Order-dependent sampling control for state estimation of uncertain fractional-order neural networks system

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Abstract. In this paper, the problem of state estimation for a fractional-order neural networks system with uncertainties is studied by a sampled-data controller. First, considering the convenience of digital field, such as anti-interference, not affected by noise, a novel sampled-data controller is designed for the fractional-order neural network system of uncertainties with changeable sampling time. In the light of the input delay approach, the sampled-data control system of fractional-order is simulated by the delay system. The main purpose of the presented method is to obtain a sampled-data controller gain K to estimate the state of neurons, which can guarantee the asymptotic stability of the closed-loop fractional-order system. Then, the fractional-order Razumishin theorem and linear matrix inequalities (LMIs) are utilized to derive the stable conditions. Improved delay-dependent and order-dependet stability conditions are given in the form of LMIs. Furthermore, the sampled-data controller can be acquired to promise the stability and stabilization for fractional-order system. Finally, two numerical examples are proposed to demonstrate the effectiveness and advantages of the provided method.

Keywords: Fractional-order systems, neural networks, sampled-data control, Lyapunov function, stability.

1. Introduction

Neural network (NNs) is a classic complicated system made of the interconnected neurons, which has been focused by many scholars. An artificial NNs is made up of artificial neurons or nodes in the modern science. The NNs is a network system consisting of interlinked nodes, which simulates the function of neurons in the brain. It is widely applied to help people solve various problems, for example, image processing, pattern recognition, convex optimization, robot manipulator in [1]-[6], etc. It should be noted that these problems have an significant relationship with their interior dynamics and the dynamical behavior of NNs, such as stability, multi-stability, synchronization, etc. Now, NNs have attracted numerous attention and become a related research field.

With the development of society, fractional calculus is introduced to replace integer order with some non-integer order. In recent years, the calculus and the control systems of fractional-order (FO) systems have attracted much attention from researchers in control theory. It is worth mentioning that the order of FO calculus is not integer, which can overcome the disadvantage that the differential equation model of integer order can not accurately describe the complex system. And fractional derivatives can better describe the dynamic behavior of neurons. Fractional derivatives have non-local characteristics, and FO has more accurate memory and genetic properties compared with integer order. Although fractional calculus has not yet emerged, it has recently attracted the attention of researchers, and it remains a great field of study. So far, some excellent results have been presented on the dynamic analysis of FO systems in [7]-[9]. Many important and interesting conclusions have been acquired in fractional-order neural networks (FONNs) systems and all kinds of issues

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have been investigated by many authors in [10]-[17]. From these results, it can be proved that the FONNs can more correctly depict and imitate the neurons than the traditional integer neural networks in the human brain. Because fractional calculus has the advantages of infinite memory and genetic characteristics when representing system models in biological engineering, neural network, fluid dynamics and other fields, it has received more and more concern. It is well known that the long memory properties have been ignored in NNs. Additionally, previous studies have indicated that electrical conductivity of biological cell membranes are FO systems [32]. Therefore, it is extraordinarily proper and accurate to use FO differential equations to simulate and study the dynamics of real NNs.

It is well known that stability is the most important issue in control systems, and it is the primary condition for the dynamical systems. For FO systems, numerous interesting conclusions have been obtained in the literature of proving system stability by Lyapunov function, containing asymptotic stability [20], consistent stability [21], stability [22], Hopf-Bifurcation research [23, 24], Mittage-Leffler stability [18, 19]. At present, the FO systems have been attracted more and more attention on the robustness and performance, specially for FONN systems with uncertainties. For instance, the delay-dependent stability and stabilization for a kind of FONNs with uncertainty and time-varying delays are discussed in [10]. The asymptotic stability condition for the FONNs is derived by the Lyapunov method, a neural network property and the FO Razumikhin theorem. In [12], sufficient conditions that ensure the stability of a estimated fractional uncertain neural network error system are established by the fractional Lyapunov method.

From the engineering point of view, it is necessary to scheme a controller to make the closed-loop system stable. In recent years, most scholars design state feedback controller to stabilize the system in previous literature. For example, asymptotic stability and stabilization of FO linear systems with time-varying structured uncertainties and time-varying delay are discussed in [7]. The stabilization criterion of the system is derived by a state feedback controller. During the implementation of state feedback control, the controller gain inevitably produces a certain degree of inaccuracy due to the degradation of the actuator, the aging of the components or the requirements of controller gain adjustment. It has been shown that relatively small perturbations in controller parameters could even destabilize the closed-loop system. Therefore, it is necessary to design a non-fragile controller that can tolerate the component variation of the controller to a certain extent. The non-fragile controller design of a class of FO linear delayed systems assumed to have structured uncertainties in both the plant and the controller is addresses in [8]. And a non-fragile feedback controller is designed to make the system asymptotically stable. Since most systems are nonlinear in real life, the linear systems are conservative in the above literatures. To solve this problem, some scholars address the delay-dependent stability and the stabilization of a class of FO memristive neural networks with time-varying delay in [10]. However, for NNs, it is often difficult to obtain the state information of all neurons due to network congestion, packet loss or measurement loss. Therefore, it is an important method to evaluate the neuronal state using the available network output. Later, some scholars estimate the state of neurons by the available output measurements, and then design feedback controllers to make FONNs asymptotically stable in [12]. However, although the above controllers effectively make the system stable, the system receives data in real time, which may increase the burden of the network channel. As high-speed computing technology develops, the sampled data controller [25] has been widely used in industry due to its low price, high reliability, convenient preservation. Compared with the state feedback control, this method greatly improves the bandwidth utilization and reduces the pressure of receiving information for system. But because the control input is invariant in the following sampling time interval, it is not easy to acquire the stability criteria for the control system. So researcher have put forward numerous ways to research it, and the input delay method [26] has been a well-prevalent way for the past few years. Through the input delay method, the system can be considered as a sequential system with time-varying delay produced by a zero-order holder (ZOH). In addition, for all we know, there are few results on the stability for state estimation of FONNs by sampled-data control. Therefore, motivated by the reasons above, we will focus on the asymptotic stability for state estimation of the FONNs with uncertainty by sampled-data controller. Compared with [12], this paper reduces the conservatism.

