

Impacts of Fracture Properties on the Formation and Development of Stimulated Reservoir Volume: a Global Sensitivity Analysis

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Abstract

Stimulated reservoir volume (SRV), the high-permeable fracture network created by hydraulic fracturing, is essential for fluid production from low-permeable reservoirs. However, the configuration of SRV and its impacting factors are largely unknown. In this work, we adopt the stochastic discrete fracture network method to mimic natural fractures in subsurface formations and conduct a global sensitivity analysis with the Sobol method. The sensitivity of different fracture properties, including geometrical properties (fracture lengths, orientations and center positions), mechanical properties (fracture roughness and fracture strength), fracture sealing properties (probabilities of open fractures and segment lengths) and the fracture intensity, are investigated in two and three-dimensional fracture networks. JRC-JCS model is adopted to identify critically stressed fractures. We find that critically stressed fractures compose the backbone of SRV, while partially open fractures can significantly enlarge the size of SRV by connecting more critically orientated fractures. The fracture roughness is the most influential factor for the total length (area) of critically stressed frac-

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tures. For the relative increase of SRV (RI) in 2D/3D fracture networks, the probability of open fractures is the most significant factor. The fracture lengths and center positions are essential factors for RI in 2D fracture networks but insignificant in 3D fracture networks. This work provides a realistic scenario of the subsurface structure and systematically investigates the influential factors of SRV, which is useful for estimating the size of SRV and predicting shale gas reservoirs' production in an accurate and physically meaningful way.

Keywords

Simulated reservoir volume; Discrete fracture network; Global sensitivity analysis; Geomechanics; Fracture sealing; Geometrical properties

Highlights

- Stimulated reservoir volume (SRV) formed in two and three-dimensional fracture networks are investigated.
- Systematic sensitivity analysis of different fracture properties on the formation and development of SRV is conducted.
- Critically stressed fractures form the backbone of SRV and partially open fractures enlarge the size of SRV significantly.

1. Introduction

1 In low permeability formations, such as shale reservoirs, natural fractures
2 and hydrofractures form complex fracture networks and provide a highly per-
3 meable pathway for fluid transportation. The complex fracture network formed
4 through hydraulic fracturing is named stimulated reservoir volume (SRV), which
5 contributes to shale gas production (Mayerhofer et al., 2010).

6 To accurately estimate SRV and figure out the key factors that impact the
 7 size of SRV is nontrivial. Currently available methods to estimate SRV include
 8 microseismic monitoring, tiltmeter measurement, electromagnetic imaging and
 9 numerical models. Microseismic monitoring records real-time seismic signals
 10 and estimate the approximate range of three-dimensional (3D) SRV from the
 11 microseismicity map (Warpinski et al., 2001; Maxwell et al., 2002; Fisher et al.,
 12 2004; Maxwell et al., 2009; Warpinski et al., 2009; Zhang et al., 2019; Liu et al.,
 13 2021). Microseismic events are created mainly by shear slippages of natural
 14 fractures around hydrofractures (Albright et al., 1982; Warpinski et al., 2001;
 15 Rutledge and Phillips, 2003). The enhanced pore pressure can reduce the effective stress and cause the critically oriented fractures to become critically stressed
 16 and slide. Tiltmeter measurement (Astakhov et al., 2012) uses a surface tilt-
 17 meter array to measure the micro-deformation of the surface, which can only
 18 provide a much coarser resolution compared with microseismicity maps. The
 19 electromagnetic imaging method monitors hydraulic fractures in reservoirs by
 20 identifying the contrasts between the electromagnetic properties of the injected
 21 proppants and the subsurface (LaBrecque et al., 2016). Nowadays, this method
 22 is at the initial state in lab experiments without field applications. Numerical
 23 models usually simulate the multi-stage multi-cluster fracturing process coupling the hydrofracture propagation, fluid flow, and the activation of natural
 24 fractures (Ren et al., 2016; Wu and Olson, 2016). Significant simplifications
 25 of the hydraulic fracturing process and geological structures must be made in
 26 numerical models to make the simulation computationally solvable. Therefore,
 27 the complexity of natural fracture networks is usually essentially relaxed.

30 The stimulated reservoir volume is mainly composed of two types of fractures
 31 activated in the hydraulic fracturing operation. One type is tensile fractures
 32 caused by elevated pore pressure, higher than the minimum principal stress. The

33 other type is shear fractures attributed to shear slippage, where the elevated pore
34 pressure does not exceed the minimum principal stress but is still large enough to
35 cause the shear failure of preexisting fractures (Maulianda et al., 2014; Wu et al.,
36 2019). Activated natural fractures (both tensile and shear failed fractures) serve
37 as the high-permeable pathway for the fluid pressure propagation. Therefore,
38 preexisting natural fractures are essential for the formation of SRV.

39 The geometrical and mechanical properties of natural fractures can impact
40 the size of SRV significantly. However, such investigations are rarely conducted.
41 One important reason is that with current technologies, such as borehole im-
42 ages (Prioul and Jocker, 2009), outcrop observations (Abouelresh and Babalola,
43 2020), 3D seismic techniques (Rijks and Jauffred, 1991) , crosswell imaging tech-
44 niques (Wilt et al., 1995; Ellefsen et al., 2002), it is almost impossible to have
45 a comprehensive mapping of natural fractures in the subsurface. Therefore,
46 a detailed configuration of SRV is also unavailable. Furthermore, fractures are
47 usually partially sealed instead of being completely sealed. The complex process
48 of crystal growth can result in different sealing patterns, such as massive sealing
49 deposits, thin rinds or veneers that line the surfaces of open fractures, and bridge
50 structures that span otherwise open fractures (Laubach et al., 2004; Lander and
51 Laubach, 2015). The impacts of partially open fractures have rarely been con-
52 sidered in the formation of SRV due to the large scale difference between the
53 fracture sealing and fracture networks (Zhu et al., 2021a). A few preliminary
54 works use the discrete fracture network method to mimic the fracture sealing
55 and the Coulomb failure criterion to distinguish the critical and non-critical
56 stressed fractures (Zhu et al., 2021a,b). They find that partially open fractures
57 can significantly enlarge the size of SRV by connecting more critically orientated
58 fractures. In this research, we further extend the study in Zhu et al. (2021b)
59 and aim to investigate the impact of different fracture properties, including ge-

60 ometrical properties, mechanical properties and fracture sealing properties, on
 61 the formation and development of SRV.

62 We adopt the stochastic discrete fracture network (SDFN) model method
 63 (Lei et al., 2017) to mimic natural fracture networks in the subsurface. By
 64 implementing the JRC-JCS model proposed by Barton (1973), we can identify
 65 the critically stressed fractures under a given global stress state. This work fo-
 66 cuses more on shear fractures and only considers one large tensile hydrofracture
 67 caused by hydraulic fracturing. In reality, there might be several tensile frac-
 68 tures in one hydraulic fracturing cluster (Marder et al., 2015; Ratner et al.,
 69 2018), but it is usually hard to predict.

We adopt the JRC-JCS model instead of the commonly used Coulomb fail-
 ure criterion (COULOMB, 1773) because for planar discontinuity surfaces, like
 a sawn or ground surface, Coulomb failure criterion is a good option to represent
 the relationship between the peak shear strength τ_p and the normal stress σ_n
 (Barton et al., 1995; Im et al., 2018; Mattila and Follin, 2019). However, a nat-
 ural fracture surface in rocks can never be smooth but rough. The undulations
 and asperities on a natural fracture surface can significantly impact the shear
 behavior of fractures. In general, a rough surface increases the shear strength
 and make it more difficult to have shear failures. Barton (1973) proposed an em-
 pirical relationship to model the shear strength of rock discontinuities (Barton,
 1973; Barton and Choubey, 1977).

$$\tau_p = (\sigma_n - P_p) \tan(\phi_r + JRC \log_{10}(\frac{JCS}{\sigma_n})), \quad (1)$$

70 where ϕ_r is the residual friction angle; JRC is the joint roughness coefficient;
 71 JCS is the joint wall compressive strength; JRC varies between 0-20, where
 72 0 refers to perfectly smooth surface and 20 is the roughest possible joint with-
 73 out actual steps. If fractures have not been weathered, i.e. fresh fracture,

74 JCS equals the uniaxial compressive strength of rocks and this value decreases
75 with increasing weathering grades. To determine the proper values for pa-
76 rameters in Eq.1 is nontrivial, which depends on many factors, such as rock
77 types, weathering grades, scales, cementations (Barton and Bandis, 1990; Mari-
78 nos et al., 2005). Furthermore, Eq. 1 ceases to have any practical meaning for
79 $(\phi_r + JRC \log_{10}(\frac{JCS}{\sigma_n}) > 70^\circ$ (Marinos et al., 2005).

80 Fracture properties considered in this work include three geometries proper-
81 ties (fracture lengths, orientations and positions of fracture centers), two factors
82 related to fracture sealing (the probability of open fractures and the segment
83 length), two mechanical properties (fracture roughness and fracture compressive
84 strength) and one factor of relative fracture intensity. Each factor is represented
85 by a key parameter in the corresponding distribution or definition. Details of
86 each factor are introduced in the next section. In total, eight factors that may
87 impact the formation and development of SRV are considered, and factors are
88 assumed to be independent of each other. A surrogate model is obtained by fit-
89 ting results of 50,000 realizations, and then a global sensitivity analysis with the
90 Sobol method is conducted. Local stress perturbations induced by interactions
91 of neighbouring fractures are neglected mainly because numerical calculations of
92 stress fields are expensive in complex discrete fracture networks with thousands
93 of realizations. In addition, fractures usually need to be close enough to have a
94 significant stress perturbation (Thomas et al., 2017).

