

Sub-MeV Electron Precipitation Driven by EMIC Waves through Nonlinear Fractional Resonances

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Key Points:

- Electrons resonate with intense quasiparallel EMIC waves at fractions of the minimum resonance energy
- Fractional resonant scattering causes significant precipitation when the wave amplitude reaches above 1% of the ambient field
- Precipitating electron flux spectrum observed by the ELFEN CubeSats supports the estimated influence of fractional resonances

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Abstract

Electromagnetic ion cyclotron waves in the Earth's outer radiation belt drive rapid electron losses through wave-particle interactions. The precipitating electron flux can be high in the 100s keV energy range, well below the typical minimum resonance energy. One of the proposed explanations relies on nonresonant scattering, which causes pitch-angle diffusion away from the fundamental cyclotron resonance. Here we propose the fractional sub-cyclotron resonance, a second-order nonlinear effect that scatters particles at resonance order $n = 1/2$, as an alternate explanation. Using test-particle simulations, we evaluate the precipitation ratios of sub-MeV electrons for wave packets with various shapes, amplitudes, and wave normal angles. We show that the nonlinear sub-cyclotron scattering produces larger ratios than the nonresonant scattering when the wave amplitude reaches sufficiently large values. The ELFIN CubeSats detected several events with precipitation ratio patterns matching our simulation, demonstrating the importance of sub-cyclotron resonances during intense precipitation events.

Plain Language Summary

High-energy electrons in the Earth's radiation belt are constantly being scattered by the ubiquitous electromagnetic plasma waves. A portion of these scattered electrons is lost to the atmosphere, where the particles deposit their energy and cause a chain of chemical reactions possibly contributing to ozone destruction. The energy and flux of the precipitating electrons depend on the nature of the wave-particle interactions in the radiation belt. The electromagnetic ion cyclotron wave (EMIC), known to be responsible for scattering relativistic electrons, has been observed to cause precipitation at energies much lower than expected by the standard theory. We numerically investigate two types of interactions, the nonresonant scattering and the nonlinear sub-cyclotron scattering, and show how both influence the relative precipitating fluxes. We demonstrate that sub-cyclotron interactions driven by intense EMIC waves can cause stronger precipitation than nonresonant scattering at sub-MeV energies. The dual ELFIN CubeSats detected precipitation profiles that match our numerical results, confirming the importance of nonlinear sub-cyclotron scattering in the analysis of intense precipitation events.

1 Introduction

The Earth’s outer radiation belt is sustained by a dynamic balance between particle injections, acceleration, transport, and losses (Shprits et al., 2008; Reeves et al., 2013; Baker et al., 2019; Li & Hudson, 2019). One major particle loss driver is the electromagnetic ion cyclotron (EMIC) waves, which interact with energetic protons and relativistic electrons (Jordanova et al., 2001; Usanova et al., 2014; Zhu et al., 2020; Lyu et al., 2022). The EMIC waves are located predominantly near the equatorial plane (Allen et al., 2015), propagating quasiparallel to the local magnetic field line (Min et al., 2012). During geomagnetically active times, intense EMIC waves occur in the noon-to-dusk sector at radial distances from 4 to 6.5 Earth’s radii (Saikin et al., 2016; Zhang et al., 2016), causing rapid pitch-angle scattering of high-energy radiation belt electrons through cyclotron resonance (Horne & Thorne, 1998). The lost electrons deposit their energy in the upper atmosphere, contributing to changes in atmospheric chemistry (Thorne, 1977; Seppälä et al., 2015).

The minimum cyclotron resonance energy of electrons interacting with EMIC waves is given by the approximate formula (Chen et al., 2019)

$$E_{\text{Rmin}} \approx mc^2 \left(\sqrt{1 + \frac{n^2 \Omega_e^2}{k_{\parallel}^2 c^2}} - 1 \right). \quad (1)$$

Here m stands for the electron mass, c is the speed of light, Ω_e is the local electron gyrofrequency, k_{\parallel} is the parallel wavenumber, and n represents the resonance order. For quasiparallel waves, the fundamental resonance $n = 1$ dominates. Electrons with pitch angle $\alpha = 0^\circ$ and kinetic energy $E_k = E_{\text{Rmin}}$ resonate with EMIC waves when propagating along the wave field. With oblique waves, strong resonance is possible in both directions (Wang et al., 2017; Hanzelka et al., 2023). The resonance energy increases with pitch angle, following the resonance curves (Summers et al., 1998). For the typical EMIC wave parameters, E_{Rmin} stays above 1 MeV (Miyoshi et al., 2008; Meredith et al., 2014). However, recent studies show growing evidence of EMIC waves causing significant precipitating electron fluxes down to hundreds of keV (Ukhorskiy et al., 2010; Clilverd et al., 2015; Hendry et al., 2019; Denton et al., 2019; Capannolo et al., 2021, 2023). According to Equation 1, sub-MeV energies can be reached by the fundamental resonance when k_{\parallel} is sufficiently high — this is possible when the electron plasma-to-gyrofrequency ratio (ω_{pe}/Ω_e) is large or when the wave frequency gets close to local ion gyrofrequencies (Li et al., 2007; Min et al., 2022). Sub-MeV electrons can also interact with EMIC waves

78 through nonresonant scattering, a type of interaction associated with wave packet mod-
 79 ulations (Chen et al., 2016). The nonresonant interaction is tied to the fundamental res-
 80 onance but can extend efficient pitch-angle diffusion down to hundreds of keV when the
 81 wave field consists of very short packets (An et al., 2022).

