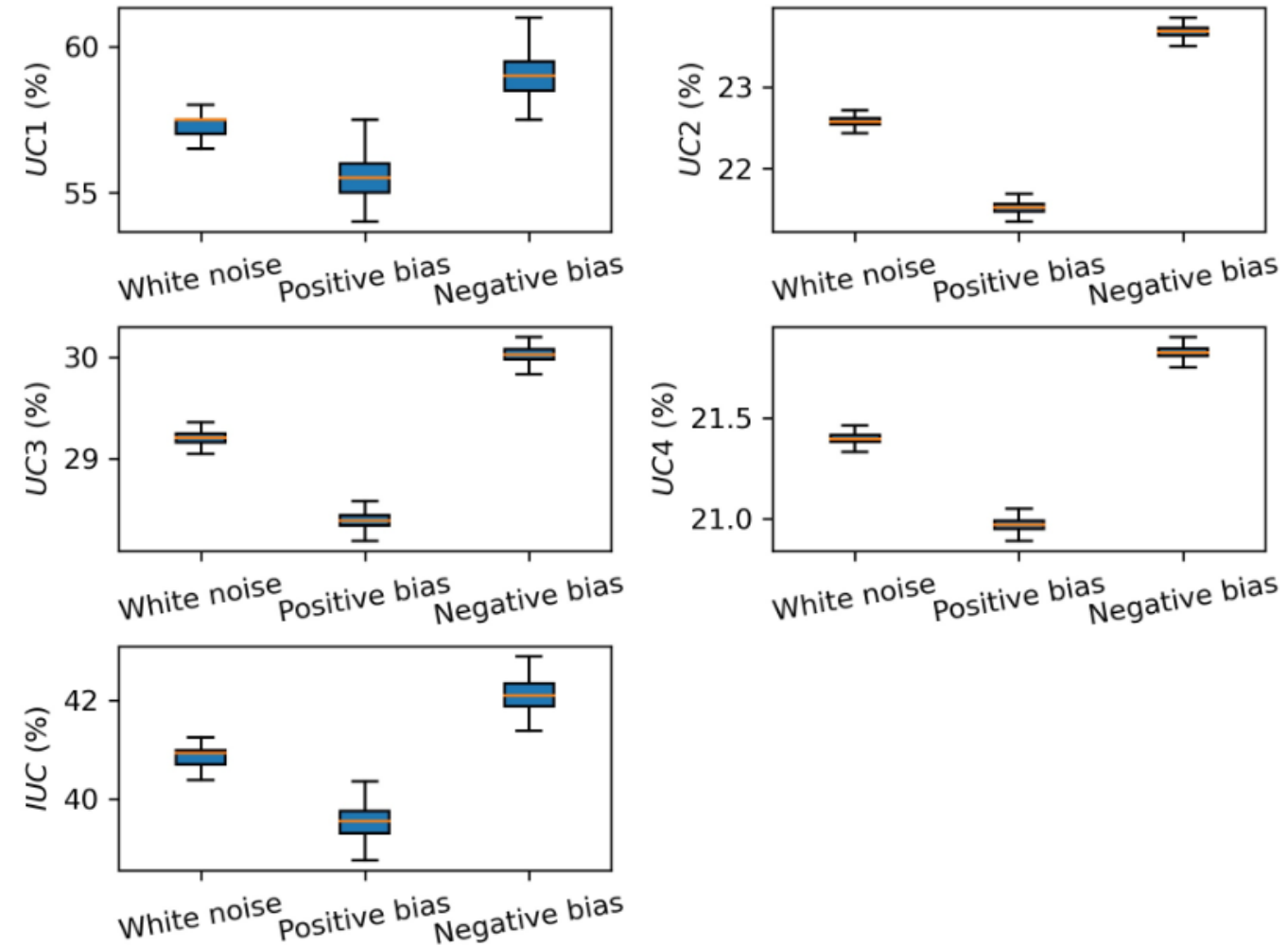
\geq Per (%)	<i>IUC</i> (%)	90% width (m)	<i>RMSE</i> (m)	Ratio of <i>sd</i>	Ratio of <i>mean</i>	<i>r</i>	<i>slope</i>
0	14.96	0.16	0.23	0.85	1.00	0.98	1.00
10	14.92	0.17	0.24	0.84	1.00	0.98	1.00
20	15.19	0.18	0.26	0.83	1.00	0.97	1.00
30	15.84	0.20	0.27	0.82	1.00	0.97	1.00
40	17.57	0.21	0.29	0.81	1.00	0.97	1.00
50	18.65	0.23	0.31	0.78	1.00	0.97	1.00
60	18.82	0.25	0.34	0.77	1.00	0.97	1.01
70	19.28	0.29	0.39	0.76	0.99	0.97	1.01
80	29.33	0.34	0.46	0.73	0.99	0.96	1.01
90	40.70	0.39	0.64	0.80	0.97	0.93	1.03



