

The Formulas of the Dimensionless Universe

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Abstract

In this paper in an elegant way will be present the formulas of the Dimensionless Universe. All these equations are simple,elegant and symmetrical in a great physical meaning. These equations are applicable for all energy scales. We will propose the Dimensionless unification of the fundamental interactions and the Dimensionless unification of atomic physics with cosmology. We find the formulas for the cosmological constant and we will prove that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy.

Keywords

Theory of everything , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant , Cosmological parameters , Cosmological constant , Unification of the microcosm and the macrocosm , Poincaré dodecahedral space

1. Introduction

In physics the mathematical constants appear almost everywhere. In [1] we presented exact and approximate expressions between the Archimedes constant π ,the golden ratio ϕ ,the Euler's number e and the imaginary number i . Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between the most fundamental numbers in mathematics:

$$e^{i\pi} + 1 = 0$$

The expression who connects the six basic mathematical constants,the number 0,the number 1,the golden ratio ϕ ,the Archimedes constant π ,the Euler's number e and the imaginary unit i is:

$$e^{\frac{i\pi}{1+\phi}} + e^{\frac{-i\pi}{1+\phi}} + e^{\frac{i\pi}{\phi}} + e^{\frac{-i\pi}{\phi}} = 0$$

The fine-structure constant α is defined as:

$$\alpha = \frac{q_e^2}{4\pi\epsilon_0\hbar c}$$

Also the fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

Dr. Rajalakshmi Heyrovská in [2] has found that the golden ratio ϕ provides a quantitative link between various known quantities in atomic physics. Fine-structure constant can also be formulated in [3] , [4] and [5] exclusively in terms of the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \quad (1)$$

with numerical values:

$$\alpha^{-1}=137.0359991647.....$$

The numerical value is the average of all the measurements. The formula is the exact formula for the fine-structure constant α . Another beautiful forms of the equations are:

$$\frac{1}{\alpha} = \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{1}{3^5\varphi^5} \quad (2)$$

$$\frac{1}{\alpha} = \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{3^{-5}}{\varphi^5} \quad (3)$$

We proposed in [5] , [6] and [7] the simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π :

$$\alpha^{-1} = \frac{2706}{43}\pi \ln 2 \quad (4)$$

with absolutely accurate numerical values:

$$\alpha^{-1}=137.035999078...$$

The equivalent expression for the fine-structure constant is:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln 2 \quad (5)$$

So the equivalent expressions for the fine-structure constant with the madelung constant $b_2(2)$ are:

$$\alpha^{-1} = -\frac{2706}{43}b_2(2) \quad (6)$$

In Physics,the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron,from the system of units. The value of μ is a solution of the equation:

$$3\cdot\mu^4-5508\cdot\mu^3-841\cdot\mu^2+10\cdot\mu-2111=0$$

We propose in [8] the exact mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu=11^{47/32}\cdot5^{5/2}\cdot9349^{5/76}\cdot\varphi^{-21/16} \quad (7)$$

So the exact mathematical expression for the proton to electron mass ratio is:

$$\mu^{32}=(\varphi^5-\varphi^{-5})^{47}\cdot(2\cdot\varphi-1)^{160}\cdot(\varphi^{19}-\varphi^{-19})^{40/19}\cdot\varphi^{-42} \quad (8)$$

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19} \quad (9)$$

with numerical value $\mu=1836.15267343...$ Also we propose the exact mathematical expression for the proton to electron mass ratio:

$$\mu = 165\sqrt[3]{\frac{\ln^{11}10}{7}} \quad (10)$$

with numerical value $\mu=1836.15267392...$ Other equivalent expressions for the proton to electron mass ratio are:

$$\begin{aligned}\mu^3 &= 7^{-1} \cdot 165^3 \cdot \ln^{11} 10 \\ 7 \cdot \mu^3 &= (3 \cdot 5 \cdot 11)^3 \cdot \ln^{11}(2 \cdot 5)\end{aligned}\quad (11)$$

Other exact mathematical expression for the proton to electron mass ratio is:

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15} \quad (12)$$

with numerical value $\mu=1836.15267343...$ Also in [8] was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α :

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\phi + 42) \quad (13)$$

$$\mu - 6 \cdot \alpha^{-1} = 360 \cdot \phi - 165 \cdot \pi + 345 \cdot e + 12 \quad (14)$$

$$\mu - 182 \cdot \alpha = 141 \cdot \phi + 495 \cdot \pi - 66 \cdot e + 231 \quad (15)$$

$$\mu - 807 \cdot \alpha = 1205 \cdot \pi - 518 \cdot \phi - 411 \cdot e \quad (16)$$

In [9] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu \cdot \alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1}) + 13^2 = 0 \quad (17)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \quad (18)$$

This exponential form can also be written with the beautiful form:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = 13^2 \cdot e^{i\pi} \quad (19)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i \quad (20)$$

So other beautiful formula that connects the fine-structure constant, the proton to electron mass ratio and the fifth power of the golden mean is:

$$5^2 \cdot (5 \cdot \phi^{-2} + \phi^{-5})^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + (5 \cdot \phi^2 - \phi^{-5})^2 = 0 \quad (21)$$

The formula that connects the fine-structure constant, the proton to electron mass ratio and the mathematical constants π, ϕ, e and i is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = (5 \cdot \phi^2 - \phi^{-5})^2 \cdot e^{i\pi} \quad (22)$$

In physics, the gravitational coupling constant α_G is a constant that characterizes the gravitational pull between a given pair of elementary particles. The gravitational coupling constant α_G is defined as:

$$\alpha_G = \frac{Gm_e^2}{\hbar c}$$

There is so far no known way to measure α_G directly. The value of the constant gravitational coupling α_G is only known in four significant digits. The approximate value of the constant gravitational coupling α_G is $\alpha_G = 1.7518099 \times 10^{-45}$. Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e} \right)^2 = \left(\alpha \frac{l_{pl}}{r_e} \right)^2 = \left(\frac{l_{pl}}{\alpha \alpha_0} \right)^2$$

The gravitational coupling constant $\alpha_{G(p)}$ for the proton is produced similar to the electron, but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton $\alpha_{G(p)}$ is defined as:

$$\alpha_{G(p)} = \frac{Gm_p^2}{\hbar c}$$

The approximate value of the constant gravitational coupling of the proton is $\alpha_{G(p)} = 5.9061512 \times 10^{-39}$. Also other expression for the gravitational coupling constant is:

$$\alpha_{G(p)} = \frac{m_p^2}{m_{pl}^2} = \mu^2 \alpha_G = \frac{\alpha \mu}{N_1} = \frac{\alpha^2}{N_1^2 \alpha_G}$$

The enormous value of the ratio of electric force to gravitational force was first pointed out by Benjamin Davis in 1904. But Weyl and Eddington suggested that the number was about 10^{40} and was related to cosmological quantities. The electric force F_c between electron and proton is defined as:

$$F_c = \frac{q_e^2}{4\pi\epsilon_0 r^2}$$

The gravitational force F_g between electron and proton is defined as:

$$F_g = \frac{Gm_e m_p}{r^2}$$

So from these expressions we have:

$$\begin{aligned} N_1 &= \frac{F_c}{F_g} \\ N_1 &= \frac{q_e^2}{4\pi\epsilon_0 Gm_e m_p} \\ N_1 &= \frac{k_e q_e^2}{Gm_p m_e} \\ N_1 &= \frac{\alpha \hbar c}{Gm_e m_p} \end{aligned}$$

So the ratio N_1 of electric force to gravitational force between electron and proton is defined as:

$$N_1 = \frac{\alpha}{\mu \alpha_G} = \frac{\alpha \mu}{\alpha_{G(p)}} = \frac{\alpha}{\sqrt{\alpha_G \alpha_{G(p)}}} = \frac{k_e q_e^2}{Gm_e m_p} = \frac{\alpha \hbar c}{Gm_e m_p}$$

The approximate value of the ratio of electric force to gravitational force between electron and proton is $N_1 = 2.26866072 \times 10^{39}$. The ratio N_1 of electric force to gravitational force between electron and proton can also be written in expression:

$$N_1 = \frac{5}{3} 2^{130} = 2,26854911 \times 10^{39}$$

According to current theories N_1 should be constant. The ratio N_2 of electric force to gravitational force between two electrons is defined as:

$$N_2 = \mu N_1 = \frac{\alpha}{\alpha_G} = \frac{N_1^2 \alpha_{G(p)}}{\alpha} = \frac{k_e q_e^2}{G m_e^2} = \frac{\alpha \hbar c}{G m_e^2}$$

The approximate value of N_2 is $N_2=4.16560745 \times 10^{42}$. According to current theories N_2 should grow with the expansion of the universe. Avogadro's number N_A is defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. The exact value of the Avogadro's number is $N_A=6.02214076 \times 10^{23}$. The value of the Avogadro's number N_A can also be written in expressions:

$$N_A=84446885^3=6.02214076 \times 10^{23}$$

$$N_A=2^{79}=6.04462909 \times 10^{23} \quad (23)$$

In [10] was presented the exact mathematical formula that connects 6 dimensionless physical constants The length Planck l_{pl} can be defined by three fundamental natural constants, the speed of light at vacuum c , the reduced Planck constant and the gravity constant G as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl} c} = \frac{h}{2\pi m_{pl} c} = \frac{m_p r_p}{4m_{pl}}$$

The Bohr radius a_0 is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius a_0 is defined as:

$$a_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi\alpha}$$

For the reduced Planck constant $\hbar$ apply:

$$\hbar = \alpha \cdot m_e \cdot a_0 \cdot c$$

So from these expressions we have:

$$\hbar^2 = \alpha^2 \cdot m_e^2 \cdot a_0^2 \cdot c^2$$

$$(\hbar \cdot G / c^3) = \alpha^2 \cdot m_e^2 \cdot a_0^2 \cdot (G / \hbar \cdot c)$$

$$(\hbar \cdot G / c^3) = \alpha^2 \cdot a_0^2 \cdot (G \cdot m_e^2 / \hbar \cdot c)$$

$$l_{pl}^2 = \alpha^2 \cdot a_G \cdot a_0^2$$

So the new formula for the Planck length l_{pl} is:

$$l_{pl} = \alpha \sqrt{a_G} a_0 \quad (24)$$

Jeff Yee proposed in [11] that the mole and charge are related by deriving Avogadro's number from three constants, the Bohr radius, the Planck length and Euler's number. The Avogadro's number N_A can be calculated from the Planck length l_{pl} , the Bohr radius a_0 and Euler's number e :

$$N_A = \frac{\alpha_0}{2e l_{pl}}$$

We will use this expression and the new formula for the Planck length l_{pl} to resulting the unity formula that connects the fine-structure constant α and the gravitational coupling constant α_G :

$$a_0 = 2 \cdot e \cdot N_A \cdot l_{pl}$$

$$\alpha_0 = 2e N_A \alpha \sqrt{\alpha_G} a_0$$

$$2e N_A \alpha \sqrt{\alpha_G} = 1$$

Therefore the unity formula that connect the fine-structure constant α , the gravitational coupling constant α_G and the Avogadro's number N_A is:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (25)$$

The unity formula is equally valid:

$$\alpha^2 \cdot \alpha_G = (2 \cdot e \cdot N_A)^{-2} \quad (26)$$

This formula is the simple unification of the electromagnetic and the gravitational interactions. So from this expression the new formula for the Avogadro number N_A is:

$$N_A = \left(2e\alpha\sqrt{\alpha_G} \right)^{-1} \quad (27)$$

The exact mathematical formula that connect the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro's number N_A , the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_G(p)$ are:

$$\alpha_G(p) = \mu^2 \cdot \alpha_G$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (28)$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G \quad (29)$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_G(p) \quad (30)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_G(p) \quad (31)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (32)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \quad (33)$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (34)$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (35)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (36)$$

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (37)$$

2. Dimensionless unification of the fundamental interactions

The value of the strong coupling constant, like other coupling constants, depends on the energy scale. As the energy increases, this constant decreases. The last measurement [12] in 23 November 2021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as $\alpha_s(m_Z) = 0.1170 \pm 0.0019$. For quarks in quantum chromodynamics, a strong interaction constant is introduced:

$$\alpha_s = \frac{q_{gg}^2}{4\pi\hbar c} = \frac{q_{gg}^2 \varepsilon_0 \alpha}{q_e^2} = \frac{\varepsilon_0 q_{gg}^2}{q_{pl}^2}$$

where q_{gg} is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks, which is achieved in high-energy particle collisions, a logarithmic reduction of α_s and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. In [13] we

presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Euler' number}}{\text{Gerford's constant}} = \frac{e}{e^\pi} = e^{1-\pi} = 0,11748.. \quad (38)$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. It fits perfectly in the measurement of the strong coupling constant of the European organization for nuclear research (CERN). Also for the value of the strong coupling constant we have the equivalent expressions:

$$\alpha_s = e \cdot e^{-\pi} = e \cdot i^{2i} = i^{-2i/n} \cdot i^{2i} = i^{2i-(2i/n)} = i^{2i(n-1)/n}$$

In the papers [14] , [15] , [16] and [17] was presented the unification of the fundamental interactions. The decays of the delta baryons is:

$$\Delta^+ \rightarrow p + \pi^0$$

The lifetime of the delta baryons is:

$$\tau_{\Delta} \approx 6 \times 10^{-24} \text{ s}$$

The decays of the sigma baryons is:

$$\Sigma^+ \rightarrow p + \pi^0$$

The lifetime of the delta baryons is:

$$\tau_{\Sigma} \approx 8 \times 10^{-11} \text{ s}$$

The coupling constant ratio can then be estimated for this situation [18]:

$$\begin{aligned} \frac{\alpha_w}{\alpha_s} &= \sqrt{\frac{\tau_{\Delta}}{\tau_{\Sigma}}} = 10^{-7} e \\ \frac{\alpha_w}{\alpha_s} &= 10^{-7} e \end{aligned} \quad (39)$$