95 The remainder of this paper is organized as follows: Section. 2 introduces
96 the techniques to construct a typical 2D and 3D fracture networks to mimic
97 subsurface formations. The identification of SRV through incorporating the
98 JRC-JCS failure criterion. The method of sensitivity analysis is introduced as
99 well. Sections. 3 presents results of Sobol sensitivity analysis of each factor
100 on the formation and development of SRV. Section. 4 discusses the insights of

101 the work on fluid transportation. Important conclusions are summarized in
 102 Section. 5.

103 2. Materials and Methods

104 This section introduces the construction of a typical two/three-dimensional
 105 subsurface formation and the procedures to identify stimulated reservoir vol-
 106 umes. The Sobol method for global sensitivity analysis is introduced.

107 2.1. 2D/3D stochastic discrete fracture networks

108 The detailed mappings of fracture networks in the subsurface are usually un-
 109 available with current technologies, such as outcrop observations, wellbore imag-
 110 ing, and 3D seismic mappings. A stochastic discrete fracture network model is
 111 a practical method to mimic the natural fracture networks with simplified ge-
 112 ometries but preserve essential topological relationships. In this research, a 2D
 113 fracture is represented by a line segment, and a square plate represents a 3D
 114 fracture for simplicity. As Jing and Stephansson (2007) pointed out, the signifi-
 115 cance of the fracture shape decreases with an increase in the fracture population
 116 size. A square plate is convenient to mimic fracture sealing introduced later.

Three main geometrical properties of fractures are considered, including frac-
 ture lengths (2D)/ sizes (3D), orientations, positions of fracture centers. Each
 geometrical property is described with a widely used statistical distribution
 (Bonnet et al., 2001). A power-law distribution is implemented to describe
 fracture lengths,

$$n(l) = \alpha l^{-a}, \quad (2)$$

117 where $n(l)dl$ is the number of fractures with lengths ranging from $[l, l + dl]$, α is
 118 the proportionality coefficient and a is the power-law exponent. The minimum
 119 and maximum fracture length used in the power-law distribution is 1 m and

100,000 m. In 3D fracture networks, we generate a unit square with its side
length equal to 1 m, then perform the scaling operation on the square with a
scale factor of l to change their sizes. Through a simplistic fractal model, we
have derived that the power-law exponent has to be larger than one (Zhu et al.,
2021c). For most cases, the exponent ranges between 2 and 3 (Bour and Davy,
1997; Bonnet et al., 2001).

The fracture orientations follow von Mises–Fisher distributions (Kemeny and
Post, 2003; Whitaker and Engelder, 2005)

$$f(\vec{x}, \vec{\mu}, \kappa) = C(\kappa) \exp(\kappa \vec{\mu}^T \vec{x}), \quad (3)$$

where $C(\kappa)$ is the normalization constant. $\vec{\mu}$ and κ are the mean direction and
concentration parameter, respectively. The parameter κ controls the concen-
tration degree of the distribution around the mean direction $\vec{\mu}$. When $\kappa = 0$,
the von Mises–Fisher distribution degenerates to a uniform distribution. When
 κ is large, the distribution is approximate to a normal distribution and con-
centrates around the angle $\vec{\mu}$ with $1/\kappa$ analogous to σ^2 . In this research, we
choose $\vec{\mu} = [1, 0]$ for 2D fracture networks and $\vec{\mu} = [1, 0, 0]$ for 3D fracture
networks. From a collection of natural outcrop maps (Zhu et al., 2021c), 2D
fracture networks usually have their orientations scattered and the correspond-
ing κ is smaller than 3. In this work, we consider a wider range of κ from 0 to
20.

The positions of fracture centers are sampled from a uniform or fractal spa-
tial density distribution. The fractal spatial density distribution (Meakin, 1991;
Darcel et al., 2003) introduces clustering effects in the network, which is charac-
terized by a fractal dimension F_D . Real fracture networks are usually clustered,
and a fractal spatial density distribution can better describe it (Darcel et al.,
2003; Zhu et al., 2021c). For 2D fracture networks, the fractal dimension F_D

varies between 1.1 and 2.0. For 3D fracture networks, the corresponding fractal dimension varies between 2.1 and 3.0.

It is difficult to estimate the fracture intensity in the subsurface from available 1D or 2D measurements (Dershowitz, 1984). However, real subsurface fracture networks should have a much higher fracture intensity than the intensity at percolation if their outcrop maps show good geometrical connectivity. In reality, most outcrop maps are well connected (Zhu et al., 2021c,f). Therefore, we check the cluster in this research and take the fracture intensity at the percolation (formation of a spanning cluster) as the reference. We considered different fracture intensities and described them by a ratio between the number of fractures at termination and the number of fractures at percolation. This ratio is denoted as FI for 2D and 3D fracture networks and varies between 0.8 and 2.6. Therefore, the fracture intensity is larger than the fracture intensity at percolation, and good global connectivity is reached in most cases.

Fig. 1 shows the examples of generated 2D and 3D fracture networks. The fracture networks are generated with an in-house built, open-source software, HatchFrac (Zhu et al., 2021d).

2.2. Identification of stimulated reservoir volume

Without losing generality, we assume a stable strike-slip stress state ($Sh_{min} < S_v < Sh_{max}$). Similar analysis can be extended to a normal or reverse stress state. The injected fluid pressure of hydraulic fracturing is set as the reference stress, i.e. $P_f = 1$. The other important stresses are: the maximum horizontal stress $Sh_{max} = 1.3P_f$, the minimum horizontal stress $Sh_{min} = 0.8P_f$, the vertical stress $S_v = 1.1P_f$; the reservoir pressure is uniformly distributed in considered domain with $P_p = 0.5P_f$. In this work, we only consider one primary hydrofracture in one hydraulic fracturing cluster (Green line segment and square in Fig. 1). The elevated pore pressure caused by hydraulic fracturing is

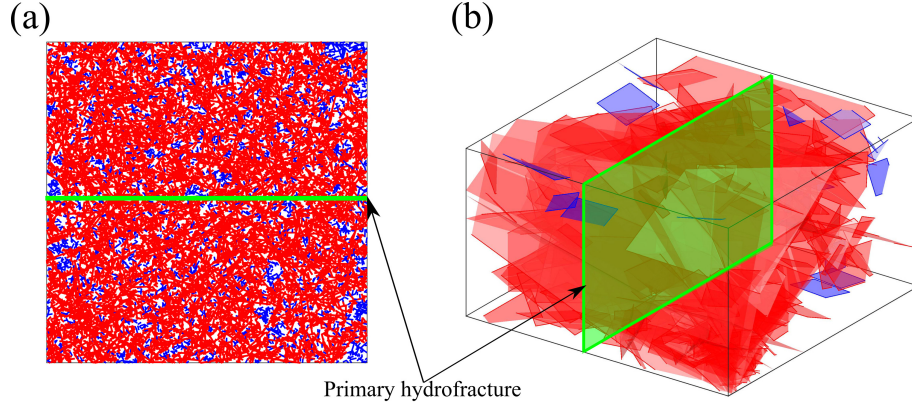


Figure 1: Examples of 2D and 3D fracture networks. For the 2D fracture network (left), the red line segments form the connected spanning cluster. The blue line segments correspond to all other locally connected clusters. The green line segment represents the primary hydrofracture. The fracture orientations follow a uniform distribution ($\kappa = 0$), lengths obey a power-law distribution ($a = 3.0$), positions of fracture centers are uniformly distributed ($F_D = 2.0$). The relative ratio of fracture intensity is 2.0. For the 3D fracture network (right), the red polygons form the connected spanning cluster. The blue squares correspond to all other locally connected clusters. The green square represents the primary hydrofracture. The fracture orientations follow a uniform distribution ($\kappa = 0$), lengths obey a power-law distribution ($a = 3.0$), positions of fracture centers follow a fractal spatial density distribution ($F_D = 2.5$). The system size is 100 for the 2D fracture network and 10 for the 3D fracture network to have a better visualization. The relative ratio of fracture intensity is 1.5.

170 assumed to be constant along the hydraulic fracture and decreases linearly with
 171 increasing distance. Stress states are critical for the formation of SRV, but they
 172 are strongly case-dependent and here we only analyze a typical scenario.

173 JRC-JCS failure criterion (Eq. 1) is implemented to identify critically/non-
 174 critically stressed fractures, where the residual friction angle ϕ_r is set as 30
 175 degrees. JRC varies between 0 (perfectly smooth) and 20 (roughest). JCS
 176 varies between $0.5P_f$ and $18.5P_f$.

Fracture sealing is simulated by dividing 2D fractures into small segments. Each small segment can be sealed and can block the flow of the fluid. The degree of fracture sealing is controlled by two parameters, the probability of open fractures (P_o) and the segment length (L_{se}). The segment length is the minimum unit of fracture sealing, which can reach a millimeter in reality, but is impractical in the numerical simulation because of the limited computation

capacity. Therefore, we choose decreasing segment lengths from 1 m to 0.2 m to show the impact of the segment length. 3D fractures are divided into small grids to mimic the fracture sealing. Detailed introduction of this method can be found in (Zhu et al., 2021a,b). The probability of open fractures is defined as:

$$P_o = \frac{L_{open}}{L_{total}}, \quad (4)$$

177 where L_{open} is the total length of open fractures and L_{total} is the total length
 178 of all fractures. In a 3D fracture network, the fracture length is replaced with
 179 the fracture area.

