Supporting Information for "Simulation of loss cone overfilling and atmospheric precipitation induced by a fine-structured chorus element"

M. Hanzelka[1,2], O. Santolík[1,3], I. Kolmašová[1,3]

[1]Department of Space Physics, Institute of Atmospheric Physics, Czech Academy of Sciences, Prague, Czech Republic

[2]Center for Space Physics, Boston University, Boston, Massachusetts, USA

[3]Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

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Introduction

Text 1 explains the anomalous behavior of resonant electrons at low pitch angles, which has a limited effect on scattering into the loss cone. Furthermore, we mention some basic properties of resonant particle motion that support the explanation of loss cone overfill-

Corresponding author: M. Hanzelka (mirekhanzelka@gmail.com)

ing from the main text. Figure S1 provides a visual accompaniment to the anomalous scattering description and complements the explanation of loss cone overfilling.

Text S1. The equations of motion for an electron interacting with a parallel whistler wave on a homogeneous magnetic field background can be written in cylindrical coordinates as (cf. Summers, Thorne, and Xiao (1998))

$$\frac{\mathrm{d}u_\parallel}{\mathrm{d}t} = \frac{\Omega_\mathrm{w} u_\perp \sin\zeta}{\gamma} \tag{1}$$

$$\frac{\mathrm{d}u_\perp}{\mathrm{d}t} = -\left(\frac{u_\parallel}{\gamma} - V_\mathrm{p}\right)\Omega_\mathrm{w}\sin\zeta \tag{2}$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \frac{\Omega_\mathrm{e}}{\gamma} - \frac{1}{u_\perp}\left(\frac{u_\parallel}{\gamma} - V_\mathrm{p}\right)\Omega_\mathrm{w}\cos\zeta - \omega + \frac{k u_\parallel}{\gamma}\,. \tag{3}$$

Here we use a normalized amplitude $\Omega_\mathrm{w} = B_\mathrm{w} e/m$ (with e and m standing for elementary charge and electron mass, respectively), and V_p stands for phase velocity of the whistler wave. As in the main text, ζ is defined as the difference between the gyrophase φ and the wave magnetic field phase ψ_B, $\zeta = \varphi - \psi_\mathrm{B}$.

When discussing the motion of resonant electrons in the frame of nonlinear theories of chorus growth (e.g. Omura (2021)), the term with $\Omega_\mathrm{w}\cos\zeta$ in Equation 3 is usually omitted because it becomes large only when $u_\perp$ is tiny and thus cannot contribute to the perpendicular resonant current that drives the nonlinear wave growth. With this term removed, the first order resonance condition $\mathrm{d}\zeta/\mathrm{d}t = 0$ leads to the resonance velocity curve

$$\frac{V_\mathrm{R}(v_\perp)}{c} = \frac{ck\omega \mp \Omega_\mathrm{e}\sqrt{(\Omega_\mathrm{e}^2 + c^2k^2)(1 - v_\perp^2/c^2) - \omega^2}}{\Omega_\mathrm{e}^2 + c^2k^2}\,. \tag{4}$$

or in momenta,

$$\frac{U_\mathrm{R}(u_\perp)}{c} = \frac{\gamma V_\mathrm{R}(v_\perp)}{c} = \frac{-ck\Omega_\mathrm{e} + \omega\sqrt{(c^2k^2 - \omega^2)(1 + u_\perp^2/c^2) + \Omega_\mathrm{e}^2}}{c^2k^2 - \omega^2}\,, \tag{5}$$

with the alternate sign coming into play only at ultrarelativistic particle velocities. V_R and U_R respectively are parallel components of the velocity $v_\parallel$ and momentum $u_\parallel$ of the resonant particles. These curves can be seen in Figures S1a and S1b. In these figures, we also plot the resonant diffusion curves for a constant frequency wave, obtained by solving the differential equation arising from a formal division of Equations 1 and 2. Comparing these curves to the contours of a bi-Maxwellian velocity distribution (assuming $\gamma = 1$ for simplicity) shows that the electrons oscillate approximately along the contours of a distribution with temperature anisotropy $A = \omega/(\Omega_\mathrm{e} - \omega)$, which is the marginal condition for anisotropy-driven linear growth (Kennel & Petschek, 1966). Therefore, as long as the anisotropy is strong enough to support linear growth, particles scattered to lower pitch angles will arrive from higher PSD regions.