From this expression we can result the world average value of the weak coupling constant α_w :

$$\alpha_w = e \cdot \alpha_s \cdot 10^{-7}$$

$$\alpha_w = e^{2-\pi} \cdot 10^{-7}$$

$$\alpha_w = e \cdot e \cdot i^{2i} \cdot 10^{-7}$$

$$\alpha_w = e^2 \cdot i^{2i} \cdot 10^{-7}$$

So the recommended theoretical current world average value for the weak coupling constant is:

$$\alpha_w = (e \cdot i)^2 \cdot 10^{-7} = 3.19310 \cdot 10^{-8} \quad (40)$$

From expression can result other equivalent expressions:

$$\alpha_w \cdot \alpha_s^{-1} = e \cdot 10^{-7}$$

$$\alpha_s \cdot \alpha_w^{-1} = e^{-1} \cdot 10^7$$

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w \quad (41)$$

From this expression apply:

$$\begin{aligned}
 e^n \cdot a_s \cdot a_s &= 10^7 \cdot a_w \\
 e^n \cdot a_s^2 &= 10^7 \cdot a_w \\
 a_s^2 &= 10^7 \cdot e^{-n} \cdot a_w \\
 a_s^2 &= i^{2i} \cdot 10^7 \cdot a_w
 \end{aligned} \tag{42}$$

From this expression and Euler's identity resulting the beautiful formulas:

$$\begin{aligned}
 e^{in} + 1 &= 0 \\
 (10^7 \cdot a_s^{-2} \cdot a_w)^i + 1 &= 0 \\
 (10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i + 1 &= 0 \\
 10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} + 1 &= 0 \\
 10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} &= i^2 \\
 a_s^{2i} &= i^2 \cdot 10^{7i} \cdot a_w^i \\
 \frac{\alpha_s^{2i}}{\alpha_w^i} &= i^2 10^{7i}
 \end{aligned} \tag{43}$$

We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear forces:

$$e \cdot a_s = 10^7 \cdot a_w \tag{44}$$

$$a_s^2 = i^{2i} \cdot 10^7 \cdot a_w \tag{45}$$

$$e^n \cdot a_s^2 = 10^7 \cdot a_w \tag{46}$$

$$a_s^{2i} = i^2 \cdot 10^{7i} \cdot a_w^i \tag{47}$$

(Dimensionless unification of the strong nuclear and the weak nuclear force interactions)

Jesús Sánchez in [19] explained that the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos a^{-1} = e^{-1} \tag{48}$$

Another elegant expression is the following exponential form equations:

$$\begin{aligned}
 e^{i/a} \cdot e^{-1} &= -e^{-i/a} + e^{-1} \\
 e^{i/a} + e^{-i/a} &= 2 \cdot e^{-1}
 \end{aligned} \tag{49}$$

Also the fine-structure constant is one of the roots of the following trigonometric equation:

$$\begin{aligned}
 \cos(10^3 \cdot a^{-1}) &= \varphi^2 \cdot e^{-1} \\
 e \cdot \cos(10^3 \cdot a^{-1}) &= \varphi^2
 \end{aligned} \tag{50}$$

Another elegant expression is the following exponential form equation:

$$e^{1000i/a} + e^{-1000i/a} = 2 \cdot \varphi^2 \cdot e^{-1} \tag{51}$$

From these expressions resulting the following equations:

$$\begin{aligned}\cos^{-1}\alpha^{-1} \cdot \cos(10^3 \cdot \alpha^{-1}) &= \varphi^2 \\ \cos(10^3 \cdot \alpha^{-1}) &= \varphi^2 \cdot \cos \alpha^{-1}\end{aligned}\tag{52}$$

We will use the expressions to resulting the unity formulas that connects the strong coupling constant α_s and the fine-structure constant α :

$$\begin{aligned}\cos \alpha^{-1} &= e^{-1} \\ \alpha_s &= e^{1-\pi} \\ \cos \alpha^{-1} &= (e^\pi \cdot \alpha_s)^{-1} \\ \cos \alpha^{-1} &= e^{-\pi} \cdot \alpha_s^{-1} \\ e^\pi \cdot \alpha_s \cdot \cos \alpha^{-1} &= 1\end{aligned}\tag{53}$$

Other forms of the equations are:

$$\begin{aligned}\cos \alpha^{-1} &= (i^{-2i} \cdot \alpha_s)^{-1} \\ i^{-2i} \cdot \alpha_s \cdot \cos \alpha^{-1} &= 1 \\ \cos \alpha^{-1} &= i^{2i} \cdot \alpha_s^{-1} \\ \alpha_s \cdot \cos \alpha^{-1} &= i^{2i}\end{aligned}\tag{54}$$

So the beautiful formulas for the strong coupling constant α_s are:

$$\begin{aligned}\alpha_s &= e^{-\pi} \cdot \cos^{-1} \alpha^{-1} \\ \alpha_s &= i^{2i} \cdot \cos^{-1} \alpha^{-1}\end{aligned}$$

Now we need to study the following equivalent equations:

$$\begin{aligned}\cos \alpha^{-1} &= \frac{e^{-\pi}}{\alpha_s} \\ \cos \alpha^{-1} &= \frac{i^{2i}}{\alpha_s} \\ \cos \alpha^{-1} &= \frac{\alpha_s^{-1}}{e^\pi} \\ \cos \alpha^{-1} &= \frac{\alpha_s^{-1}}{i^{-2i}}\end{aligned}$$

From expressions resulting the formulas that connects the strong coupling constant α_s and the fine-structure constant α :

$$\begin{aligned}e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot e^{-1} \\ e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot (e^\pi \cdot \alpha_s)^{-1} \\ e^{i/\alpha} - (e^\pi \cdot \alpha_s)^{-1} &= -e^{-i/\alpha} + (e^\pi \cdot \alpha_s)^{-1}\end{aligned}$$

$$e^{i/a} + e^{-i/a} = 2 \cdot e^{-n} \cdot a_s^{-1}$$

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \quad (55)$$

Other forms of the equations are:

$$e^{i/a} + e^{-i/a} = 2 \cdot (i^{-2i} \cdot a_s)^{-1}$$

$$e^{i/a} + e^{-i/a} = 2 \cdot i^{2i} \cdot a_s^{-1}$$

$$e^{i/a} \cdot i^{2i} \cdot a_s^{-1} = -e^{-i/a} + i^{2i} \cdot a_s^{-1}$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i} \quad (56)$$

These equations are applicable for all energy scales. So the beautiful formulas for the strong coupling constant a_s are:

$$a_s = 2 \cdot e^{-n} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

$$a_s = 2 \cdot i^{2i} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$a_s \cdot \cos a^{-1} = i^{2i} \quad (57)$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i} \quad (58)$$

$$e^n \cdot a_s \cdot \cos a^{-1} = 1 \quad (59)$$

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \quad (60)$$

$$a_s^i \cdot (e^{i/a} + e^{-i/a})^i = 2^i \cdot i^2 \quad (61)$$

(Dimensionless unification of the strong nuclear and the electromagnetic interactions)

We will use the expressions to resulting the unity formula that connect the weak coupling constant a_w and the fine-structure constant α :

$$e \cdot a_s = 10^7 \cdot a_w$$

$$e^n \cdot a_s \cdot \cos a^{-1} = 1$$

$$e^n \cdot 10^7 \cdot a_w \cdot \cos a^{-1} = e$$

$$e^{n-1} \cdot 10^7 \cdot a_w \cdot \cos a^{-1} = 1$$

$$10^7 \cdot a_w \cdot \cos a^{-1} = e^{1-n} \quad (62)$$

Other forms of the equations are:

$$a_w \cdot \cos a^{-1} = e \cdot i^{2i} \cdot 10^{-7}$$

$$10^7 \cdot a_w \cdot \cos a^{-1} = e \cdot i^{2i} \quad (63)$$

So the formulas for the weak coupling constant a_w are:

$$a_w = (e^{n-1} \cdot 10^7 \cdot \cos a^{-1})^{-1}$$

$$a_w = e^{1-n} \cdot 10^{-7} \cdot \cos^{-1} a^{-1}$$

$$a_w = e \cdot i^{2i} \cdot (10^7 \cdot \cos a^{-1})^{-1}$$

$$a_w = e \cdot i^{2i} \cdot 10^{-7} \cdot \cos^{-1} a^{-1}$$

Resulting the unity formulas that connects weak coupling constant a_w and the fine-structure constant a :

$$e \cdot a_s = 10^7 \cdot a_w$$

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2$$

$$e^n \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e$$

$$e^{i/a} + e^{-i/a} = 2 \cdot (e^{n-1} \cdot 10^7 \cdot a_w)^{-1}$$

$$e^{i/a} - (e^{n-1} \cdot 10^7 \cdot a_w)^{-1} = -e^{-i/a} + (e^{n-1} \cdot 10^7 \cdot a_w)^{-1}$$

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e^{1-n} \quad (64)$$

Other form of the equations is:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i} \quad (65)$$

So the formulas for the weak coupling constant a_w are:

$$a_w = 2 \cdot [e^{n-1} \cdot 10^7 \cdot (e^{i/a} + e^{-i/a})]^{-1}$$

$$a_w = 2 \cdot e^{1-n} \cdot 10^{-7} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

$$a_w = 2 \cdot e \cdot i^{2i} \cdot [10^7 \cdot (e^{i/a} + e^{-i/a})]^{-1}$$

$$a_w = 2 \cdot e \cdot i^{2i} \cdot 10^{-7} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

$$10^7 \cdot a_w \cdot \cos a^{-1} = e \cdot i^{2i} \quad (66)$$

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i} \quad (67)$$

$$10^7 \cdot e^n \cdot a_w \cdot \cos a^{-1} = e \quad (68)$$

$$10^7 \cdot e^n \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \quad (69)$$

(Dimensionless unification of the weak nuclear and the electromagnetic interactions)

We will use the expressions to find the expression that connects the strong coupling constant a_s , the weak coupling constant a_w and the fine-structure constant a :

$$e \cdot a_s = 10^7 \cdot a_w$$

$$\cos a^{-1} = e^{-1}$$

$$\cos a^{-1} = a_s \cdot a_w^{-1} \cdot 10^{-7}$$

So the unity formula that connects the strong coupling constant a_s , the weak coupling constant a_w and the fine-structure constant a is:

$$10^7 \cdot \alpha_w \cdot \cos \alpha^{-1} = \alpha_s \quad (70)$$

Now we need to study the following equivalent equations:

$$\cos \alpha^{-1} = \frac{10^{-7} \alpha_w^{-1}}{\alpha_s^{-1}}$$

$$\cos \alpha^{-1} = \frac{\alpha_s}{10^7 \alpha_w}$$

$$10^7 \cos \alpha^{-1} = \frac{\alpha_s}{\alpha_w}$$

$$\cos \alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7}$$

Resulting the beautiful formulas that connects the strong coupling constant α_s , the weak coupling constant α_w and the fine-structure constant α :

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot e^{-1}$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot 10^{-7} \cdot \alpha_s \cdot \alpha_w^{-1}$$

$$\alpha_w \cdot \alpha_s^{-1} \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot 10^{-7}$$

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot \alpha_s \quad (71)$$

These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic forces:

$$10^7 \cdot \alpha_w \cdot \cos \alpha^{-1} = \alpha_s \quad (72)$$

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot \alpha_s \quad (73)$$

(Dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions)

It was presented in [10] the mathematical formulas that connects the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro's number N_A , the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_G(p)$:

$$\alpha_G(p) = \mu^2 \cdot \alpha_G \quad (74)$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G \quad (75)$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_G(p) \quad (76)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_G(p) \quad (77)$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (78)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (79)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \quad (80)$$

$$4 \cdot e^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (81)$$

$$\mu^3 = 4 \cdot e^2 \cdot a \cdot a_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (82)$$

$$\mu^2 = 4 \cdot e^2 \cdot a_G \cdot a_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (83)$$

$$\mu = 4 \cdot e^2 \cdot a \cdot a_G \cdot a_G(p) \cdot N_A^2 \cdot N_1 \quad (84)$$

Also resulting the expressions:

$$\cos(a^{-1}) = e^{-1}$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$4 \cdot a^2 \cdot a_G \cdot N_A^2 = e^{-2}$$

$$\cos^2 a^{-1} = 4 \cdot a^2 \cdot a_G \cdot N_A^2$$

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot N_A^2 \quad (85)$$

This unity formula is equally valid:

$$\alpha^{-1} \cos \alpha^{-1} = 2N_A \sqrt{\alpha_G} \quad (86)$$

Also resulting another elegant exponential form equations:

$$e^{i/a} + e^{-i/a} = 2 \cdot e^{-1}$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$4 \cdot a^2 \cdot a_G \cdot N_A^2 = e^{-2}$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2 \quad (87)$$

This unity formula is equally valid:

$$\alpha^{-1} \left(e^{\frac{i}{\alpha}} + e^{\frac{-i}{\alpha}} \right) = 4N_A \sqrt{\alpha_G} \quad (88)$$

Also resulting the expression with power of two:

$$2^{160} \cdot e^2 \cdot a^2 \cdot a_G = 1 \quad (89)$$

$$a^{-2} \cdot \cos^2 a^{-1} = 2^{160} \cdot a_G \quad (90)$$

$$2^{162} \cdot a^2 \cdot a_G = (e^{i/a} + e^{-i/a})^2 \quad (91)$$

Other form of the equations is:

$$\alpha^{-1} \left(e^{\frac{i}{\alpha}} + e^{\frac{-i}{\alpha}} \right) = 2^{34} \sqrt{\alpha_G} \quad (92)$$