180 In this research, we consider a $100\text{ m} \times 100\text{ m}$ square domain for 2D for-
 181 mations and a $50\text{ m} \times 50\text{ m} \times 50\text{ m}$ cubic system for 3D formations. From mi-
 182 croseismicity observations, the SRV usually shows an elongated shape in most
 183 cases (Shaffner et al., 2011; Rateman et al., 2018). Therefore, we assume the
 184 farthest distance where the injected fluid pressure can propagate is 20% of the
 185 system size on each side of the hydrofracture.

186 After generating discrete fracture networks, we can identify stimulated reser-
 187 voir volume (SRV) based on the given stress state and JRC-JCS criterion. The
 188 SRV comprises two main parts: one is the critically stressed fractures, and the
 189 other is the partially open fractures. Critically stressed fractures form the back-
 190 bone of SRV, while partially open fractures can further enlarge SRV. Fig. 2(a)
 191 shows the SRV composed of critically stressed fractures. Red fractures are crit-
 192 ically stressed fractures, and they are connected to the primary hydrofracture
 193 directly or indirectly. Therefore, the elevated pore pressure in the hydrofracture
 194 can propagate to those fractures. The purple fractures are critically orientated
 195 fractures because they are not connected to the primary hydrofracture, and the
 196 high fluid pressure cannot be transmitted to purple fractures. Fig. 2(c) shows
 197 the SRV composed of both critically stressed fractures and partially fractures.

198 Partially open fractures have enlarged the SRV by connecting more critically
 199 orientated fractures to the hydrofracture and making them critically stressed.
 200 The total lengths of permeable fractures in Figs. 2(a) and (c) are denoted as
 201 L_{cs} and L_{cso} , respectively. L_{cs} is 2026 m, and L_{cso} is 5078 m. Fig. 2(b) show
 202 the SRV composed of critically stressed fractures in 3D. Fig. 2(d) show the SRV
 203 composed of critically stressed fractures plus partially open fractures. The total
 204 areas of permeable fractures (red fractures) in Figs. 2(b) and (d) are denoted
 205 as A_{cs} and A_{cso} . The system size is 10 m in Figs. 2 (b) and (d) for demonstra-
 206 tion because it is difficult to visualize a 3D fracture network with thousands of
 207 fractures in a large system. A_{cs} is 513 m², and A_{cso} is 1810 m². In linear flow,
 208 the flux from the matrix to fractures is proportional to the fracture area (Bello
 209 et al., 2010; Haider et al., 2020), suggesting that partially sealed fractures can
 210 increase reservoir production by enlarging the stimulated reservoir volume.

211 In this research, we demonstrate the contribution of partially open fractures
 212 by performing a full-scale, embedded discrete fracture network model simula-
 213 tion with UNCONG software (Li et al., 2015). For simplicity, all critically
 214 stressed fractures and partially open fractures are assigned with a permeabil-
 215 ity of 10 darcies, and sealed fractures are impermeable. The matrix has a low
 216 permeability of 0.05 micro darcies. The primary hydrofracture is replaced with
 217 a horizontal production well to implement boundary conditions and schedule
 218 control conveniently. The initial reservoir pressure is set as 300 bar, and the
 219 bottomhole pressure is set as 100 bar and kept constant. We simulate the
 220 production for ten days and compare the production difference with and with-
 221 out partially open fractures. Detailed input parameters are listed in Table. 1.
 222 Fig. 3(a) shows changes of the gas formation volume factor and gas viscosity
 223 with pressure. Figs. 3 (b) and (c) show the relative permeability curves in the
 224 matrix and fractures, respectively.

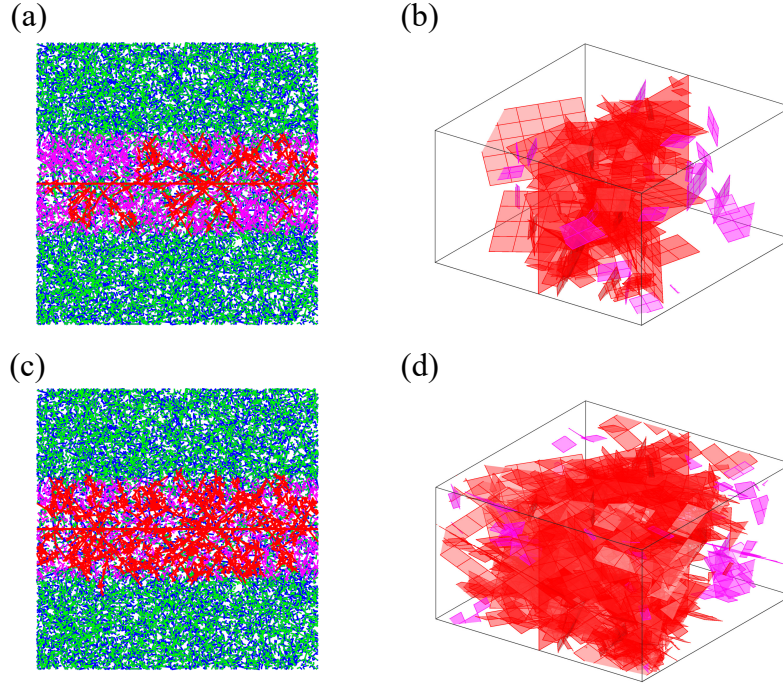


Figure 2: Demonstration of 2D/3D SRV composed of only critically stressed fractures (a,b) or critically stressed fractures plus partially open fractures (c,d). The probability of open fractures is 0.5 for both 2D and 3D fracture networks. The fracture surface is assumed to be smooth ($JRC = 0$). In 2D fracture networks (a,c), the red line segments form the SRV. The purple fractures are critically orientated fractures. The green line segments are open fractures, and the blue line segments are sealed fractures. In 3D fracture networks (b, d), open and sealed fractures are not shown for better visualization. The red squares form the SRV, and the purple squares are critically orientated fractures.

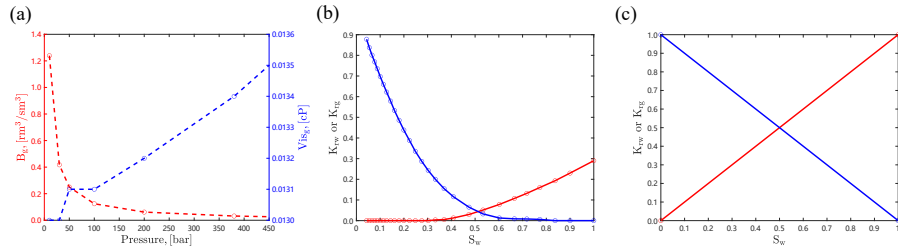


Figure 3: (a) Changes of the formation gas volume factor (B_g) and gas viscosity (Vis_g) with pressure; (b) The relative permeability curve in the matrix; (c) The relative permeability curve in fractures

225 Figs. 4 (a) and (b) show the pressure distribution of two scenarios after
 226 ten days of production, where one only consider the critically stressed fractures

Table 1: Input parameters for the simulation	
Matrix permeability, k_m , [μd]	0.05
Matrix porosity	0.05
Fracture permeability, k_f , [d]	10
Fracture porosity	1.0
The coefficient of water compressibility, [bar^{-1}]	$3.15\text{e-}6$
The coefficient of water viscosity compressibility, [$\text{cP} \cdot \text{bar}^{-1}$]	$2.10\text{e-}6$
Initial water saturation	0.5
Initial reservoir pressure, P_i , [bar]	300
Constant bottomhole pressure, [bar],	100

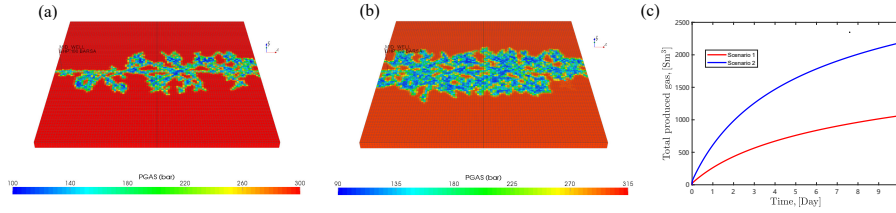


Figure 4: (a) Pressure distribution in Scenario 1, where the SRV comprises critically stressed fractures only. (b) Pressure distribution in Scenario 2, where the SRV comprises critically stressed plus partially open fractures. (c) The comparison of the total production of gas in two scenarios

and the other one consider both critically stressed and partially open fractures. Figs. 4 (c) shows the comparison of the total production of gas in these two scenarios. The simplified simulation is different from an actual production case. However, it straightforwardly demonstrates the significant contribution of partially open fractures by connecting more critically orientated fractures and enlarging the size of SRV. The total length in the first scenario is 2026 m, while this value is 5078 m in the second scenario. The ratio of the total fracture length between the two scenarios is 2.51, and the ratio of the total production of gas in the two scenarios is 2.05. Therefore, the shale gas production is proportional to the fracture length in the connected cluster. In 3D, the production is proportional to the total fracture area.

