221914: Biennial-Aligned Lunisolar-Forcing of ENSO: Implications for Simplified Climate Models

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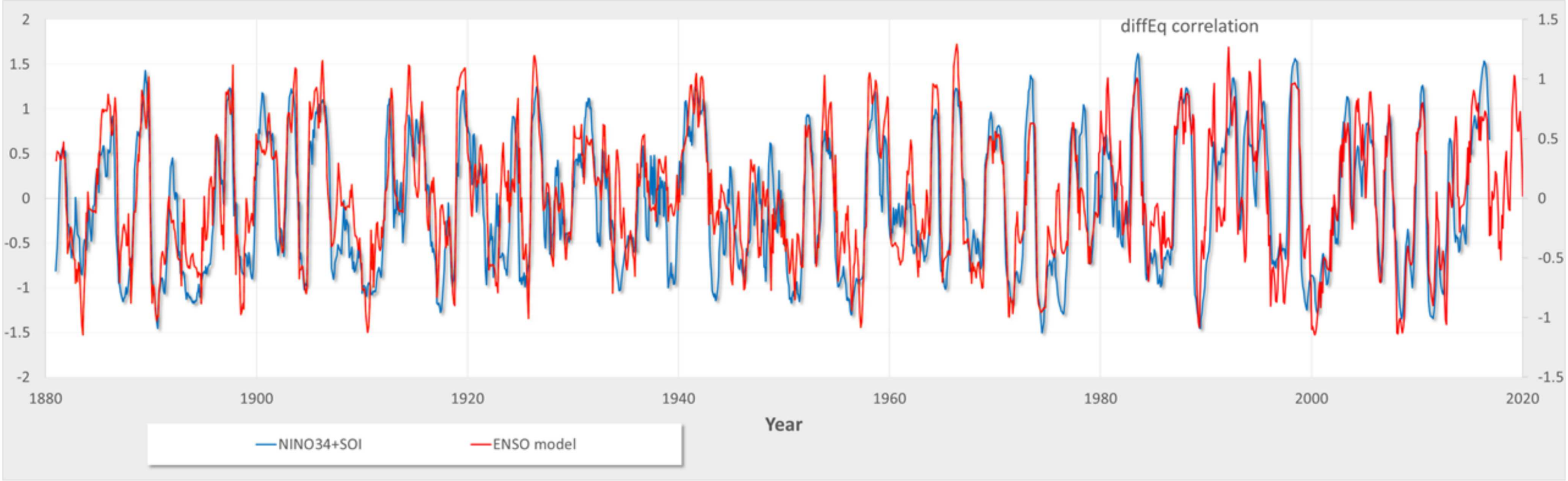
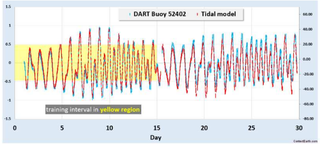
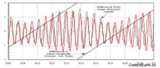


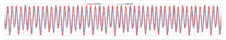
It's well known that lunar gravitational forces lead to ocean tides and deep ocean mixing, but why not the oscillation in the equatorial Pacific Ocean thermocline? If a seasonal impulse that exaggerates the draconic and anomalistic lunar cycles is applied to Laplace's tidal equations, the result shown above is obtained.

Model is very similar to conventional tidal analysis but operates on a long-period basis due to the seasonal impulse influence.



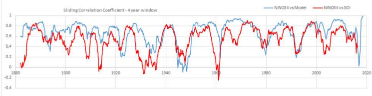


Precise modeling of draconic and anomalistic periods required to align seasonal impulse

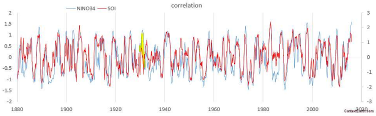


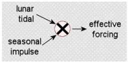
These second-order effects are mainly due to the synodic influence on the draconic and anomalistic cycles.

The model fit is good considering that only four known periods are applied (draconic, anomalistic, synodic, and annual).. Regions that don't align well are associated with discrepancies observed between the NINO34 and SOI time series..



Correlation likely limited by noise in the SOI signal, but perhaps more high-resolution work is needed to establich what is signal versus noise.





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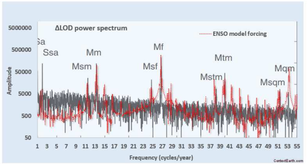
Any sloshing model for ENSO implies angular momentum changes. The forcing for the ENSO model aligns perfectly with measured LOD-based changes in the earth's angular momentum



Laplace developed his namesake tidal equations to mathematically explain the behavior of tides by applying straightforward Newtonian physics. In their expanded form, known as the primitive equations, Laplace's starting formulation is used as the basis of almost all detailed climate models. The concise derivation for a model of ENSO depends on reducing Laplace's tidal equations along the equator.