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (93)$$

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot N_A^2 \quad (94)$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2 \quad (95)$$

(Dimensionless unification of the gravitational and the electromagnetic interactions)

We will use the previous expressions to resulting the unity formulas that connect the strong coupling constant α_s , the fine-structure constant α and the gravitational coupling constant α_G :

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$4 \cdot e^2 \cdot (e^n \cdot \alpha_s)^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (96)$$

Other form of the equation is:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \quad (97)$$

Also resulting the mathematical formulas that connects the strong coupling constant α_s , the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A , the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_G(p)$:

$$4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (98)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (99)$$

$$\mu \cdot N_1 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^3 \cdot N_A^2 \quad (100)$$

$$4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (101)$$

$$\mu^3 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (102)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (103)$$

$$\mu = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (104)$$

Other equivalent forms of the equations are:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \quad (105)$$

$$i^{4i} \cdot \mu = \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (106)$$

$$i^{4i} \cdot \mu \cdot N_1 = 4 \cdot \alpha_s^2 \cdot \alpha^3 \cdot N_A^2 \quad (107)$$

$$4 \cdot \alpha_s^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = i^{4i} \quad (108)$$

$$i^{4i} \cdot \mu^3 = 4 \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (109)$$

$$i^{4i} \cdot \mu^2 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (110)$$

$$i^{4i} \cdot \mu = 4 \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (111)$$

$$\alpha_s \cdot \cos \alpha^{-1} = i^{2i}$$

$$2 \cdot N_A \cdot \alpha_G^{1/2} = \alpha^{-1} \cdot \cos \alpha^{-1}$$

$$2 \cdot \alpha_s \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = i^{2i}$$

$$2 \cdot \alpha_s \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} \cdot \alpha_s \cdot \cos \alpha^{-1} = i^{2i} \cdot i^{2i}$$

$$2 \cdot \alpha \cdot \cos \alpha^{-1} \cdot \alpha_s^2 \cdot \alpha_G^{1/2} \cdot N_A = i^{4i}$$

$$4 \cdot \alpha^2 \cdot \cos^2 \alpha^{-1} \cdot \alpha_s^4 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (112)$$

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i}$$

$$\begin{aligned}
2 \cdot N_A \cdot a_G^{1/2} &= a^{-1} \cdot (e^{i/a} + e^{-i/a}) \\
2 \cdot a_s \cdot a \cdot N_A \cdot a_G^{1/2} &= i^{2i} \\
a_s \cdot (e^{i/a} + e^{-i/a}) \cdot 2 \cdot a_s \cdot a \cdot N_A \cdot a_G^{1/2} &= 2 \cdot i^{2i} \cdot i^{2i} \\
a \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^2 \cdot a_G^{1/2} \cdot N_A &= i^{4i} \\
a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 &= i^{8i}
\end{aligned} \tag{113}$$

Also resulting the expressions with power of two:

$$\begin{aligned}
2^{80} \cdot a_s \cdot a \cdot a_G^{1/2} &= i^{2i} \\
2^{160} \cdot a_s^2 \cdot a^2 \cdot a_G &= i^{4i}
\end{aligned} \tag{114}$$

$$\begin{aligned}
2^{80} \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^2 \cdot a_G^{1/2} &= i^{4i} \\
2^{160} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_s^4 \cdot a_G &= i^{8i}
\end{aligned} \tag{115}$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \tag{116}$$

$$2 \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i} \tag{117}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i} \tag{118}$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 1 \tag{119}$$

$$2 \cdot e^{4n} \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot N_A^2 = 1 \tag{120}$$

$$e^{4n} \cdot a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = 1 \tag{121}$$

(Dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions)

We will use the previous expressions to resulting the unity formulas that connects the weak coupling constant a_w , the fine-structure constant a and the gravitational coupling constant a_G :

$$e \cdot a_s = 10^7 \cdot a_w$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 1$$

$$2 \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = i^{2i}$$

$$2 \cdot e^n \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot N_A = e$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = e^2 \tag{122}$$

$$2 \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot N_A = i^{2i} \cdot e$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \cdot e^2 \tag{123}$$

Also resulting the mathematical formulas that connects the weak coupling constant a_w , the proton to electron mass ratio μ , the fine-structure constant a , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro's number N_A , the gravitational coupling constant a_G of the electron and the gravitational coupling constant of the proton $a_G(p)$:

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = e^2 \tag{124}$$

$$e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot a_G(p) \cdot NA^2 \quad (125)$$

$$e^2 \cdot \mu \cdot N_1 = 4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^3 \cdot NA^2 \quad (126)$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a \cdot \mu \cdot a_G^2 \cdot NA^2 \cdot N_1 = e^2 \quad (127)$$

$$e^2 \cdot \mu^3 = 4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a \cdot a_G(p)^2 \cdot NA^2 \cdot N_1 \quad (128)$$

$$e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a_G \cdot a_G(p)^2 \cdot NA^2 \cdot N_1^2 \quad (129)$$

$$e^2 \cdot \mu = 4 \cdot e^n \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N_1 \quad (130)$$

Other equivalent forms of the equations are:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2 \quad (131)$$

$$i^{4i} \cdot e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G(p) \cdot NA^2 \quad (132)$$

$$i^{4i} \cdot e^2 \cdot \mu \cdot N_1 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^3 \cdot NA^2 \quad (133)$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot \mu \cdot a_G^2 \cdot NA^2 \cdot N_1 = i^{4i} \cdot e^2 \quad (134)$$

$$i^{4i} \cdot e^2 \cdot \mu^3 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G(p)^2 \cdot NA^2 \cdot N_1 \quad (135)$$

$$i^{4i} \cdot e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a_G \cdot a_G(p)^2 \cdot NA^2 \cdot N_1^2 \quad (136)$$

$$i^{4i} \cdot e^2 \cdot \mu = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N_1 \quad (137)$$

$$a_w^{-1} \cdot a_s^2 = i^{2i} \cdot 10^7$$

$$a_s^2 = i^{2i} \cdot 10^7 \cdot a_w$$

$$2 \cdot a \cdot \cos a^{-1} \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$2 \cdot a \cdot \cos a^{-1} \cdot i^{2i} \cdot 10^7 \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$2 \cdot 10^7 \cdot a \cdot \cos a^{-1} \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{2i}$$

$$4 \cdot 10^{14} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (138)$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot i^{2i} \cdot 10^7 \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$10^7 \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{2i}$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (139)$$

Also resulting the expression with power of two:

$$2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} = i^{2i} \cdot e$$

$$2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G = i^{4i} \cdot e^2 \quad (140)$$

$$2^{80} \cdot 10^7 \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_w \cdot a_G^{1/2} = i^{2i}$$

$$2^{160} \cdot 10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G = i^{4i} \quad (141)$$

We reached the conclusion of the dimensionless unification of the weak nuclear, the gravitational and electromagnetic forces:

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2 \quad (142)$$

$$4 \cdot 10^{14} \cdot \alpha^2 \cdot \cos^2 \alpha^{-1} \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (143)$$

$$10^{14} \cdot \alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha})^2 \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (144)$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^2 \quad (145)$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot \alpha^2 \cdot \cos^2 \alpha^{-1} \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (146)$$

$$10^{14} \cdot e^{4n} \cdot \alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha})^2 \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (147)$$

(Dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions)

We will use the previous expressions to resulting the unity formulas that connects the strong coupling constant α_s , the weak coupling constant α_w , the fine-structure constant α and the gravitational coupling constant α_G :

$$\alpha_w^{-1} \cdot \alpha_s^2 = i^{2i} \cdot 10^7$$

$$2 \cdot \alpha_s \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = i^{2i}$$

$$\alpha_w^{-1} \cdot \alpha_s^2 = 2 \cdot 10^7 \cdot \alpha_s \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2}$$

$$\alpha_w^{-1} \cdot \alpha_s = 2 \cdot 10^7 \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2}$$

$$2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot \alpha_s^{-1} = 1$$

$$\alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot \alpha_s^{-1} = (2 \cdot 10^7 \cdot N_A)^{-1} \quad (148)$$

$$2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} = \alpha_s$$

$$\alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot \alpha_s^{-2} = (2 \cdot 10^7 \cdot N_A)^{-2} \quad (149)$$

So the beautiful unity formula that connects the strong coupling constant α_s , the weak coupling constant α_w , the fine-structure constant α and the gravitational coupling constant α_G is:

$$(2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha)^2 \cdot \alpha_G = \alpha_s^2$$

$$4 \cdot 10^{14} \cdot N_A^2 \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G = \alpha_s^2 \quad (150)$$

Sometimes the gravitational coupling constant for the proton $\alpha_{G(p)}$ is used instead of the gravitational coupling constant α_G for the electron:

$$\alpha_{G(p)} = \mu^2 \cdot \alpha_G$$

$$\alpha_G^{1/2} = \alpha_{G(p)}^{1/2} \cdot \mu^{-1}$$

$$\alpha_s \cdot \mu \cdot (\alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2})^{-1} = 2 \cdot 10^7 \cdot N_A$$

$$\alpha_s \cdot \mu = 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2}$$

$$2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2} \cdot \alpha_s^{-1} \cdot \mu^{-1} = 1$$

$$2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2} \cdot \alpha_s^{-1} = \mu \cdot \alpha_s$$

$$\alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2} \cdot \alpha_s^{-1} = (2 \cdot 10^7 \cdot N_A)^{-1} \cdot \mu \quad (151)$$

So the beautiful unity formula that connect the strong coupling constant α_s , weak coupling constant α_w , the fine-structure constant α and the gravitational coupling constant $\alpha_{G(p)}$ for the proton is:

$$\begin{aligned}(2 \cdot 10^7 \cdot N_A \cdot a_w \cdot a)^2 \cdot a_G(p) &= \mu^2 \cdot a_s^2 \\ 4 \cdot 10^{14} \cdot N_A^2 \cdot a_w^2 \cdot a^2 \cdot a_G(p) &= \mu^2 \cdot a_s^2\end{aligned}\quad (152)$$

$$\cos a^{-1} = 2 \cdot a \cdot a_G^{1/2} \cdot N_A$$

$$2 \cdot 10^7 \cdot N_A \cdot a_w \cdot a \cdot a_G^{1/2} = a_s$$

$$2 \cdot a \cdot a_G^{1/2} \cdot N_A \cdot 2 \cdot 10^7 \cdot N_A \cdot a_w \cdot a \cdot a_G^{1/2} = a_s \cos a^{-1}$$

$$4 \cdot 10^7 \cdot a^2 \cdot a_G \cdot a_w \cdot N_A^2 = a_s \cos a^{-1}$$

$$a^{-1} \cdot (e^{i/a} + e^{-i/a}) = 4 \cdot N_A \cdot a_G^{1/2}$$

$$2 \cdot 10^7 \cdot N_A \cdot a_w \cdot a \cdot a_G^{1/2} = a_s$$

$$2 \cdot 10^7 \cdot N_A \cdot a_w \cdot a \cdot a_G^{1/2} \cdot 4 \cdot N_A \cdot a_G^{1/2} = a_s \cdot a^{-1} \cdot (e^{i/a} + e^{-i/a})$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a})$$

Resulting the mathematical formulas that connects the strong coupling constant a_s , the weak coupling constant a_w , the proton to electron mass ratio μ , the fine-structure constant a , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro's number N_A , the gravitational coupling constant a_G of the electron and the gravitational coupling constant of the proton $a_G(p)$:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (153)$$

$$\mu^2 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G(p) \cdot N_A^2 \quad (154)$$

$$\mu \cdot N_1 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^3 \cdot N_A^2 \quad (155)$$

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 \quad (156)$$

$$\mu^3 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (157)$$

$$\mu \cdot a_s = 4 \cdot 10^{14} \cdot a_w^2 \cdot a_G \cdot G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (158)$$

$$\mu \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot N_A^2 \cdot N_1 \quad (159)$$

Also resulting the expressions with power of two:

$$2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot a_s^{-1} = 1$$

$$2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} = 1 \quad (160)$$

$$a_s = 2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2}$$

$$a_s^2 = 2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \quad (161)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (162)$$

$$a_s \cdot \cos a^{-1} = 4 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G \quad (163)$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a}) \quad (164)$$

(Dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions)

In [20] and [21] it presented the theoretical value of the Gravitational constant $G=6.67448 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$. This value is very close to the CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. For Milne, the space was not a structured object but merely a frame of reference in which relations such as this could accommodate Einstein's conclusions:

$$G = \frac{c^3}{M_U} T_U$$

According to this relationship, G increases with time. Dirac hypothesized that the constant of universal attraction G varies with time. Dirac's hypothesis went so far as to claim that such coincidences could be explained if the very physical constants changed with T_U , especially the gravitational constant G , which must decrease with time:

$$G \approx \frac{1}{t}$$

The gravitational constant is defined as:

$$G = \alpha_G \frac{\hbar c}{m_e^2}$$

The expressions for the gravitational constant G in terms of Planck units are:

$$G = \frac{c^3 l_{pl}^2}{\hbar} = \frac{\hbar c}{m_{pl}^2} = \frac{l_{pl} c^2}{m_{pl}} = \frac{c^5 t_{pl}^2}{\hbar}$$

