Supporting Information for "Characteristics of Blue Corona Discharges observed by ASIM"

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Introduction The supplementary information describes the process of calculating rise times and durations for the blue events, how to correct for satellite parallax and the derivation of the cloud depth coefficient. Additionally, there is a figure of the geographical distribution of events without smoothing applied and a figure with the better estimate for the cloud diffusion coefficient.

1. Rise time and Duration Calculation

Calculating the rise time and duration of the blue flashes in the ASIM photometers uses the standard definition of rise time and duration. The rise time is defined as the time it takes for the event pulse to increase from 10% of the amplitude to 90% of the amplitude while the duration is defined as the rise time plus the time it takes for the event to subside from 90% of the amplitude to 10% of the amplitude. In practice, this is done by counting the number of samples within the interval defined above.

2. Parallax Correction

Satellite images of clouds far away from the local zenith will artificially displace cloud locations due to the viewing angle. Satellite images always assume the target is nadir which is not the case. To mitigate this error the parallax must be corrected by assuming an average cloud top height. Given the satellite height h, the radius of the earth r, the cloud height C_h and the event latitude l, one can generate a triangle which corrects for this parallax. The hypotenuse of this triangle by the law of cosines is:

$$d = \sqrt{(h+r)^2 + r^2 - 2R(h+r)\cos(l)} \tag{1}$$

Using this, we can find the angle, A of the triangle generated between the cloud and the ground as:

$$\phi = \sin^{-1}\left(\frac{(r+h)\sin(l)}{d}\right), A = \pi - \phi \tag{2}$$

and finally the length of the parallax p as

$$p = C_h \tan(A) \tag{3}$$

Once the length of the parallax is found, the event is moved length p on the surface of the Earth along the line between the satellite position and the event position.

3. Photon Intensity Derivation

Starting from equation 27 in (Luque et al., 2020) we have:

$$F(t) = \frac{\exp\left(-\frac{t}{\tau_A} - \frac{\tau_D}{t}\right)}{\sqrt{\pi}\tau_D}\left(\frac{t}{\tau_D}\right)^{-\frac{3}{2}} \tag{4}$$

where $\tau_D = \frac{L^2}{4D}$ and τ_A is the absorption. Introducing $\alpha = \frac{\tau_D}{t}$ and neglecting absorption we have

$$F(t) = F_\alpha(\alpha) = \frac{\exp(-\alpha)\,\alpha^{\frac{3}{2}}}{\sqrt{\pi}\tau_D} \tag{5}$$

The derivative is then:

$$F'_\alpha(\alpha) = \frac{1}{\sqrt{\pi}\tau_D}\exp(-\alpha)\left(-\alpha^{\frac{3}{2}} + \frac{3}{2}\sqrt{\alpha}\right) = \frac{\exp(-\alpha)\sqrt{\alpha}}{\sqrt{\pi}\tau_D}\left(\frac{3}{2} - \alpha\right). \tag{6}$$

The maximum is reached at $\alpha = \frac{3}{2}$ and is given by:

$$F_{max} = \frac{\exp\left(-\frac{3}{2}\right)\left(\frac{3}{2}\right)^{\frac{3}{2}}}{\sqrt{\pi}\tau_D} \tag{7}$$

And the flux can the be written in the original variable as

$$\frac{F(t)}{F_{max}} = \frac{\exp\left(\frac{3}{2} - \frac{\tau_D}{t}\right)}{\left(\frac{3}{2}\frac{t}{\tau_D}\right)^{\frac{3}{2}}} \tag{8}$$

which is equation 2 from the paper.

We note that it a function of only $\frac{t}{\tau_D}$ which makes it easy to solve for the rise time. 10% of the maximum is reached when $t_{10} = 0.171174\tau_D$ and 90% of the maximum is reached when $t_{90} = 0.468486\tau_D$ the 10% to 90% rise time is then $t_{90} - t_{10} = 0.297312$.