82 Here we propose that nonlinear effects can contribute to sub-MeV electron precip-
 83 itation through resonant scattering at the $n = 1/2$ fraction of gyrofrequency. This type
 84 of resonance was studied by Fu et al. (2015) in the case of whistler-mode waves and, more
 85 recently, by Hanzelka et al. (2023) within the frame of the electron-EMIC interactions.
 86 Assuming a fundamental resonance energy of ~ 1 MeV, the $n = 1/2$ resonance affects
 87 electrons at around 400 keV, making it a plausible explanation for sub-MeV precipita-
 88 tion. Hanzelka et al. (2023) numerically demonstrated that the $n = 1/2$ resonance ap-
 89 pears only during oblique propagation (wave normal angle $\theta_k > 0$). They also provided
 90 a simplified analytical derivation indicating that the standard deviation in energy and
 91 pitch angle grows with the second power of wave amplitude, B_w^2 , unlike the $n = 1$ re-
 92 sonance, which scales with the first power (where B_w is the wave magnetic field ampli-
 93 tude). Therefore, this fractional resonance is expected to be efficient only during high-
 94 amplitude events.

95 To evaluate nonresonant and nonlinear sub-cyclotron scattering of electrons in the
 96 $E_k < 1$ MeV range, we perform test-particle simulations using EMIC wave models with
 97 various packet shapes, amplitudes, and wave normal angles (WNA). In Section 2.1, we
 98 describe our numerical method and input parameters. Section 2.2 briefly describes the
 99 dataset of ELFIN CubeSats particle measurements. In Section 3, we present a paramet-
 100 ric analysis of pitch-angle diffusion and precipitation fluxes and compare the results with
 101 selected events from ELFIN observations. In Section 4, we discuss and summarize our
 102 findings.

103 **2 Methods**

104 **2.1 Test-particle Simulations**

105 We employ the test-particle simulation method following Hanzelka et al. (2023).
 106 The particles are evolved by the relativistic Boris algorithm with phase angle correction
 107 (Zenitani & Umeda, 2018). The integration time step adapts to the background mag-
 108 netic field, sampling the local electron gyroperiod with 128 points. All simulations are

109 performed in one spatial dimension along a dipole field line $L = 6$ with background mag-
 110 netic field $B_0 = 143.5 \text{ nT}$ at the equatorial minimum. The ratio of equatorial plasma
 111 frequency to gyrofrequency is set to $\omega_{pe0}/\Omega_{e0} = 15$, and the relative concentrations of
 112 protons, He+ ions, and O+ ions are in a ratio of 90 : 5 : 5.

113 The wave model assumes hydrogen band EMIC waves with a constant frequency
 114 $\omega = 0.66\Omega_{p0}$, where Ω_{p0} is the equatorial proton gyrofrequency. Solving the cold plasma
 115 dispersion relation with wave normal angle $\theta_k = 0^\circ$ gives us wavelength $\lambda_w \approx 210 \text{ km}$,
 116 and plugging the corresponding wavenumber into Equation 1 results in minimum res-
 117 onance energy $E_{Rmin} \approx 1.0 \text{ MeV}$. Three amplitude profiles are used: a single packet
 118 with a field-aligned length $h_{wp} = 16\lambda_w$, two packets each $h_{wp}/2$ long, and four pack-
 119 ets each $h_{wp}/4$ long. Individual subpackets are modeled with $\cos^2(\pi h/(2d_e))$, where d_e
 120 represents the half-width. The smallest value, $d_e = 2\lambda_w$, matches with the shortest sub-
 121 packets reported by Chen et al. (2016) and An et al. (2022). Each wave profile can have
 122 four amplitude values, $B_{w0} = \{0.005, 0.01, 0.02, 0.04\}B_0$, and three WNA values, $\theta_k =$
 123 $\{0^\circ, 15^\circ, 30^\circ\}$. The largest amplitude is comparable to the extremely strong EMIC waves
 124 reported by Engebretson et al. (2015), which peaked at $B_{w0} \approx 0.06B_0$. Due to short
 125 simulation times, group velocity motion is not included.

126 To obtain the pitch angle diffusion coefficient, we launch particles from $h = 0$ and
 127 let them propagate to $h = 16\lambda_w$ and record the pitch angle variations. The initial en-
 128 ergies are sampled logarithmically from 0.3 MeV to 3 MeV with 96 points, the pitch an-
 129 gles are sampled uniformly from 0° to 45° with 45 points, and the gyrophases are sam-
 130 pled uniformly with $N_\varphi = 72$ points over the full angle. The changes in equatorial pitch
 131 angle α_{eq} are calculated for all particles in each energy-pitch-angle bin and combined
 132 into the diffusion coefficient

$$D_{\alpha\alpha} = \frac{1}{2\tau_{qb}} \frac{1}{N_\varphi} \sum_{i=1}^{N_\varphi} \left(\alpha_{eqi} - \frac{1}{N_\varphi} \sum_{j=1}^{N_\varphi} \alpha_{eqj} \right)^2, \quad (2)$$

133 where τ_{qb} is the quarter-bounce period. As the propagation is quasiparallel, interactions
 134 during electron motion back toward the equator are not considered. For visualization
 135 of trajectories, pitch angle evolution is sampled with four points per gyroperiod.