In the following sections, we will systematically analyze the impact of each fracture property by implementing the global sensitivity analysis. The factors

include three geometrical properties (the exponent of the power-law distribution (a), the fractal dimension of the fractal spatial density distribution (F_D), the concentration parameter in a von Mises–Fisher distribution (κ)), two factors related to the fracture sealing (the probability of open fractures (P_o) and the segment length (L_{se})), two factors related to the mechanical properties of fracture surfaces (joint roughness coefficient (JRC) and joint wall compressive strength (JCS)) and one relative fracture intensity (FI). In 2D fracture networks, three essential parameters are selected as the response parameters: the total length of connected critically stressed fractures, L_{cs} , the total length of connected critically stressed fractures plus partially open fractures, L_{cso} , and the relative increase of fracture length, RI_{2D} . For 3D fracture networks, the fracture area replaces the fracture length, and the corresponding response parameters are A_{cs} , A_{cso} and RI_{3D} .

The relative increase is defined in Eq. 5, which represents the contribution of partially open fracture on enlarging the size of SRV.

$$RI_{2D} = \frac{L_{cso} - L_{cs}}{L_{cs}} \quad (5)$$

In 3D fracture networks, a similar formula is applied with the fracture length replaced with the fracture area. A detailed summary of factors and responses are listed in Tables. 2 and 3. Five hundred cases with each factor randomly chosen from the given interval (Table. 2) are simulated for the following sensitivity analysis. For each considered case, the results are averaged over 100 random realizations for stabilization.

2.3. Global sensitivity analysis

This research evaluates the impact of each factor and their interactions with the Sobol’ indices (IM, 1993). To simplify the notation without losing generality,

Table 2: Factors in the global sensitivity analysis

Factor	Range	Property	Definition
a	$[2, 3]^{2/3D}$	Fracture length	The exponent of a power-law distribution
F_D	$[1.1, 2]^{2D}, [2.1, 3]^{3D}$	Position of fracture centers	The fractal dimension of a fractal spatial density distribution
κ	$[0, 20]^{2/3D}$	Fracture orientation	The concentration parameter in a von Mises–Fisher distribution
FI	$[0.8, 2.6]^{2/3D}$	Fracture intensity	The ratio between the number of fractures at termination and at percolation
P_o	$[0.2, 0.8]^{2/3D}$	Fracture sealing	The ratio of the total length/area of open fractures and total fractures
L_{se}	$[0.2 \text{ m}, 1 \text{ m}]^{2D}, 1 \text{ m}^{3D}$	Fracture sealing	The minimum unit of fracture sealing
JRC	$[0, 20]^{2/3D}$	Fracture roughness	Joint roughness coefficient in the JRC-JCS model
JCS	$[0.5, 18.5]P_f^{2/3D}$	Fracture strength	Joint wall compressive strength in the JRC-JCS model

Note: superscripts, $2D$, $3D$ and $2/3D$, refer to 2D fracture networks, 3D fracture networks, and both 2D and 3D fracture networks.

Table 3: Responses in the global sensitivity analysis

Response	Dimension	Definition
L_{cs}	2D	The total length of critically stressed fractures
L_{cso}	2D	The total length of critically stressed plus partially open fractures
RI_{2D}	2D	Ratio between $(L_{cso} - L_{cs})$ and L_{cs}
A_{cs}	3D	The total area of critically stressed fractures
A_{cso}	3D	The total area of critically stressed plus partially open fractures
RI_{3D}	3D	Ratio between $(A_{cso} - A_{cs})$ and A_{cs}

we assume the input variables are uniformly distributed in $[0,1]$. Therefore, the support of the input set with n variables is a n -dimensional unit hypercube $\mathcal{S} = [0, 1]^n$. The Sobol method is a variance-based method, which represents a deterministic model, $Y = f(\mathbf{X})$, as a sum of elementary functions:

$$f(x_1, x_2, \dots, x_n) = f_0 + \sum_{i=1}^n f_i(x_i) + \sum_{1 \leq i < j < n} f_{ij}(x_i, x_j) + \dots + f_{1,2,\dots,n}(x_1, x_2, \dots, x_n) \quad (6)$$

This expansion is unique under conditions:

$$\int_0^1 f_{i_1 \dots i_s} dx_{i_k} = 0, \quad 1 \leq k \leq s, \quad \{i_1, \dots, i_s\} \subseteq \{1, \dots, d\}, \quad (7)$$

260 This means f_0 is constant, which equals to the expected value of $f(\mathbf{X})$. \mathbf{X} is the
 261 input vector composed of n random variables ($\mathbf{X} = \{x_1, x_2, \dots, x_n\}$), which
 262 are mutually independent.

Analogously, the model's total variance can be decomposed as the sum of the variances of the summands.

$$Var(Y) = \sum_{i=1}^n D_i(Y) + \sum_{1 \leq i < j < n} D_{ij}(Y) + \dots + D_{1,2,\dots,n}(Y), \quad (8)$$

where $D_i(Y) = Var[\mathbb{E}(Y \mid x_i)]$, $D_{ij}(Y) = Var[\mathbb{E}(Y \mid x_i, x_j)] - D_i(Y) - D_j(Y)$ and so on for higher order interactions. The decomposition of the variance leads to the Sobol' indices as follows, which can be adopted as a sensitivity measure.

$$S_i = \frac{D_i(Y)}{Var(Y)}, \quad S_{ij} = \frac{D_{ij}(Y)}{Var(Y)}, \quad \dots \quad (9)$$

263 The Sobol' indices represent the relative contribution of each factor or their
 264 combinations. The index concerning individual factor x_i is called the first-order
 265 Sobol' index (S_i). Multiple-term indices, e.g. S_{ij} , $i \neq j$, are referred to as
 266 higher-order Sobol' indices (interaction indices), which account for the effects

267 of interactions of the factor pair x_i and x_j .

The total Sobol' index of input factor x_i , denoted S_i^T , is the sum of all the Sobol' indices involving this factor:

$$S_i^T = S_i + \sum_{i \neq j} S_{ij} + \sum_{j \neq i, k \neq i, j < k} S_{ijk} + \dots = \sum_{l \in \#i} S_l \quad (10)$$

268 where $\#i$ are all the subsets of $\{1, \dots, n\}$ including i . In practice, when n is
 269 large, only the total Sobol' indices (total effects), the first-order Sobol' indices
 270 (the main effects) and the second-order Sobol indices (the interaction effects)
 271 are computed.

272 In this work, we use a surrogate model to represent the deterministic model
 273 described in Section 2.2. The surrogate model is a third-order polynomial func-
 274 tion obtained through an ordinary least squares regression with 500 cases. After
 275 obtaining the surrogate model, 250,000 samples are collected with a Latin hy-
 276 percube sampling method to evaluate the global sensitivity of the response con-
 277 cerning each factor and interactions between factors. The analysis is conducted
 278 with an open-source MATLAB software, UQLAB (Marelli and Sudret, 2014).

279 3. Results

280 This section analyses the impact of each factor and interactions between
 281 different factors on the formation and development of SRV. The formation and
 282 development of SRV is represented by three response parameters (L_{cs} , L_{cso} and
 283 RI_{2D} in 2D fracture networks and A_{cs} , A_{cso} and RI_{3D} in 3D fracture networks).
 284 The sensitivity analysis with each factor as the response is conducted separately.

285 3.1. Sensitivity analysis in 2D fracture networks

286 The variations of L_{cs} , L_{cso} and RI_{2D} in 500 cases are shown in Figs. 5 and
 287 6. In Fig. 5, L_{cs} and the corresponding L_{cso} are linked with a line segment to

288 demonstrate the difference between these two values in each case. The mean
 289 value of L_{cs} and L_{cso} are 398.3 m and 1762.5 m. Therefore, it is obvious
 290 that partially open fractures can enlarge the size of SRV and contribute to
 291 production. The mean value of the relative increase of SRV is about 4. However,
 292 the mean value can be sensitive to extreme values. A median value may be closer
 to reality, which is 0.42 in 500 cases.

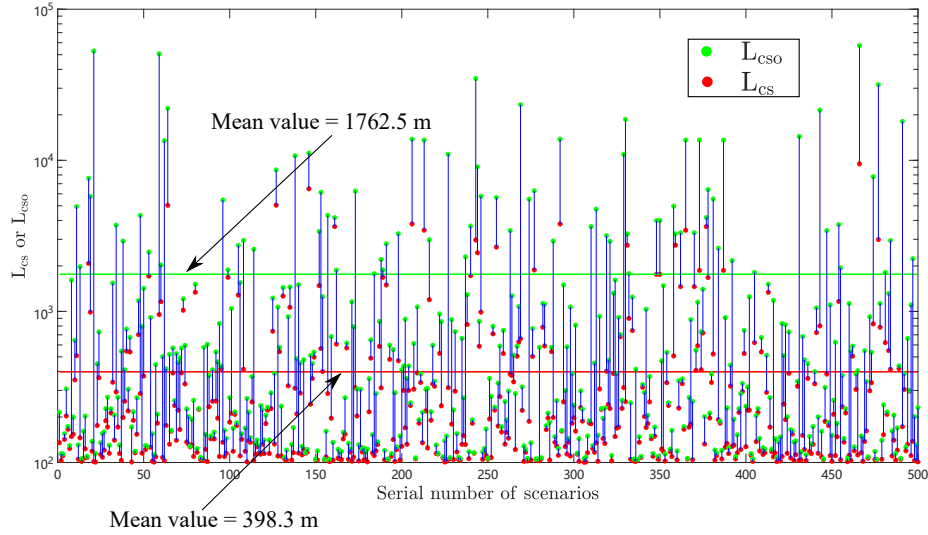


Figure 5: L_{cs} and L_{cso} variations in 500 scenarios

293

294 Fig. 7 shows results of the global sensitivity analysis with the total length
 295 of critically stressed fractures (L_{cs}) as the response. Fig. 7(a) shows the good-
 296 ness of the multivariate polynomial fit, which has an R-square value of 0.95
 297 between the simulation results and predictions. The first order Sobol' indices
 298 (Fig. 7(c)) reflect the sensitivity of the individual factor. For the total length
 299 of critically stressed fractures, L_{cs} , the fracture roughness (JRC), the expo-
 300 nent of the power-law distribution (a) and the concentration parameter in the
 301 von Mises-Fisher distribution (κ) are the most influential factors. The frac-
 302 ture sealing factors (P_o and L_{se}) are irrelevant to L_{cs} because critically stressed
 303 fractures only depend on the JRC-JCS model. The fracture sealing can impact

