An Assessment of Errors Introduced in Estimation of Surface Wave Group Velocities Due to Incorrect Assumptions

Akash Kharita, Sagarika Mukhopadhyay*

Department of Earth Sciences, IIT Roorkee, Roorkee – 247667, India

***Corresponding author email: sagarfes@gmail.com**

Code availability: The code is available at https://github.com/Sagarikaiitr/Sagar.git. It is open-source (with an MIT license).

Data Availability: This is purely theoretical work. No data was used for this work.

Authorship statement: SM formulated the problem and advised AK on what needed to be done. AK developed the necessary computer codes and did all the calculations and plotting. SM wrote the manuscript.

Highlights:

i) For estimation of surface wave group velocity, it is assumed that surface waves start from the source/epicenter. This is a wrong assumption.

ii) We developed a method by which it is possible to quickly check what is the minimum epicentral distance from which data can be used for velocity estimation using the same wrong assumption without incurring significant error in the result, especially while using regional earthquake data.

iii) It is shown that for regions having thin/normal crusts, it is possible to use even local earthquake data for surface wave analysis without incurring significant errors in estimated group velocity caused by wrong assumptions.

Declaration of competing interests: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Abstract

For estimation of surface wave group velocity at a given period (T), the epicentral distance is divided by the difference in the arrival time of the corresponding group and the origin time of the earthquake. Hence, it is assumed that such waves are generated at the source/epicenter of the earthquake. However, this assumption is not correct. This work describes an effort to understand and quantify the amount of error that can creep into the estimated surface-wave group velocity values due to wrong assumptions used during their estimation. The error may affect the group velocity values especially when regional earthquake data is used for velocity estimation. The analysis was carried out using a horizontal layer over the halfspace model. Errors were estimated at different epicentral distances and periods for different layer thicknesses (H). Also, focal depth (h) was varied from 5 km to just 5 km above the layer boundary for each model. It is observed that for any combination of h, H, and T, error in estimated group velocity decrease rapidly with epicentral distance. The present work gives us some idea about what is the minimum epicentral distance from which data can be included for estimation of group velocity without adding significant error with such a wrong assumption. It is observed that the minimum epicentral distance at which error becomes less than or equal to a given percentage error decreases with increasing focal depth, i.e. lower is the difference between crustal thickness and focal depth, the lower is the error at a given epicentral distance and period. This means that when the difference between crustal thickness and the focal depth is low, even local earthquake data may be used without adding much error in the estimated group velocity values. For a given value of crustal thickness, focal depth, and epicentral distance, error increase with increasing period.

Keywords: Surface wave; Group velocity; Wrong assumption; Regional earthquake; Epicentral distance.

1. Introduction

Surface waves constitute most of the long period energy observed in earthquake records. These waves are generated from the constructive interference of body waves and travel along the surface on the great circle path between the source and the receiver. In an isotropic medium, two types of surface waves exist independently – Rayleigh waves, generated from the constructive interference of P and vertically polarized SV waves, and Love waves, generated from the constructive interference of multiply reflected horizontally polarized SH waves. The lowest mode of surface wave is termed as fundamental mode. The fundamental mode appears as a distinct low frequency and high amplitude pulse well separated on a seismogram from the higher modes or overtones. Surface waves show dispersive nature in a vertically heterogeneous structure where velocities depend on their frequencies, and waves of a longer period usually travel faster as they are more sensitive to deeper structures. Two types of velocities of propagation of surface waves are defined considering their dispersive nature- (i) phase velocities – the velocity of a phase of a specific frequency component of the surface wave train and (ii) group velocities – the velocity with which the energy propagates. For a given period, the phase velocity is usually greater than the group velocities. Both the phase and group velocity dispersion curves (i.e. variation of speeds with periods) of the fundamental mode of the surface waves have been extensively used to study the crust and upper mantle structure.