Part 1 : Deriving a closed-form solution

For a fluid sheet of average thickness D, the vertical tidal elevation ζ, as well as the horizontal velocity components u and v (in the latitude φ and longitude λ directions).

This is the set of Laplace's tidal equations (wikipedia). Along the equator, for φ at zero we can reduce this.

$$\frac{\partial \zeta}{\partial t} + \frac{1}{a\cos(\varphi)}\left[\frac{\partial}{\partial \lambda}(uD) + \frac{\partial}{\partial \varphi}(vD\cos(\varphi))\right] = 0,$$
$$\frac{\partial u}{\partial t} - v(2\Omega\sin(\varphi)) + \frac{1}{a\cos(\varphi)}\frac{\partial}{\partial \lambda}(g\zeta + U) = 0,$$
$$\frac{\partial v}{\partial t} + u(2\Omega\sin(\varphi)) + \frac{1}{a}\frac{\partial}{\partial \varphi}(g\zeta + U) = 0,$$

where Ω is the angular frequency of the planet's rotation, g is the planet's gravitational acceleration at the mean ocean surface, a is the planetary radius, and U is the external gravitational tidal-forcing potential.

The main candidates for removal due to the small-angle approximation along the equator are the second terms in the second and third equation. The plan is to then substitute the isolated u and v terms into the first equation, after taking another derivative of that equation with respect to t.

$$\frac{\partial \zeta}{\partial t} + \frac{1}{a}\left[\frac{\partial}{\partial \lambda}(uD) + \frac{\partial}{\partial \varphi}(vD)\right] = 0,$$
$$\frac{\partial u}{\partial t} + \frac{1}{a}\frac{\partial}{\partial \lambda}(g\zeta + U) = 0,$$
$$\frac{\partial v}{\partial t} + \frac{1}{a}\frac{\partial}{\partial \varphi}(g\zeta + U) = 0,$$

Taking another derivative of the first equation:

$$a\frac{\partial^2 \zeta}{\partial t^2} + \frac{\partial}{\partial t}\left[\frac{\partial}{\partial \lambda}(uD) + \frac{\partial}{\partial \varphi}(vD)\right] = 0,$$

Next, on the bracketed pair we invert the order of derivatives (and pull out the constant D)

$$a\frac{\partial^2 \zeta}{\partial t^2} + D\left[\frac{\partial}{\partial \lambda}(\frac{\partial u}{\partial t}) + \frac{\partial}{\partial \varphi}(\frac{\partial v}{\partial t})\right] = 0,$$

Notice now that the bracketed terms can be replaced by the 2nd and 3rd of Laplace's equations

$$\frac{\partial u}{\partial t} = -\frac{1}{a}\frac{\partial}{\partial \lambda}(g\zeta + U)$$
$$\frac{\partial v}{\partial t} = -\frac{1}{a}\frac{\partial}{\partial \varphi}(g\zeta + U)$$

as

$$a^2\frac{\partial^2 \zeta}{\partial t^2} - D\left[\frac{\partial}{\partial \lambda}(\frac{\partial}{\partial \lambda}(g\zeta + U)) + \frac{\partial}{\partial \varphi}(\frac{\partial}{\partial \varphi}(g\zeta + U))\right] = 0$$

The λ terms are in longitude so that we can use a separation of variables approach and create a spatial standing wave for ENSO (i.e. the sloshing model), $SW(s)$ where s is a wavenumber.

$$\frac{\partial^2 \zeta}{\partial t^2} - D\left[(SW(s)\zeta) + \frac{\partial}{\partial \varphi}(\frac{\partial}{\partial \varphi}(g\zeta + U))\right] = 0$$

$$AZ(t) + \frac{1}{\omega\cos(\omega t)} \times \frac{\partial}{\partial t}\frac{Z'(t)}{\omega\cos(\omega t)} = 0$$

Sturm-Liouville equation:

$$\frac{d}{dt}(\sec(t\,\omega)\,Z'(t)) + A\,\omega^2\cos(t\,\omega)\,Z(t) = 0$$

Differential equation solution:

$$Z(t) = c_2\sin(\sqrt{A}\,\sin(t\,\omega)) + c_1\cos(\sqrt{A}\,\sin(t\,\omega))$$

That is essentially close to a variation of a Sturm-Liouville equation (see right). To solve this for a plausible set of boundary conditions, we make a connection between a change in latitudinal forcing with a temporal change:

$$\frac{\partial \zeta}{\partial \varphi} = \frac{\partial \zeta}{\partial t}\frac{\partial t}{\partial \varphi}$$