136 The perturbations in phase space density are obtained from backward-in-time sim-
 137 ulations with Liouville mapping (Hanzelka et al., 2023). The initial energetic electron
 138 distribution is a sum of five relativistic bi-Maxwellian distributions with parallel ther-

mal velocities $U_{t\parallel}/c = \{0.2, 0.5, 1.0, 2.5, 9.0\}$, perpendicular thermal velocities $U_{t\perp}/c = \{0.3, 0.75, 1.5, 3.75, 14.5\}$, and relative hot plasma densities $n_{\text{hot}}/n_{\text{cold}} = \{0.05, 0.005, 5 \cdot 10^{-4}, 5 \cdot 10^{-5}, 5 \cdot 10^{-9}\}$. As we seek relative changes in phase space density and EMIC waves affect the electron energy negligibly, the exact values of hot plasma density and thermal velocities are unimportant, leaving anisotropy $A = U_{t\perp}^2/U_{t\parallel}^2 - 1 = 1.25$ as the sole relevant parameter. The loss cone content of the perturbed PSD $f_p(h_{\text{wp}})$ at each energy level is divided by the PSD just outside the loss cone, resulting in the precipitation ratio

$$\frac{f_{\text{in}}}{f_{\text{out}}} = \frac{\int_{\alpha_{1c}}^{\alpha_{\text{sc}90}} \sin \alpha \, d\alpha}{\int_0^{\alpha_{1c}} \sin \alpha \, d\alpha} \frac{\int_0^{\alpha_{1c}} f_p \sin \alpha \, d\alpha}{\int_{\alpha_{1c}}^{\alpha_{\text{sc}90}} f_p \sin \alpha \, d\alpha}. \quad (3)$$

The loss cone angle α_{1c} is about 2.9° at the end of the wave packet, and $\alpha_{\text{sc}90}$ is the pitch angle of an electron at h_{wp} with a mirroring point at the altitude of the low Earth orbit (LEO) spacecraft. In other words, particles in the range $(\alpha_{1c}, \alpha_{\text{sc}90})$ constitute the population that would be observed as quasi-trapped by the spacecraft. For the purpose of PSD simulations, the pitch angles are sampled from 0° to 15° with 90 points.

2.2 ELFIN Spacecraft Flux Data

The dual ELFIN CubeSats (on a polar LEO at 300-450 km of altitude) provided electron flux data from July 2019 to September 2022. The energetic particle detector (EPDE) measured electrons over the range of 50 to 5000 keV sampled by 16 approximately logarithmic bins (Angelopoulos et al., 2020, 2023). The satellites had a spin period of ~ 3 seconds and twice per spin provided flux measurements nominally over the whole 180° range of pitch angles with 22.5° spin phase resolution, allowing for differentiation of particles inside and outside the local loss cone.

We utilize the dataset of 144 EMIC-driven precipitation events identified by Capannolo et al. (2023) using precipitation signatures from proton measurements by NOAA POES when conjugate to ELFIN as a proxy for EMIC waves. Events with electron precipitation ratio decreasing with energy in the low hundreds of keV range are not part of this dataset, as such profiles indicate the presence of whistler waves (Ma et al., 2016; Angelopoulos et al., 2023). However, the coexistence of other waves with EMICs cannot be completely ruled out without direct observations; refer to Capannolo et al. (2023) for details. Based on simulation results from Section 3, we visually selected 6 events that show patterns related to nonlinear fractional resonance. In the selection process, we sought pre-

169 precipitation profiles where a large precipitation ratio (> 0.75) is reached at energies be-
 170 low 1.5 MeV but above 0.7 MeV; that is, in a range of energies that can be affected by
 171 the fundamental cyclotron resonance under favorable conditions (see Equation 1 and the
 172 paragraph below it). The lowest energy bin with a ratio > 0.75 was labeled E_k^* . We then
 173 checked if a peak or plateau appeared near $E_k^*/2$. Unlike in Capannolo et al. (2023), we
 174 did not subtract backscattered electrons from the precipitating ones, and we used the
 175 full resolution half-spin data (one pitch-angle sweep per ~ 1.5 seconds) instead of the
 176 full-spin data. Time stamps for the selected events can be found in the Supporting In-
 177 formation, Table S1.

178 **3 Results**

179 In Figure 1, we show particle trajectories and diffusion coefficients $D_{\alpha\alpha}$ for three
 180 selected combinations of wave parameters. In the first case, we choose a long, unmod-
 181 ulated wave packet ($d_e = 8\lambda_w$) with a moderate amplitude ($B_{w0}/B_0 = 1\%$) and 0°
 182 wave normal angle. The trajectories in Figure 1a show the equatorial pitch angle evo-
 183 lution of particles undergoing a combination of phase trapping and phase bunching, a
 184 typical behavior of electrons in resonance with large amplitude EMICs (Albert & Bort-
 185 nnik, 2009). This is an example of nonlinear effects driven by the fundamental cyclotron
 186 resonance. Figure 1b shows that the associated $D_{\alpha\alpha}$ drops to negligible values below 700 keV.
 187 The weakening of diffusion near $E_k = 1$ MeV, $\alpha_{eq} = 0^\circ$, is caused by anomalous scat-
 188 tering, an advective process affecting low- α resonant electrons (Bortnik et al., 2022; Hanzelka
 189 et al., 2023).

190 Figures 1c and 1d demonstrate diffusive behavior in an extremely strong ($B_{w0}/B_0 =$
 191 4%) parallel-propagating EMIC wave with short subpacket modulations ($d_e = 2\lambda_w$).
 192 The trajectories in Figure 1c show that after passing through a subpacket, the electrons
 193 spread out in pitch angle, even though their energy $E_k = 0.594$ MeV is far from $E_{Rmin} \approx$
 194 1.0 MeV. The pitch angle change depends not only on the amplitude gradient but also
 195 on the subpacket length, as electrons are released at different phases of their oscillatory
 196 motion (Chen et al., 2016). This is an example of nonresonant scattering. Diffusion co-
 197 efficients in Figure 1d remain significant down to 300 keV, and high values of > 0.1 s $^{-1}$
 198 are seen in a much wider range of energies than for long, weaker amplitude, unmodu-
 199 lated wave packets. The stripe structures in $D_{\alpha\alpha}$ arise due to the aforementioned phase
 200 dependence.