1.1 Review of Previous Work

Earlier, in the 1950s and 1960s, dispersion curves for phase and group velocities have been extensively computed using the single-station method (**Figure 1a**). This involved applying a multiple filtering technique (MFT) to the surface waves obtained from the earthquakes (Dziewonski et al. 1969). However, the estimated phase velocities are sensitive to errors in the earthquake source location and must account for source effects (Foster et al., 2014; Trampert and Woodhouse, 1995; Satˆo 1955; Brune and Dorman 1963; Pilant and Knopoff 1964; Knopoff et al. 1966; Bloch and Hales 1968). Hence, for the phase velocity dispersion curve, the two-station method (**Figure 1b**) is used now as it removes the dependency of the effect of source characteristics in the computation of surface wave speeds. This also removes the effect of the medium outside an array (Foster et al., 2014; Brisbourne and Stuart 1998). This method may also be used for the estimation of group velocity. However, this can be used only when two stations lie in the same great circle path with the earthquake source (**Figure 1b**). In practice, that may be difficult to get, and hence stations lying in great circle paths with the source that is within a small angular deviation from each other are also used (**Figure 1c**), sometimes with a correction factor for the deviation incorporated (Foster et al., 2014; Brisbourne and Stuart 1998). The problem with using the two-station method for earthquake data is that quite often two stations may not lie along a great circle path or along two close great circle paths (**Figure 1d**). In that case, it is common to estimate group velocity for each individual station and then combine the results of nearby stations to produce average group velocity estimates for a region (Singh, 2005; Erduran et al., 2007; Mitra et al., 2011; Bhattacharyya et al., 2013; Suresh et al., 2014; Kumar et al., 2017, 2018a, b) or the individual estimates are used to derive a tomographic image of group velocity at different periods for a given region (Ditmar and Yanovskaya, 1987; Ritzwoller and Levshin, 1998; Ritzwoller et al., 1998; Yanovskaya et al., 1998, 2003; Yao et al., 2006; González et al., 2007; Acton et al., 2010; Kumar et al., 2019; Kumar et al., 2021).

Figure 1

In the 1970s, the MFT was perfected as a frequency-time analysis (FTAN) technique (Levshin and Ritzwoller 2001). It involves two steps – (i) the input signal is first converted into the frequency domain and a series of Gaussian filters centered on different frequencies are applied and (ii) the signal is converted back into the time domain to form a smooth 2D envelope function as a function of time and central frequency. The ridge of this 2D envelope function is tracked and the time axis is converted into the velocity axis using the distance from the source.

In the early 2000s, theoretical (Lobkis and Weaver 2001; Larose et al. 2005; Wapenaar 2004) and experimental studies (Shapiro et al. 2005; Roux et al. 2005; Sabra et al. 2005) showed that the Green's function of the medium between two stations can be successfully extracted from the cross-correlation of the ambient noise recorded at two stations. Green's functions obtained from the cross-correlation were similar to those obtained at one station if the earthquake would originate at the other station in the cross-correlation pair. This discovery has since revolutionized the way of computation of surface wave group and phase velocities due in part because of the improved resolution it provides and because it reduces the dependency on the earthquakes that have limited spatial and temporal spread (e.g. Yang et al. 2007; Bensen et al. 2007). The dispersion curves between the station pair for smaller periods are now extensively computed by applying FTAN on the ambient noise cross-correlation functions.

Single station method is still the only reliable method of computation of surface wave velocities in the geographically extreme regions where the density of seismometers is very less due to physical barriers. For example, the extra-terrestrial bodies of the Mars (Trebi-Ollennu et al. 2018; Banerdt et al. 2020) and the Moon (Lammlein et al. 1974; Nakamura et al. 1974) have only one seismometer installed on them. In such regions, single-station method of computation is one of the few available methods of computation of surface wave group and phase velocities.