Determining the sampling period is an important issue for the stability of FONNs system by using sampled data controller. We note that a large sampling period may be advantageous, for instance, fewer informational communication, less controller drivers, and fewer channel occupancy. Then, a large sampling period is obtained by using some methods to stabilize the FONNs system. In most cases, because the characteristics of the plant are hard to judge precisely, the internal parameters of the system are unpredictable, which can change with the variations of the outside situation. Consequently, FONNs system has parameter uncertainty and interference. And the stability is easily destroyed for FONNs system with parameter uncertainty and interference. Although much effort has been made in the existing literature. The research is rarely studied at present about sampled data controller for FONNs systems with uncertainties, which is also the purpose of this paper.

In this paper, some new stability criteria are proposed for state estimation of FONNs system with uncertainties under the sampled data controller. The sampling periods are changeable and smaller than a known maximal admissible upper bound. The order-dependent criteria with lower conservativeness are gained by a set of LMIs. Compared with the now available studies, the main contributions of this study are as follows:

i) In practical application, the influence of parameter uncertainties always exist, which affects robust stability. Since most system states are not measurable, state information is not available. Therefore, the state estimation of uncertain FONNs is studied and a sampled-data controller with variable period is designed to stabilize the FONNs in this paper. Different from the previous methods, instead of continuous measurement, the method of sampling measurement is used to estimate the state of neurons. By the input delay method, the sampled-data controller can be used to assure that the uncertain FONNs is asymptotically stable.

ii) Order-dependent and delay-dependent stability criteria are developed for uncertain FONNs by Razumikhin theorem and LMIs. Moreover, in order to stabilize the FONNs system, a design method of the sampled-data control is presented.

iii) The sampling intervals generated by ZOH are time-varying stochastically for FONNs. The alterable sampling peculiarity can be displayed in the simulation example.

The organization of the article is shown as follows: The model, definition and lemmas needed for the stability of FONNs system with uncertainties are given in the section II; The stability conditions for FONNs system with uncertainties are proposed and a sampled-data controller is designed in the section III; A numerical example confirms the validity of the theoretical approach in the section IV and the conclusion is reached in the section V.

2. Problem statement and preliminary

Consider the following n-dimensional FONNs system with parameter uncertainties described by

$$\begin{cases} D^{\gamma}x(t) = -\widetilde{A}x(t) + \widetilde{B}f(x(t)), \\ \quad y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) = (x_1(t), \dots, x_n(t))^T \in R^n$ represents the state vector; $f(x(t)) = (f_1(x(t)), ..., f_n(x(t)))^T \in R^n$ represents the activation function; $y(t) = (y_1(t), ..., y_m(t))^T \in R^m$ denotes the measurement output; $\gamma \in (0, 1)$; $\widetilde{A} = A + \Delta A(t)$; $\widetilde{B} = B + \Delta B(t)$; $A = diag\{a_1, a_2, ..., a_n\}$ is the positive definite diagonal matrix; $B \in R^{n\times n}$ is the constant matrix; C is a constant matrice of appropriate dimensions; $\Delta A(t)$, $\Delta B(t)$ are time-varying matrices with compatible dimensions satisfying:

$$[\Delta A(t) \quad \Delta B(t)] = ME(t)[N_1 \quad N_2], \tag{2}$$

where M, N_1 and N_2 are given constant matrices; unidentified matrix function $E(t)$ is fulfilled as follows:

$$E^T(t)E(t) \leq I. \tag{3}$$

Firstly, in order to facilitate the calculation of the derivative of order γ, we propose the definition as follows:
Definition 1. [27]. The Caputo derivative of FO α of function $x(t)$ is described as follows:

$$D^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-\tau)^{n-\alpha-1} x^{(n)}(\tau) d\tau, \tag{4}$$

where $n-1 < \alpha < n \in Z^+$. $\Gamma(\cdot)$ is the Gamma function $\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt$.
The aim of this paper is to propose an efficient estimation algorithm to observe the state of neurons from the available network output. Therefore, the following state estimator is proposed:

$$\begin{cases} D^\gamma \hat{x}(t) = -A\hat{x}(t) + Bf(\hat{x}(t)) + u(t), \\ \quad \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{5}$$

where $\hat{x}(t) \in R^n$ stands for the estimator of $x(t)$; $u(t) = (u_1(t), ..., u_n(t))^T$ denotes control input; $f(\hat{x}(t)) = (f_1(\hat{x}(t)), ..., f_n(\hat{x}(t)))^T \in R^n$; $\hat{y}(t) = (\hat{y}_1(t), ..., \hat{y}_m(t))^T \in R^m$ is the estimated output $y(t)$.
We use $e(t) = x(t) - \hat{x}(t)$ to denote the estimation error, and get the following estimation error system from (1) and (5):

$$D^\gamma e(t) = -Ae(t) - \Delta A(t)x(t) + Bf(e(t)) + \Delta B(t) f(x(t)) - u(t), \tag{6}$$

where $f(e(t)) = f(x(t)) - f(\hat{x}(t))$.

