Power Law Distribution of Independent Basin Areas in Fluvial Landscapes

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Key Points:

- The distribution of basin areas in fluvial landscapes, as studied across 25 diverse islands, exhibits a power law relationship
- This emergent behavior is attributed to the headward growth nature of drainage networks evolution demonstrated using a probabilistic model
- The power law exponent is found correlated with landscape boundary characteristics notably, fractal dimension and compactness coefficient

Abstract

River networks around the world exhibit statistical scaling laws, including the distribution of independent basin sizes in landscapes. The widespread occurrence of these patterns in various landscapes suggests that there are fundamental, but not yet fully understood, processes responsible for these power law distributions. This study investigates the distribution of independent basin areas across 25 islands worldwide, revealing a clear adherence to a power law pattern. The research suggests that the power law exponent is influenced by landscape boundary characteristics, such as the compactness coefficient and fractal dimension, with the exponent value increasing with these factors. Furthermore, the study demonstrates the development of power law patterns in basin areas using a probabilistic network growth model. This model, based on a preferential headward growth mechanism, underscores the significant roles of boundary conditions and headward growth dynamics in the self-organization of power law patterns in fluvial landscapes.

Plain Language Summary

Fluvial landscapes exhibit several intriguing statistical patterns, among which we have explored how the basin areas of rivers are organized in a power law distribution. We analyzed 25 different island landscapes and observed a consistent pattern in basin area distribution. While the mechanisms behind these common structures are complex and not fully understood, our findings suggest a significant influence of the island's boundary shape on these patterns. To delve deeper, we employed a computer model to simulate channel network growth from their sources towards the landscape edges, which helped explain the emergence of the power-law pattern. This research enhances our understanding of natural landscape formation, highlighting the interplay between an island's shape and channel network development in shaping these distinct patterns.

1 Introduction

Power law distributions manifest in a wide range of natural phenomena, spanning disciplines such as demography, economy, geosciences, biology, among others (Corral & González, 2019; Marquet et al., 2005; Newman, 2005). Such patterns serve as signatures of complexity and can provide us important insights about the self-organizing processes leading to such patterns. The existence of power laws is sometimes attributed to evolution of system towards particular state such as self-organized criticality, energy or cost minimization, highly optimized tolerance or highly resilient system (Bak, 1996; Carlson & Doyle, 1999; Marković & Gros, 2014; A. Rinaldo et al., 1993; Ronellenfitsch & Katifori, 2019). Thus, the study of such patterns of river networks can help us in understanding the mechanisms involved in the fluvial landscape evolution (Andrea Rinaldo et al., 2014; Rodriguez-Iturbe Ignacio, 1997).

The river networks have been widely studied for their fractal patterns and certain scaling laws which they follow with quite narrow range of exponents irrespective of the differences in the underlined geography or climate (Maritan et al., 1996; Pelletier, 1999; Rodriguez-Iturbe Ignacio, 1997). The similar scaling laws are also found in martian fluvial networks as well (Stepinski et al., 2002). These statistical laws include Hack’s law relating the scaling between channel length and basin area, distribution of drainage area and upstream length for a basin. These patterns suggest that river networks are unique type of tree networks that might be arising due to certain similar geomorphic processes leading to such configuration. These scaling laws are also used by the river network simulation models to compare the modelled networks with real networks by comparing the exponents (Borse & Biswal, 2023; Hooshyar et al., 2020; Paik & Kumar, 2008; Rodríguez-Iturbe et al., 1992). The quantitative understanding of such networks helps us to understand the underlined principles hidden in the pattern formation. Moreover, the understanding of network structures has several applications in studying the hydrological modelling as well as ecological networks (Biswal & Singh, 2017; Larsen et al., 2017; Ranjbar et al., 2020; Rodríguez-Iturbe et al., 1982; Sarker et al., 2019). Thus, developing quantitative indices to characterize the landscapes would be beneficial for the better understanding of landscape organization (Nowosad & Stepinski, 2019).

Numerous efforts, including experimental studies (Cheraghi et al., 2018; Pelletier & Turcotte, 2000), have been made to model the evolution of river networks, utilizing either statistical frameworks (Meakin et al., 1991; Andrea Rinaldo et al., 1998) or physical mass balance equations (Tucker & Hancock, 2010; Willgoose et al., 1991), with the objective of modelling landscapes and elucidating the underlying principles of such scaling laws. Some studies suggested that such organization is effect of river networks evolving to minimize their energy expenditure (Rodríguez-Iturbe et al., 1992). However, some studies suggested that emergence of such patterns is consequence of inherent randomness in the landscape (Paik & Kumar, 2008). Some studies simulated channel network evolution following headward growth of channels in statistical manner (Howard, 1971). Thus, our understanding of why river networks organize to such unique patterns is still evolving.