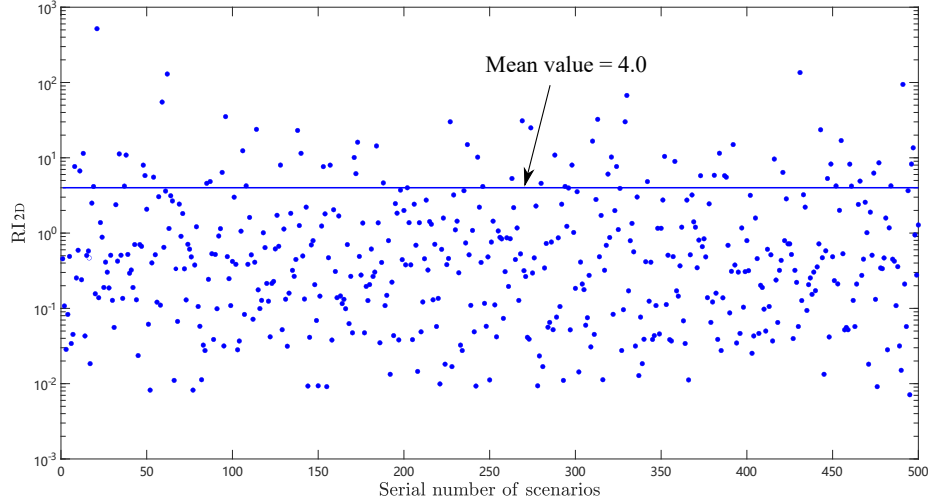


Figure 6: RI_{2D} variations in 500 scenarios

the shear slippage if it significantly changes rock strength at the failure plane. However, in this research, factors are assumed to be independent of each other. The second-order Sobol' indices show interactions between factors and the top five pairs of factors are shown in Fig. 7(d), including $a - JRC$, $a - \kappa$, $FI - \kappa$, $FI - JRC$ and $a - JCS$. After considering interactions between factors, the total Sobol' indices shows the total effect of each factor in Fig. 7(b). The Sobol' index of each factor has increased due to interactions between factors, but the relative ranking is the same. The fracture roughness has an essential impact on the total length of critically stressed fractures. From correlation analysis, the correlation coefficient between JRC and L_{cs} is -0.34. Therefore, JRC has a negative correlation with L_{sc} . A rougher fracture surface (a larger JRC value) can enlarge the general friction angle as shown in the JRC-JCS model, making the failure harder.

Fig. 8 shows results of the global sensitivity analysis with the total length of critically stressed plus partially open fractures (L_{cso}) as the response. Fig. 8(a) shows the goodness of the multivariate polynomial fit, which has an R-square

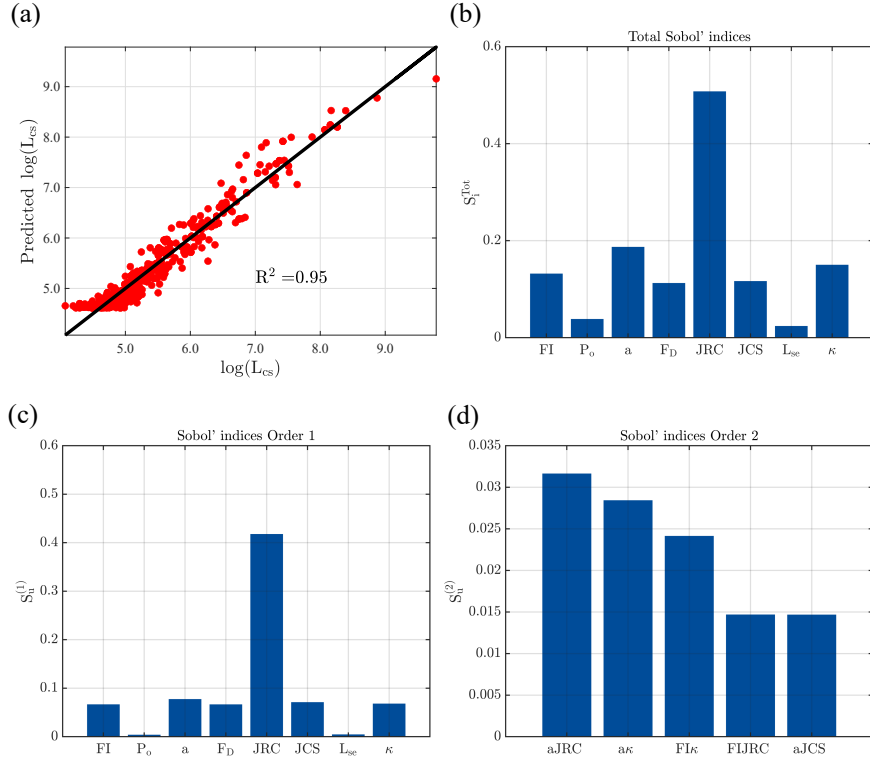


Figure 7: Sensitivity analysis with L_{cs} as the response. (a) The multivariate polynomial fit of L_{cs} in 500 cases. (b) The total Sobol' indices show the total effect of each factor. (c) The first-order Sobol' indices show the main effect of each factor. (d) The second-order Sobol' indices show the interaction effects of factors.

value of 0.95 between the simulation results and predictions. According to the first order Sobol' indices (Fig. 8(c)), the exponent of the power-law distribution (a), the fractal dimension of the fractal spatial density distribution (F_D) and the probability of open fractures (P_o) are the most influential factors. The least influential factors are the concentration parameter (κ), joint compressive strength (JCS), and the segment length (L_{se}). The top five pairs of factors are shown in Fig. 8(d), including $P_o - a$, $a - F_D$, $FI - P_o$, $P_o - L_{se}$ and $FI - a$. After considering the interactions between factors, the total Sobol' indices shows the total effect of each factor in Fig. 8(b). The Sobol' index of each factor has increased due to the interaction between factors, but the relative ranking is

the same. The exponent of the power-law distribution has the essential impact on L_{CSO} . A larger exponent means more small fractures dominate the system. From simple correlation analysis, we find the correlation coefficient is 0.34, which means that the exponent has a positive correlation with L_{CSO} and more small fractures can make L_{CSO} larger.

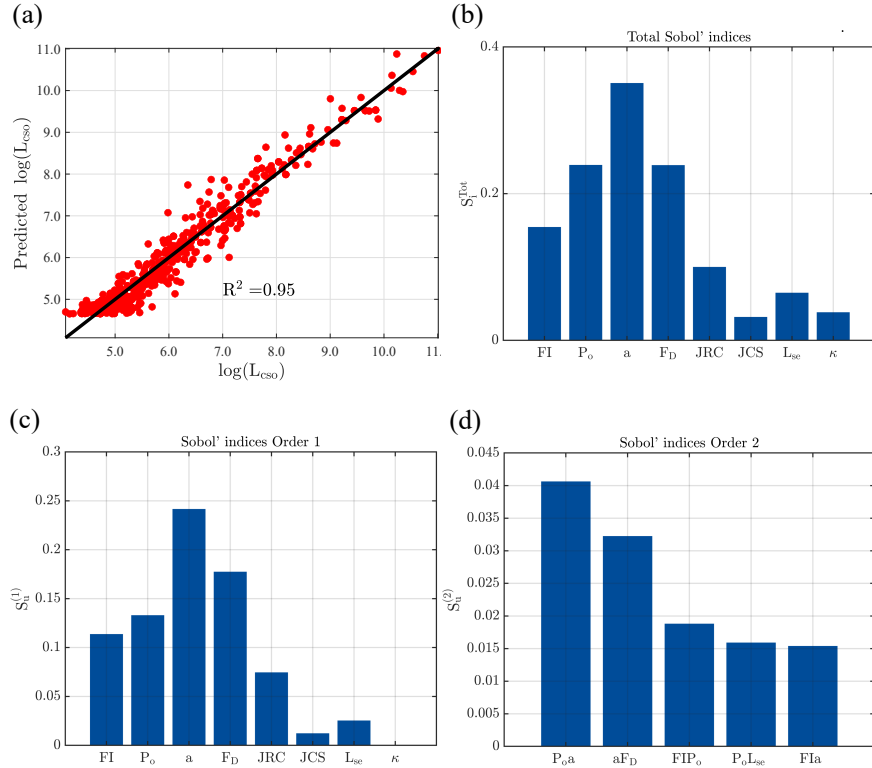


Figure 8: Sensitivity analysis with L_{CSO} as the response

334

Fig. 9 shows results of the global sensitivity analysis with the relative increase of SRV (RI_{2D}) as the response. Fig. 8(a) shows the goodness of the multivariate polynomial fit, which has an R-square value of 0.95 between the simulation results and predictions. Based on the first order Sobol' indices (Fig. 8(c)), the exponent of the power-law distribution (a), the fractal dimension of the fractal spatial density distribution (F_D) and the probability of open fractures (P_o) are

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the most influential factors. The least influential factors are the concentration parameter (κ), the joint compressive strength (JCS), and the segment length (L_{se}). The top five pairs of factors are shown in Fig. 8(d), including $P_o - \kappa$, $a - JRC$, $P_o - a$, $a - F_D$ and $P_o - L_{se}$. After considering the interactions between factors, the total Sobol' indices shows the total effect of each factor in Fig. 8(b). Compared with the result in Fig. 8, impacts of each factor on RI_{2D} are almost the same as impacts on L_{cso} . The interactions between factors do not change the sensitivity ranking. The exponent of the power-law distribution has the essential impact on RI_{2D} . The correlation coefficient between a and RI is 0.16, indicating a positive correlation. Therefore, the contribution from partially open fractures is more significant in fracture networks dominated by small fractures.