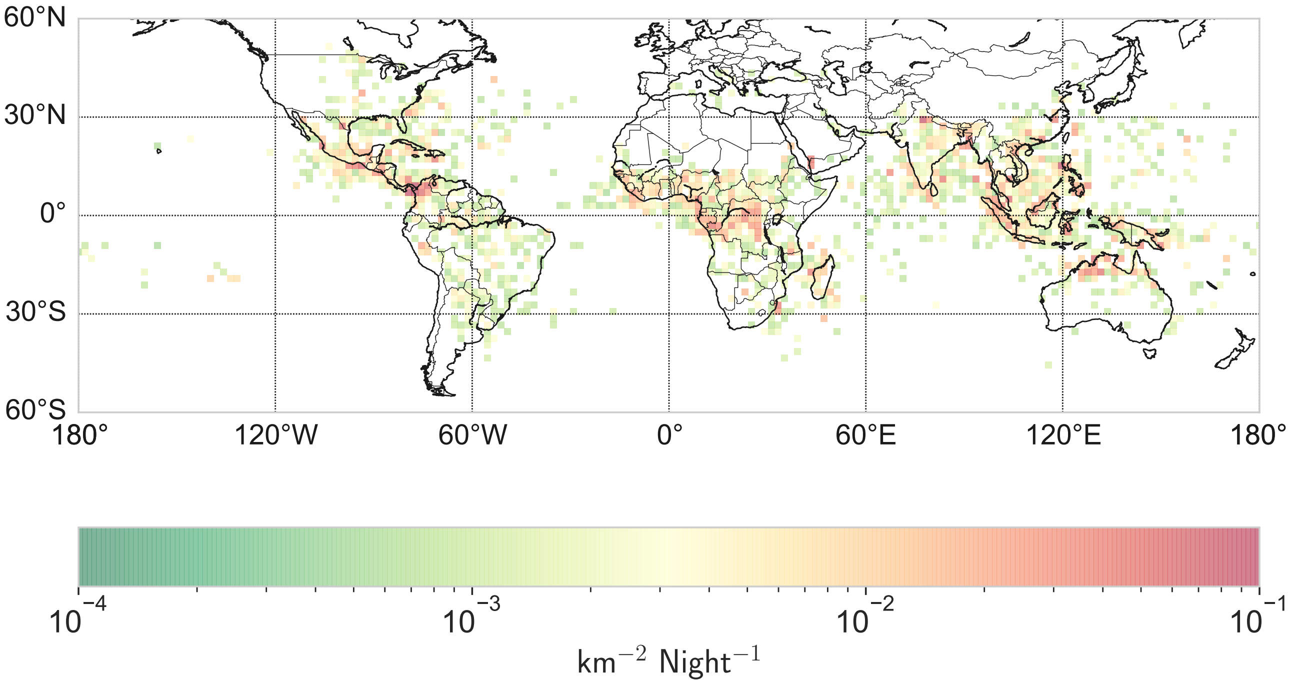

Figure S1. Non-smooth geographical distribution of BCD events with the same color scale as in the main paper with bins of size $2° \times 2°$.

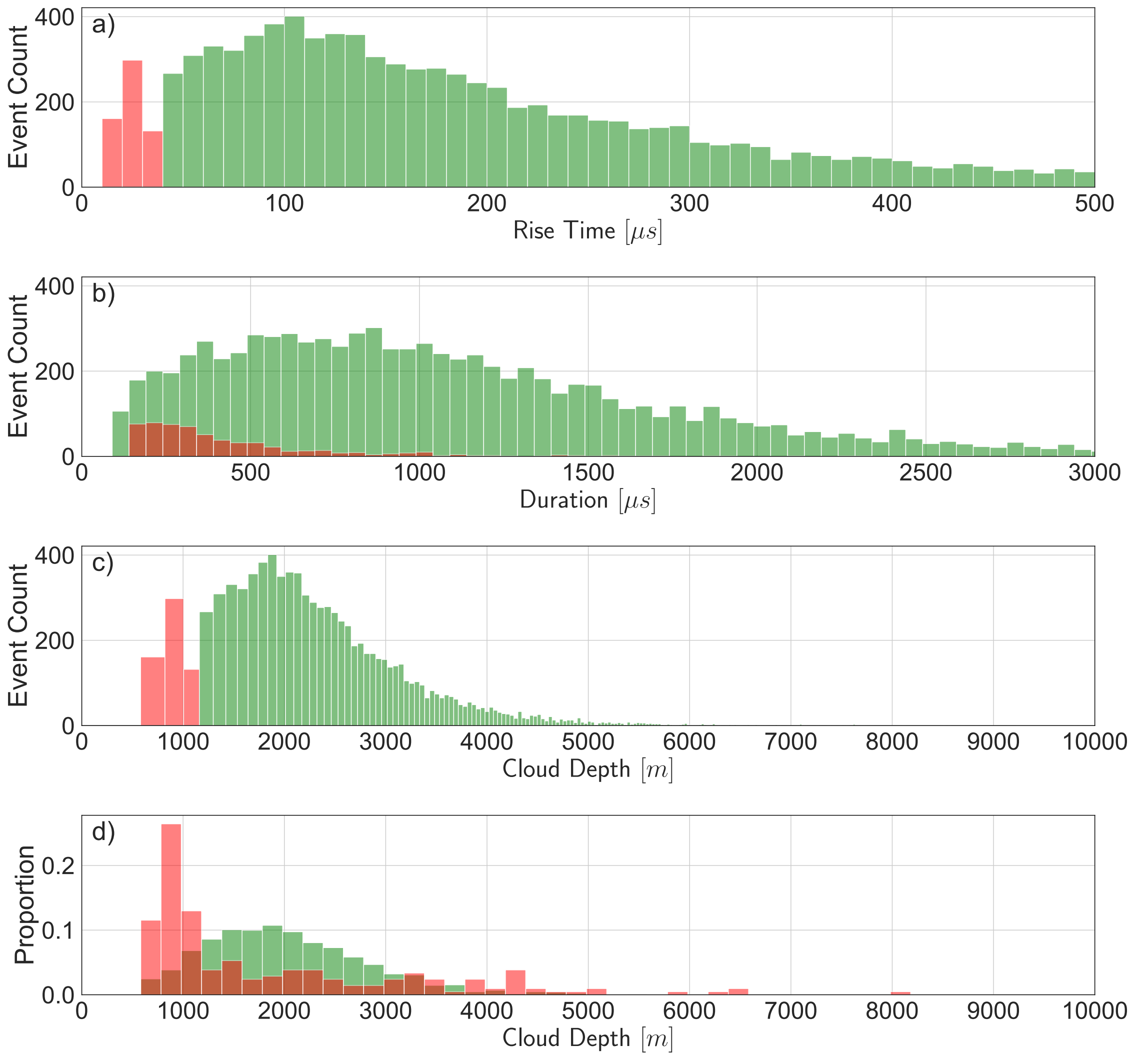

Figure S2. Recreation of figure 4 with $D = 2.5 \times 10^9 \mathrm{m}^2\mathrm{s}^{-1}$.