One of the power laws observed in landscapes is the Basin Area Distribution (BAD) of independent basin areas within a landscape. Although river networks have been extensively researched in terms of other statistical laws, the understanding of independent basin area distribution remains limited to date. Previous studies have explored the power law behavior in Basin Area Distribution (BAD). For instance, using a minimum energy dissipation model, Sun & Meakin (1994) not only reported power law behavior in BAD but also suggested a direct relationship between the BAD exponent and the fractal dimension of landscape boundaries, as indicated by $\tau = 1 + D/2$, where τ is the power law exponent and D is the fractal dimension. Similarly, La Barbera & Lanza (2001), through a case study in Italy, confirmed that the distribution of independent basin areas follows a power law pattern with an exponent of 0.7 and also noted its relation to fractal dimension. Oliveira et al., (2019) extended these observations to Martian and lunar landscapes, demonstrating the universality of power law distribution in BAD. Despite these advancements, the underlying mechanism of this organization remains largely unknown, and comprehensive studies encompassing a wide variety of real-world landscapes have been lacking. In this study, we investigate the BAD across various islands globally and examine its relationship with boundary properties. Additionally, we employ a probabilistic model to shed light on the mechanisms driving BAD.

2 Materials and Methods

2.1 Basin Area Distribution (BAD) for Islands

For this study, we selected 25 islands of varying sizes, ranging from 60 km² to 115,000 km², located in diverse geographical regions. The digital elevation terrain data was sourced from the USGS Earth Explorer SRTM dataset, available at a 1 arc-second resolution. Watersheds were delineated using the hydrology toolbox in ArcMap. We studied BAD using exceeding probability distribution to avoid the issue of bin sizing. We can obtain the exceedance probability of basin area as $P(N_A \geq A)$ representing the number of basins in landscape having basin areas greater than A or the probability that a basin chosen at random will have a basin area greater than A. The relationship can be expressed as $P(N_A \geq A) \propto A^{-\tau}$. We then used maximum likelihood estimation to calculate the best fit power law exponent (τ) for that distribution as it is reported to be more accurate (Goldstein et al., 2004). We used the threshold area of 0.05 km^2 to delineate basins for the power law distribution analysis.

2.2 Characterizing Shape of Landscape Boundary

Fractal dimension (FD) have been widely used to study several geomorphological characteristics (Breyer & Scott Snow, 1992; Fehr et al., 2011; Khanbabaei et al., 2013). The FD value for a 2-dimensional boundary characterizes the boundary's roughness or irregularity. We calculated the FD of boundaries for 25 islands using the Box-counting method. This method involves overlaying the island images with a grid and counting the number of grid boxes intersecting the island boundary. The process is repeated with progressively finer grids, increasing resolution and thus more accurately capturing the boundary's intricate structure. The fractal dimension is determined by plotting log(N) against log(r) and calculating the slope of the resulting line, where $FD = \frac{log(N)}{log(r)}$. Here N is number of box counts and *r* is resolution that is 1/box size. Note that minimum box size here will be the resolution of original DEM i.e. around 30 meters.

Furthemore, Gravelius compactness (GC) is employed to assess the shapes of islands by quantifying their spatial compactness. GC is calculated as the ratio of the island perimeter to the perimeter of a circle with equivalent area. A lower GC indicates a more compact shape, resembling a circular or regular form, while a higher GC suggests more elongated or irregular boundaries. Specifically, when GC equals 1, it represents a perfectly circular shape. This metric facilitates the quantitative evaluation and comparison of spatial organization in island boundaries.

3 Results: Basin Area Distribution Analysis Across Islands

The delineated basins of Hawaii, Taiwan, and Tasmania islands are illustrated in Figure 1, along with their respective BAD plots. The power law distribution can be infurred from the linear trend on log-log scale. This scaling is consistent for a large portion of the data, which includes smaller basins, but it deviates for the very large basins. With the sample example of 3 islands shown here; it can be observed that as the roughness of the island boundaries is increasing from Hawaii to Taiwan to Tasmania which is reflected in the increasing value of Fractal dimension. Additionally, the boundaries become increasingly irregular and less compact, which is reflected in the rising Gravelius compactness (GC) coefficient values. The value of the BAD exponent (τ) is also increases alongside the FD and GC values. Table 1 presents the calculated values for all 25 islands. Figures 1d and 1e demonstrate a positive correlation between τ and both FD and GC.