When the ζ-dependent term is retained, the resonance momentum curve must also depend on ζ:

$$\frac{U_\mathrm{R}^\zeta(u_\perp)}{c} = \frac{-ck\Omega_\mathrm{e} + \omega\sqrt{\Omega_\mathrm{e}^2 + (c^2k^2 - \omega^2)(1 + u_\perp^2/c^2)(1 - \tilde{\Omega}_\zeta)^2}}{(c^2k^2 - \omega^2)(1 - \tilde{\Omega}_\zeta)}\,, \tag{6}$$

$$\tilde{\Omega}_\zeta \equiv \Omega_\mathrm{w} \cos\zeta/(ku_\perp)\,, \tag{7}$$

This means that for a fixed value of $u_\perp$, electrons can reach the exact resonance at various values of $u_\parallel$, and they also can become trapped in the gyrating frame. In Figures S1c, S1d and S1e, we show a plot of the modified resonance momentum along with examples of particle trajectories in the $(u_\parallel, u_\perp)$ space, $(\zeta, u_\parallel)$ space and $(\zeta, u_\perp)$ space. While the trajectories lose their usual symmetry along U_R, the resonant diffusion curves remain unchanged (as they do not depend on Equation 3 at all). Therefore, the only important effect related to the scattering is the large range of pitch angles covered by the resonant particles, leading to significant variations in phase space density along the particle trajectory.

Movie S1. Evolution of phase space density in the loss cone, normalized to the maximum value of $f_{0\max} = f(0,0)$. Frames between simulation time steps are obtained by linear interpolation. Otherwise, each frame has the same format as panels a) and b) in Figure 3 of the main text (resonance velocity curves were removed).

Movie S2. Evolution of energy distribution inside the loss cone. Frames have the same format as panels c) and d) in Figure 3 of the main text.