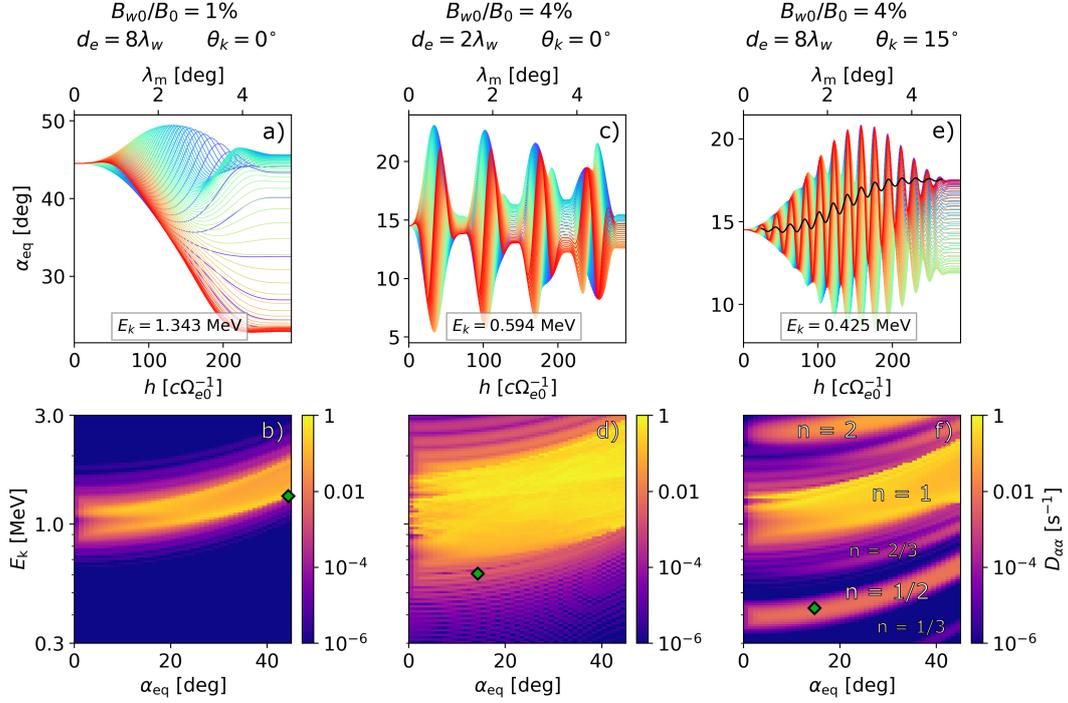


Figure 1. Selected examples of resonant and nonresonant interactions. a) Trajectories of electrons starting at pitch angle $\alpha_{\text{eq}} = 44.5^\circ$ with energy $E_k = 1.343$ MeV, illustrating the phase trapping and bunching behavior near the fundamental cyclotron resonance $n = 1$. Line colors represent the uniformly sampled initial gyrophase φ , and λ_m is the magnetic latitude. b) Diffusion coefficient in energy–pitch-angle space. The green square shows the α_{eq} , E_k values used in the trajectory plot. c,d) Similar to panels a and b but with a higher wave amplitude and sub-packet modulations, demonstrating nonresonant scattering. e,f) Similar to panels a and b but the wave is strong and $\theta_k = 15^\circ$. The trajectories demonstrate nonlinear sub-cyclotron scattering, with the black line showing a moving average of particle trajectory with initial gyrophase $\varphi = 0^\circ$ (red line). Various resonance orders are labeled in the diffusion plot.

201 The last couple of panels, Figures 1e and 1f, retain the extremely high amplitude
 202 $B_{w0}/B_0 = 4\%$, but the packet is long ($d_e = 8\lambda_w$), and the wave propagation direc-
 203 tion is slightly oblique ($\theta_k = 15^\circ$). The trajectories in Figure 1e now exhibit large vari-
 204 ations in pitch angle even at the low energy of 0.425 MeV. Examining a single trajec-
 205 tory, such as the red curve with $\varphi(h = 0) = 0^\circ$, reveals that oscillations slowly grow
 206 and then fade again, excluding nonresonant scattering. The moving average of this tra-
 207 jectory (black line) reveals a slow drift toward higher pitch angles, while the combined
 208 effect on all trajectories is a symmetric spread in α_{eq} . This behavior is caused by the non-
 209 linear resonance of fractional order $n = 1/2$. Figure 1f confirms the resonant nature of
 210 this diffusive behavior, with high $D_{\alpha\alpha}$ values localized along the $n = 1/2$ resonance curve,
 211 distinct from the nonresonant widening of the fundamental resonance. The diffusion plot
 212 also captures the $n = 2$ harmonic resonance and two minor fractional resonances with
 213 orders $n = 2/3$ and $n = 1/3$, which are of little significance for sub-MeV precipita-
 214 tion.