In actual applications, some hardware communication is transmitted by digital signals with the popularity of digital signals. In order to make full use of networking technique, a sampled data controller is designed for the stability of FONNs (6). Obviously, the sampled-data controller is given as follows:

$$u(t) = K(y(t_k) - \hat{y}(t_k)) = KCe(t_k), \tag{7}$$

where K is controller gain matrix.
In this paper, the control message is generated by ZOH function at the sampling instant $0 = t_0 \leq t_1 \leq \cdots \leq \lim_{k\to+\infty} t_k = +\infty$. In addition, the sampling period is changeable and time-variable. In other words, the sampling period is described as follows:

$$0 < t_{k+1} - t_k = d_k \leq d, \quad \forall k \geq 0, \tag{8}$$

where $d > 0$ represents the upper bound of sampling periods. Define $d(t) = t - t_k$, $t \in [t_k, t_{k+1}]$. Then, the state feedback controller can be rewritten as:

$$u(t) = KCe(t - d(t)), \quad t_k \leq t < t_{k+1}. \tag{9}$$

Let $r(t) = [x^T(t), e^T(t)]^T$ be the augmented vector, and the augmented system can be obtained from (1), (6) and (9):

$$D^\gamma r(t) = -\bar{A}r(t) + \bar{B}g(r(t)) - \bar{K}r(t - d(t)), \tag{10}$$

where $g(r(t))=[f^T(x(t)), f^T(e(t))]^T$, $\bar{A}=\begin{bmatrix} A+\Delta A(t) & 0 \\ \Delta A(t) & A \end{bmatrix}, \bar{B}=\begin{bmatrix} B+\Delta B(t) & 0 \\ \Delta B(t) & B \end{bmatrix}, \bar{K}=\begin{bmatrix} 0 & 0 \\ 0 & KC \end{bmatrix}$.
Here, it is supposed that activation function $f_i(.)$ meets the following assumption.
Assumption 1. [31] For any $a, b \in R$, $a \neq b$, we have

$$l_i^- < \frac{f_i(b)-f_i(a)}{b-a} < l_i^+, \tag{11}$$

where l_i^-, l_i^+ are known constants.

Under the sampled-data controller, the stability is guaranteed for FONNs system (8). To this aim, the following lemmas are presented firstly.
Lemma 1. [28]. Let $x(t) \in R^n$ be a differentiable vector value function. Then, for any time instant $t \geq t_0$, we have

$$D^\alpha(x^T(t)Px(t)) \leq (x^T(t)P)D^\alpha x(t) + (D^\alpha x(t))^T Px(t).$$

Lemma 2. [29]. For given matrices $Q = Q^T, E, H$ and $F(t)$ with compatible dimensions, the following inequality holds:

$$Q + HF(t)E + E^T F^T(t) H^T < 0.$$

Then, for all $F(t)$ satisfying $F^T(t)F(t) \leq I$, if and only if there exist a scalar $\lambda > 0$ such that

$$Q + \lambda^{-1} HH^T + \lambda E^T E < 0,$$

or, equivalently,

$$\begin{bmatrix} Q & \lambda H & E^T \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} < 0.$$

Lemma 3. [30]. Assume that there exist three positive constants $\alpha_1, \alpha_2, \alpha_3$ and a quadratic Lyapunov function $V(.)$: $R^+ \times R^n \to R^+$ such that
(i) $\alpha_1 \parallel x(t)\|^2 \leq V(t, x(t)) \leq \alpha_2 \parallel x(t)\|^2$,
(ii) $D_t^\alpha V(t, x(t)) \leq -\alpha_3 \parallel x(t)\|^2$,
whenever $V(t+\theta, x(t+\theta)) < \rho V(t, x(t)), \forall \theta \in [-h, 0], t \geq 0$, for some $\rho > 1$, then under the zero initial condition, FO system $D_t^\alpha x(t) = f(t, x_t), \alpha \in (0, 1)$ is asymptotically stable.

3. Main results

In this section, the stability is discussed for the FONNs system (10) by the sampled-date controller (9). The delay-dependent and order-dependent stability criteria in the form of LMIs for system (10) are proposed, which are given in the next theorem.
Theorem 1. Given scalars d, $\gamma \in (0, 1]$ and matrix K, if there exist matrices $P > 0$, $\bar{X} > 0$, $\bar{Z} > 0$, $W_1 > 0$, $W_2 > 0$ and any suitable matrix $\bar{Y}$, a scalar $\lambda_1 > 0$ such that the next LMIs hold:

$$\psi = \begin{bmatrix} R & U & \lambda_1 G^T \\ * & -\lambda_1 I & 0 \\ * & * & -\lambda_1 I \end{bmatrix} < 0, \tag{12}$$

$$\Pi = \begin{bmatrix} \bar{X} & \bar{Y} \\ \bar{Y^T} & \bar{Z} \end{bmatrix} \geq 0, \tag{13}$$

where

$$R = \begin{bmatrix} \psi_{11}+L_1W_1L_1 & 0 & \psi_{13} & 0 & 0 & 0 & \psi_{17} & 0 \\ * & \psi_{22}+L_2W_2L_2 & 0 & \psi_{24} & 0 & \psi_{26} & 0 & \psi_{28} \\ * & * & -W_1 & 0 & 0 & 0 & \psi_{37} & 0 \\ * & * & * & -W_2 & 0 & 0 & 0 & \psi_{48} \\ * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & \psi_{68} \\ * & * & * & * & * & * & \psi_{77} & 0 \\ * & * & * & * & * & * & * & \psi_{88} \end{bmatrix},$$