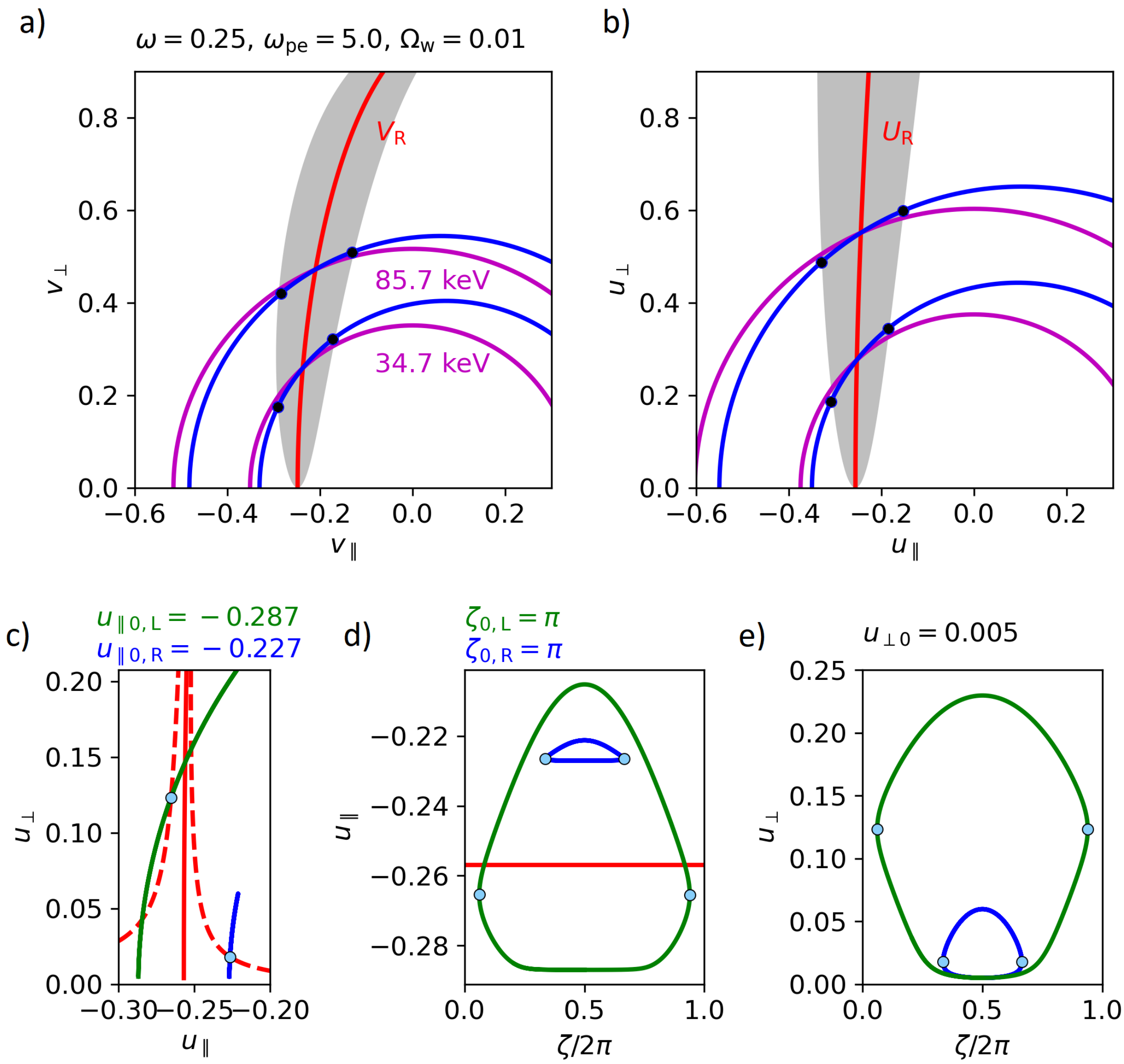
a)
$\omega = 0.25$, $\omega_{pe} = 5.0$, $\Omega_w = 0.01$
V_R
85.7 keV
34.7 keV
$v_\perp$
$v_\parallel$
b)
U_R
$u_\perp$
$u_\parallel$
c)
$u_{\parallel 0,L} = -0.287$
$u_{\parallel 0,R} = -0.227$
$u_\perp$
$u_\parallel$
d)
$\zeta_{0,L} = \pi$
$\zeta_{0,R} = \pi$
$u_\parallel$
$\zeta/2\pi$
e)
$u_{\perp 0} = 0.005$
$u_\perp$
$\zeta/2\pi$

Figure S1.

Figure S1. a) Particle motion in velocity space, with the input wave and plasma parameters written above the panel (normalized with $\Omega_{e0} = 1$, $c = 1$). The red line represents the resonance velocity. Blue lines show two examples of resonant diffusion curves; particles that start their motion on these lines will remain on them unless the wave or plasma parameters change. The grey region covers the maximum extent of the resonance island computed in the approximation of large $u_\perp$; black circles mark the edges of the unapproximated trapping region. Magenta lines are constant energy curves passing through the intersection of the resonance velocity curve and the resonant diffusion curves. b) Similar to panel a), but in momentum space. c-e) Two trajectories (green and blue) of trapped particles in three coordinate spaces: $(u_\parallel, u_\perp)$, $(u_\parallel, \zeta)$ and $(u_\perp, \zeta)$. The initial parallel momenta and relative phase angles are above the panel in corresponding colours; the initial perpendicular momentum is always $u_{\perp 0} = 0.005$. The solid red line represents the standard resonance momentum, $U_\mathrm{R} \equiv U_\mathrm{R}^{\zeta=\pi/2}$. The dashed red lines are $U_\mathrm{R}^{\zeta=0}$ and $U_\mathrm{R}^{\zeta=\pi}$, left to right (not plotted below $u_\perp = 0.01$).

References

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