1.2 Purpose of the Work

For estimation of surface wave group velocity using earthquake data, epicentral distance is divided by group travel time (Dzienwoski, 1969; Levshin et al., 1972; Herrmann and Amon, 2002). It is assumed that the surface wave has originated at the source/epicenter of the earthquake, hence the distance it has traveled is equal to the epicentral distance. Also, the travel time of a particular group of the surface wave is taken to be its arrival time at a station minus the origin time of the causative earthquake. However, both the assumptions are incorrect, as a surface wave is not generated at the source. When a body wave falls on an interface at an angle greater than the critical angle, the refracted wave becomes evanescent, i.e. its amplitude decreases exponentially with depth. When such evanescent waves constructively interfere with each other they produce surface waves. Hence, surface waves are generated at a place away from the source. The resultant error that crops up in the calculation of surface wave group velocity can be insignificantly small when teleseismic events are used for their estimation, but when regional events are used, the error may be large. This may also lead to significant error in the estimated S wave velocity of the medium obtained by inversion of such group velocity estimates. Hence, it is necessary to have an idea about the minimum epicentral distance from which data can be used for the estimation of surface wave group velocity.

In this work, we have estimated the percentage error that can creep in the estimated surface wave group velocities for the Love wave with the help of model analysis. **Figure 2** shows a simple schematic earth model having a single layer over halfspace, representative of a homogeneous crust overlying the mantle. The minimum distance from the source at which surface wave can be generated is X_{cr}. Hence, the maximum distance the surface wave can travel in this case is epicentral distance X minus X_{cr}. The time when the surface wave is generated will also be different from the origin time of the earthquake. We have used the Love wave as an example to do the necessary derivation.

Figure 2

2. Theoretical Development

In **Figure 2**, the model parameters are as follows: H is the thickness of the layer, h is the focal depth and X is the epicentral distance. $_1$, $_1$, $_1$ are the S-wave velocity, density, and shear modulus respectively of the layer and $_2$, $_2$, $_2$ are the corresponding values of the halfspace medium. Let T_0 be the origin time of the earthquake, T_{cr} be the time taken by the wave to travel from the source to the reflector at an angle j_c (critical angle of reflection) and the corresponding horizontal distance is X_{cr}. According to Snell's law critical angel j_c:

$$\sin(j_c) = \frac{\beta_1}{\beta_2} \quad (1)$$

Then $X_{\text{cr}} = (H-h)\tan(j_c) = \frac{(H-h)\beta_1}{\sqrt{{\beta_2}^2 - {\beta_1}^2}}$ (2)

and $T_{\text{cr}} = \frac{(H-h)}{\cos(j_c)\beta_1} = \frac{(H-h)\beta_2}{\sqrt{{\beta_2}^2 - {\beta_1}^2}\beta_1}$ (3)

When angle of incidence becomes slightly higher than j_c, waves refracted in the second medium becomes evanescent. This leads to a situation where Love wave can develop. Taking the minimum distance from the source from where Love wave can start to be approximately equal to X_{cr}, the time taken by a phase with frequency , whose actual phase velocity for the given model is $C( )_{actual}$ can be given as:

$$Tc(\omega)_{\text{actual}} = \frac{(X - X_{\text{cr}})}{C(\omega)_{\text{actual}}} \quad (4)$$

Corresponding arrival time at a station at an epicentral distance of X is given by:

$$\text{Tc}_{\text{arrival}} = T_0 + T_{\text{cr}} + \text{Tc}(\omega)_{\text{actual}} \quad (5)$$

However, in practice phase velocity is estimated with the assumption that Love wave travels along the surface from the source (epicenter) to the station. Hence, estimated phase velocity is given by:

$$C(\omega)_{\text{estimated}} = \frac{X}{Tc(\omega)_{\text{arrival}} - T_0} \quad (6)$$

This will lead to error in estimated phase velocity:

$$\%\ \text{error} = \frac{(C(\omega)_{\text{actual}} - C(\omega)_{\text{estimated}})}{C(\omega)_{\text{actual}}} \text{ x } 100 \quad (7)$$

Love wave phase velocities (C( )) at different angular frequencies , in a homogeneous single layered over half space medium (**Figure 2**) is obtained by solving the Love wave dispersion equation (Lay and Wallace, 1995):

$$\tan\left(\text{H}\sqrt{\frac{1}{{\beta^2}_1} - \frac{1}{C(\omega)^2}}\right) = \frac{\mu_2\sqrt{\frac{1}{C(\omega)^2} - \frac{1}{\beta_2^2}}}{\mu_1\sqrt{\frac{1}{{\beta^2}_1} - \frac{1}{C(\omega)^2}}} \quad (8)$$

The graphs for both sides of equation 8 are plotted (**Figure 3**). Their first intersection from the left-hand side represents the fundamental mode. Corresponding values are used to obtain fundamental mode phase velocity at a particular angular frequency.