Water stage predictions of TX-1 model

Results and Discussion

➤ Impact of various measurement errors

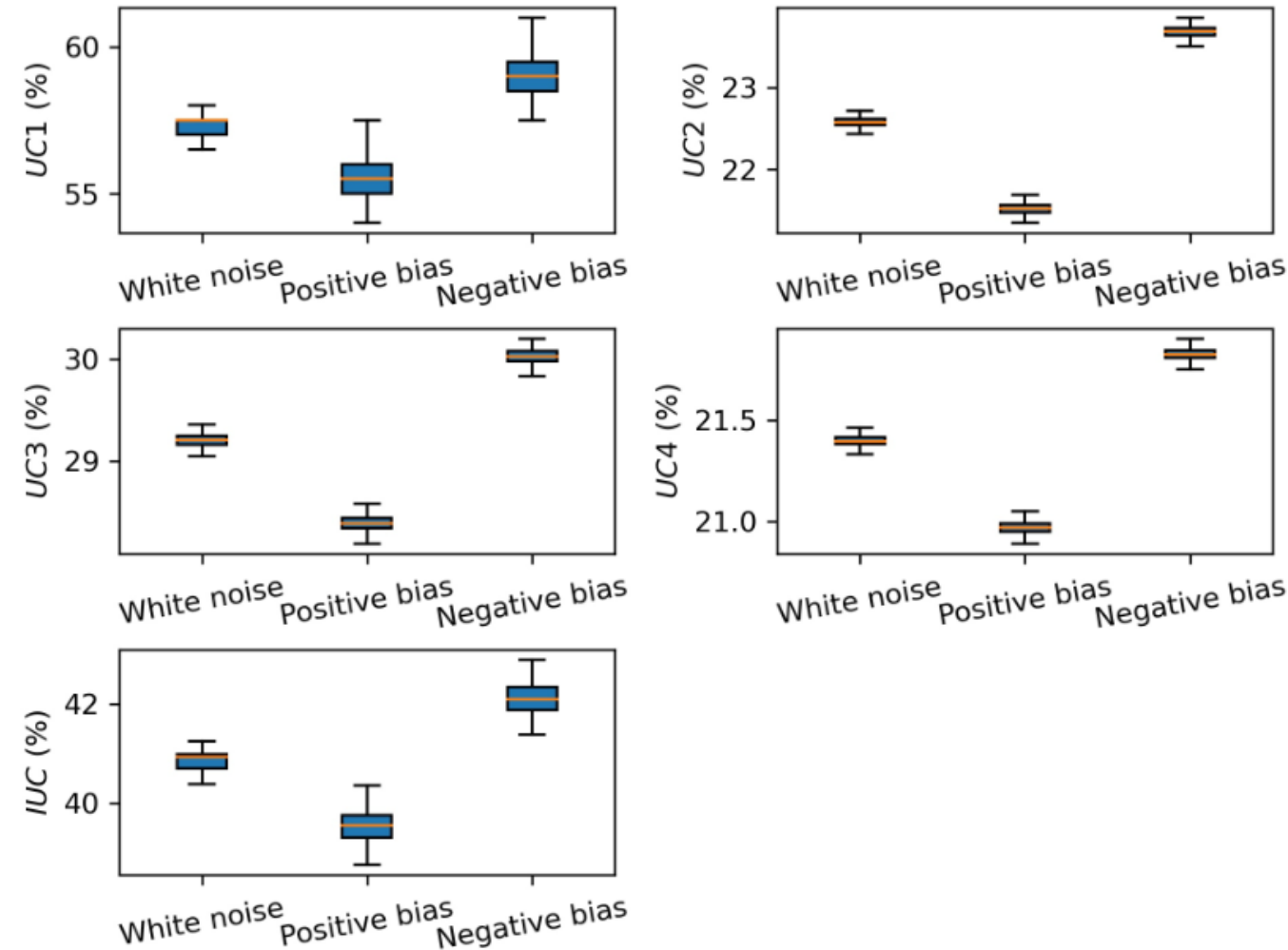


Water stage predictions of IN-4 model

UC distributions of IN-4 model

Results and Discussion

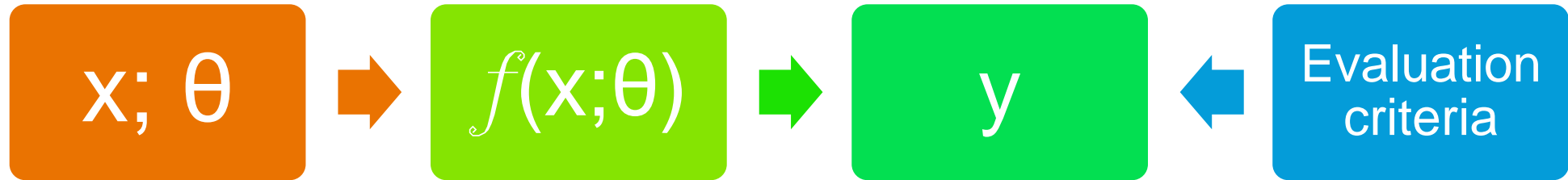
➤ Impact of various measurement errors



UC distributions of IN-4 model

Type of errors	Type of UC	Mean (%)	Relative change (%)	Type of UC	Mean (%)	Relative change (%)
No errors	UC1	57.50	/	UC2	22.57	/
WN		57.35	-0.26		22.58	0.04
PB		55.36	-3.72		21.52	-4.65
NB		59.11	2.80		23.69	4.96
No errors	UC3	29.20	/	UC4	21.40	/
WN		29.20	0.00		21.40	0.00
PB		28.39	-2.77		20.97	-2.01
NB		30.03	2.84		21.83	2.01
No errors	IUC	40.95	/			
WN		40.87	-0.20			
PB		39.49	-3.57			
NB		42.15	2.93			

Conclusions



- A uniform prior in the GLUE analysis is adequate for the uncertainty quantification in the absence of solid prior knowledge.
- Evaluation metrics (*UCs*) are random variables: conditional on a specific flow scenario; present a statistical distribution.
- White-noise measurement errors have the least impact on *UCs*.
- The integrated evaluation framework based on GLUE can be applied to any other hydrologic variables.

THANK YOU

“No one trusts a model except the man who wrote it;
everyone trusts an observation except the man who
made it.”

– Harlow Shapely

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