$\psi_{11} = -PA - AP + d^\gamma\gamma^{-1}(X - YA - AY^T) + P, \psi_{13} = PB + d^\gamma\gamma^{-1}YB, \psi_{17} = -d^\gamma\gamma^{-1}A^TZ,$
$\psi_{19} = -(P + d^\gamma\gamma^{-1}Y)M, \psi_{22} = \psi_{11} + L_2W_2L_2, \psi_{24} = \psi_{13}, \psi_{26} = -PKC - d^\gamma\gamma^{-1}YKC, \psi_{28} = \psi_{17},$
$\psi_{29} = \psi_{19}, \psi_{37} = d^\gamma\gamma^{-1}B^TZ, \psi_{48} = \psi_{37}, \psi_{68} = -d^\gamma\gamma^{-1}C^TK^TZ, \psi_{77} = -d^\gamma\gamma^{-1}Z, \psi_{79} = -d^\gamma\gamma^{-1}ZM,$
$\psi_{88} = \psi_{77}, \psi_{89} = \psi_{79}, \bar{X} = diag\{X, X\}, \bar{Y} = diag\{Y, Y\}, \bar{Z} = diag\{Z, Z\}.$
Then, the closed-loop system (10) is asymptotically stable.

Proof: Constructing the Lyapunov function as $V(r(t)) = r^T(t)\bar{P}r(t)$ and computing the derivative of $V(r(t))$ along system (10) yield in light of Lemma 1

$$\begin{aligned} D^\gamma V(r(t)) &\leq r^T(t)\bar{P}D^\gamma r(t) + (D^\gamma r(t))^T\bar{P}r(t) \\ &= r^T(t)(-\bar{P}\bar{A} - \bar{A}\bar{P})r(t) + 2r^T(t)\bar{P}\bar{B}g(r(t)) + 2r^T(t)\bar{P}\bar{K}r(t-d(t)), \end{aligned} \tag{14}$$

where $\bar{P} = diag\{P, P\}$.

There exist any real matrices $\bar{X} = \bar{X}^T > 0, \bar{Y}$ and $\bar{Z} = \bar{Z}^T > 0$, satisfying (13). Then, the following result holds,

$$d^\gamma\gamma^{-1}\upsilon^T(t)\Pi\upsilon(t) - \int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon^T(t)\Pi\upsilon(t)ds \geq 0, \tag{15}$$

where $\upsilon(t) = \left[r^T(t), (D^\gamma r(t))^T\right]^T$.
Under the assumption 1, the following inequality holds for any diagonal matrix $W = diag\{W_1, W_2\} > 0$,

$$r^T(t)LWLr(t) - g^T(r(t))Wg(r(t)) \geq 0, \tag{16}$$

where $L = diag\{L_1, L_2\}$.
By Lemma 3, for any real number $\rho > 1$, we suppose that

$$V(t+\theta, r(t+\theta)) < \rho V(t, r(t)).$$

Then, we can obtain

$$\rho r^T(t)\bar{P}r(t) - r^T(t-d(t))\bar{P}r(t-d(t)) > 0. \tag{17}$$

Combining (14)-(17), we have

$$\begin{aligned} D^\gamma V(r(t)) \leq\ & r^T(t)(-\bar{P}\bar{A} - \bar{A}\bar{P} + LWL + \rho\bar{P} + d^\gamma\gamma^{-1}(\bar{X} - \bar{Y}\bar{A} - \bar{A}\bar{Y}^T + \bar{A}\bar{Z}\bar{A}))r(t) \\ &+ 2r^T(t)(\bar{P}\bar{B} + d^\gamma\gamma^{-1}(\bar{Y}\bar{B} - \bar{A}\bar{Z}\bar{B}))g(r(t)) + g^T(r(t))(-W + d^\gamma\gamma^{-1}\bar{B}^T\bar{Z}\bar{B})g(r(t)) \\ &+ 2r^T(t)(\bar{P}\bar{K} + d^\gamma\gamma^{-1}(\bar{Y}\bar{K} - \bar{A}\bar{Z}\bar{K}))r(t-d(t)) + 2g^T(r(t))d^\gamma\gamma^{-1}\bar{B}^T\bar{Z}\bar{K}r(t-d(t)) \\ &+ r^T(t-d(t))(-\bar{P} + d^\gamma\gamma^{-1}\bar{K}^T\bar{Z}\bar{K})r(t-d(t)) - \int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon(t)\Pi\upsilon(t)^Tds \\ =\ & \vartheta^T(t)\phi\vartheta(t) - \int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon^T(t)\Pi\upsilon(t)ds, \end{aligned} \tag{18}$$

where $\vartheta(t) = [r^T(t), g^T(r(t)), r^T(t-d(t))]^T$,

$$\phi = \begin{bmatrix} \phi_{11} + \rho\bar{P} + d^\gamma\gamma^{-1}\bar{A}\bar{Z}\bar{A} & \phi_{12} - d^\gamma\gamma^{-1}\bar{A}\bar{Z}\bar{B} & \phi_{13} + d^\gamma\gamma^{-1}\bar{A}\bar{Z}\bar{K} \\ * & -W + d^\gamma\gamma^{-1}\bar{B}^T\bar{Z}\bar{B} & -d^\gamma\gamma^{-1}\bar{B}^T\bar{Z}\bar{K} \\ * & * & -\bar{P} + d^\gamma\gamma^{-1}\bar{K}^T\bar{Z}\bar{K} \end{bmatrix},$$