Figure 3

In practice, single station method is mostly used for the calculation of group velocity. They can also be used for estimation of phase velocity. The relation between phase velocity C( ) and group velocity U( ) is as follows (Lay and Wallace, 1995):

$$\mathrm{U}(\ ) = \frac{C(\omega)}{\left(1 - \frac{\omega}{C(\omega)}\left(\frac{\mathrm{dC}(\omega)}{\mathrm{d}}\right)\right)} \quad (9)$$

In this case the corresponding parameters for group velocity may be given as:

$$T_U(\omega)_{\text{actual}} = \frac{(X - X_{\text{cr}})}{U(\omega)_{\text{actual}}} \quad (10)$$

Corresponding arrival time at a station at an epicentral distance of X is given by:

$$T_{U_{\text{arrival}}} = T_0 + T_{\text{cr}} + T_U(\omega)_{\text{actual}} \quad (11)$$

However, in practice like for phase velocity, group velocity is also estimated with the assumption that Love wave travels along the surface from the source (epicenter) to the station. Hence, estimated group velocity is given by:

$$U(\omega)_{\text{estimated}} = \frac{X}{T_U(\omega)_{\text{arrival}} - T_0} \quad (12)$$

This will lead to error in estimated group velocity:

$$\%\ \text{error} = \frac{\left(U(\omega)_{\text{actual}} - U(\omega)_{\text{estimated}}\right)}{U(\omega)_{\text{actual}}} \ \text{x}\ 100 \quad (13)$$

where $\mathrm{T_U}(\ )_{\text{actual}}$ = Love wave group travel time for frequency , $\mathrm{T_U}(\ )_{\text{arrival}}$ = Love wave group arrival time for frequency .

3. Model Analysis

Let us take a model having a layer with thickness H overlying a homogeneous halfspace with earthquake source located within the layer where depth of source is h and epicentral distance is X. Let $_1$ and $_2$ be the shear wave velocities in the layer and the halfspace respectively. We have used the values of velocity, density and shear moduli given in Stein and Wysession (2003). The values are as follows: $_1$ = 3.9 km/s, $_2$ = 4.6 km/s, $_1$ = 2.8 gm/cm^3, $_2$ = 3.3 gm/cm^3. **Tables 1** and **2** show actual phase and group velocity values, respectively for different time periods and different layer thickness values varying from 10 km to 70 km.

Source depth (h) values were taken between 0 km to 5 km above the interface between the layer and the halfspace with a step of 5 km. Phase velocity values for different combinations of H and T, where T = 2 / , were obtained using the value of the dashed line (**Figure 3**), representing the right-hand side of the equation 8, where it crosses the first solid line from the left. Corresponding group velocity values were obtained using equation 9. Percentage errors in estimated group velocities at different epicentral distances X for different time periods T were calculated for all combinations of H and h. The code developed for such calculations is given in https://github.com/Sagarikaiitr/Sagar.git. Model parameters can be changed in this code to include crustal and mantle models for different areas which are more appropriate for those areas. Based on the model calculations we have determined the minimum epicentral distance at which the percentage error is equal to 5% for group velocity estimates for the models used in this analysis (**Table 3**).