In summary, mechanical properties, such as fracture roughness (JRC) and fracture strength (JCS), and fracture orientations (κ) are essential to trigger shear slippage of natural fractures and form the backbone of SRV. Partially open fractures can connect more critically orientated fractures and enlarge the size of SRV. After considering partially open fractures, fracture sealing properties and geometrical properties of fractures become essential, such as P_o (the probability of open fractures), a (the fracture length) and F_D (the fracture center positions).

3.2. Sensitivity analysis in 3D fracture networks

In 3D fracture networks, we consider seven factors, excluding the segment length because of the limited computational capacity. In addition, from the analysis in 2D fracture networks, the segment length does not significantly impact the formation and development of SRV.

The variations of A_{cs} , A_{cso} and RI are shown in Figs. 10 and 11. In Fig. 10, A_{cs} and the corresponding A_{cso} are linked with a line segment in each case to show the difference between these two values. Compared with 2D cases,

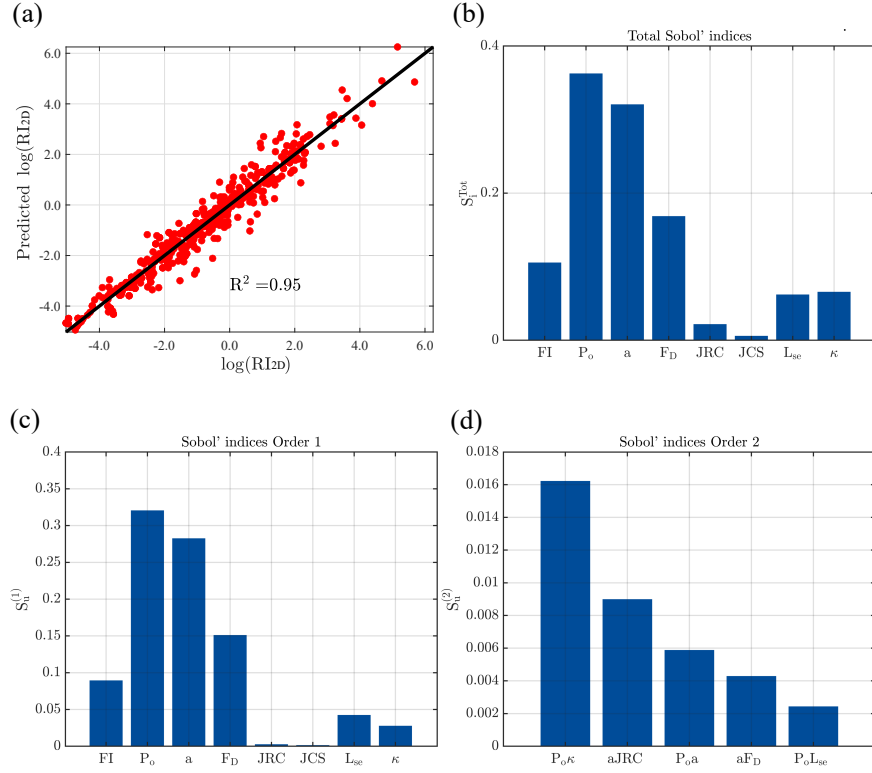


Figure 9: Sensitivity analysis with RI_{2D} as the response

the partially open fractures have more significant enlargement of SRV in 3D fracture networks. The mean value of A_{cs} and A_{cso} are 2962 m^2 and 34688 m^2 . Therefore, partially open fractures can significantly enlarge the size of SRV and contribute to production. The mean value of the relative increase of SRV is about 11, and the median value is 8.5 in 500 cases, which are much higher than the values in 2D fracture networks.

Fig. 12 shows results of the global sensitivity analysis with the total area of critically stressed fractures (A_{cs}) as the response. Fig. 12(a) shows the goodness of the multivariate polynomial fit, which has an R-square value of 0.96 between the simulation results and predictions. The first order Sobol' indices (Fig. 12(c)) reflect the sensitivity of individual factors. The most influential factors are

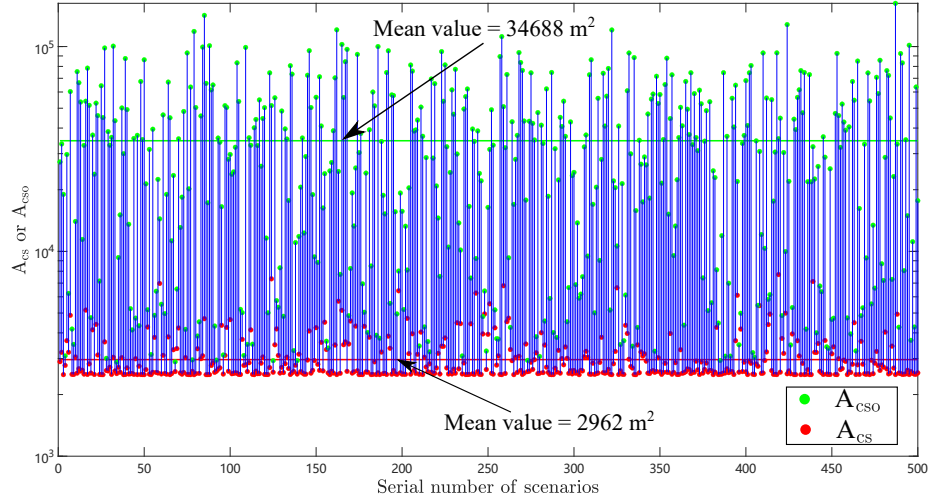


Figure 10: A_{cs} and A_{cso} variations in 500 scenarios

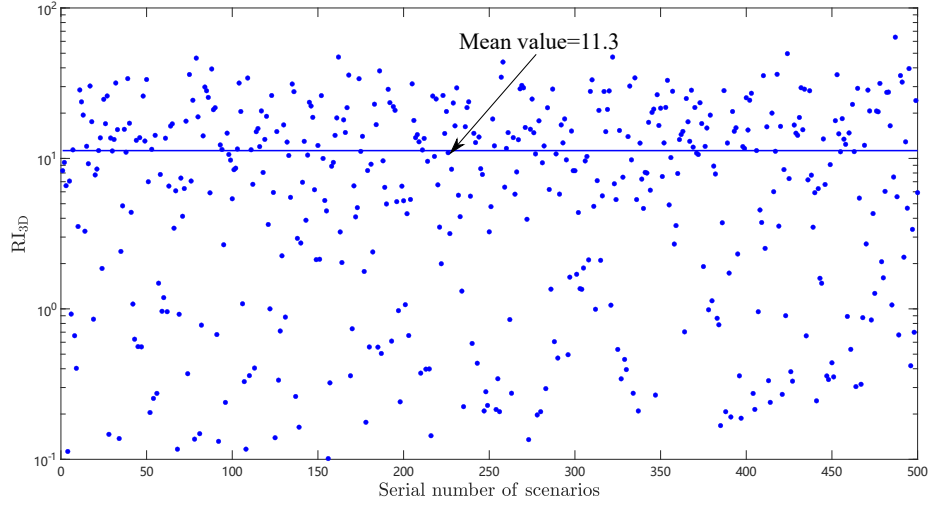


Figure 11: RI_{3D} variations in 500 scenarios

379 the fracture roughness (JRC), the concentration parameter (κ), and the joint
 380 compressive strength (JCS). This observation is similar to the result in 2D
 381 fracture networks. The probability of open fractures (P_o) is irrelevant to the
 382 critically stressed fractures because the JRC-JCS model completely constrains
 383 the critically stressed states. In contrast with 2D fracture networks, fracture

384 geometrical properties, a and F_D , do not change the response significantly. The
 385 second-order Sobol' indices show the interactions between factors, and the top
 386 five pairs of factors are shown in Fig. 12(d), including $\kappa - JRC$, $FI - JRC$,
 387 $JRC - JCS$, $\kappa - JCS$ and $FI - \kappa$. After considering the interactions between
 388 factors, the total Sobol' indices shows the total effect of each factor in Fig. 12(b).
 389 The Sobol' index of each factor has increased due to the interaction between
 390 factors, but the relative ranking has not changed. The fracture roughness has
 391 an essential impact on the total length of critically stressed fractures. From
 392 correlation analysis, the correlation coefficient between JRC and A_{cs} is -0.57.
 393 Therefore, JRC has a negative correlation with A_{sc} . A rougher fracture surface
 394 (a larger JRC value) can enlarge the general friction angle as shown in the
 395 JRC-JCS model, making the failure harder.