After properly applying the chain rule, this reduces the equation to a function of $\zeta(t)$ and $\varphi(t)$, along with a constant *A*. The *A* subsumes the wavenumber *SW(s)* portion so there will be multiple solutions for the various standing waves, which will be used in fitting the model to the data.

$$A\zeta(t) + \frac{1}{\frac{\partial\varphi}{\partial t}} \cdot \frac{\partial}{\partial t}\frac{\zeta'(t)}{\frac{\partial\varphi}{\partial t}} = 0$$

So if we fix $\varphi(t)$ to a periodic function with a long-term mean of zero

$$\frac{\partial \varphi}{\partial t} = \sum_{i=1}^{i=N} k_i\omega_i\cos(\omega_i t)$$

to describe the tractive latitudinal displacement terms near the equator, then the solution is the following:

$$\zeta(t) = \sin(\sqrt{A}\sum_{i=1}^{i=N} k_i\sin(\omega_i t) + \theta_0)$$

where A is an aggregate of the constants of the differential equation and θ_0 represents the fixed phase offset necessary for aligning on a seasonal peak. This approximation of a tangential tractive force as a cyclic displacement for $\varphi(t)$ is a subtle yet very effective means of eliminating a big unknown in the dynamics (this is essentially similar to a Berry phase applied as a cyclic adiabatic process, with the phase being the internal sin modulation). Not including this approximation leaves us with an indeterminate set of equations.

Now consider that the ENSO itself is precisely the $\frac{\partial v}{\partial t}$ term – the horizontal longitudinal acceleration of the fluid, i.e. leading to the observed sloshing in the thermocline – which can be derived from the above by applying the solution to Laplace's third tidal equation in simplified form above:

$$\frac{\partial v}{\partial t} = \cos(\sqrt{A}\sum_{i=1}^{i=N} k_i\sin(\omega_i t) + \theta_0)$$

There is also a *cosh* solution for when *A* is negative

$$\frac{\partial v}{\partial t} = \cosh(\sqrt{A}\sum_{i=1}^{i=N} k_i\sin(\omega_i t) + \theta_0)$$

Essentially this result is simply a *F=ma* response to a tractive gravitational forcing which obeys Laplace's tidal equations. The *cos* solutions show a negative feedback while the *cosh* solutions are the positive feedback variation.

Part 2 : Deriving the lunar forcing periods

One of the features of the ENSO time-series is a strong biennial component. To model this characteristic, we apply a seasonal aliasing of the lunar gravitational pull to generate the terms needed as a forcing stimulus. This turns into a set of harmonics which we can fit the data to.

The starting premise is that a known lunar tidal forcing signal is periodic

$$L(t) = k \cdot sin(\omega_L t + \phi)$$

The seasonal signal is likely a strong periodic delta function, which peaks at a specific time of the year. This can be approximated as a Fourier series of period 2π.

$$s(t) = \sum_{i=1}^{n} a_i sin(2\pi ti + \theta_i)$$

For now, the exact form of this doesn't matter, as what we are trying to show is how the aliasing comes about.

The forcing is then a combination of the lunar cycles $L(t)$ amplified in some way by the strongly cyclically peaked seasonal signal $s(t)$.

$$f(t) = s(t)L(t)$$

Multiplying this out, and pulling the lunar factor into the sum

$$f(t) = k\sum_{i=1}^{n} a_i sin(\omega_L t + \phi) sin(2\pi ti + \theta_i)$$

then with the trig identity

$$sin(x)sin(y) = \tfrac{1}{2}(cos(x-y) - cos(x+y))$$

Expanding the lower frequency difference terms and ignoring the higher frequency additive terms

$$f(t) = k/2\sum_{i=1}^{n} a_i sin((\omega_L - 2\pi i)t + \psi_i) + ...$$

Thus we can now understand how the high frequency lunar tidal ω_L terms get reduced in frequency by multiples of 2π, until it nears the period of the seasonal cycle. These are the Fourier harmonics of the aliased lunar cycles that comprise the forcing. The aliasing explains why we do not see the ENSO cycles at the monthly scale, but instead observed cycles at the multi-year scale.

In a more precise fashion, we can apply the known gravitational forcing from the lunar orbit and the interaction with the sun's yearly (lunar-perturbed) cycle. This works very effectively, and the closer one can get to the precise orbital path, the better resolved the fit.