Table 1

Table 2

Table 3

3. Results and Discussion

There are a few interesting facts that show up in **Table 3**. For example, for a given period and layer thickness, minimum epicentral distance at which error becomes less than or equal to a given percentage error (here we have taken 5%) decreases with increasing focal depth; so much so that for deeper crustal earthquakes records from even local earthquakes may be used for estimation of group velocities for normal crustal structures (H~40 km or less) without incurring significant error caused by the wrong assumption whose effect we are investigating in this work. For oceanic crust even for very shallow focus earthquakes, local earthquake data may be used for the same purpose. For a very thick crust like the one in Tibetan plateau (H~70 km), local earthquake data can be used for estimation of group velocities of Love wave for periods up to 80 s when the distance between focus and the crust-mantle boundary (H-h) is less than 10 km. Here we assume local earthquake means those whose epicentral distance is less than or about 100 km. If we take the epicentral distance range for local earthquakes up to about 150 km then data from events with even H-h of 15 km may be utilized. This goes to show that under certain situations data from even local earthquakes may be used for estimation of group velocities.

Figure 4 show plots of percentage errors versus epicentral distances for focal depths varying from 0 km to just 5 km above the interface for group velocities from epicentral distance greater than critical distance. Figures are plotted to show variation of errors with epicentral distances for three different layer thicknesses, viz. 10 km, 40 km and 70 km which can be taken as representatives for oceanic crust, normal continental crust and the thickest continental crust present in the Tibetan plateau, respectively. Such plots as well as information provided in **Table 3** will be useful in determining data from which stations should not be taken while estimating group velocities using regional earthquake data and when even local earthquake data may be used. It is observed from **Table 3** and **Figure 4** that for fixed values of H and h, X at which error remains same increases with increasing value of T. This means that in order to estimate group velocity at longer periods, one should take data from stations at larger epicentral distances to avoid significant error caused by wrong assumption. The other interesting observation is that, for a fixed focal depth, minimum epicentral distance where error level drops to 5% increases with increasing layer thickness (**Table 3**, **Figure 4**). It is observed that there is a decrease in percentage error with increase in epicentral distances. At a given epicentral distance percentage error is larger for larger time periods for a given depth of focus. Whereas, percentage error for a given epicentral distance and time period decreases with increasing depth of focus.

Figure 4

4. Conclusions

i) When regional earthquake data are used for estimation of group velocity of surface waves, records of earthquakes with epicentral distances below a certain minimum value should not be used to avoid large amount of error creeping in due to incorrect assumption used for estimation of such velocity values.

ii) For thinner crust, data from even local earthquakes may be used for such purpose.

iii) The error caused by incorrect assumption decreases with increasing epicentral distance for a given period and a given focal depth.

iv) The error caused by wrong assumption decreases with increasing focal depth for a given period and a given epicentral distance.

v) The error caused by incorrect assumption increases with increasing period for a given epicentral distance and a given focal depth.

vi) We have assumed that the minimum distance from the source at which surface wave can be generated is X_{cr}. However, for generation of Love wave both upgoing and downgoing evanescent waves are required. In that case the minimum distance from the source where surface wave will be generated will increase. As a consequence, the estimated errors in surface wave group velocity will also increase further if data is taken from smaller epicentral distance.

vii) This is the first approach ever to quantify the errors in the estimation of Love wave velocities using single station method.

viii) Equations have been developed to assess such errors for any combination of period, thickness of layer, focal depth, epicentral distance and shear wave velocities.

Acknowledgements

This is a purely theoretical work. No data from any organization or repository has been used for this work. This is an outcome of undergraduate project carried out by AK under the guidance of SM, who acknowledges the Head of the Department for providing all the facilities. No fund was received from any funding agency or organization for carrying out this work.

References

Acton, C.E., Priestley, K., Gaur, V.K., Rai, S.S., 2010. Group velocity tomography of the Indo-Eurasian collision zone. J. Geophys. Res. Solid Earth 115, 1–16. http://dx.doi.org/10.1029/2009JB007021.

Banerdt, W. Bruce, Suzanne E. Smrekar, Don Banfield, Domenico Giardini, Matthew Golombek, Catherine L. Johnson, Philippe Lognonné, et al. 2020. "Initial Results from the InSight Mission on Mars." *Nature Geoscience 2020 13:3* 13 (3): 183–89. https://doi.org/10.1038/s41561-020-0544-y.