396 Fig. 13 shows results of the global sensitivity analysis with the total length
 397 of critically stressed fractures plus the partial open fractures (A_{cso}) as the re-
 398 sponse. Fig. 13(a) shows the goodness of the multivariate polynomial fit, which
 399 has an R-square value of 0.98 between the simulation results and predictions.
 400 According to the first order Sobol' indices (Fig. 13(c)), the probability of open
 401 fractures (P_o), the relative fracture intensity (FI) and the concentration pa-
 402 rameter (κ) are the most influential factors. However, P_o is the most dominant
 403 factor compared with all other six factors. The top five pairs of factors with
 404 interaction effects are shown in Fig. 13(d), including $P_o - \kappa$, $FI - P_o$, $P_o - JRC$,
 405 $P_o - a$ and $a - F_D$. After considering the interactions between factors, the total
 406 Sobol' indices shows the total effect of each factor in Fig. 13(b). The results
 407 are not changed with the first order Sobol' indices. The correlation coefficient
 408 between P_o and A_{cso} is 0.73, indicating a strong positive correlation between
 409 these two parameters, and more open fractures lead to a larger SRV.

410 Fig. 14 shows results of the global sensitivity analysis with the relative in-

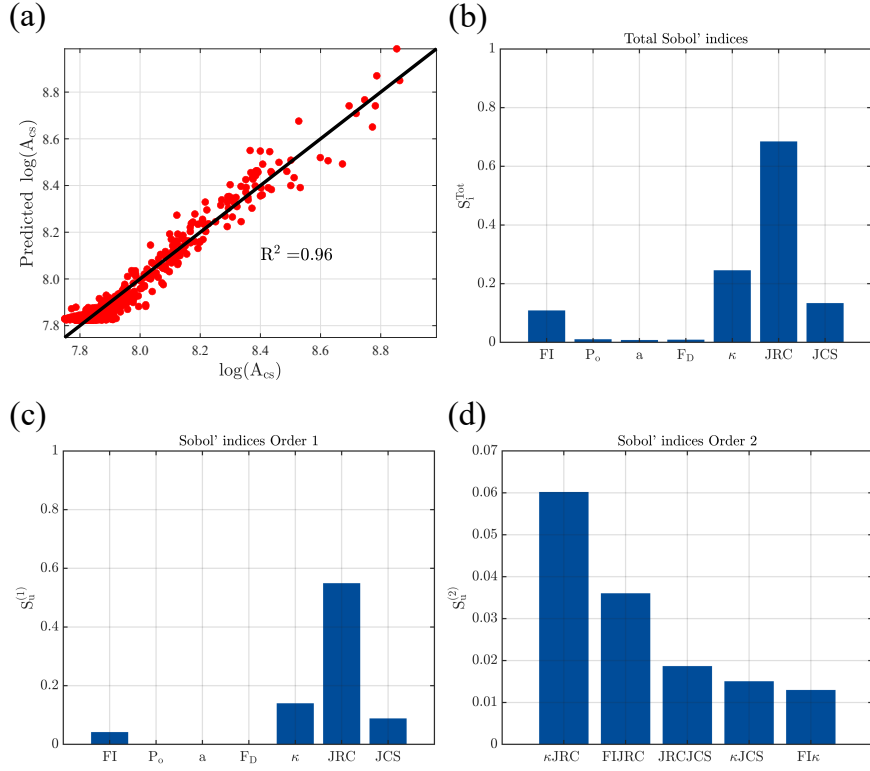


Figure 12: Sensitivity analysis with A_{cs} as the response

crease of SRV (RI_{3D}) as the response. Fig. 14(a) shows the goodness of the multivariate polynomial fit, which has an R-square value of 0.99 between the simulation results and predictions. The results are similar to the results of A_{cso} . The second-order Sobol' indices are small, indicating negligible interactions between factors. The probability of open fractures is the most significant factor in the relative increase of SRV. All other six factors are insignificant. The correlation coefficient between P_o and RI_{3D} is 0.71, indicating a strong positive correlation.

In summary, mechanical properties of fractures, fracture roughness (JRC) and fracture strength (JCS), and fracture orientations (κ) are essential to the formation of critically stressed fractures. Partially open fractures can signifi-

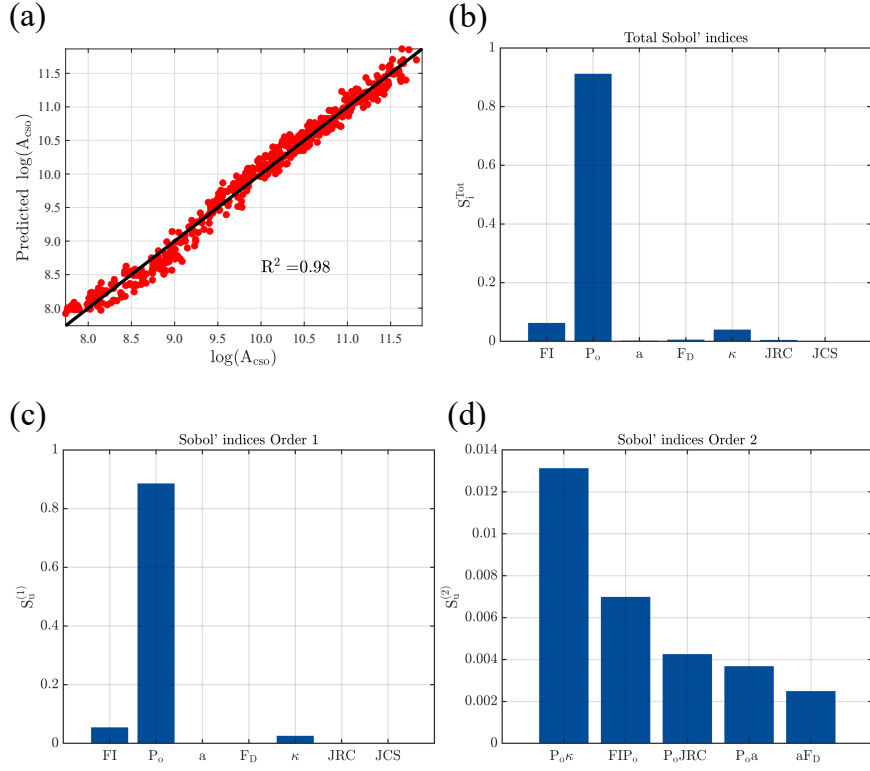


Figure 13: Sensitivity analysis with A_{CSO} as the response

422 cantly enlarge the SRV by connecting more critically orientated fractures. The
 423 important factors include the probability of open fractures (P_o), fracture in-
 424 tensity (FI), and fracture orientations (κ). The probability of open fractures
 425 (P_o) is the most dominant factor among all six factors. Fracture geometrical
 426 properties, fracture length (a), and center positions (F_D) are insignificant for
 427 SRV enlargement in 3D fracture networks.

428 3.3. Comparison of results in 2D and 3D fracture networks

429 The global sensitivity analysis results are similar for both 2D and 3D frac-
 430 ture networks in terms of the total length (area in 3D) of critically stressed
 431 fractures. The fracture roughness (JRC), fracture orientations (κ), fracture

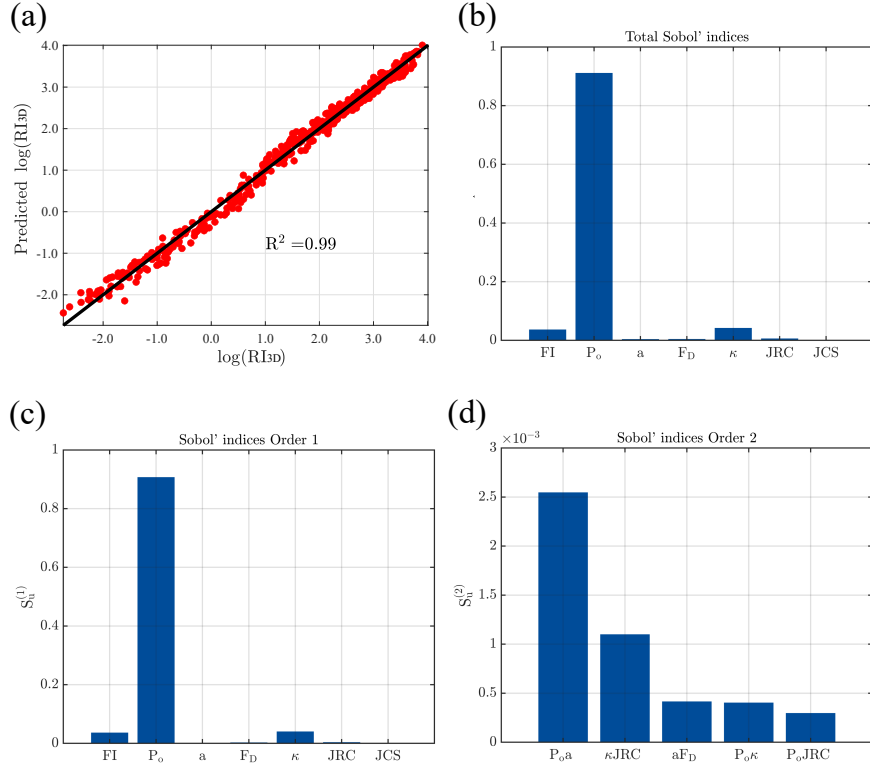


Figure 14: Sensitivity analysis with RI as the response

432 strength (JCS) are important for the formation of critically stressed fractures.
 433 For the contribution of partially open fractures, represented by L_{CSO} , A_{CSO} , RI_{2D}
 434 and RI_{3D} , the probability of open fractures (P_o) are the most significant factor
 435 for both 2D and 3D fracture networks. However, important geometrical prop-
 436 erties of fractures, i.e. fracture length (a) and fracture center positions (F_D),
 437 have significantly different behaviors in 2D and 3D fracture networks. This phe-
 438 nomenon is partially due to the different effects of geometrical properties on the
 439 connectivity of 2D and 3D fracture networks.