215 To visually represent diffusion coefficients for all 36 combinations of wave param-
 216 eters, we plot them in Figure 2 as one-dimensional line plots for two selected initial pitch
 217 angles $\alpha_{eq} = 2.5^\circ$ and $\alpha_{eq} = 14.5^\circ$. The first value represents the bin closest to the
 218 loss cone; the second was chosen to limit the influence of anomalous scattering on $D_{\alpha\alpha}$.
 219 A 3-point moving average over energies was used to suppress the stripe structure related
 220 to nonresonant scattering that appeared in Figure 1d. Parallel propagation results in Fig-
 221 ures 2a-d reveal that the resonance peak widens with amplitude. $D_{\alpha\alpha}$ at energies far-
 222 ther from the resonance increases by approximately an order of magnitude from $d_e =$
 223 $8\lambda_w$ to $d_e = 4\lambda_w$, and another order of magnitude from $d_e = 4\lambda_w$ to $d_e = 2\lambda_w$. The
 224 $D_{\alpha\alpha}$ peak near $E_k = 1$ MeV grows with wave amplitude at $\alpha_{eq} = 14.5^\circ$, but this trend
 225 is less clear at the loss cone, where anomalous scattering effects are prominent.

226 The $D_{\alpha\alpha}$ peak associated with the $n = 1/2$ resonance appears near $E_k = 400$ keV
 227 when we increase the WNA to $\theta_k = 15^\circ$ (Figures 2e-h). For the unmodulated wave packet
 228 and with $\alpha_{eq} = 14.5^\circ$ (blue dashed line), the peak grows about 16 times with each dou-
 229 bling of wave amplitude, confirming the B_w^4 scaling expected for this fractional resonance.
 230 The model with short subpackets predicts a slightly lower $D_{\alpha\alpha}$ than the single long packet
 231 but widens the resonance peak. Consequently, the modulated packet can cause a nearly
 232 constant diffusion rate between 300 and 700 keV (Figure 2h). This result of combined
 233 nonresonant and nonlinear scattering predicts vastly different diffusion rates compared

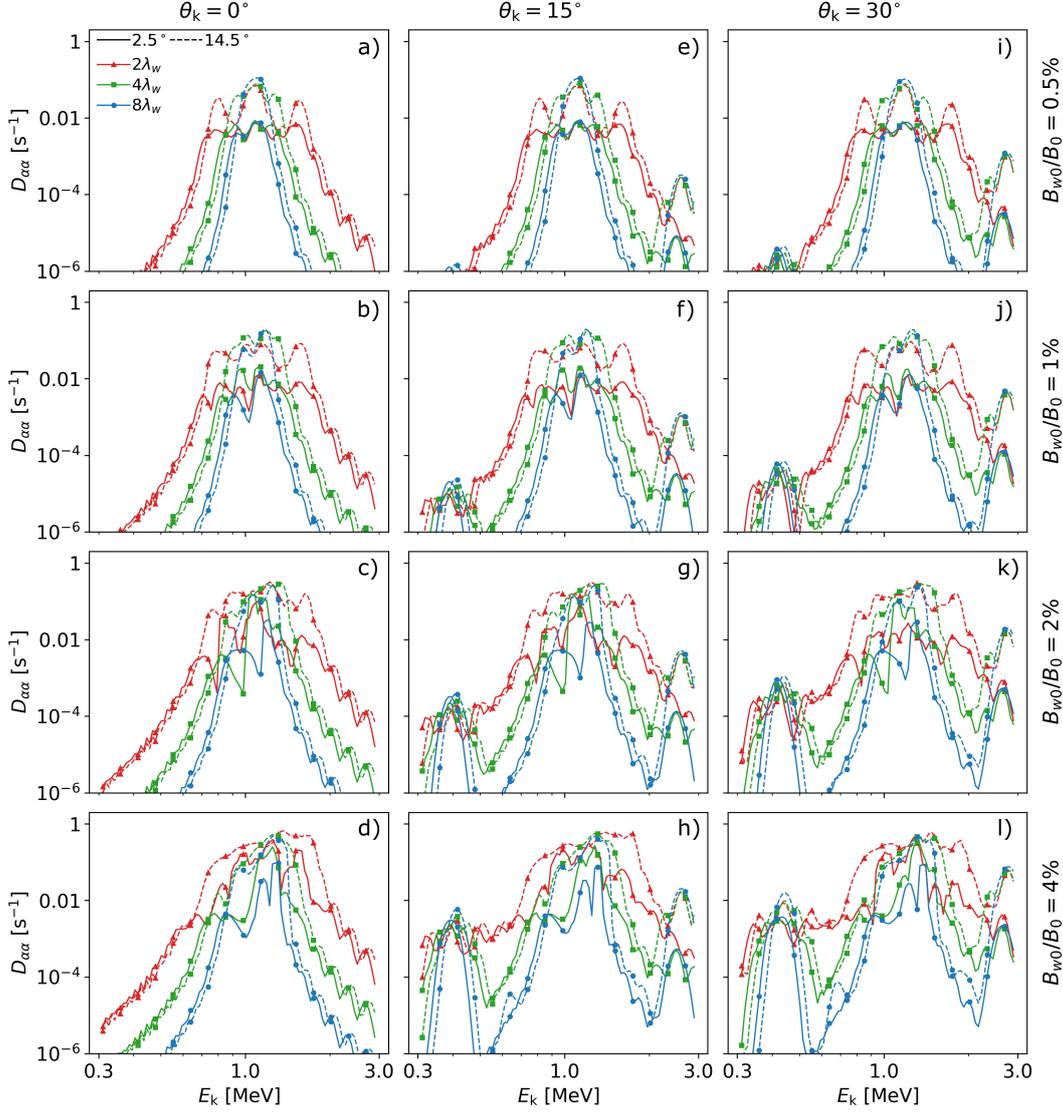


Figure 2. Pitch-angle diffusion coefficients calculated for various wave parameters. The solid and dashed lines represent $D_{\alpha\alpha}$ for pitch angles $\alpha_{eq} = 2.5^\circ$ and $\alpha_{eq} = 14.5^\circ$, respectively. Amplitude modulations of the wave packets are captured by different line colors and markers: red triangle for $d_e = 2\lambda_w$, green square for $d_e = 4\lambda_w$, and blue circle for $d_e = 8\lambda_w$. a-d) Diffusion coefficients for parallel-propagating waves, with amplitude increased by the factor of two in each row, going from $B_{w0}/B_0 = 0.5\%$ to $B_{w0}/B_0 = 4\%$. e-h) Diffusion coefficients for wave normal angle $\theta_k = 15^\circ$. i-l) Diffusion coefficients for wave normal angle $\theta_k = 30^\circ$.