$\phi_{11} = -\bar{P}\bar{A} - \bar{A}\bar{P} + LWL + d^\gamma\gamma^{-1}(\bar{X} - \bar{Y}\bar{A} - \bar{A}\bar{Y}^T), \phi_{12} = \bar{P}\bar{B} + d^\gamma\gamma^{-1}\bar{Y}\bar{B}, \phi_{13} = -\bar{P}\bar{K} - d^\gamma\gamma^{-1}\bar{Y}\bar{K}$.
By Schur complement, $\phi < 0$ is equivalent to

$$\begin{bmatrix} \phi_{11} + \rho\bar{P} & \phi_{12} & \phi_{13} & \widetilde{\psi}_{14} \\ * & -W & 0 & \widetilde{\psi}_{24} \\ * & * & -\bar{P} & \widetilde{\psi}_{34} \\ * & * & * & \widetilde{\psi}_{44} \end{bmatrix} < 0, \tag{19}$$

where $\widetilde{\psi}_{14} = -d^\gamma\gamma^{-1}\bar{A}\bar{Z}, \widetilde{\psi}_{24} = d^\gamma\gamma^{-1}\bar{B}^T\bar{Z}, \widetilde{\psi}_{34} = -d^\gamma\gamma^{-1}\bar{K}^T\bar{Z}, \widetilde{\psi}_{44} = -d^\gamma\gamma^{-1}\bar{Z}$.
Replacing $\Delta A(t)$ and $\Delta B(t)$ in (19) with $ME(t)N_1$ and $ME(t)N_2$, respectively, one has

$$\bar{R} + He(UH(t)G) < 0, \tag{20}$$

where

$$\bar{R} = \begin{bmatrix} \bar{\psi}_{11} + L_1W_1L_1 & 0 & \psi_{13} & 0 & 0 & 0 & \psi_{17} & 0 \\ * & \bar{\psi}_{22} + L_2W_2L_2 & 0 & \psi_{24} & 0 & \psi_{26} & 0 & \psi_{28} \\ * & * & -W_1 & 0 & 0 & 0 & \psi_{37} & 0 \\ * & * & * & -W_2 & 0 & 0 & 0 & \psi_{48} \\ * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & \psi_{68} \\ * & * & * & * & * & * & \psi_{77} & 0 \\ * & * & * & * & * & * & * & \psi_{88} \end{bmatrix},$$

$\bar{\psi}_{11} = -PA - AP + d^\gamma\gamma^{-1}(X - YA - AY^T) + \rho P, \bar{\psi}_{22} = \bar{\psi}_{11}$,

$U = \begin{bmatrix} \psi_{19}^T & \psi_{29}^T & 0 & 0 & 0 & 0 & \psi_{79}^T & \psi_{89}^T \end{bmatrix}^T$,

$G = \begin{bmatrix} N_1 & -N_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Applying Lemma 2 to (20), there exists a positive scalar λ_1, such that

$$\bar{R} + \lambda_1^{-1}UU^T + \lambda_1 G^T G < 0.$$

Taking $\rho \to 1^+$, ψ is equivalent to (12) by Schur complement. Since $\int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon(t)\Pi\upsilon(t)^T ds > 0$, we have $D^\gamma V(r(t)) < \vartheta^T(t)\psi\vartheta(t)$. From (12), we have $D^\gamma V(r(t)) < -\int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon(t)\Pi\upsilon(t)^T ds$, then, condition (ii) in Lemma 3 is also satisfied. Thus, system (10) with sampled-data controler K is asymptotically stable.

When the controller gain matrix K is unknown, it is evident that the (12) is not an LMI because some crosses of these determined parameters are described in (12) in a nonlinear term PK. However, it can be transformed into an LMI by the following theorem.

Theorem 2. Given scalars d, $\gamma \in (0, 1]$. If there exist $P > 0$, $X > 0$, $W_1 > 0$, $W_2 > 0$ and any suitable matrix J, a scalar $\lambda_1 > 0$ such that the next LMIs hold

$$\begin{bmatrix} \hat{R} & \hat{U} & \lambda_1 G^T \\ * & -\lambda_1 I & 0 \\ * & * & -\lambda_1 I \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} \bar{X} & 0 \\ 0 & \bar{P} \end{bmatrix} \geq 0, \tag{22}$$

where

$$\hat{R} = \begin{bmatrix} \hat{\psi}_{11} + L_1W_1L_1 & 0 & PB & 0 & 0 & 0 & \hat{\psi}_{17} & 0 \\ * & \hat{\psi}_{22} + L_2W_2L_2 & 0 & PB & 0 & -JC & 0 & \hat{\psi}_{28} \\ * & * & -W_1 & 0 & 0 & 0 & \hat{\psi}_{37} & 0 \\ * & * & * & -W_2 & 0 & 0 & 0 & \hat{\psi}_{48} \\ * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & \hat{\psi}_{68} \\ * & * & * & * & * & * & \hat{\psi}_{77} & 0 \\ * & * & * & * & * & * & * & \hat{\psi}_{88} \end{bmatrix},$$

$\hat{U} = \begin{bmatrix} -M^TP^T & -M^TP^T & 0 & 0 & 0 & 0 & \hat{\psi}_{79}^T & \hat{\psi}_{89}^T \end{bmatrix}^T$,
$\hat{\psi}_{11} = -PA - AP + L_1W_1L_1 + P + d^\gamma\gamma^{-1}X, \hat{\psi}_{17} = -d^\gamma\gamma^{-1}A^TP, \hat{\psi}_{22} = \hat{\psi}_{11}, \hat{\psi}_{28} = \hat{\psi}_{17}, \hat{\psi}_{37} = d^\gamma\gamma^{-1}B^TP,$
$\hat{\psi}_{48} = \hat{\psi}_{37}, \hat{\psi}_{68} = -d^\gamma\gamma^{-1}C^TJ^T, \hat{\psi}_{77} = -d^\gamma\gamma^{-1}P, \hat{\psi}_{79} = -d^\gamma\gamma^{-1}PM, \hat{\psi}_{88} = \hat{\psi}_{77}, \hat{\psi}_{89} = \hat{\psi}_{79}$. Then, the closed-loop system (10) is asymptotically stable. Moreover, the controller gain matrix is given by $K = P^{-1}J$.