A surprisingly close relationship between gravity and the electrostatic interaction. The gravitational constant G and the Coulomb constant k_e are expressed in terms of Planck units as:

$$G = \frac{K_e q_e^2}{a m_{pl}^2}$$

Also another beautiful expression that proves the close relationship between gravity and electrostatic interaction is:

$$G = \frac{\alpha c^4 l_{pl}^2}{K_e q_e^2}$$

The 2018 CODATA recommended value of gravitational constant is $G=6.67430 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ with standard uncertainty $0.00015 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ and relative standard uncertainty 2.2×10^{-5} . In August 2018, a Chinese research group announced new measurements based on torsion balances, $6.674184(78) \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ and $6.674484(78) \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ based on two different methods [22]. Now we will find the formulas for the gravitational constant G using the unity formulas for the coupling constants that we calculated. The gravitational coupling constant α_G can be written in the form:

$$4 \cdot e^2 \cdot N_A^2 \cdot a^2 \cdot \alpha_G = 1$$

$$\alpha_G = (2 \cdot e \cdot a \cdot N_A)^{-2} \quad (165)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2e a N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (166)$$

The gravitational coupling constant α_G can be written in the forms:

$$4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$\alpha_G = (2 \cdot e^n \cdot a_s \cdot a \cdot N_A)^{-2} \quad (167)$$

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i}$$

$$a_G = i^{4i} \cdot (2 \cdot a_s \cdot a \cdot N_A)^{-2} \quad (168)$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^\pi \alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (169)$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (170)$$

The gravitational coupling constant a_G can be written in the form:

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = e^2$$

$$a_G = (2 \cdot e^{n-1} \cdot 10^7 \cdot a_w \cdot a \cdot N_A)^{-2} \quad (171)$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$a_G = i^{4i} \cdot e^2 \cdot (2 \cdot 10^7 \cdot a_w \cdot a \cdot N_A)^{-2} \quad (172)$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^{\pi-1} 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (173)$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (174)$$

The gravitational coupling constant a_G can be written in the form:

$$4 \cdot 10^{14} \cdot N_A^2 \cdot a_w^2 \cdot a^2 \cdot a_G = a_s^2$$

$$a_G = a_s^2 \cdot (2 \cdot 10^7 \cdot a_w \cdot a \cdot N_A)^{-2} \quad (175)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (176)$$

Now we will find the theoretical value of the Gravitational constant G using the unity formulas for the coupling constants that we calculated. The gravitational coupling constant a_G can be written in the form:

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot N_A^2$$

$$a_G = (2 \cdot a \cdot N_A)^{-2} \cdot \cos^2 a^{-1} \quad (177)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2\alpha N_A)^{-2} \cos^2 \alpha^{-1} \frac{\hbar c}{m_e^2} \quad (178)$$

Using the 2018 CODATA recommended value of the the fundamental constants resulting the theoretical value of the Gravitational constant G :

$$G = 6.67448 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2 \quad (179)$$

3. Dimensionless unification of atomic physics and cosmology

In [22] and [23] resulting in the dimensionless unification of atomic physics and cosmology. The new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0$$

The fine-structure constant equals:

$$\alpha^2 = \frac{r_e}{a_0}$$

From these expressions we have:

$$l_{pl} = \frac{\alpha\sqrt{\alpha_G}r_e}{\alpha^2}$$

$$l_{pl} = \frac{\sqrt{\alpha_G}}{\alpha}r_e$$

$$\frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

The gravitational fine structure constant α_g is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3}$$

$$\alpha_g = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

$$\alpha_g = \sqrt{\frac{\alpha_G^3}{\alpha^6}} \quad (180)$$

with numerical value:

$$\alpha_g = 1.886837 \times 10^{-61}$$

Also equals:

$$\alpha_g^2 \cdot a^6 = \alpha_G^3$$

$$\alpha_g^2 = \alpha_G^3 \cdot a^{-6}$$

$$\alpha_g^2 = \left(\frac{\alpha_G}{a^2} \right)^3$$

Two approaches for Archimedes constant π are:

$$6 \cdot 7^{103} \cdot \pi^5 \approx 2^{300} \quad (181)$$

$$6 \cdot \pi^5 \approx 150^{3/2} - 1 \quad (182)$$

A approach for the Gelfond's constant e^π is:

$$e^\pi \simeq \frac{55}{\pi} \sqrt{\frac{2}{\ln \pi}} \quad (183)$$

A approximation expression that connects the golden ratio ϕ , the Archimedes constant π and the Euler's number e is:

$$2^2 11^2 e \simeq 3^4 \phi^5 \sqrt[3]{\pi} \quad (184)$$

Two approximations expressions that connects the golden ratio ϕ , the Archimedes constant π , the Euler's number e and the Euler's constant γ are:

$$4e^2 \gamma \ln^2(2\pi) \simeq \sqrt{3^3} \phi^5 \quad (185)$$

$$\sqrt{3^5} e \gamma \ln(2\pi) \sqrt[3]{\pi} \simeq 11^2 \quad (186)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant π , the Euler's number e and the Euler's constant γ is:

$$\alpha_g = [e \cdot \gamma \cdot \ln^2(2 \cdot \pi)]^{-1} \times 10^{-60} = 1.886837 \times 10^{-61} \quad (187)$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio ϕ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt[3]{3}\phi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (188)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant π is:

$$\alpha_g = \frac{\sqrt{3^5} \sqrt[3]{\pi}}{11^2} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (189)$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio ϕ and the Euler's constant γ is:

$$\alpha_g = \frac{7\phi\gamma^2}{2} \times 10^{-60} = 1,886826 \times 10^{-61} \quad (190)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant and the golden ratio ϕ is:

$$\alpha_g = \frac{2\pi}{3\phi^5} \times 10^{-60} = 1,888514 \times 10^{-61} \quad (191)$$

Resulting the unity formula for the gravitational fine-structure constant α_g :

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3} \quad (192)$$

Also apply the expressions:

$$(2 \cdot e \cdot a^2 \cdot N_A)^3 \cdot \alpha_g = 1$$

$$8 \cdot e^3 \cdot a^6 \cdot \alpha_g \cdot N_A^3 = 1$$

Resulting the unity formula for the gravitational fine-structure constant α_g :

$$a_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-3} \quad (193)$$

Also apply the expression:

$$(2 \cdot a_s \cdot a^2 \cdot NA)^3 \cdot a_g = i^{6i}$$

$$8 \cdot a_s^3 \cdot a^6 \cdot a_g \cdot NA^3 = i^{6i}$$

Resulting the unity formula for the gravitational fine-structure constant a_g :

$$a_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^{-3} \quad (194)$$

Also apply the expression:

$$(2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^3 \cdot a_g = i^{6i} \cdot e^3$$

$$8 \cdot 10^{21} \cdot a_w^3 \cdot a^9 \cdot a_g \cdot NA^3 = i^{6i} \cdot e^3$$

Resulting the unity formulas for the gravitational fine-structure constant a_g :

$$a_g = (10^7 \cdot a_w \cdot a_G^{1/2} \cdot e^{-1} \cdot a_s^{-1} \cdot a^{-1})^3 \quad (195)$$

Also apply the expressions:

$$a_g = 10^{21} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-3} \cdot a^{-3} \cdot e^{-3}$$

$$a_g \cdot a_s^3 \cdot a^3 \cdot e^3 = 10^{21} \cdot a_w^3 \cdot a_G^{3/2}$$

So the unity formula for the gravitational fine-structure constant a_g is:

$$a_g^2 = (10^{14} \cdot a_w^2 \cdot a_G \cdot e^{-2} \cdot a_s^{-2} \cdot a^{-2})^3 \quad (196)$$

Also apply the expressions:

$$a_g^2 = 10^{42} \cdot a_w^6 \cdot a_G^3 \cdot e^{-6} \cdot a_s^{-6} \cdot a^{-6}$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot a_g^2 = 10^{42} \cdot a_w^6 \cdot a_G^3$$

$$a_g^2 \cdot (e \cdot a_s \cdot a)^6 = (10^{14} \cdot a_w^2 \cdot a_G)^3$$

Resulting the unity formula for the gravitational fine-structure constant a_g :

$$a_g = i^{6i} \cdot (10^7 \cdot a_w \cdot a_G^{1/2} \cdot a_s^{-2} \cdot a^{-1})^3$$

$$a_g = 10^{21} \cdot i^{6i} \cdot (a_w \cdot a_G^{1/2} \cdot a_s^{-2} \cdot a^{-1})^3$$

$$a_g = 10^{21} \cdot i^{6i} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-6} \cdot a^{-3} \quad (197)$$

Also apply the expressions:

$$a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} = i^{2i} \cdot 10^7$$

$$a_g \cdot a_s^6 \cdot a^3 = 10^{21} \cdot i^{6i} \cdot a_w^3 \cdot a_G^{3/2}$$

So the unity formulas for the gravitational fine-structure constant a_g are:

$$a_g^2 = i^{6i} \cdot (10^{14} \cdot a_w^2 \cdot a_G \cdot a_s^{-4} \cdot a^{-2})^3$$

$$a_g^2 = 10^{42} \cdot i^{12i} \cdot (a_w^2 \cdot a_G \cdot a_s^{-4} \cdot a^{-2})^3$$

$$a_g^2 = 10^{42} \cdot i^{12i} \cdot a_w^6 \cdot a_G^3 \cdot a_s^{-12} \cdot a^{-6} \quad (198)$$

Also apply the expressions:

$$\alpha_g^2 \cdot \alpha_s^{12} \cdot a^6 \cdot a_w^{-6} \cdot a_G^{-3} = i^{12i} \cdot 10^{42}$$

$$(\alpha_s^6 \cdot a^3 \cdot \alpha_g)^2 = (10^{14} \cdot i^{4i} \cdot a_w^2 \cdot a_G)^3$$

$$\alpha_s^{12} \cdot a^6 \cdot \alpha_g^2 = 10^{42} \cdot i^{12i} \cdot a_w^6 \cdot a_G^3$$

So the unity formulas for the gravitational fine-structure constant α_g are:

$$\alpha_g = \left(\frac{10^7 \alpha_w \sqrt{\alpha_G}}{e \alpha_s a} \right)^3 \quad (199)$$

$$\alpha_g^2 = 10^{42} \left(\frac{\alpha_G a_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \quad (200)$$

$$\alpha_g = 10^{21} i^{6i} \left(\frac{\alpha_w \sqrt{\alpha_G}}{\alpha_s^2 a} \right)^3 \quad (201)$$

$$\alpha_g^2 = 10^{42} i^{12i} \left(\frac{\alpha_G a_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (202)$$

Laurent Nottale in [24] suggests a large-number relation:

$$\alpha \frac{m_{pl}}{m_e} = \left(\frac{L}{l_{pl}} \right)^{\frac{1}{3}}$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant α_g :

$$\alpha \frac{m_{pl}}{m_e} = \left(l_{pl} \sqrt{\Lambda} \right)^{-\frac{1}{3}}$$

$$\alpha_g = l_{pl} \sqrt{\Lambda}$$

$$\alpha_g = \sqrt{\frac{G \hbar \Lambda}{c^3}}$$

So the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

$$\Lambda = \alpha_g^2 \frac{c^3}{G\hbar}$$

$$\Lambda = \frac{G}{\hbar^4} \left(\frac{m_e}{a} \right)^6$$

Resulting the dimensionless unification of the atomic physics and the cosmology:

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (203)$$

$$(2 \cdot e \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = 1 \quad (204)$$

Now we will use the unity formulas of the dimensionless unification of atomic physics and cosmology to find the equations of the cosmological constant. For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (205)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (206)$$

$$(2 \cdot a_s \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \quad (207)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} \left(2\alpha_s a^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (208)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6} \quad (209)$$

$$(2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \quad (210)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 \left(2 \cdot 10^7 \alpha_w a^3 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (211)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} \left(\frac{\alpha_G a_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3$$

$$l_{pl}^2 \Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \quad (212)$$

$$e^6 \cdot \alpha_s^6 \cdot \alpha^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot \alpha_w^6 \quad (213)$$

For the cosmological constant equals:

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \frac{c^3}{G \hbar} \quad (214)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

$$l_{pl}^2 \Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (215)$$

$$\alpha_s^{12} \cdot \alpha^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6 \quad (216)$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar} \quad (217)$$

In [25] we presented the Equation of the Universe:

$$\frac{\Lambda G \hbar}{c^3} = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (218)$$

$$e^{6\pi} \frac{\Lambda G \hbar}{c^3} = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (219)$$

4. Dimensionless Unification of the Microcosm and the Macrocosm

In [26] , [27] and [28] we presented the law of the gravitational fine-structure constant α_g followed by ratios of maximum and minimum theoretical values for natural quantities. This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length l , time t , speed u and temperature T have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}} \quad (220)$$

Energy E , mass M , action A , momentum P and entropy S have another min/max ratio, which is the square of α_g :

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}} \quad (221)$$

Force F has min/max ratio which is α_g^4 :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \quad (222)$$

Mass density has min/max ratio which is α_g^5 :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \quad (223)$$

In [29] we presented the Unification of the Microcosm and the Macrocosm. The length Planck l_{pl} defined as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl}c} = \frac{h}{2\pi m_{pl}c} = \frac{m_p r_p}{4m_{pl}}$$

The classical electron radius is given as:

$$r_e = \alpha^2 \alpha_0 = \frac{\hbar \alpha}{m_e c} = \frac{\lambda_c \alpha}{m_e c^2} = \frac{\mu_0}{4\pi} \frac{q_e^2}{m_e} = \frac{k_e q_e^2}{m_e c^2} = \frac{\alpha^3}{4\pi R_\infty}$$

The Bohr radius α_0 is defined as:

$$\alpha_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi \alpha}$$

Thus respectively the Compton wavelength λ_c of the electron with mass m_e is given by the formula:

$$\lambda_c = \frac{2\pi r_e}{\alpha} = \frac{h}{m_e c}$$

The fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi \alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e} \right)^2 = \left(\alpha \frac{l_{pl}}{r_e} \right)^2 = \left(\frac{l_{pl}}{\alpha \alpha_0} \right)^2$$