234 to those caused by pure nonresonant scattering in the parallel-propagating case. A fur-
 235 ther increase of WNA to $\theta_k = 30^\circ$ enhances the peak nonlinear diffusion rate 2-3 times
 236 and shifts the minimum resonance energy to slightly higher values (Figures 2i-l). Due
 237 to the high absolute values of $D_{\alpha\alpha}$ near $E_k = 450$ keV in Figure 2l, the differences be-
 238 tween $\alpha_{\text{eq}} = 2.5^\circ$ and $\alpha_{\text{eq}} = 14.5^\circ$ related to anomalous scattering start becoming ap-
 239 parent. Otherwise, the increase in WNA does not bring qualitative changes.

240 Using the backward-in-time simulation method outlined in Section 2.1, we obtained
 241 the PSD of particles pushed into the loss cone and then calculated precipitation ratios
 242 using Equation 3 with α_{sc90} value corresponding to the ELFIN spacecraft orbit (altitude
 243 of 450 km). The resulting precipitation ratios are shown in Figure 3. A comparison be-
 244 tween nonresonant scattering by modulated waves in Figures 3a and 2a reveals that the
 245 precipitation profile near the fundamental resonance aligns more closely with the diffu-
 246 sion coefficients at higher pitch angles ($\alpha_{\text{eq}} = 14.5^\circ$) than those near the loss cone ($\alpha_{\text{eq}} =$
 247 2.5°). This suggests that precipitating particles predominantly originate at higher pitch
 248 angles. In the high-amplitude cases (Figures 3b-d), the diffusion coefficients near the fun-
 249 damental resonance are always large enough ($D_{\alpha\alpha} > 0.01 \text{ s}^{-1}$) to completely fill the loss
 250 cone (precipitation ratios close to 1). As expected, strong nonresonant scattering extends
 251 the energy range in which the loss cone is full and can cause nonnegligible precipitation
 252 down to about 400 keV.

253 The precipitation caused by nonlinear sub-cyclotron resonance is significant only
 254 when the amplitude rises above 1%, as demonstrated in Figures 3e-h. In agreement with
 255 the diffusion coefficient calculation, the interplay of nonresonant and nonlinear resonances
 256 yields a flat precipitation profile. Increase of WNA from 15° to 30° further enhances the
 257 precipitation, reaching a ratio of ~ 1 in Figure 3l. Further increase in obliquity to 45°
 258 has only minor effect on the resulting precipitation and is therefore not plotted here.

259 The numerical prediction of precipitation ratios can be compared to electron flux
 260 observations provided by the ELFIN spacecraft. Figure 4a shows the EMIC-driven elec-
 261 tron precipitation event of March 31, 2021, detected by the ELFIN-A spacecraft in the
 262 northern hemisphere. Precipitation ratios close to one were detected at energies near 1 MeV,
 263 with nonnegligible values (> 0.1) reaching down to about 200 keV. In the time inter-
 264 val of presumed EMIC activity (dashed magenta lines), the trapped electron flux in Fig-
 265 ure 4b steadily decreases with energy. Note that we remove low-count data by requir-