Proof: To simplify the structure of the LMI, let $\bar{Y} = 0, \bar{Z} = \bar{P}$ in (12). The proof is completed.

In order to show the advantages of the controller (9), the following result without uncertainties is given for comparison. Excepting for the uncertainties, the system (10) is describe by

$$D^\gamma r(t) = -\hat{A}r(t) + \hat{B}g(r(t)) - \hat{K}r(t - d(t)), \tag{23}$$

where $\hat{A} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \hat{B} = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix}, \hat{K} = \begin{bmatrix} 0 & 0 \\ 0 & KC \end{bmatrix}$.
From Theorem 1, we can get the following results for FONNs system (23).
Corollary 1. Given scalars d, $\gamma \in (0, 1]$ and matrix K. If there exist $P > 0$, $\bar{X} > 0$, $W_1 > 0$, $W_2 > 0$, $\bar{Z} > 0$, and any suitable matrix $\bar{Y}$, such that the next LMIs hold

$$R < 0, \tag{24}$$

$$\Pi = \begin{bmatrix} \bar{X} & \bar{Y} \\ \bar{Y^T} & \bar{Z} \end{bmatrix} \geq 0. \tag{25}$$

Then, the closed-loop system (23) is asymptotically stable without parameter uncertainty.
Corollary 2. Given scalars d, $\gamma \in (0, 1]$. If there exist $P > 0$, $W_1 > 0$, $W_2 > 0$, $X > 0$ and any suitable matrix J, such that the next LMIs hold

$$\hat{R} < 0, \tag{26}$$

$$\begin{bmatrix} \bar{X} & 0 \\ 0 & \bar{P} \end{bmatrix} \geq 0. \tag{27}$$

Then, the closed-loop system (23) is asymptotically stable. Moreover, the controller gain matrix is given by $K = P^{-1}J$.

Remark 1. The conditions in criteria can guarantee the stabilization of the FONNs under a sampled-data controller. In this paper, by using the FO Razumikin theorem, appropriate inequalities are established. Then, the order-dependent stability conclusions are obtained for the FONNs with sampled-data control. Finally, the LMI toolbox can be used to overcome the issue quickly. In addition, the conclusions are easy to be applied to engineering applications. In existing papers, the authors have reflected on the issue of stabilization for regions or conditions without sampled-data control in [11]-[17]. For all we know, there are few results on the stability of FONNs by sampled-data control. Hence, we take the sampled-data controller into account.

Remark 2. When $\gamma = 1$, the FONNS system (1) with sampled-data control will degenerate to integer order. Accordingly, the asymptotic stability and stability results are still effective for integer order neural network models in [10], [12] and [13].

Remark 3. In this paper, the sampled-data controller is designed and the time delay is considered in the system by the input delay method, which makes the system closer to the actual system. Different from [17], sampled-data controller and time delay are added in this paper, while controller and the time delay are not added in [17]. Therefore, compared with [17], this paper is less conservative. In addition, Eq.(15) is added to reduce the conservatism of the system based on the free weight matrix. The stability criteria of delay-dependent and order-dependent are derived by the FO Razumikhin theorem. Moreover, in [17], the authors used Lyapunov direct method to derive an order-independent analysis criterion for FONNs. Therefore, compared with [17], this paper is less conservative.

4. Numerical Example

In this section, two examples are displayed to testify the availability of the proposed method.

Example 1. Consider the following uncertain FONNs system (23) based on sampled-data control with parameters given by:

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}, B = \begin{bmatrix} 2 & -1.2 & 0 \\ 1.8 & 1.71 & 1.15 \\ -4.75 & 0 & 1.1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Setting the activation functions $f_1 = f_2 = f_3 = tanh(s)$ with $l_1^+ = l_2^+ = l_3^+ = 1$, the following matrix is given:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

By using the MATLAB toolbox calculating the conditions in Corollary 2, the maximum sampling interval d can be obtained under different γ, which are listed in Table 1. Letting $\gamma = 0.98$, the corresponding maximum d is 0.17. Then, the sampled-data controller gain matix K is designed as:

$$K = \begin{bmatrix} 3.8193 & -0.0463 & -0.0266 \\ -0.2556 & 2.8515 & 0.0960 \\ -0.6520 & 0.4267 & 1.3798 \end{bmatrix}.$$

Table 1: The simulation results of the sampling interval with different γ for Example 1.

γ	0.90	0.92	0.95	0.98
d	0.13	0.14	0.15	0.17

Applying the above gain matrix with the initial conditions $r(t) = [0.9, 0.8, 0.9]^T$, the state response curves of system (23) are displayed in Fig. 1. The history of the sampled-data control input $u(t)$ can be shown in Fig. 2. From Fig. 1, it is clear to see that the FONNs system can achieve the stability in a short time. Form Fig. 2, we can see the discrete characteristic of the sampled-data controller.