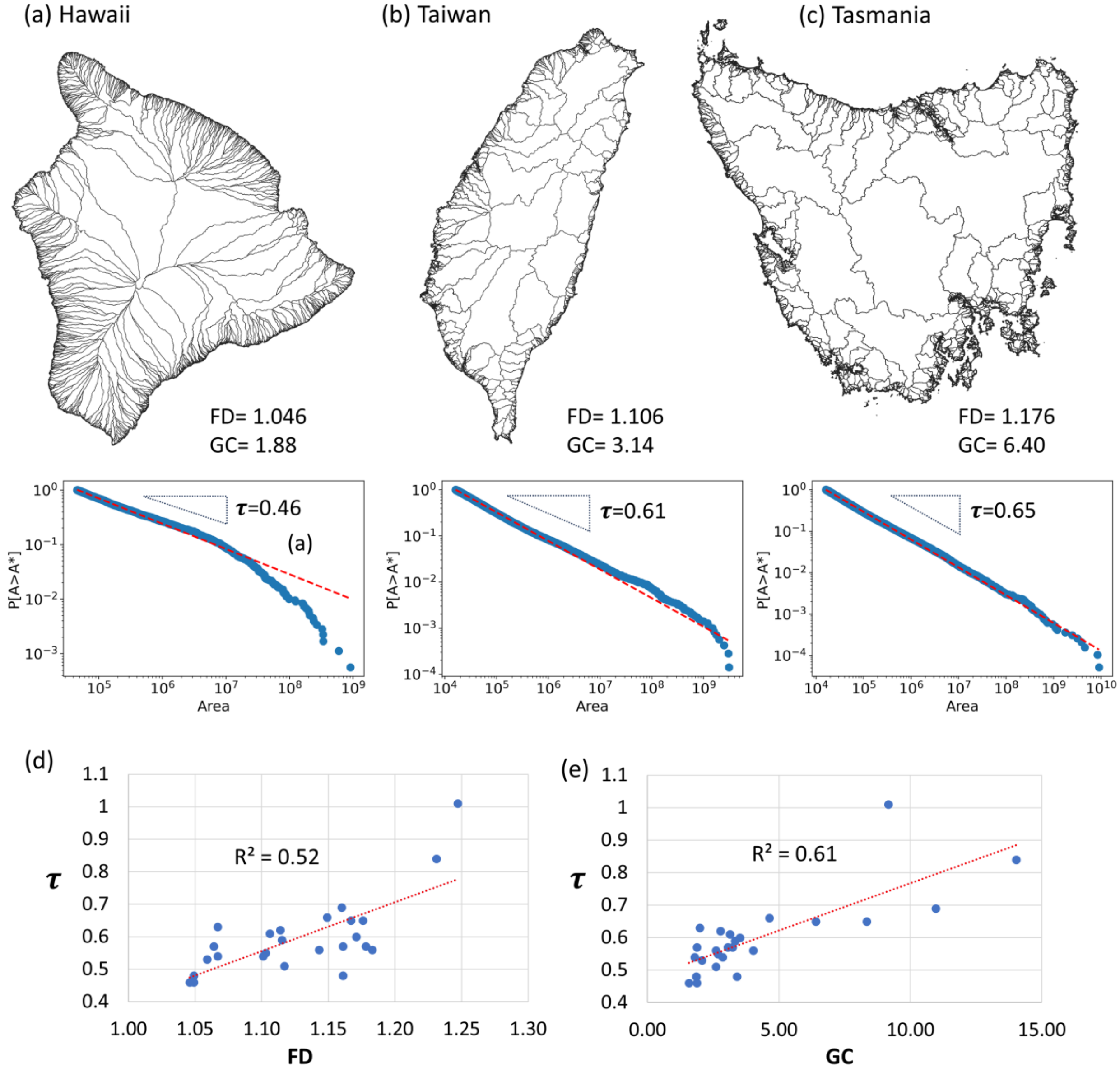

Figure 1. Basin area distribution of islands. **(a), (b)** and **(c)** shows the delineated basin of Hawaii, Taiwan and Tasmania respectively alongwith the corresponding basin area distributions. **(d)** and **(e)** shows that τ is positively correlated to fractal dimension and Gravelius compactness coefficient of the landscape boundary of 25 islands.