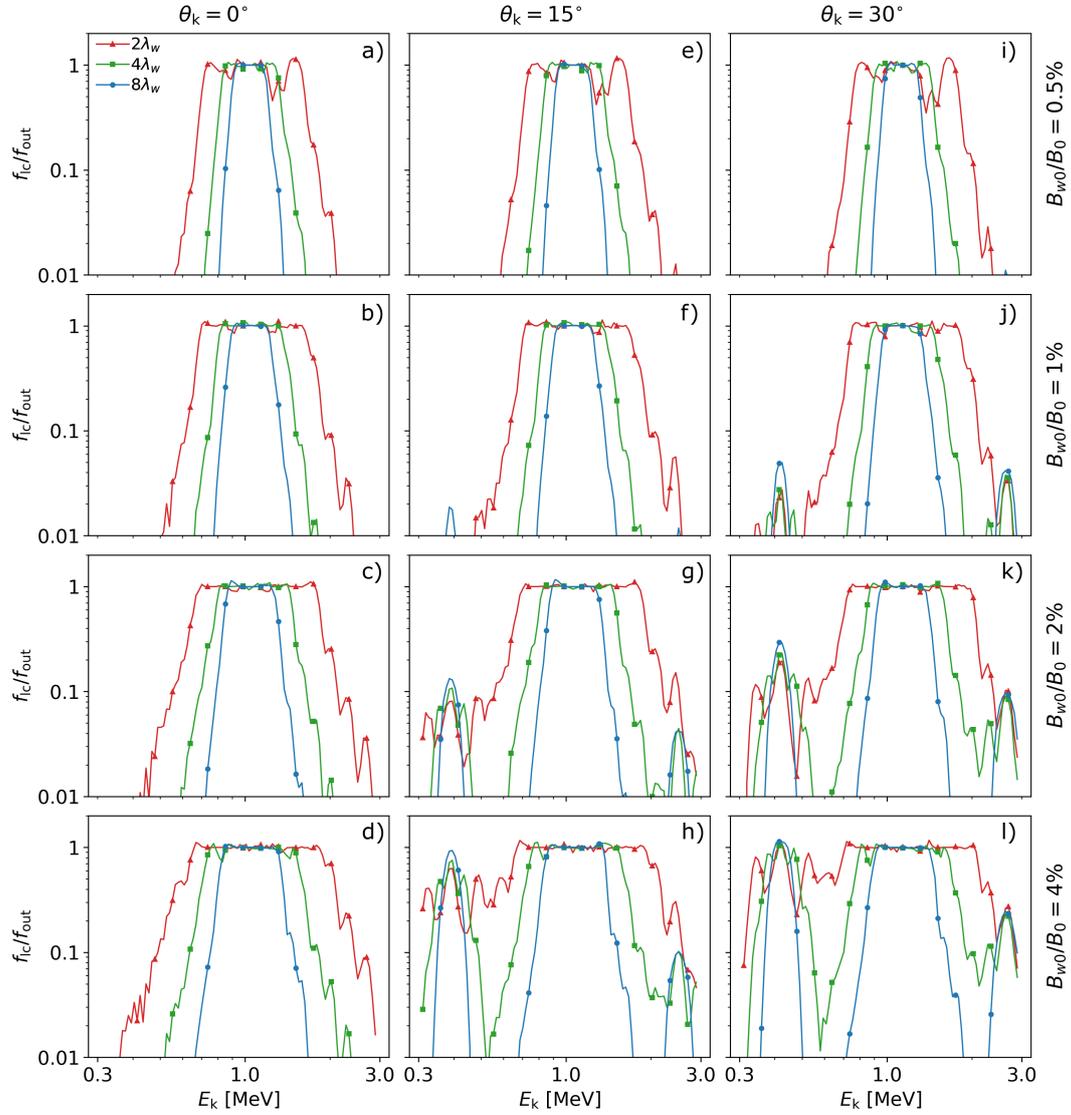


Figure 3. Electron precipitation ratios for various wave parameters. The panel layout and line colors are the same as in Figure 2.

266 ing a maximum percentage error of 50%. The detailed plots of precipitation ratios dur-
 267 ing each half-spin in Figure 4c display a large variability, but a main peak around 1 MeV
 268 and a secondary peak close to 400 keV can be discerned.

269 Figure 4d displays averaged precipitation ratios from six selected events (details
 270 in Section 2.2 and Table S1 in the Supporting Information). We recall that $E_k/E_k^* =$
 271 1 is the lowest energy bin with a high precipitation ratio > 0.75 . Simulation results in
 272 Figure 3 show that the minimum resonance energy E_{Rmin} is typically slightly higher than
 273 E_k^* . On the other hand, the $n = 1/2$ resonance does not appear at $E_{Rmin}/2$ but instead
 274 somewhere between $0.3E_{Rmin}$ and $0.4E_{Rmin}$ (for E_{Rmin} near 1 MeV). Hence, the secondary
 275 peak or plateau from the fractional resonance should appear roughly in the $E_k/E_k^* =$
 276 0.3 to 0.7 range (highlighted in blue). The enhanced precipitation near $E_k/E_k^* = 0.5$
 277 greatly differs from the steep decrease of precipitation ratio predicted by nonresonant
 278 scattering (Figures 3a-d). These observations can be explained either by a low-power EMIC
 279 wave component with extremely low E_{Rmin} , or by the nonlinear sub-cyclotron scatter-
 280 ing, or their combination.

281 We must point out that according to the simulation with extremely large wave am-
 282 plitudes and short subpackets (Figures 3h,l), the criterion $f_{in}/f_{out} > 0.75$ could be met
 283 by the sub-cyclotron resonance peak. However, the observations show ratios as large as
 284 $f_{in}/f_{out} > 1$ in the $E_k/E_k^* > 1$ range, which goes even above the simulated extreme
 285 case. Moreover, since the selection process behind the (Capannolo et al., 2023) dataset
 286 aimed to exclude contributions from whistler-mode waves (see Section 2.2), we would
 287 have no explanation for the significant f_{in}/f_{out} values below E_k^* . The faint $n = 1/3$ res-
 288 onance in Figure 1f is too weak for substantial precipitation. Thus, it is safe to assume
 289 that the peak above E_k^* originates in the fundamental resonance.

290 4 Discussion and Conclusion

291 We have demonstrated that the nonlinear sub-cyclotron scattering can contribute
 292 to EMIC-driven electron precipitation in the energy range of hundreds of keV. Precip-
 293 itation ratios remain low unless wave amplitudes exceed $0.01B_0$, explaining the scarcity
 294 of ELFIN-detected events with clear nonlinear patterns. Despite these low ratios, the
 295 precipitating flux can be substantial. Taking the ELFIN-A measurements of trapped elec-
 296 tron flux from Figure 4b, the values at $E_{k1} = 300$ keV are ten times larger than at $E_{k2} =$