440 The connectivity of a fracture network is essential for the formation of SRV.
 441 Fracture geometries can impact the connectivity of fracture networks at perco-
 442 lation (Zhu et al., 2021e). Here, the percolation state refers to the formation

443 of a spanning cluster in the fracture system. This work defines relative frac-
 444 ture intensity as the ratio between the total number of fractures at termination
 445 and at percolation. This value varies from 0.8 to 2.6. Therefore, for most
 446 cases, the global connectivity of the fracture network is good since a spanning
 447 cluster is formed in the system. From our recent analysis of 80 natural out-
 448 crop maps (Zhu et al., 2021c,f), we found that 3D subsurface fractures have to
 449 be pervasive if their outcrop maps show good geometrical connectivity. Most
 450 natural outcrop maps have a spanning cluster formed and show good geomet-
 451 rical connectivity(Zhu et al., 2021c). Therefore, it is close to reality that the
 452 real subsurface fracture networks have their RI larger than 1, which means the
 453 three-dimensional fracture networks have a much higher fracture intensity than
 454 the intensity at percolation. It is also meaningful to discuss SRV when fractures
 455 are well-developed because formations with sparse fractures cannot yield good
 456 production after the hydraulic fracturing operation.

457 Clustering effects and small fractures usually have negative impacts on the
 458 global connectivity of fracture networks (Zhu et al., 2021e). However, in this
 459 work, the correlation coefficients of $a - RI_{2D}$ and $F_D - RI_{2D}$ are 0.16 and -
 460 0.07 (slightly negative), which means that small fractures and clustering effects
 461 can enlarge the size of SRV. This inconsistency is possibly due to differences
 462 between global and local connectivity. For SRV, the local connectivity close to
 463 the hydrofracture is more important than the global connectivity of the entire
 464 system. For global connectivity, large fractures are essential for long-distance
 465 interactions, especially for sparse fracture systems. However, for local connec-
 466 tivity, especially in this work, where most fracture networks have a spanning
 467 cluster formed, the impact of large fractures are not significant. Clustering
 468 effects cannot contribute much to global connectivity but can enhance local
 469 connectivity(Zhu et al., 2018). Therefore, small fractures and clustering effects

are beneficial to the relative increase of SRV (RI_{2D}) in 2D fracture networks.

For 3D fracture networks, both fracture lengths and clustering effects are insignificant for the relative increase of SRV (RI_{3D}). 3D fracture networks are not sensitive to clustering effects as observed in Zhu et al. (2021e). Therefore, their impact on the formation of SRV is also insignificant. Variations of fracture lengths in 3D fracture networks are also negligible but significant in 2D fracture networks. This phenomenon is partially caused by the convenient interactions between three-dimensional fractures because they can intersect the other fractures in a volume, but 2D fractures are constrained in the same plane. For 3D fracture networks with a high fracture intensity, fracture lengths and center positions of the fracture network are not significant to enhance the local connectivity. Instead, the probability of open fractures P_o is significant for SRV development because it determines the hydraulic connectivity of fracture networks. Fracture orientations are also crucial because they affect the mechanical response of natural fractures.

4. Discussions on the fluid transportation

For formations with ultra-low permeability, such as shale gas reservoirs or enhanced geothermal systems, the hydraulic fracturing operation is necessary to extract fluid economically. Hydrofractures and stimulated natural fractures (SRV) provide the main permeable pathway for fluid flow. Real fractures are complex in terms of their irregular shapes, complex rough surfaces, the tortuosity of flow paths in fractures and stress impacts on the hydraulic apertures. However, among all complexities, the configuration of SRV has the most significant impact on fluid transportation because it quantifies the connected permeable fractures. However, the detailed configuration of SRV is unavailable with current technologies. The commonly adopted idealized configuration of

496 SRV is an orthogonal fracture network around the hydraulic fractures (Fisher
497 et al., 2002, 2004). This configuration is not physically meaningful for most
498 stress states because natural fractures perpendicular to hydraulic fractures are
499 parallel to the minimum principal stress. Substantial fluid pressure is required
500 to overcome the maximum principal stress and make those natural fractures
501 critically stressed. In reality, it is almost impossible to reach such a significant
502 condition of fluid pressure.

503 This work provides a preliminary framework for identifying SRV configu-
504 rations by constructing subsurface formations with stochastic discrete fracture
505 networks and implementing the JRC-JCS failure criterion. Although actual
506 subsurface structures are largely unknown, two aspects of efforts can be empha-
507 sized to make the model more realistic and trustworthy. One is to constrain
508 the discrete fracture network model with more available data, including geolog-
509 ical and geomechanical data. Geological data include outcrop observations and
510 seismic maps. If rock types and structural settings of the surface outcrops and
511 subsurface formations are similar, outcrops can be regarded as relevant to the
512 subsurface formation, and the statistical rules summarized from outcrop maps
513 can constrain the model of subsurface formations. Seismic maps can provide in-
514 formation of large faults (at least tens of meters) due to their limited resolution.
515 However, the associated small-scale damage zones around the fault can be esti-
516 mated based on the self-similarity of fault segments and statistical distributions
517 of inner and outer damage zones (Kim et al., 2004; De Joussineau and Aydin,
518 2007). Geomechanical data include current stress states and stress histories.
519 The current stress state is essential for identifying the critically stressed natural
520 fractures, while stress histories can further constrain the fracture properties,
521 such as fracture orientations and sealing degrees. The other aspect is to find
522 the appropriate failure criterion for particular formations. For example, differ-

ent rock types for different reservoirs should be considered, such as shale and granite for shale gas reservoirs and geothermal systems. The thermal impact on the failure of a specific rock type can also be significant due to the large temperature difference between the injected fluid and formation rocks. Interactions between fractures are neglected, considering the computation cost in this work. However, this impact is significant if fractures are close to each other, especially for formations with abundant natural fractures. In addition, global and local stress states can significantly change the hydraulic aperture and impact fracture conductance.

A potential application of this work is to estimate the size of SRV and predict shale gas production in an accurate and physically meaningful way. Production prediction is one of the essential issues in shale gas development. However, in currently available methods, such as empirical methods (Arps, 1945), analytical methods (Clarkson and Pedersen, 2010) and numerical simulation methods (Shabro et al., 2011), detailed SRV structures are neglected or significantly simplified, which brings enormous uncertainties on the results and difficulties in analyzing sensitivities. After optimizing the model with the two aspects mentioned above and combining existing SRV estimation methods, such as microseismic monitoring and electromagnetic imaging, this work provides detailed procedures to construct realistic structures of the subsurface formation and identify SRV under a given stress state. With the realistic SRV configuration and appropriate up-scaling methods, the numerical simulation or analytical solutions (Patzek et al., 2013) on a reservoir scale is possible and can be optimized to be more physically meaningful. The knowledge on influential factors from this work can further guide the history match of production data and analyze the well performance.

549 5. Conclusions

550 In this work, we mimic typical 2D and 3D formations with a stochastic dis-
551 crete fracture network modelling method. By implementing the JRC-JCS failure
552 criterion, we identify the SRV under a given stress state. We further system-
553 atically investigate the impact of different fracture properties on the formation
554 and development of SRV. The fracture properties include geometrical proper-
555 ties (fracture lengths, center positions, and fracture orientations), mechanical
556 properties (fracture roughness and strength), fracture sealing properties (the
557 probability of open fractures and the segment length) and fracture intensity.
558 Key conclusions are summarized below.

- 559 • Critically stressed fractures compose the backbone of SRV. Partially open
560 fractures can enlarge the size of SRV by connecting more critically orien-
561 tated fractures and contribute to the production significantly.
- 562 • For the total length (area in 3D) of critically stressed fractures, mechanical
563 properties (fracture roughness (JRC) and strength (JCS)) and fracture
564 orientations (κ) are the most important factors. Geometrical properties
565 (fracture length (a) and center positions (F_D)) are important for SRV in
566 2D fracture networks, but insignificant in 3D fracture networks.
- 567 • For the total length of critically stressed fractures plus partially open
568 fractures and the relative increase of SRV in 2D fracture networks, the
569 probability of open fractures (P_o), fracture length (a), and center posi-
570 tions (F_D) are the most important factors. For 3D fracture networks, the
571 probability of open fractures (P_o), the fracture intensity (FI) and fracture
572 orientation (κ) are the essential factors.
- 573 • SRV formed in 2D fracture networks are sensitive to fracture lengths (a)
574 and positions of fracture centers (F_D), but SRV formed in 3D fracture

575 networks are insensitive to these geometrical properties. Real fracture
576 networks are always three-dimensional instead of two dimensional. There-
577 fore, to accurately estimate SRV or have a good production prediction, it
578 is particularly important to accurately assess the fracture sealing degree,
579 fracture intensity, and fracture orientations of the subsurface fracture net-
580 works.

581 **Declaration of competing interest**

582 The authors report no conflicts of interest. The authors alone are responsible
583 for the content and writing of this article.

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588 cases.

589 **Data Availability**

590 All the data are generated by the in-house built DFN modelling software,
591 HatchFrac. The C++ code of important algorithms are available online (Zhu
592 et al., 2021d)(<https://data.mendeley.com/datasets/zhs97tsdry/1>)

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