Bensen, G. D., M. H. Ritzwoller, M. P. Barmin, A. L. Levshin, F. Lin, M. P. Moschetti, N. M. Shapiro, and Y. Yang. 2007. "Processing Seismic Ambient Noise Data to Obtain Reliable Broad-Band Surface Wave Dispersion Measurements." *Geophysical Journal International* 169 (3): 1239–60

Bhattacharya, S.N., Mitra, S., Suresh, G., 2013. The shear wave velocity of the upper mantle beneath the Bay of Bengal, northeast Indian ocean from interstation phase velocities of surface waves. Geophys. J. Int. 193, 1506–1514. http://dx.doi.org/10.1093/gji/ggt007.

Bloch S, Hales A (1968) New techniques for the determination of surface wave phase velocities, Bull. Seism. Soc. Am., 58, 1021–1034

Brisbourne AM, Stuart GW (1998) Shear-wave velocity structure beneath North Island, New Zealand, from Rayleigh-wave interstation phase velocities Geophys. J. Int. 133, 175–184.

Brune J, Dorman J (1963) Seismic waves and earth structure in the Canadian shield, Bull. seism. Soc. Am., 53, 167–210

Ditmar, P. G., and Yanovskaya, T. B. (1987). A Generalization of the Backus-Gilbert Method for Estimation of Lateral Variations of Surface Wave Velocity (In Russian). Izv.Akad.Nauk Sssir, Fiz. Zemli 6, 30–60.

Dziewonski A, Bloch S, Landisman M (1969) A technique for the analysis of transient seismic signals. Bulletin of Seismological Society of America 59:427-444

Erduran, M., Cakir, O., Tezel, T., Sahin, S., Alptekin, O., 2007. Anatolian surface wave evaluated at GEOFON station ISP Isparta, Turkey. Tectonophysics 434, 39–54. http://dx.doi.org/10.1016/j.tecto.2007.02.005.

Foster A, Ekstr¨om G, Nettles M (2014) Surface wave phase velocities of theWestern United States from a two-station method. Geophys. J. Int. 196, 1189–1206

González, O. L., Alvarez, L., Guidarelli, M., and Panza, G. F. (2007). Crust and Upper Mantle Structure in the Caribbean Region by Group Velocity Tomography and Regionalization. Pure Appl. Geophys. 164, 1985–2007. doi:10.1007/s00024-007-0259-7

Herrmann RB, Ammon CJ (2002) Computer Programs in Seismology. Department of Earth and Atmospheric Sciences, St Louis University, St Louis, MO USA

Knopoff L, Mueller S, Pilant WL (1966) Structure of the crust and upper mantle in the Alps from the phase velocity of Rayleigh waves, Bull. Seism. Soc. Am., 56, 1009–1044

Kumar A, Kumar N, Mukhopadhyay S, Klemperer SL (2021) Tomographic Image of Shear Wave Structure of NE India Based on Analysis of Rayleigh Wave Data. Front. Earth Sci. 9:680361. doi: 10.3389/feart.2021.680361

Kumar, N., Aoudia, A., Guidarelli, M., Babu, V. G., Hazarika, D., and Yadav, D. K. (2019). Delineation of Lithosphere Structure and Characterization of the Moho Geometry under the Himalaya-Karakoram-Tibet Collision Zone Using Surface-Wave Tomography. Geol. Soc. Lond. Spec. Publications 481, 19–40. doi:10.1144/SP481-2017-172

Kumar A, Kumar N, Mukhopadhyay S (2018a) Investigation of azimuthal variation in seismic surface waves group velocity in the western part of Himalaya-Tibet Indo-Gangetic plains region. Himalayan Geology, 39: 33-46.