Island name	Location	FD	Perimeter (Km)	Area (Sq. Km)	GC	BAD (τ) Exponent
Andaman	Bay of Bengal, Indian Ocean	1.160	2898.70	5571.53	10.96	0.69
Auckland	Pacific Ocean	1.167	10016.99	114931.02	8.34	0.65
Barbados	Caribbean Sea	1.064	138.49	435.07	1.87	0.57

Belle Île	Atlantic Ocean	1.114	93.21	89.67	2.78	0.62
Bermuda	Atlantic Ocean	1.247	252.67	60.52	9.16	1.01
Cyprus	Mediterranean Sea	1.067	973.70	9261.18	2.85	0.54
Fiji	Pacific Ocean	1.149	1697.31	10683.85	4.63	0.66
Grenada	Caribbean Sea	1.143	164.05	317.46	2.60	0.56
Socotra	Arabian Sea	1.067	421.86	3610.46	1.98	0.63
Hawaii	Central Pacific Ocean	1.046	680.46	10462.94	1.88	0.46
Oahu	Central Pacific Ocean	1.178	426.72	1562.74	3.05	0.57
Sicily	Mediterranean Sea	1.115	1883.08	25431.83	3.33	0.59
Jeju	Korea Strait	1.183	614.10	1858.76	4.02	0.56
Kauai	Central Pacific Ocean	1.101	241.27	1440.80	1.79	0.54
Mataram	Bali Sea and Indian Ocean	1.171	839.11	4555.57	3.51	0.6
Maui	Central Pacific Ocean	1.161	523.53	1888.44	3.40	0.48
Mauritius	Indian Ocean	1.117	399.21	1869.09	2.61	0.51
Grande Comore	Indian Ocean	1.049	209.85	1021.39	1.85	0.48
Sumba	Indian Ocean	1.059	769.77	10939.17	2.08	0.53
French Southern and Antarctic lands	Antarctic Ocean	1.231	4239.12	7281.61	14.02	0.84
Cabo Verde-Praia	Atlantic Ocean	1.161	362.44	1004.73	3.23	0.57
Reunion	Indian Ocean	1.049	280.22	2524.23	1.57	0.46
Taiwan	Western Pacific Ocean	1.106	2108.27	35973.34	3.14	0.61
Tasmania	Southern Ocean	1.176	5785.95	65076.21	6.40	0.65
Tereciara	Atlantic Ocean	1.103	191.35	406.68	2.68	0.55

Table 1. The BAD exponent (τ) and the boundary characteristics FD and GC for islands

3 Probabilistic Drainage Network Evolution Model to Explain BAD

To understand the power law organization of basins, we employed the probabilistic network evolution model by Borse & Biswal (2023) applied within realistic boundary shapes. The model aims to simulate the evolution and organization of drainage networks into self-similar, tree-like patterns, incorporating physically meaningful variables into its statistical modeling framework. The modelled networks follow statistical scaling laws similar to real river networks. This model introduces new parameters that offer flexibility in generating networks with varying shapes and characteristics. The model follows preferential headward growth of channels from boundary or outlets, where the preferences is probabilistically determined based on the downstream length of the evolving streams. Refer to (Borse & Biswal, 2023) for detailed information about the model. The parameter α characterizes this preferential headward growth and plays a pivotal role in shaping the overall sizes of the basins. As α increases, the competition among growing networks is restricted, providing networks with a faster growth rate to expand more. Consequently, networks with lower α values exhibit increased elongation due to heightened competition in network growth. Thus larger basins forms with higher α and smaller elongated basins are formed for lower α (Figure 2c and 2d).

Figure 2a-c illustrates the evolution of drainage networks from boundary outlets in a headward direction, using a sample simulation with $\alpha = 0$, where networks grow from all sides with equal probability. Figure 2d shows the resulting network with $\alpha = 1$ where single larger basin is formed. Our findings indicate that the modeled basins adhere to power laws for Basin Area Distribution (BAD), with the exponent varying according to the shape of the landscape boundary. Figure 2e-g shows basins obtained using the model within the Hawaii's boundary with different α values. While the growth mechanism characterized by α significantly affects the sizes of the largest basins formed, the majority of smaller basins maintain uniform scaling laws, as evidenced by similar scaling exponents and the overlap in the BAD plot across different α values (Figure 2h). However, keeping α constant and varying boundary conditions reveals noticeable differences in scaling exponents. Figures 2f, 2i, and 2j display networks generated from sample simulations with the same α value (0.5), but under different boundary conditions. The corresponding BAD plots in Figure 2k distinctly highlight the differences in slopes for the three diverse landscape boundaries. Figure 2l demonstrates that τ is influenced by the shape of the landscape boundary, rather than the chosen α value. To ensure computational efficiency of the model, we resampled the original island boundaries to a smaller grid sizes. We have applied flow accumulation threshold of 50 for delineating networks as well as plotting basins.