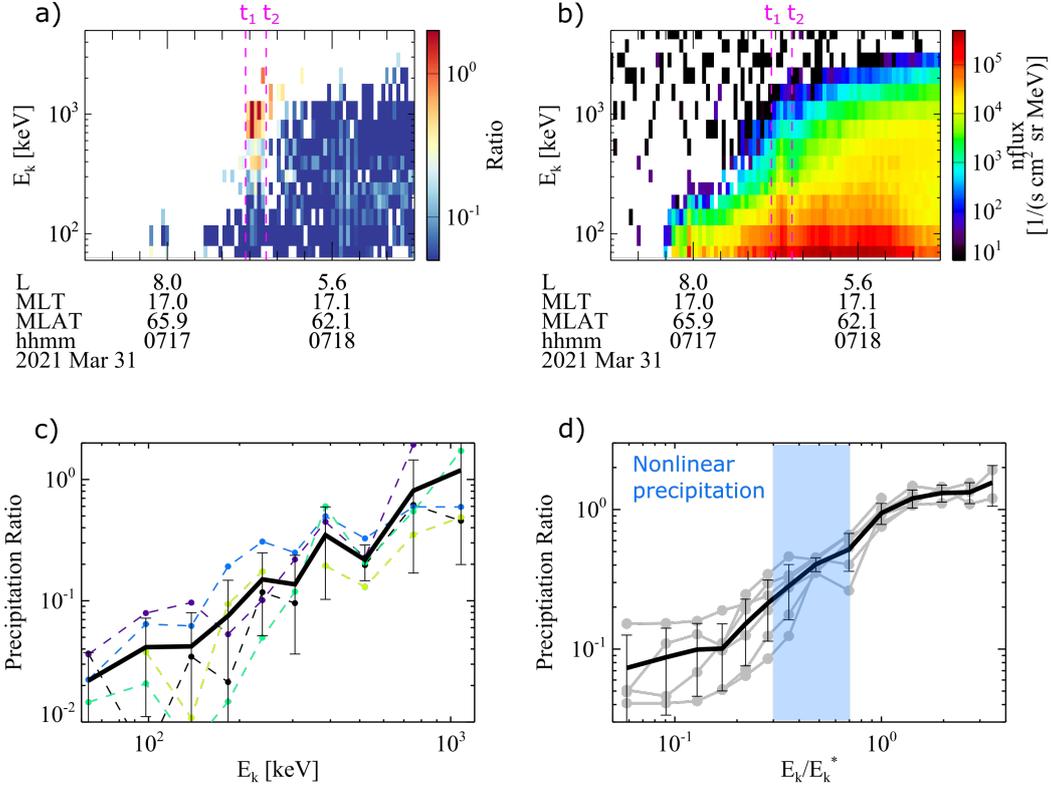


Figure 4. Electron precipitation detected by the ELFIN CubeSats. a) Precipitation ratios detected by ELFIN-A on March 31, 2021 in the northern hemisphere, with a typical EMIC-driven precipitation pattern shown between $t_1 = 07:17:30$ and $t_2 = 07:17:33$ (dashed magenta lines). b) Trapped electron number flux. c) Line plots of precipitation ratios between t_1 and t_2 , with each dashed colored line representing a single half-spin and the thick black line showing the average with standard deviations as errorbars. d) Statistical precipitation ratios in selected events plotted against normalized energy (see Section 2.2 for the definition of E_k^* and event selection). Grey lines represent averages over individual events, the black line is the sample average with errorbars showing the standard deviations. The light blue area highlights the energy range where the strongest effects from the $n = 1/2$ nonlinear resonance are expected.

297 1 MeV. A precipitation ratio of 0.1 at E_{k1} then yields a comparable precipitating flux
298 to a ratio of 1.0 at E_{k2} . This observation aligns with the predominance of precipitating
299 flux peaks at energies $E_k < 1, \text{MeV}$ in the dataset of Hendry et al. (2017).

300 Our numerical predictions are largely insensitive to variations in the initial pitch-
301 angle distribution, which we confirmed by recalculating results in Figure 3 using an isotropic
302 initial PSD ($U_{t\perp} = U_{t\parallel}$). Our choice of wave model is supported only by case studies
303 as those of Nakamura et al. (2015) and Ojha et al. (2021), since a statistical analysis of
304 amplitude modulations in near-equatorial EMIC waves is not available in published lit-
305 erature. An alternate model with a $16\lambda_w$ long subpacket with sharp edges results in sup-
306 pression of nonresonant spreading of the $n = 1/2$ precipitation peak, but the overall
307 picture remains the same. And while interactions with wave packets extending to higher
308 latitudes might broaden resonance peaks towards higher energies, obtaining realistic re-
309 sults would require considering the latitude-dependent evolution of B_w and θ_k . To ad-
310 dress these complexities, we plan to construct improved EMIC wave models for future
311 investigations.

312 In summary, our simulations reveal that nonlinear sub-cyclotron resonance of elec-
313 trons with quasiparallel EMIC waves substantially amplifies precipitation fluxes at en-
314 ergies below the minimum resonance energy. Together with nonresonant scattering, these
315 two effects can be used to explain the enhanced precipitation ratios observed by ELFIN
316 in the sub-MeV part of the energy spectrum. To conclusively confirm the importance
317 of these two scattering processes in energetic electron precipitation, a dataset of conju-
318 gate measurements between equatorial radiation belt probes and LEO spacecraft is needed.
319 We hope that such data will become more abundant in the future thanks to the emer-
320 gence of low-cost CubeSat missions.

321 Open Research Section

322 Processed data from the test-particle simulations are available at [https://doi.org/](https://doi.org/10.6084/m9.figshare.23960964)
323 [10.6084/m9.figshare.23960964](https://doi.org/10.6084/m9.figshare.23960964). The list of precipitation events from Capannolo et
324 al. (2023) can be accessed at <https://doi.org/10.5281/zenodo.7697272>. ELFIN data
325 are available at <https://data.elfin.ucla.edu/> and processed using SPEDAS routines
326 specifically written for processing ELFIN data by the ELFIN UCLA team. The SPEDAS

327 library is publicly available at <http://spedas.org/wiki/index.php?title=Downloads>
328 [_and_Installation](#).

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