We proposed to be a lattice structure, in which its unit cells have sides of length $2 \cdot e \cdot l_{pl}$. Perhaps for the minimum distance l_{min} apply:

$$l_{min} = 2 \cdot e \cdot l_{pl} \quad (224)$$

From expressions apply:

$$\begin{aligned} \cos \alpha^{-1} &= e^{-1} \\ \cos \alpha^{-1} \cdot l_{min} &= 2 \cdot l_{pl} \\ \cos \alpha^{-1} &= \frac{2l_{pl}}{l_{min}} \end{aligned} \quad (225)$$

For the Bohr radius α_0 apply:

$$\alpha_0 = N_A \cdot l_{\min}$$

$$\alpha_0 = 2 \cdot e \cdot N_A \cdot |p| \quad (226)$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H = c \cdot H_0^{-1}$$

For the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{L_H^2}{R_d^2}$$

$$\Omega_\Lambda = \left(\frac{L_H}{R_d} \right)^2$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\%$$

So from this expression apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (227)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (228)$$

The maximum distance $l_{\max}$ corresponds to the distance of the universe $l_U = c \cdot H_0^{-1}$. Therefore:

$$l_{\max} = l_U = c \cdot H_0^{-1}$$

Length l has the max/min ratio which is:

$$\alpha_g = \frac{l_{\min}}{l_{\max}} \quad (229)$$

The maximum distance $l_{\max}$ corresponds to the distance of the universe:

$$l_{\max} = L_H = c \cdot H_0^{-1} = \alpha_g^{-1} \cdot l_{\min} \quad (230)$$

The value of the maximum distance $l_{\max}$ is $l_{\max}=4.656933 \times 10^{26}$ m. In [30] , [31] , [32] and [33] we presented the Dimensionless theory for everything. In [34] we presented the New Large Number Hypothesis of the universe. The diameter of the observable universe will be calculated to be equal to the product of the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot R_U = N_1 \cdot \lambda_c \quad (231)$$

Diameter of the universe = ratio of electric force to gravitational force $\times$ reduced Compton wavelength of the electron

So the expressions for the radius of the observable universe are:

$$\frac{2R_U}{r_e} = \frac{N_1}{\alpha} \quad (232)$$

$$\frac{2R_U}{r_e} = \frac{1}{\mu \alpha_G} \quad (233)$$

$$\frac{2R_U}{\alpha_0} = \alpha N_1 \quad (234)$$

$$\frac{2R_U}{l_{\min}} = \alpha N_1 N_A \quad (235)$$

The expression between the radius of the observable universe R_U with the Planck length l_p is:

$$R_U = e \cdot \alpha \cdot N_1 \cdot N_A \cdot l_p \quad (236)$$

The expression between the radius of the observable universe R_U with the minimum distance $l_{\min}$ is:

$$2 \cdot R_U = \alpha \cdot N_1 \cdot N_A \cdot l_{\min} \quad (237)$$

So apply the expressions for the radius of the observable universe:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 \quad (238)$$

$$R_U = \frac{N_1}{2\alpha} r_e \quad (239)$$

$$R_U = \frac{1}{2\mu \alpha_G} r_e \quad (240)$$

$$R_U = \frac{m_{pl}^2 r_e}{2m_e m_p} \quad (241)$$

$$R_U = \frac{\hbar c r_e}{2G m_e m_p} \quad (242)$$

$$R_U = \frac{\alpha \hbar}{2G m_e^2 m_p} \quad (243)$$

For the value of the radius of the universe apply $R_U = 4.38 \times 10^{26}$ m. The expressions for the gravitational constant are:

$$G = \frac{\hbar c r_e}{2m_e m_p} \frac{1}{R_U} \quad (244)$$

$$G = \frac{\alpha \hbar}{2m_e^2 m_p} \frac{1}{R_U} \quad (245)$$

The mass Planck m_{pl} can be defined by three fundamental natural constants, the speed of light in vacuum c , the reduced Planck constant $\hbar$ and the gravity constant G as:

$$m_{pl} = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{l_{pl} c} = \frac{\mu_0 q_{pl}^2}{4\pi l_{pl}}$$

In [35] J. Forsythe and T. Valev found an extended mass relation for seven fundamental masses. Six of these masses are successfully identified as mass of the observable universe, Eddington mass limit of the most massive stars, mass of hypothetical quantum “Gravity Atom” whose gravitational potential is equal to electrostatic potential, Planck mass, Hubble mass and mass dimension constant relating masses of stable particles with coupling constants of fundamental interactions. In [36] we found a similar mass relation for seven fundamental masses:

$$M_n = a^{-1} \cdot a_g^{(2-n)/3} \cdot m_e \quad (246)$$

$n=0,1,2,3,4,5,6$

For $n=0$ M_0 is the minimum mass M_{min} :

$$M_0 = a^{-1} \cdot a_g^{(2-0)/3} \cdot m_e$$

$$M_0 = a^{-1} \cdot a_g^{2/3} \cdot m_e \quad (247)$$

For $n=1$ M_1 is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass, most likely a yet unobserved light particle:

$$M_1 = a^{-1} \cdot a_g^{(2-1)/3} \cdot m_e$$

$$M_1 = a^{-1} \cdot a_g^{1/3} \cdot m_e \quad (248)$$

For $n=2$ M_2 is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass:

$$M_2 = a^{-1} \cdot a_g^{(2-2)/3} \cdot m_e$$

$$M_2 = a^{-1} \cdot m_e \quad (249)$$

For $n=3$ M_3 is the Planck mass:

$$M_3 = a^{-1} \cdot a_g^{(2-3)/3} \cdot m_e$$

$$M_3 = a^{-1} \cdot a_g^{-1/3} \cdot m_e \quad (250)$$

For $n=4$ is the central mass of a hypothetical quantum “Gravity Atom”.

$$M_4 = a^{-1} \cdot a_g^{(2-4)/3} \cdot m_e$$

$$M_4 = a^{-1} \cdot a_g^{-2/3} \cdot m_e \quad (251)$$

For $n=5$ is of the order of the Eddington mass limit of the most massive stars:

$$M_5 = a^{-1} \cdot a_g^{(2-5)/3} \cdot m_e$$

$$M_5 = a^{-1} \cdot a_g^{-1} \cdot m_e \quad (252)$$

For n=6 is the mass of the Hubble sphere and the mass of the observable universe.

$$M_6 = a^{-1} \cdot a_g^{(2-5)/3} \cdot m_e$$

$$M_6 = a^{-1} \cdot a_g^{-4/3} \cdot m_e \quad (253)$$

The similar mass relation for seven fundamental masses is:

$$M_n = a_g^{-n/3} \cdot M_{\min} \quad (254)$$

$$n = 0, 1, 2, 3, 4, 5, 6$$

For n=0 M₀ is the minimum mass:

$$M_0 = M_{\min} \quad (255)$$

For n=1 M₁ is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass, most likely a yet unobserved light particle:

$$M_1 = a_g^{-1/3} \cdot M_{\min} \quad (256)$$

For n=2 M₂ is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass:

$$M_2 = a_g^{-2/3} \cdot M_{\min} \quad (257)$$

For n=3 M₃ is the Planck mass m_{pl} :

$$M_3 = a_g^{-1} \cdot M_{\min} \quad (258)$$

For n=4 is the central mass of a hypothetical quantum “Gravity Atom”.

$$M_4 = a_g^{-4/3} \cdot M_{\min} \quad (259)$$

For n=5 is of the order of the Eddington mass limit of the most massive stars:

$$M_5 = a_g^{-5/3} \cdot M_{\min} \quad (260)$$

For n=6 is the mass of the Hubble sphere and the mass of the observable universe.

$$M_6 = a_g^{-2} \cdot M_{\min} \quad (261)$$

The following applies to the minimum mass M_{min}:

$$M_{\min} c^2 = \frac{\hbar}{t_{\max}}$$

$$M_{\min} c^2 = \hbar H_0$$

$$M_{\min} = \frac{\hbar H_0}{c^2} \quad (262)$$

$$M_{\min} = \frac{\hbar}{cl_{\max}} \quad (263)$$

So apply the expressions:

$$M_{\min} = \frac{\hbar}{c} \sqrt{\Lambda} \quad (264)$$

$$M_{\min} = \frac{m_{pl}^2}{M_{\max}} \quad (265)$$

$$M_{\min} = \frac{m_{pl}^2}{M_{\max}} \quad (266)$$

Therefore for the minimum mass $M_{\min}$ apply:

$$M_{\min} = \alpha_g m_{pl} \quad (267)$$

$$M_{\min} = \frac{\alpha_G}{\alpha^3} m_e \quad (268)$$

$$M_{\min} = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \quad (269)$$

$$M_{\min} = (2 \cdot e \cdot N_A)^{-2} \cdot \alpha^{-1} \cdot m_e \quad (270)$$

For the value of the minimum mass $M_{\min}$ apply:

$$M_{\min} = 4.06578 \times 10^{-69} \text{ kg}$$

Mass M have max/min ratio, which is the square of α_g :

$$\alpha_g^2 = \frac{M_{\min}}{M_{\max}} \quad (271)$$

For the maximum mass $M_{\max}$ applies:

$$M_{\max} = \frac{F_{\max} l_{\max}}{c^2} \quad (272)$$

$$M_{\max} = \frac{m_{pl}^2}{M_{\min}} \quad (273)$$

$$M_{\max} = \alpha^{-1} \cdot \alpha_g^{-4/3} \cdot m_e \quad (274)$$

$$M_{\max} = \alpha^3 \cdot \alpha_G^{-2} \cdot m_e \quad (275)$$

For the value of the maximum mass $M_{\max}$ apply:

$$M_{\max} = 1.153482 \times 10^{53} \text{ kg}$$

Also apply the expressions:

$$m_{pl} \cdot L_{\max} = m_{\max} \cdot l_{pl} \quad (276)$$

$$l_{\max}^2 \cdot M_{\min} = l_{\min}^2 \cdot M_{\max} \quad (277)$$

R. Adler in [37] calculated the energy ratio in cosmology, the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies, the rest energy of the electron E_e , and the binding energy of the hydrogen atom E_H . The rest energy of the electron E_e is defined as:

$$E_e = m_e c^2$$

The binding energy of the hydrogen atom E_H is defined as:

$$E_H = \frac{m_e e^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:

$$\frac{E_H}{E_e} = \frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales, the Planck energy density ρ_{pl} , and the density of the dark energy ρ_Λ . The Planck energy density is defined as:

$$\rho_{pl} = \frac{E_{pl}}{l_{pl}} = \frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density ρ_Λ is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

So for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2 \varphi^{-5}}{3^3 \pi \varphi^5} \times 10^{-120} \quad (278)$$

The Planck time t_{pl} is defined as:

$$t_{pl} = \frac{l_{pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} = \frac{\hbar}{m_{pl} c^2}$$

For the minimum distance l_{min} apply:

$$l_{min} = 2 \cdot e \cdot l_{pl}$$

So for the minimum time t_{min} apply:

$$t_{min} = \frac{l_{min}}{c}$$

$$t_{\min} = \frac{2el_{pl}}{c}$$

$$t_{\min} = 2 \cdot e \cdot t_{pl}$$

From expressions apply:

$$\cos \alpha^{-1} = e^{-1}$$

$$\cos \alpha^{-1} \cdot t_{\min} = 2 \cdot t_{pl}$$

$$\cos \alpha^{-1} = \frac{2t_{pl}}{t_{\min}} \quad (279)$$

The maximum time period $t_{\max}$ is the time from the time of Bing Bang to the present day. This time period corresponds to the time of the universe $t_U = H_0^{-1}$. Therefore:

$$t_{\max} = t_U = H_0^{-1}$$

Time t has the min/max ratio which is.

$$\alpha_g = \frac{t_{\min}}{t_{\max}}$$

$$\alpha_g = t_{\min} \cdot H_0 \quad (280)$$

$$\alpha_g = 2 \cdot e \cdot t_{pl} \cdot H_0 \quad (281)$$

From [38] the gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. The time quantum in the brain t_B , the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \quad (282)$$

For the age of the universe apply:

$$T_U = \frac{R_U}{c} \quad (283)$$

$$T_U = \frac{N_1 r_e}{2ac} \quad (284)$$

$$T_U = \frac{r_e}{2\mu\alpha_G c} \quad (285)$$

$$T_U = \frac{\alpha N_1 \alpha_0}{2c} \quad (286)$$

$$T_U = \frac{\alpha \hbar}{2c G m_e^2 m_p} \quad (287)$$

$$T_U = \frac{\hbar r_e}{2Gm_e m_p} \quad (288)$$

For the value of the age of the universe apply $T_U = 1.46 \times 10^{18}$ s. The expressions for the gravitational constant are:

$$G = \frac{\alpha \hbar}{2cm_e^2 m_p} \frac{1}{T_U} \quad (289)$$

$$G = \frac{\hbar r_e}{2m_e m_p} \frac{1}{T_U} \quad (290)$$

Laurent Nottale assumed a large-number relation:

$$\alpha \frac{m_{pl}}{m_e} = \left(\frac{L}{l_{pl}} \right)^{\frac{1}{3}}$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant α_g :

$$\alpha \frac{m_{pl}}{m_e} = \left(l_{pl} \sqrt{\Lambda} \right)^{-\frac{1}{3}}$$

$$\alpha_g = l_{pl} \sqrt{\Lambda}$$

$$\alpha_g = \sqrt{\frac{G\hbar\Lambda}{c^3}}$$

In the papers [39] was presented the theoretical value for the Hubble Constant. The density parameter for dark energy is defined as:

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$

Also for the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{c^2}{R_d^2 H_0^2}$$

So for the density parameter for dark energy apply:

$$\Omega_{\Lambda} = \left(\frac{L_H}{R_d} \right)^2 = \frac{L_H^2}{R_d^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is $\Omega_{\Lambda}=2 \cdot e^{-1}=0.73576=73.57\%$. So from this expression apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (291)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (292)$$

For the cosmological constant Λ equals:

$$\Lambda = \frac{6H_0^2}{ec^2} \quad (293)$$

So apply the expressions:

$$\frac{6H_0^2}{\Lambda c^2} = e \quad (294)$$

$$\frac{H_0^2}{\Lambda c^2} = \frac{e}{6} \quad (295)$$

$$\cos \alpha^{-1} = \frac{\Lambda c^2}{6H_0^2} \quad (296)$$

So the formula for the Hubble Constant is:

$$H_0 = c \sqrt{\frac{e}{6} \Lambda} \quad (297)$$

For the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

So the formulas for the Hubble Constant are:

$$H_0 = \frac{\alpha_g}{t_{pl}} \sqrt{\frac{e}{6}} \quad (298)$$

$$H_0 = \frac{\alpha_g c}{l_{pl}} \sqrt{\frac{e}{6}} \quad (299)$$

Also apply the expression:

$$(H_0 t_{pl})^2 = \frac{e}{6} \alpha_g^2 \quad (300)$$

These equations calculate the theoretical value of the Hubble Constant $H_0 = 2.355683 \times 10^{-18} \text{ s}^{-1} = 72.69 \text{ (km/s)/Mpc}$.
The cosmological constant Λ equals:

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

So the formula for the Hubble Constant is:

$$H_0 = \frac{cl_{pl}^2}{r_e^2} \sqrt{\frac{e}{6}} \quad (301)$$

Also the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 \frac{c^3}{G\hbar}$$

So the formula for the Hubble Constant is:

$$H_0 = \alpha_g \sqrt{\frac{ec^5}{6G\hbar}} \quad (302)$$

Also apply the expression:

$$\frac{G\hbar H_0^2}{c^5} = \frac{e}{6} \alpha_g^2 \quad (303)$$

The cosmological constant Λ equals:

$$\Lambda = \frac{G}{\hbar^4} \left(\frac{m_e}{a} \right)^6$$

So the formula for the Hubble Constant is:

$$H_0 = \frac{cm_e^3}{\alpha^3 \hbar^2} \sqrt{\frac{eG}{6}} \quad (304)$$

From the dimensionless unification of the atomic physics and the cosmology apply:

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (305)$$

$$(2 \cdot e \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = 1 \quad (306)$$

For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (307)$$

So the formula for the Hubble Constant is:

$$H_0 = \frac{1}{\left(2e\alpha^2 N_A \right)^3} \sqrt{\frac{ec^5}{6G\hbar}} \quad (308)$$

Also apply the expression:

$$\frac{G\hbar H_0^2}{c^5} = \frac{1}{6e^5 (2\alpha^2 N_A)^6} \quad (309)$$

From the dimensionless unification of atomic physics and cosmology apply:

$$a_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (310)$$

$$(2 \cdot a_s \cdot a^2 \cdot N_A)^6 \cdot |p|^2 \cdot \Lambda = i^{12i} \quad (311)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} (2\alpha_s a^2 N_A)^{-6} \frac{c^3}{G\hbar} \quad (312)$$

So the formulas for the Hubble Constant are:

$$H_0 = \frac{i^{6i}}{(2\alpha_s a^2 N_A)^3} \sqrt{\frac{ec^5}{6G\hbar}} \quad (313)$$

$$H_0 = \frac{1}{(2e^\pi \alpha_s a^2 N_A)^3} \sqrt{\frac{ec^5}{6G\hbar}} \quad (314)$$

Also apply the expression:

$$\frac{G\hbar H_0^2}{c^5} = \frac{e}{48 (e^\pi \alpha_s a^2 N_A)^3} \quad (315)$$

From the dimensionless unification of atomic physics and cosmology apply:

$$a_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-3}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6} \quad (316)$$

$$(2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^6 \cdot |p|^2 \cdot \Lambda = i^{12i} \cdot e^6 \quad (317)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 (2 \cdot 10^7 \alpha_w a^3 N_A)^{-6} \frac{c^3}{G\hbar} \quad (318)$$

So the formulas for the Hubble Constant are:

$$H_0 = \frac{i^{6i} e^3}{(2 \cdot 10^7 \alpha_w a^3 N_A)^3} \sqrt{\frac{ec^5}{6G\hbar}} \quad (319)$$

$$H_0 = \frac{1}{(2 \cdot 10^7 e^{\pi-1} \alpha_w a^3 N_A)^3} \sqrt{\frac{ec^3}{6G\hbar}} \quad (320)$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g^2 = 10^{42} \left(\frac{\alpha_G a_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \quad (321)$$

$$l_{pl}^2 \Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \quad (322)$$

$$e^6 \cdot \alpha s^6 \cdot \alpha^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot \alpha G^3 \cdot \alpha w^6 \quad (323)$$

For the cosmological constant equals:

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \frac{c^3}{G \hbar} \quad (324)$$

So the formula for the Hubble Constant is:

$$H_0 = 10^{21} \left(\frac{\alpha_w \sqrt{\alpha_G}}{e \alpha_s \alpha} \right)^3 \sqrt{\frac{ec^5}{6G\hbar}} \quad (325)$$

Also apply the expression:

$$\frac{G\hbar H_0^2}{c^5} = \frac{10^{42}}{6e^5} \left(\frac{\alpha_w^2 \alpha_G}{\alpha_s^2 \alpha^2} \right)^3 \quad (326)$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g^2 = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (327)$$

$$l_{pl}^2 \Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (328)$$

$$\alpha s^{12} \cdot \alpha^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha G^3 \cdot \alpha w^6 \quad (329)$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar} \quad (330)$$

So the formulas for the Hubble Constant are:

$$H_0 = \left(\frac{10^7 i^{2i} \alpha_w \sqrt{\alpha_G}}{\alpha \alpha_s^2} \right)^3 \sqrt{\frac{ec^5}{6G\hbar}} \quad (331)$$

$$H_0 = \left(\frac{10^7 \alpha_w \sqrt{\alpha_G}}{e^\pi \alpha \alpha_s^2} \right)^3 \sqrt{\frac{ec^5}{6G\hbar}} \quad (332)$$

Also apply the expression:

$$\frac{6G\hbar H_0^2}{ec^5} = \left(\frac{10^{14} \alpha_w^2 \alpha_G}{e^{2\pi} \alpha^2 \alpha_s^4} \right)^3 \quad (333)$$

$$6e^{6\pi} \frac{G\hbar H_0^2}{c^5} = e \left(\frac{10^{14} \alpha_w^2 \alpha_G}{\alpha^2 \alpha_s^4} \right)^3 \quad (334)$$

$$6e^{5\pi} \frac{G\hbar H_0^2}{c^5} = \frac{1}{\alpha_s^{11}} \left(\frac{10^{14} \alpha_w^2 \alpha_G}{\alpha^2} \right)^3 \quad (335)$$

So the Equations of the Universe are:

$$6e^{5\pi} \frac{G\hbar H_0^2}{c^5} = 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{11}} \quad (336)$$

$$e^{7\pi} \frac{G\hbar \Lambda^2}{cH_0^2} = 6 \cdot 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{13}} \quad (337)$$

5. Poincaré dodecahedral space

In [40] and [41] we proved that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter, dark matter and dark energy. In 2003 J.-P. Luminet in [42] proved that the long-wavelength modes tend to be relatively lowered only in a special family of finite, multi connected spaces that are called “well-proportioned spaces” because they have a similar extent in all three dimensions. The sum of the contributions to the total density parameter Ω_0 at the current time is $\Omega_0 = 1.02 \pm 0.02$. Current observations suggest that we live in a dark energy dominated Universe with $\Omega_\Lambda = 0.73$, $\Omega_D = 0.23$ and $\Omega_B = 0.04$ [43]. The assessment of baryonic matter at the current time was assessed by WMAP to be $\Omega_B = 0.044 \pm 0.004$. From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B = e^{-\pi} = i^{2i} = 0.0432 = 4.32\% \quad (338)$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^i + 1 = 0 \quad (339)$$

$$\Omega_B^i = i^2 \quad (340)$$

$$\Omega_B^{2i} = 1 \quad (341)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B = e^{-1} \cdot \alpha_s \quad (342)$$

$$\Omega_B = a_w^{-1} \cdot \alpha_s^2 \cdot 10^{-7} \quad (343)$$

$$\Omega_B = 2^{-1} \cdot \alpha_s \cdot (e^{i/a} + e^{-i/a}) \quad (344)$$

$$\Omega_B = 2 \cdot N_A \cdot \alpha_s \cdot a \cdot \alpha_G^{1/2} \quad (345)$$

$$\Omega_B = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (346)$$

$$\Omega_B = 2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot \alpha_G^{1/2} \quad (347)$$

$$\Omega_B = 10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} \quad (348)$$

In [44] we presented the solution for the Density Parameter of Dark Energy. From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\% \quad (349)$$

So apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (350)$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot \cos \alpha^{-1} \quad (351)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{\Omega_\Lambda}{2} \quad (352)$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda = e^{i/a} + e^{-i/a} \quad (353)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (354)$$

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (355)$$

$$\Omega_\Lambda = 2 \cdot i^{2i} \cdot a_s^{-1} \quad (356)$$

$$\Omega_\Lambda = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1} \quad (357)$$

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (358)$$

$$\Omega_\Lambda = 4 \cdot a \cdot a_G^{1/2} \cdot N_A \quad (359)$$

$$\Omega_\Lambda = i^{8i} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot N_A^{-2} \quad (360)$$

$$\Omega_\Lambda = 10^7 \cdot i^{4i} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot N_A^{-1} \quad (361)$$

$$\Omega_\Lambda = 8 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G \cdot a_s^{-1} \quad (362)$$

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter $\Omega_D = 0.23$. From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D = 2 \cdot e^{1-n} = 2 \cdot e \cdot i^{2i} = 0.2349 = 23.49\% \quad (363)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D = 2 \cdot a_s \quad (364)$$

$$\Omega_D = 2 \cdot 10^7 \cdot e^{-1} \cdot a_w \quad (365)$$

$$\Omega_D = 2 \cdot (i^{2i} \cdot 10^7 \cdot a_w)^{1/2} \quad (366)$$

$$\Omega_D = 4 \cdot i^{2i} \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (367)$$

$$\Omega_D = 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (368)$$

$$\Omega_D = 4 \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot N_A \quad (369)$$

$$\Omega_D = 16 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (370)$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D = 2 \cdot e \cdot \Omega_B \quad (371)$$

The relationship between the density parameter of dark energy, dark matter and baryonic matter is:

$$\Omega_D \cdot \Omega_\Lambda = 4 \cdot \Omega_B \quad (372)$$

From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda$$

$$\Omega_0 = e^{-n} + 2 \cdot e^{1-n} + 2 \cdot e^{-1} \quad (373)$$

$$\Omega_0 = 1.0139 \quad (374)$$

In [45] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J.-P. Uzan presents a simple geometrical model of a finite, positively curved space, the Poincaré dodecahedral space – which accounts for WMAP's observations with no fine-tuning required. Circle searching (Cornish, Spergel and Starkman, 1998) may confirm the model's topological predictions, while upcoming Planck Surveyor data may confirm its predicted density of:

$$\Omega_0 = 1.013 > 1$$

In [46] we proposed a possible solution for the cosmological parameters. The density parameter for normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0.04321 = 4.32\% \quad (375)$$

The density parameter for dark matter is:

$$\Omega_D = 6 \cdot e^{-n} = 6 \cdot i^{2i} = 0.25928 = 25.92\% \quad (376)$$

The density parameter for the dark energy is:

$$\Omega_\Lambda = 17 \cdot e^{-n} = 17 \cdot i^{2i} = 0.73463 = 73.46\% \quad (377)$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = 24 \cdot e^{-n} = 24 \cdot i^{2i} = 1.03713 \quad (378)$$

In [47] we proposed a possible solution for the Equation of state in cosmology. In cosmology, the equation of state of a perfect fluid is characterized by a dimensionless number w , equal to the ratio of its pressure p to its energy density ρ :

$$w = \frac{p}{\rho}$$

From the dimensionless unification of the fundamental interactions the state equation w has value:

$$w = -24 \cdot e^{-n} = -24 \cdot i^{2i} = -1.037134 \quad (379)$$

From the dimensionless unification of the fundamental interactions for the measurable ordinary energy E(O) apply:

$$E(O) = i^{2i} \cdot m \cdot c^2$$

Also from the dimensionless unification of the fundamental interactions for the sum of the dark energy with the dark matter density of the universe E(D) apply:

$$E(D) = 23 \cdot i^{2i} \cdot m \cdot c^2$$

So for the total energy E apply:

$$E = K \cdot m \cdot c^2$$

$$E = E(O) + E(D)$$

$$E = i^{2i} \cdot m \cdot c^2 + 23 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = (i^{2i} + 23 \cdot i^{2i}) \cdot m \cdot c^2$$

$$E = 24 \cdot i^{2i} \cdot m \cdot c^2 \quad (380)$$

Other forms of the equation are:

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 + 12 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 - i^2 \cdot 12 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 - 12 \cdot i^{2i} \cdot m \cdot (i \cdot c)^2$$

$$12 \cdot i^{2i} \cdot m \cdot (i \cdot c)^2 + E = 12 \cdot i^{2i} \cdot m \cdot c^2 \quad (381)$$