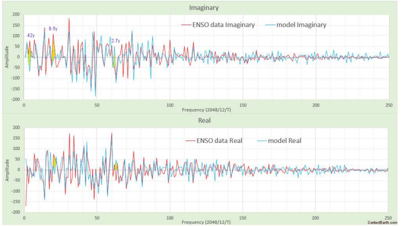
An interesting mathematical consideration is how best to adopt the biennial modulation. Either of these mechanisms work equivalently to some degree: (1) alternate the sign of the biennial forcing impulse, (2) apply a 2-year Mathieu modulation to the DiffEq, or (3) apply a 1 year-delay as a delayed differential equation. These are equivalent in the sense that similar results can be obtained if only one of these are used or they are all used, but each scaled by 1/3. For the Laplace formulation, approach (1) provides the most convenient approach because it does not require a rework to the closed-form solution.

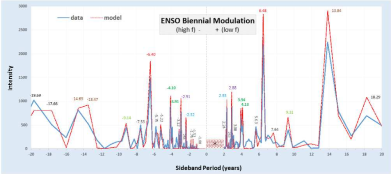
Application

The **Part 1** derivation provides the closed-form natural response and **Part 2** provides the boundary condition forcing terms due to the lunar nodal cycle. As an example of a typical fit with this approach, we apply the gravitational forcing described here:

$$F(t) \propto -\frac{a'(t)+d'(t)}{(R_0+a(t)+d(t))^3}$$

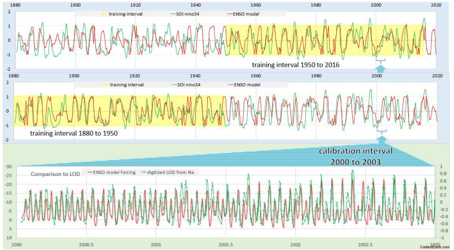
where *a(t)* and *d(t)* are the composed monthly+fortnightly anomalistic and draconic lunar cycles. This generates a rich set of harmonics that expand as a Fourier series used as input to the Laplace solution.



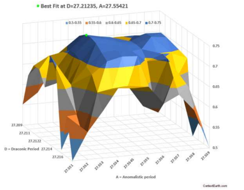


Much of tidal analysis has been performed by Fourier analysis, whereby one can straightforwardly deduce the frequency components arising from the various lunar and solar orbital factors. But in a non-linear world such as ENSO where the tidal forces interact with the seasonal cycle via modulated feedbacks the picture is quite different. What happens is that the cycles interact and get folded multiple times until what originally were three cycles (yearly plus draconic and anomalistic lunar periods) end up appearing as above. Further, most of the peak positions in Fourier space are easily related to the physical aliasing, as the biennial mode splits each peak into two paired satellite peaks (a high f and low f value)

The fitting process shows good cross-validation robustness

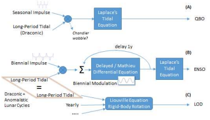


Over-fitting is reduced by constraining the tidal cycles to match other observations such as LOD and 2nd-order shaping.



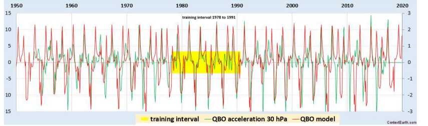
A good fit is sensitive to the precise values of the draconic and tropical periods. Any deviation results in degraded correlation

	vary draconic 27.2	27.205	27.21	27.21222	27.215	27.22	27.3
vary anomalistic 27.5	0.4208	0.3536	0.5856	0.55481	0.4866	0.3252	0.3059
27.545	0.3584	0.2578	0.3864	0.578559	0.5438	0.3837	0.3769
27.55	0.4487	0.5308	0.5486	0.5312	0.5192	0.5114	0.4273
27.55455	0.5151	0.5297	0.643268	0.6604	0.62944	0.5203	0.4974
27.555	0.5262	0.5028	0.632	0.65486	0.6207	0.4916	0.4275
27.56	0.4333	0.4362	0.4675	0.56948	0.515	0.5338	0.5002
27.6	0.3973	0.4321	0.5468	0.519975	0.4546	0.5201	0.4501

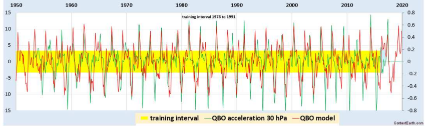


The data flow diagram for the forcing mechanism is shown above. There are two other system-wide processes that likely result from tidal forcing, that of QBO and the Chandler wobble.

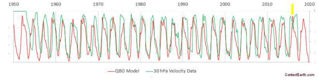
The same appoach for ENSO can be used to model QBO. Only the draconic cycle is used as forcing leading to regularity



This shows excellent cross-validattion with a small training interval



The full QBO is recovered by integrating the acceleration



The Chandler wobble provides more evidence that the draconic cycle controls the angular variations, and not a resonance