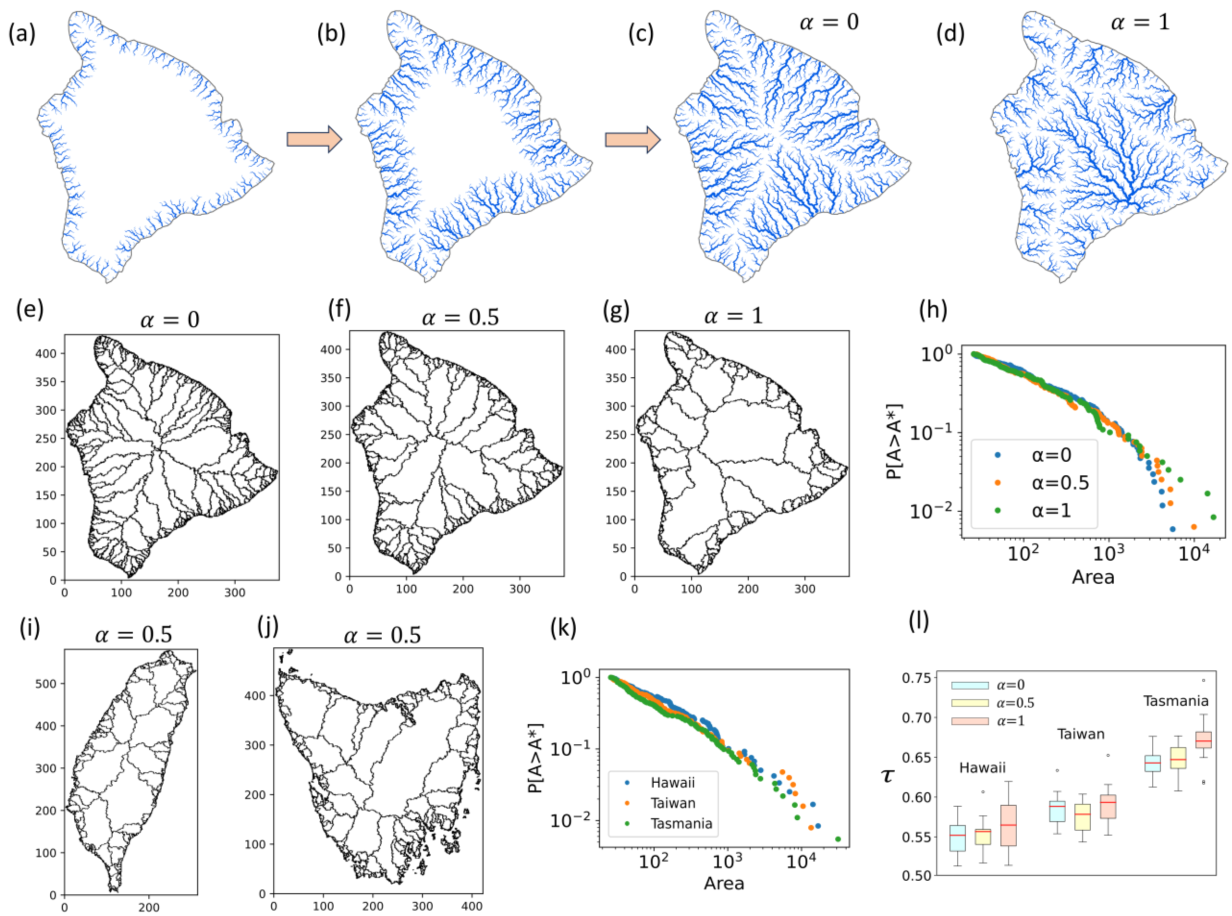

Figure 2. Modelled basins using probabilistic Network Evolution Model. **(a)**, **(b)** and **(c)** shows intermediate snapshots of network evolution using probabilistic model with $\alpha = 0$ where network grows uniformly from all directions. **(d)** presents a final network sample for $\alpha = 1$, illustrating the formation of larger basins. **(e)**, **(f)** and **(g)** are modelled basins within Hawaii boundary with different α values. **(h)** shows almost overlapping BAD plots corresponding to modelled Hawaii basins despite different α values. **(i)** and **(j)** show modeled basins for the Taiwan and Tasmania boundaries, respectively, obtained with $\alpha = 0.5$. **(k)** shows the sample BAD plots obtained with the same α but different boundary conditions highlighting the differences in the slopes. **(i)** presents boxplots for each ensemble of 20 simulations, clearly demonstrating that despite varying α values, the boundary shape significantly influences the τ value.