Example 2. Consider the following FONNs with hub structure [33], the parameters are given as follow:

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Setting the activation functions $f_1 = f_2 = f_3 = tanh(s)$ with $l_1^+ = l_2^+ = l_3^+ = 1$, the following matrix is given:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

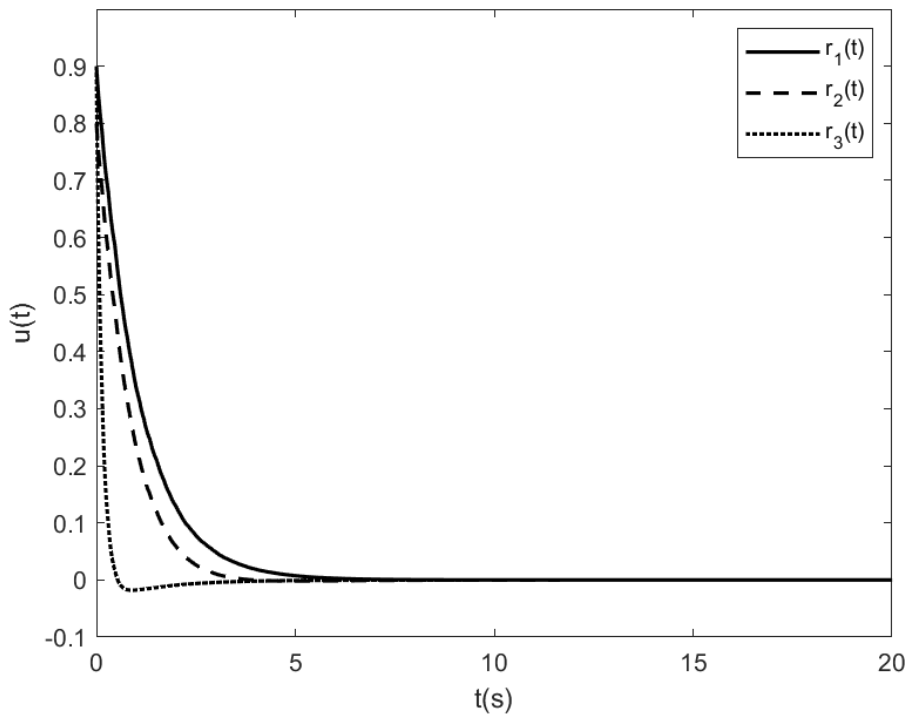

Figure 1: State $r(t)$ in the Example

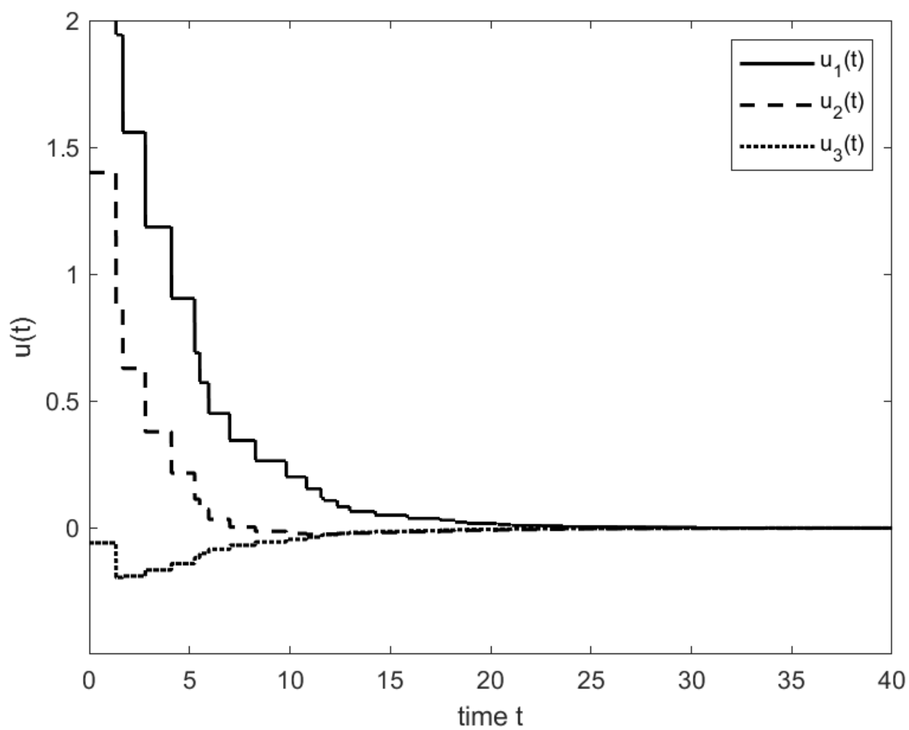

Figure 2: Sampled-data control input $u(t)$ in the Example

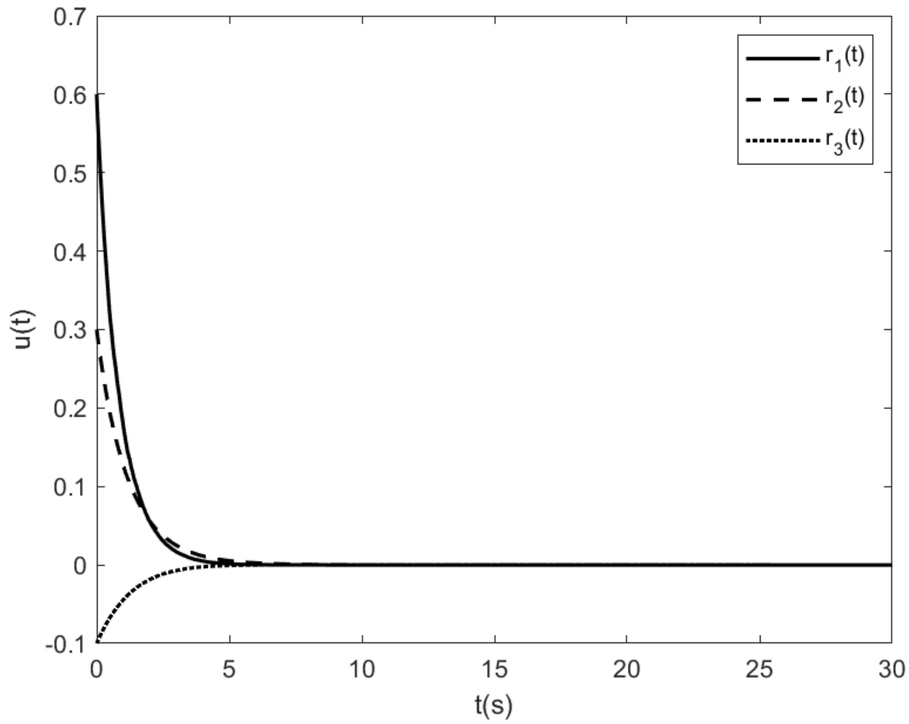

Figure 3: State $r(t)$ in the Example

Then, the FONNs system (23) can be rewritten as:

$$\begin{cases} D^{\gamma} r_1(t) = -6r_1(t) + 3\tanh(r_1(t)) - 2\tanh(r_2(t)) - 2\tanh(r_3(t)), \\ D^{\gamma} r_2(t) = -2r_2(t) + \tanh(r_1(t)) + \tanh(r_2(t)), \\ D^{\gamma} r_3(t) = -2r_3(t) + \tanh(r_1(t)) + \tanh(r_3(t)). \end{cases}$$

By using the MATLAB toolbox calculating the conditions in Corollary 2, the maximum sampling interval d can be obtained under different γ, which are listed in Table 2. Letting $\gamma = 0.98$, the corresponding maximum d is 0.16. Then, the sampled-data controller gain matix K is designed as:

$$K = \begin{bmatrix} 4.4253 & -0.0335 & -0.0335 \\ -0.0614 & 1.2292 & 0.1199 \\ -0.0614 & 0.1370 & 1.2041 \end{bmatrix}.$$

Applying the above gain matrix with the initial conditions $r(t) = [0.6, 0.3, -0.1]^T$, the state response curves