Kumar A, Mukhopadhyay S, Kumar N, Baidya, PR (2018b) Lateral Variation in Crustal and Mantle Structure in Bay of Bengal Based on Surface Wave Data. J. Geodynamics, 113: 32-42, DOI10.1016/j.jog.2017.11.006

Kumar A, Kumar N, Mukhopadhyay S, Baidya PR (2017) Crustal and Uppermost Mantle Structures in the Frontal Himalaya and Indo-Gangetic Basin Using Surface Wave: Tectonic Implications. Quart. Int., 462: 34-49. http://dx.doi.org/10.1016/j.quaint.2017.02.035

Lammlein, David R., Gary V. Latham, James Dorman, Yosio Nakamura, and Maurice Ewing. 1974. "Lunar Seismicity, Structure, and Tectonics." *Reviews of Geophysics* 12 (1): 1–21. https://doi.org/10.1029/RG012I001P00001.

Larose, Eric, Arnaud Derode, Dominique Clorennec, Ludovic Margerin, and Michel Campillo. 2005. "Passive Retrieval of Rayleigh Waves in Disordered Elastic Media." *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics* 72 (4). https://doi.org/10.1103/PHYSREVE.72.046607.

Lay T, Wallace TC (1995) Modern Global Seismology. Academic Press. pp. 535

Levshin A, Pisarenko V, Pogrebinsky G (1972) On frequency-time analysis of oscillations. Annals Geophysics 28:211-218

Levshin, A. L., and M. H. Ritzwoller. 2001. "Automated Detection, Extraction, and Measurement of Regional Surface Waves." *Pure and Applied Geophysics* 158 (8): 1531–45. https://doi.org/10.1007/PL00001233.

Lobkis, Oleg I., and Richard L. Weaver. 2001. "On the Emergence of the Green's Function in the Correlations of a Diffuse Field." *The Journal of the Acoustical Society of America* 110 (6): 3011–17. https://doi.org/10.1121/1.1417528.

Mitra, S., Priestley, K., Acton, C., Gaur, V.K., 2011. Anomalous surface wave dispersion and the enigma of continental-like structure for the Bay of Bengal. J.

Asian Earth Sci. 42, 1243–1255. http://dx.doi.org/10.1016/j.jseaes.2011.07.008.

Nakamura, Yosio, Gary Latham, David Lammlein, Maurice Ewing, Frederick Duennebier, and James Dorman. 1974. "Deep Lunar Interior Inferred from Recent Seismic Data." *Geophysical Research Letters* 1 (3): 137–40. https://doi.org/10.1029/GL001I003P00137.

Pilant WL, Knopoff L (1964) Observations of multiple seismic events, Bull. Seism. Soc. Am., 54, 19–39

Ritzwoller, M. H., and Levshin, A. L. (1998). Eurasian Surface Wave Tomography: Group Velocities. J. Geophys. Res. 103 (B3), 4839–4878. doi:10.1029/97jb02622

Ritzwoller, M. H., Levshin, A. L., Ratnikova, L. I., and Egorkin, A. A. (1998). Intermediate Period Group Velocity Maps across central Asia, Western China and Parts of Middle-East. Geophys. J. Int. 134, 315–328.

Roux, Philippe, Karim G. Sabra, Peter Gerstoft, W. A. Kuperman, and Michael C. Fehler. 2005. "P-Waves from Cross-Correlation of Seismic Noise." *Geophysical Research Letters* 32 (19): 1–4. https://doi.org/10.1029/2005GL023803.

Sabra, Karim G., Peter Gerstoft, Philippe Roux, W. A. Kuperman, and Michael C. Fehler. 2005. "Extracting Time-Domain Green's Function Estimates from Ambient Seismic Noise." *Geophysical Research Letters* 32 (3): 1–5. https://doi.org/10.1029/2004GL021862.

Satˆo Y, 1955. Analysis of dispersed surface waves by means of Fourier Transform I, Bull. Eqk. Res. Inst., Tokyo Univ., 33, 33–48

Shapiro, Nikolai M., Michel Campillo, Laurent Stehly, and Michael H. Ritzwoller. 2005. "High-Resolution Surface-Wave Tomography from Ambient Seismic Noise." *Science* 307 (5715): 1615–18. https://doi.org/10.1126/SCIENCE.1108339.