4 Discussion and Conclusion

The emergence of power laws in natural systems has been attributed to a variety of mechanisms. The popularly known generic mechanism is network growth by adding new vertices with preferential attachment to the well-connected sites given by Barabási & Albert (1999). Another mechanism by Caldarelli et al. (2002) suggests that sites with larger intrinsic fitness are more likely to become highly connected also known as good-get-richer mechanism. However, the understanding of processes leading to the scale free organization patterns in case of river networks is still evolving. The study by Sun & Meakin (1994) shown that computer generated minimum energy dissipation networks follow power law with respect to BAD. However, their model does not provide further insights into the specific processes leading to the emergence of power law patterns in BAD. Similarly, other studies, including those by La Barbera & Lanza

(2001) and Oliveira et al. (2019), report power law behavior in BAD but do not elaborate on the role of boundary conditions or the potential mechanisms behind this emergence.

The headward growth in our probabilistic model, which is proportional to the downstream length of the growing stream, is anologous to a network with sites expanding through preferential attachment. This mechanism may contribute to the emergence of power laws. In this context of BAD, a higher value of τ indicates a more uneven distribution of basin areas, meaning the probability decreases rapidly as the area increases. Now, when the boundary is more irregular, it leads to disproportionate access for the growing networks. This creates an uneven distribution, which in turn results in a higher BAD power law exponent. This pattern is observed in both real-world and modeled results. The scaling of basin area distribution is observed to break for the larger basin areas. This could be attributed to the significant influence of prevailing geological conditions on basin organization (Andrea Rinaldo et al., 2014). Landscape evolution is shaped by various factors, including climatic conditions, geological characteristics, and tectonic activities. Thus, while fluvial erosion can be one of the factor behind the evolution of networks following scaling laws (Cheraghi et al., 2018), the size of larger basins might be determined by these diverse factors and not solely by headward growth due to fluvial erosion.

We would like to highlight some intricacies involved in the present study for example the calculation of FD depends on the method used. We employed the commonly used box-counting method, but it's important to note that different methods, such as the self-similarity-based FD calculation proposed by La Barbera & Lanza (2001), might yield different FD values. Similarly, the threshold value used for basin delineation slightly influences the obtained τ value. To maintain consistency, we used a threshold of 0.05 Km^2, which is considered suitable for basin delineation and has been endorsed by (Reddy et al., 2018). This threshold strikes a balance: it is large enough to avoid the limitations imposed by the spatial resolution of the Digital Elevation Model (DEM), yet small enough to include data from smaller basins. Choosing a much smaller threshold would not be advisable due to the DEM’s resolution constraints, which limit the accuracy of the basin delineation algorithm at finer scales.

Given these methodological nuances, direct comparison of the τ values between real and modeled basins within the same boundary is not sensible due to our use of resampled grids at a coarser resolution for modeling. A more appropriate approach involves comparing modeled boundaries amongst themselves, acknowledging the significant resolution differences and resultant alterations in boundary characteristics. Nevertheless, despite slight shifts in τ values for modeled basins or variations due to the threshold used, the overall trends in our observation remain consistent. For instance, the influence of landscape boundary characteristics on τ is clearly evident in Figure 2i.

In conclusion, our study offers valuable insights into the emergence of power-law behavior in basin area distributions across varied landscapes. We have discovered that landscape boundary characteristics, particularly compactness and fractal dimension, significantly influence the power law exponent. Additionally, the headward growth of river networks is identified as a crucial factor in shaping these distribution patterns, thereby underscoring the importance of adaptive dynamic processes in landscape evolution. This research elucidates the complex interplay among boundary characteristics, network growth mechanisms, and self-organizing behaviors in fluvial landscapes and enhancing our comprehension of complex natural systems.

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Open Research

Digital elevation of islands was obtained from Shuttle Radar Topography Mission's 1 Arc-Second Global dataset from U.S. Geological Survey (DOI: /10.5066/F7PR7TFT). All codes with sample data are available at Borse (2023).

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