Table 2: The simulation results of the sampling interval with different γ for Example 2.

γ	0.90	0.92	0.95	0.98
d	0.12	0.13	0.15	0.16

of system (23) are displayed in Fig. 3. The history of the sampled-data control input $u(t)$ can be shown in Fig. 4. From Fig. 3, it is clear to see that the FONNs system can achieve the stability in a short time. Form Fig. 4, we can see the discrete characteristic of the sampled-data controller.

5. Conclusion

In this paper, the stability and stabilization issue of uncertain FONNs system is investigated based on sampled-data control. By using input delay approach, the sampling system can be described as the time-delay system.

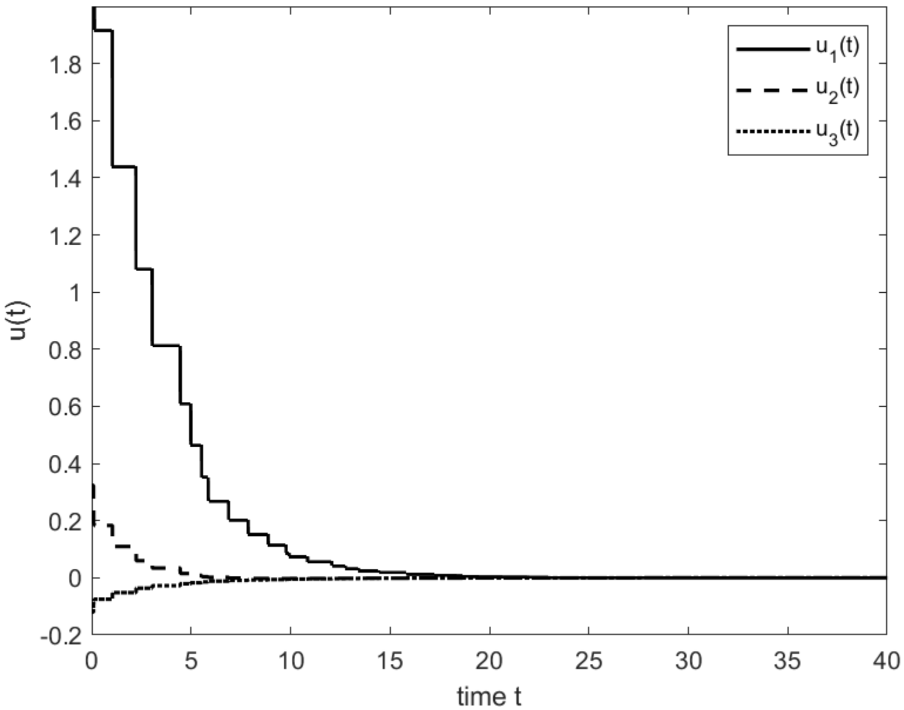

Figure 4: Sampled-data control input $u(t)$ in the Example

The designed sampled-data controller can be used to stabilize the FONNs system. The efficient conditions are derived with FO Razumikhin theorem. The proposed delay-dependent and order-dependent stability results, which are expressed in the form of LMIs, can be easily checked via MATLAB LMI toolbox. Finally, two numerical examples are given to illustrate the effectiveness and superiority of the derived criteria. One of the future research topics would be an extension of the present results to more industrial oriented cases, for example, the case with complex dynamical networks, the case with time varying delays and external noise effect.

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