Singh, D.D., 2005. Rayleigh-wave group-velocity studies beneath Indian ocean. Bull. Seismol. Soc. Am. 95, 502–511.

Stein S, Wysession M (2003) An Introduction to Seismology, Earthquakes, and Earth Structure. Blackwell Publishing. pp. 498

Suresh G, Teotia SS, Bhattacharya SN (2014) Crustal velocity structure of the Deccan Volcanic Province, Indian Peninsula, from observed surface wave dispersion. Annals Geophysics, 57, S0431 - 8; doi:10.4401/ag-6540

Trampert J, Woodhouse, JH (1995) Global phase velocity maps of Love and Rayleigh waves between 40 and 150 seconds, Geophys. J. Int., 122, 675–690

Trebi-Ollennu, A., Won Kim, Khaled Ali, Omair Khan, Cristina Sorice, Philip Bailey, Jeffrey Umland, et al. 2018. "InSight Mars Lander Robotics Instrument Deployment System." *Space Sci. Rev.* 214 (5): 93. https://doi.org/10.1007/s11214-018-0520-7.Wapenaar, Kees. 2004.

"Retrieving the Elastodynamic Green's Function of an Arbitrary Inhomogeneous Medium by Cross Correlation." *Physical Review Letters* 93 (25). https://doi.org/10.1103/PHYSREVLETT.93.254301.Yang, Yingjie, Michael H. Ritzwoller, Anatoli L. Levshin, and Nikolai M. Shapiro. 2007. "Ambient Noise Rayleigh Wave Tomography across Europe." *Geophysical Journal International* 168 (1): 259–74. https://doi.org/10.1111/J.1365-246X.2006.03203.X/3/168-1-259-FIG010.JPEG.Yanovskaya, T. B., Kizima, E. S., and Antonova, L. M. (1998). Structure of the Crust in the Black Sea and Adjoining Regions from Surface Wave Data. J. Seismology 2, 303–316. doi:10.1023/A:1009716017960

Yanovskaya, T. B., and Kozhevnikov, V. M. (2003). 3D S-Wave Velocity Pattern in the Upper Mantle beneath the Continent of Asia from Rayleigh Wave Data. Phys. Earth Planet. Interiors 138, 263–278. doi:10.1016/S0031-9201(03)00154-7

Yao, H., Van Der Hilst, R. D., and De Hoop, M. V. (2006). Surface-wave Array Tomography in SE Tibet from Ambient Seismic Noise and Two-Station Analysis - I. Phase Velocity Maps. Geophys. J. Int. 166, 732–744. doi:10.1111/j.1365-246x.2006.03028.x

Figure Caption

Figure 1: Schematic diagram showing situations where single station or two station data may be used for estimation of phase/group velocities. Star represents earthquake location, triangles represent station location. a) Ideal situation for implementation of two-station method. b) More realistic situation where two station method is implemented. c) Case when only one station is present (single station method). d) Case when multiple stations are present (single station method).

Figure 2: Simple schematic representation of an earth model. The red star represents the focus of an earthquake. The inverted triangle represents a recording station. Bold lines with arrows represent ray paths for a critically refracted ray.

Figure 3: Representation of equation 8. Solution of this equation is possible where the plot with dashed line that represents the right-hand side of the equation crosses the solid lines that represent the left-hand side. Here q = $H\sqrt{\left(\frac{1}{\beta_1^2} - \frac{1}{c^2}\right)}$ (after Lay and Wallace, 1995).

Figure 4: Plot of percentage error in group velocity estimate versus epicentral distance in kilometers. The color scale represents the period at which group velocity is estimated. H is the crustal thickness and h is focal depth.

Table Caption

Table 1: Phase velocities C( ) (in km/s) of Love wave for model parameters

mentioned in the text for different values of layer thickness H in km and period T in second.

Table 2: Group velocity U( ) (in km/s) of Love wave for model parameters mentioned in the text for different values of layer thickness H in km and period T in second.

Table 3: Minimum epicentral distance X (in km) at which percentage error in estimated group velocity falls to 5% for a different combination of layer thickness H, focal depth h, and period T.