6. Euler's identity in unification of the fundamental interactions

In the paper [48] was presented the article Euler's identity in unification of the fundamental interactions. We presented the recommended value for the strong coupling constant $\alpha_s = \alpha_s(M_Z) = e^{1-n} = 0.11748...$ This value is the current world average value for the coupling evaluated at the Z-boson mass scale. It fits perfectly in the measurement of the strong coupling constant of the European organization for nuclear research (CERN). Also for the value of the strong coupling constant we have the equivalent expressions $\alpha_s = \alpha_s(M_Z) = e \cdot e^{-n} = e \cdot i^{2i} = i^{-2i/n} \cdot i^{2i} = i^{2i-(2i/n)} = i^{2i(n-1)/n}$. So apply the expressions:

$$e^n \cdot \alpha_s = e \quad (382)$$

$$e^n = e \cdot \alpha_s^{-1} \quad (383)$$

$$e^{-n} = e^{-1} \cdot \alpha_s \quad (384)$$

From Euler's identity and the equation $e^n = e \cdot \alpha_s^{-1}$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(e \cdot \alpha_s^{-1})^i + 1 = 0 \quad (385)$$

$$e^i \cdot \alpha_s^{-i} + 1 = 0 \quad (386)$$

$$e^i \cdot a s^{-i} = -1 \quad (387)$$

$$e^i \cdot a s^{-i} = i^2 \quad (388)$$

$$e^i = i^2 \cdot a s^i \quad (389)$$

$$e^i = -a s^i \quad (390)$$

$$e^i + a s^i = 0 \quad (391)$$

Also from Euler's identity and the equation $e^{-n} = e^{-1} \cdot a s$ resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$(e^{-1} \cdot a s)^i + 1 = 0 \quad (392)$$

$$e^{-i} \cdot a s^i + 1 = 0 \quad (393)$$

$$e^{-i} \cdot a s^i = -1 \quad (394)$$

$$e^{-i} \cdot a s^i = i^2 \quad (395)$$

$$a s^i = -e^i \quad (396)$$

$$a s^i + e^i = 0 \quad (397)$$

$$e^i \cdot a s^{-i} = i^2 \quad (398)$$

$$a s^i = i^2 \cdot e^i \quad (399)$$

From the dimensionless unification of the strong nuclear and the weak nuclear interactions equals:

$$e^n \cdot a s^2 = 10^7 \cdot a w \quad (400)$$

From Euler's identity and the equation $e^n \cdot a s^2 = 10^7 \cdot a w$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(10^7 \cdot a s^{-2} \cdot a w)^i + 1 = 0 \quad (401)$$

$$10^{7i} \cdot a s^{-2i} \cdot a w^i + 1 = 0 \quad (402)$$

$$10^{7i} \cdot a s^{-2i} \cdot a w^i = -1 \quad (403)$$

$$10^{7i} \cdot a s^{-2i} \cdot a w^i = i^2 \quad (404)$$

$$a s^{2i} = i^2 \cdot 10^{7i} \cdot a w^i \quad (405)$$

Also from Euler's identity resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$(10^{-7} \cdot a s^2 \cdot a w^{-1})^i + 1 = 0 \quad (406)$$

$$10^{-7i} \cdot a s^{2i} \cdot a w^{-i} + 1 = 0 \quad (407)$$

$$10^{-7i} \cdot a s^{2i} \cdot a w^{-i} = -1 \quad (408)$$

$$10^{-7i} \cdot a s^{2i} \cdot a w^{-i} = i^2 \quad (409)$$

$$a s^{2i} = i^2 \cdot 10^{7i} \cdot a w^i \quad (410)$$

From the dimensionless unification of the strong nuclear and the electromagnetic interactions equals:

$$e^n \cdot a_s \cdot \cos a^{-1} = 1 \quad (411)$$

From Euler's identity and the equation $e^n \cdot a_s \cdot \cos a^{-1} = 1$ resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$(a_s \cdot \cos a^{-1})^i + 1 = 0 \quad (412)$$

$$a_s^i \cdot (\cos a^{-1})^i + 1 = 0 \quad (413)$$

$$a_s^i \cdot (\cos a^{-1})^i = -1 \quad (414)$$

$$a_s^i \cdot (\cos a^{-1})^i = i^2 \quad (415)$$

$$a_s^i = i^2 \cdot (\cos a^{-1})^{-i} \quad (416)$$

$$a_s^i = -(\cos a^{-1})^{-i} \quad (417)$$

$$a_s^i + (\cos a^{-1})^{-i} = 0 \quad (418)$$

From the dimensionless unification of the strong nuclear and the electromagnetic interactions equals:

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \quad (419)$$

So from Euler's identity resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$[2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})]^i + 1 = 0 \quad (420)$$

$$2^{-i} \cdot a_s^i \cdot (e^{i/a} + e^{-i/a})^i + 1 = 0 \quad (421)$$

$$2^{-i} \cdot a_s^i \cdot (e^{i/a} + e^{-i/a})^i = -1 \quad (422)$$

$$2^{-i} \cdot a_s^i \cdot (e^{i/a} + e^{-i/a})^i = i^2 \quad (423)$$

$$a_s^i \cdot (e^{i/a} + e^{-i/a})^i = 2^i \cdot i^2 \quad (424)$$

$$a_s^i = 2^i \cdot i^2 \cdot (e^{i/a} + e^{-i/a})^{-i} \quad (425)$$

$$a_s^i = -2^i \cdot (e^{i/a} + e^{-i/a})^{-i} \quad (426)$$

$$a_s^i + 2^i \cdot (e^{i/a} + e^{-i/a})^{-i} = 0 \quad (427)$$

From the dimensionless unification of the weak nuclear and the electromagnetic interactions equals:

$$10^7 \cdot e^n \cdot a_w \cdot \cos a^{-1} = e \quad (428)$$

$$10^7 \cdot e^n \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \quad (429)$$

So from Euler's identity resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$(10^7 \cdot e^{-1} \cdot a_w \cdot \cos a^{-1})^i + 1 = 0 \quad (430)$$

$$10^{7i} \cdot e^{-i} \cdot a_w^i \cdot (\cos a^{-1})^i + 1 = 0 \quad (431)$$

$$10^{7i} \cdot e^{-i} \cdot a_w^i \cdot (\cos a^{-1})^i = -1 \quad (432)$$

$$10^{7i} \cdot e^{-i} \cdot a w^i \cdot (\cos a^{-1})^i = i^2 \quad (433)$$

$$10^{7i} \cdot a w^i \cdot (\cos a^{-1})^i = i^2 \cdot e^i \quad (434)$$

$$10^{7i} \cdot a w^i \cdot (\cos a^{-1})^i = -e^i \quad (435)$$

$$10^{7i} \cdot a w^i \cdot (\cos a^{-1})^i + e^i = 0 \quad (436)$$

From the dimensionless unification of the gravitational and the electromagnetic force equals:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (437)$$

So from Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^{in} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (438)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = -e^{in} \quad (439)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^2 \cdot e^{in} \quad (440)$$

$$4 \cdot a^2 \cdot a_G \cdot N_A^2 = i^2 \cdot e^{in-2} \quad (441)$$

From Euler's identity, the equation $4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$ and the equation $e^n \cdot a_s = e$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^n \cdot a_s = e$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$e^{in} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (442)$$

$$(e \cdot a_s^{-1})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (443)$$

$$e^i \cdot a_s^{-i} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (444)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = -e^i \cdot a_s^{-i} \quad (445)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^2 \cdot e^i \cdot a_s^{-i} \quad (446)$$

$$4 \cdot e^{2-i} \cdot a^2 \cdot a_G \cdot N_A^2 = i^2 \cdot a_s^{-i} \quad (447)$$

$$4 \cdot e^{2-i} \cdot a^2 \cdot a_G \cdot a_s^i \cdot N_A^2 = i^2 \quad (448)$$

From Euler's identity, the equations $4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$ and $e^n \cdot a_s^2 = 10^7 \cdot a_w$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^n \cdot a_s^2 = 10^7 \cdot a_w$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$e^{in} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (449)$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (450)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (451)$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i = -4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (452)$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i = 4 \cdot i^2 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (453)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} = -4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (454)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} = 4 \cdot i^2 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (455)$$

$$a_s^{2i} = 4 \cdot i^2 \cdot 10^{7i} \cdot e^2 \cdot a^2 \cdot a_G \cdot a_w^i \cdot N_A^2 \quad (456)$$

Also from Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^n \cdot a_s = e$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$(e^{-1} \cdot a_s)^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (457)$$

$$(a_w^{-1} \cdot a_s^2 \cdot 10^{-7})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (458)$$

$$[2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})]^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (459)$$

$$(2 \cdot N_A \cdot a_s \cdot a \cdot a_G^{1/2})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (460)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})]^i + 14 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (461)$$

$$(2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (462)$$

$$(10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (463)$$

$$10^{-7i} \cdot a_g^{i/3} \cdot a_s^{2i} \cdot a^i \cdot a_w^{-i} \cdot a_G^{-i/2} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 0 \quad (464)$$

$$10^{-7i} \cdot a_g^{i/3} \cdot a_s^{2i} \cdot a^i \cdot a_w^{-i} \cdot a_G^{-i/2} = -4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (465)$$

$$10^{-7i} \cdot a_g^{i/3} \cdot a_s^{2i} \cdot a^i \cdot a_w^{-i} \cdot a_G^{-i/2} = 4 \cdot i^2 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 \quad (466)$$

$$4 \cdot i^2 \cdot 10^{7i} \cdot e^2 \cdot a^{2-i} \cdot a_w^i \cdot a_G^{1+(i/2)} \cdot N_A^2 = a_g^{i/3} \cdot a_s^{2i} \quad (467)$$

From the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions equals:

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 1 \quad (468)$$

So from Euler's identity resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 1$$

$$e^{in} + 2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 0 \quad (469)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = -e^{in} \quad (470)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = i^2 \cdot e^{in} \quad (471)$$

$$2 \cdot e^{n(1-i)} \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = i^2 \quad (472)$$

From Euler's identity, the equations $2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 1$ and $e^n \cdot a_s = e$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$(e \cdot a_s^{-1})^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (473)$$

$$e^i \cdot a_s^{-i} + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (474)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=-e^i \cdot a_s^{-i} \quad (475)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=i^2 \cdot e^i \cdot a_s^{-i} \quad (476)$$

$$2 \cdot e^{n-i} \cdot a_s^{1-i} \cdot a \cdot aG^{1/2} \cdot NA=i^2 \quad (477)$$

Also from Euler's identity, the equations $2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$ and $e^n \cdot a_s^2=10^7 \cdot a_w$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$e^n \cdot a_s^2=10^7 \cdot a_w$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (478)$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (479)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (480)$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (481)$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i = 2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (482)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (483)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} = 2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (484)$$

$$a_s^{2i} = 2 \cdot i^2 \cdot e^n \cdot 10^{7i} \cdot a_s \cdot a \cdot aG^{1/2} \cdot a_w^i \cdot NA \quad (485)$$

$$a_s^{2i-1} = 2 \cdot i^2 \cdot e^n \cdot 10^{7i} \cdot a \cdot aG^{1/2} \cdot a_w^i \cdot NA \quad (486)$$

Also from Euler's identity, the equations $2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$ and $4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2=1$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2=1$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$(2 \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA)^i + 4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2=0 \quad (487)$$

$$4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2 = -(2 \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA)^i \quad (488)$$

$$4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2 = i^2 \cdot (2 \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA)^i \quad (489)$$

$$a_s^i \cdot a^{i-2} \cdot aG^{(1/2i)-1} \cdot NA^{i-2} = i^2 \cdot 2^{2-i} \cdot e^2 \quad (490)$$

From the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions equals:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot NA^2 \quad (491)$$

From Euler's identity, the equations $as^2=4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot NA^2$ and $e^n \cdot as=e$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$e^n \cdot as=e$$

$$4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=1$$

$$e^{in}+4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=0 \quad (492)$$

$$(e^{-1} \cdot as)^i+4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=0 \quad (493)$$

$$e^{-i} \cdot as^i+4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=0 \quad (494)$$

$$e^{-i} \cdot as^i=-4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2 \quad (495)$$

$$e^{-i} \cdot as^i=4 \cdot i^2 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2 \quad (496)$$

$$as^{i+2}=4 \cdot i^2 \cdot e^i \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot NA^2 \quad (497)$$

From Euler's identity, the equations $as^2=4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot NA^2$ and $e^n \cdot as^2=10^7 \cdot aw$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$e^n \cdot as^2=10^7 \cdot aw$$

$$4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=1$$

$$e^{in}+4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=0$$

$$(10^7 \cdot as^{-2} \cdot aw)^i+4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=0 \quad (498)$$

$$(10^{-7} \cdot as^2 \cdot aw^{-1})^i+4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=0 \quad (499)$$

$$10^{-7i} \cdot as^{2i} \cdot aw^{-i}+4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=0 \quad (500)$$

$$4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=-10^{-7i} \cdot as^{2i} \cdot aw^{-i} \quad (501)$$

$$4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot as^{-2} \cdot NA^2=i^2 \cdot 10^{-7i} \cdot as^{2i} \cdot aw^{-i} \quad (502)$$

$$as^{2(i+1)}=4 \cdot i^2 \cdot 10^{14+7i} \cdot aw^{2+i} \cdot a^2 \cdot aG \cdot NA^2 \quad (503)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in}+1=0$$

$$2 \cdot e^n \cdot as \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$(e^{-1} \cdot as)^i+2 \cdot e^n \cdot as \cdot a \cdot aG^{1/2} \cdot NA=0$$

$$(aw^{-1} \cdot as^2 \cdot 10^{-7})^i+2 \cdot e^n \cdot as \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (504)$$

$$[2^{-1} \cdot as \cdot (e^{i/a}+e^{-i/a})]^i+2 \cdot e^n \cdot as \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (505)$$

$$(2 \cdot NA \cdot as \cdot a \cdot aG^{1/2})^i+2 \cdot e^n \cdot as \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (506)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a}+e^{-i/a})]^i+2 \cdot e^n \cdot as \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (507)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^i+2 \cdot e^n \cdot as \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (508)$$

$$(10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2})^i + 2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA = 0 \quad (509)$$

So resulting the formulas:

$$(e^{-1} \cdot a_s)^i = -2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (510)$$

$$(a_w^{-1} \cdot a_s^2 \cdot 10^{-7})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (511)$$

$$[2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})]^i = -2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (512)$$

$$(2 \cdot NA \cdot a_s \cdot a \cdot a_G^{1/2})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (513)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})]^i = -2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (514)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (515)$$

$$(10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (516)$$

$$(10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2})^i = 2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (517)$$

$$10^{-7i} \cdot a_g^{1/3i} \cdot a_s^{2i} \cdot a^i \cdot a_w^{-i} \cdot a_G^{-1/2i} = 2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA \quad (518)$$

$$a_g^{1/3i} \cdot a_s^{2i-1} \cdot a^{i-1} \cdot a_w^{-i} \cdot a_G^{-(1+i)/2i} = 2 \cdot i^2 \cdot e^n \cdot 10^{7i} \cdot NA \quad (519)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 1$$

$$(e \cdot a_s^{-1})^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 0 \quad (520)$$

$$(a_w \cdot a_s^{-2} \cdot 10^7)^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 0 \quad (521)$$

$$[2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 0 \quad (522)$$

$$[(2 \cdot NA \cdot a_s \cdot a \cdot a_G^{1/2})^{-1}]^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 0 \quad (523)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 0 \quad (524)$$

$$[(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^{-1}]^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 0 \quad (525)$$

$$(10^7 \cdot a_g^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot a_w \cdot a_G^{1/2})^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 = 0 \quad (526)$$

So resulting the formulas:

$$(e \cdot a_s^{-1})^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (527)$$

$$(a_w \cdot a_s^{-2} \cdot 10^7)^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (528)$$

$$[2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (529)$$

$$[(2 \cdot NA \cdot a_s \cdot a \cdot a_G^{1/2})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (530)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (531)$$

$$[(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (532)$$

$$(10^7 \cdot a_g^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot a_w \cdot a_G^{1/2})^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (533)$$

So resulting the formulas:

$$e^i \cdot a_s^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (534)$$

$$a_w^i \cdot a_s^{-2i} \cdot 10^{7i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (535)$$

$$2^i \cdot a_s^{-i} \cdot (e^{i/a} + e^{-i/a})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (536)$$

$$(2 \cdot NA \cdot a_s \cdot a \cdot a_G^{1/2})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (537)$$

$$2^{-i} \cdot e^{-i} \cdot 10^{7i} \cdot a_w^i \cdot (e^{i/a} + e^{-i/a})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (538)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (539)$$

$$10^{7i} \cdot a_G^{-i/3} \cdot a_s^{-2i} \cdot a^{-i} \cdot a_w^i \cdot a_G^{i/2} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (540)$$

$$a_G^{-i/3} \cdot a_s^{2-2i} \cdot a^{-i-2} \cdot a_w^{i-2} \cdot a_G^{(i-2)/2} = 4 \cdot i^2 \cdot 10^{14-7i} \cdot NA^2 \quad (541)$$

7. The cosine of angle in α^{-1} radians

The fine-structure constant is one of the most fundamental constants of physics. It describes the strength of the force of electromagnetism between elementary particles in what is known as the standard model of particle physics. For the cosine of angle in α^{-1} radians equals:

$$\cos \alpha^{-1} = i^{2i} \cdot a_s^{-1} \quad (542)$$

$$\cos \alpha^{-1} = e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1} \quad (543)$$

$$\cos \alpha^{-1} = 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (544)$$

$$\cos \alpha^{-1} = 2 \cdot a \cdot a_G^{1/2} \cdot NA \quad (545)$$

$$\cos \alpha^{-1} = 2^{-1} \cdot i^{8i} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot NA^{-2} \quad (546)$$

$$\cos \alpha^{-1} = 2^{-1} \cdot 10^7 \cdot i^{4i} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot NA^{-1} \quad (547)$$

$$\cos \alpha^{-1} = e^{-n} \cdot a_s^{-1} \quad (548)$$

$$\cos \alpha^{-1} = 10^{-7} \cdot e^{-1-n} \cdot a_w^{-1} \quad (549)$$

$$\cos \alpha^{-1} = 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (550)$$

$$\cos \alpha^{-1} = 2 \cdot a \cdot a_G^{1/2} \cdot NA \quad (551)$$

$$\cos \alpha^{-1} = 2^{-1} \cdot e^{-4n} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot NA^{-2} \quad (552)$$

$$\cos \alpha^{-1} = 2^{-1} \cdot 10^7 \cdot e^{-2n} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot NA^{-1} \quad (553)$$

$$\cos \alpha^{-1} = 4 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G \cdot a_s^{-1} \quad (554)$$

Also for the cosine of angle in α^{-1} radians equals:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (555)$$

$$\cos \alpha^{-1} = \frac{2l_{pl}}{l_{min}} \quad (556)$$

$$\cos \alpha^{-1} = \frac{2N_A l_{pl}}{\alpha_0} \quad (557)$$

$$\cos \alpha^{-1} = \frac{2t_{pl}}{t_{min}} \quad (558)$$

$$\cos \alpha^{-1} = \frac{\alpha N_1 N_A l_{pl}}{R_U} \quad (559)$$

$$\cos \alpha^{-1} = \frac{\Lambda c^2}{6H_0} \quad (560)$$

Also for the cosine of angle in α^{-1} radians equals:

$$\cos \alpha^{-1} = \frac{\Omega_B}{\alpha_s} \quad (561)$$

$$\cos \alpha^{-1} = \frac{\Omega_\Lambda}{2} \quad (562)$$

$$\cos \alpha^{-1} = \frac{2\Omega_D^{-1}}{e^\pi} \quad (563)$$

$$\cos \alpha^{-1} = \frac{2\Omega_B}{\Omega_D} \quad (564)$$

So resulting the formula:

$$\left(\frac{L_H}{2R_d} \right)^2 = \frac{l_{pl}}{l_{min}} \quad (565)$$

From the Dimensionless unification of the fundamental interactions resulting the expressions for the Gelfond's constant:

$$e^n = (e^{in})^{-i}$$

$$e^n = (-1)^{-i}$$

$$e^n = i^{-2i}$$

$$e^n = e \cdot a s^{-1} \quad (566)$$

$$e^n = 10^7 \cdot a_w \cdot a s^{-2} \quad (567)$$

$$e^n = 2 \cdot a s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (568)$$

$$e^n = (2 \cdot N_A \cdot a s \cdot a \cdot a G^{1/2})^{-1} \quad (569)$$

$$e^n = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (570)$$

$$e^n = (2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a G^{1/2})^{-1} \quad (571)$$

$$e^n = 10^7 \cdot a_g^{-1/3} \cdot a s^{-2} \cdot a^{-1} \cdot a_w \cdot a G^{1/2} \quad (572)$$

So resulting the formulas:

$$a_g^{1/3} \cdot a_s^2 \cdot a \cdot e^n = 10^7 \cdot a_w \cdot a_G^{1/2} \quad (573)$$

$$a_g^2 \cdot a_s^{12} \cdot a^6 \cdot e^{6n} = 10^{42} \cdot a_w^6 \cdot a_G^3 \quad (574)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(e^n)^i + 1 = 0$$

$$(e \cdot a_s^{-1})^i + 1 = 0 \quad (575)$$

$$(a_w \cdot a_s^{-2} \cdot 10^7)^i + 1 = 0 \quad (576)$$

$$[2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 1 = 0 \quad (577)$$

$$[(2 \cdot N_A \cdot a_s \cdot a \cdot a_G^{1/2})^{-1}]^i + 1 = 0 \quad (578)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 1 = 0 \quad (579)$$

$$[(2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^{-1}]^i + 1 = 0 \quad (580)$$

$$(10^7 \cdot a_g^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot a_w \cdot a_G^{1/2})^i + 1 = 0 \quad (581)$$

Also from the Dimensionless unification of the fundamental interactions resulting the expressions:

$$e^{-n} = e^{-1} \cdot a_s \quad (582)$$

$$e^{-n} = a_w^{-1} \cdot a_s^2 \cdot 10^{-7} \quad (583)$$

$$e^{-n} = 2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a}) \quad (584)$$

$$e^{-n} = 2 \cdot N_A \cdot a_s \cdot a \cdot a_G^{1/2} \quad (585)$$

$$e^{-n} = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (586)$$

$$e^{-n} = 2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2} \quad (587)$$

$$e^{-n} = 10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} \quad (588)$$

From Euler's identity resulting the beautiful formulas:

$$(e^{-1} \cdot a_s)^i + 1 = 0 \quad (589)$$

$$(a_w^{-1} \cdot a_s^2 \cdot 10^{-7})^i + 1 = 0 \quad (590)$$

$$[2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})]^i + 1 = 0 \quad (591)$$

$$(2 \cdot N_A \cdot a_s \cdot a \cdot a_G^{1/2})^i + 1 = 0 \quad (592)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})]^i + 1 = 0 \quad (593)$$

$$(2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^i + 1 = 0 \quad (594)$$

$$(10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2})^i + 1 = 0 \quad (595)$$

Also from Euler's identity resulting the beautiful formulas:

$$(a_w \cdot a_s^{-2} \cdot 10^7)^i = i^2 \quad (596)$$

$$[2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = i^2 \quad (597)$$

$$(2 \cdot NA \cdot as \cdot a \cdot aG^{1/2})^i = i^2 \quad (598)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = i^2 \quad (599)$$

$$[(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^{-1}]^i = i^2 \quad (600)$$

$$(10^7 \cdot ag^{-1/3} \cdot as^{-2} \cdot a^{-1} \cdot aw \cdot aG^{1/2})^i = i^2 \quad (601)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(e^{-1} \cdot as)^i = i^2$$

$$(aw^{-1} \cdot as^2 \cdot 10^{-7})^i = i^2 \quad (602)$$

$$[2^{-1} \cdot as \cdot (e^{i/a} + e^{-i/a})]^i = i^2 \quad (603)$$

$$(2 \cdot NA \cdot as \cdot a \cdot aG^{1/2})^i = i^2 \quad (604)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})]^i = i^2 \quad (605)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^i = i^2 \quad (606)$$

$$(10^{-7} \cdot ag^{1/3} \cdot as^2 \cdot a \cdot aw^{-1} \cdot aG^{-1/2})^i = i^2 \quad (607)$$

$$10^{-7i} \cdot ag^{i/3} \cdot as^{2i} \cdot a^i \cdot aw^{-i} \cdot aG^{-i/2} = i^2 \quad (608)$$

$$ag^{i/3} \cdot as^{2i} \cdot a^i = 10^{7i} \cdot i^2 \cdot aw^i \cdot aG^{i/2} \quad (609)$$

$$ag^{2i} \cdot as^{12i} \cdot a^{6i} = 10^{7i} \cdot i^2 \cdot aw^i \cdot aG^{i/2} \quad (610)$$

Archimedes constant π appears in many types in all fields of mathematics and physics. It is found in many types of trigonometry and geometry, especially in terms of circles, ellipses or spheres. It is also found in various types from other disciplines, such as Cosmology, numbers, Statistics, fractals, thermodynamics, engineering, and electromagnetism. Also Archimedes constant π appears in the cosmological constant, Heisenberg's uncertainty principle, Einstein's field equation of general relativity, Coulomb's law for the electric force in vacuum, Magnetic permeability of free space, Period of a simple pendulum with small amplitude, Kepler's third law of planetary motion, the buckling formula, etc. From the Dimensionless unification of the fundamental interactions for the Archimedes constant π equals:

$$\pi = \ln(e \cdot as^{-1}) \quad (611)$$

$$\pi = \ln(10^7 \cdot aw \cdot as^{-2}) \quad (612)$$

$$\pi = \ln[2 \cdot as^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}] \quad (613)$$

$$\pi = -\ln(2 \cdot NA \cdot as \cdot a \cdot aG^{1/2}) \quad (614)$$

$$\pi = \ln[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})^{-1}] \quad (615)$$

$$\pi = -\ln(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2}) \quad (616)$$

$$\pi = \ln(10^7 \cdot ag^{-1/3} \cdot as^{-2} \cdot a^{-1} \cdot aw \cdot aG^{1/2}) \quad (617)$$

8. Conclusions

We presented new exact formula for the fine-structure constant α in terms of the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}\cdot 2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}$$

We propose in a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π :

$$\alpha^{-1}=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}\cdot \pi\cdot \ln 2$$

We propose the exact equivalent mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19}$$

We propose the exact mathematical expressions for the proton to electron mass ratio:

$$\mu^3=7^{-1}\cdot 165^3\cdot \ln^{11}10$$

$$\mu=6\cdot \pi^5+\pi^{-3}+2\cdot \pi^{-6}+2\cdot \pi^{-8}+2\cdot \pi^{-10}+2\cdot \pi^{-13}+\pi^{-15}$$

We present the exact mathematical expressions that connect the proton to electron mass ratio and the fine-structure constant:

$$9\cdot \mu-119\cdot \alpha^{-1}=5\cdot (\varphi+42)$$

$$\mu-6\cdot \alpha^{-1}=360\cdot \varphi-165\cdot \pi+345\cdot e+12$$

$$\mu-182\cdot \alpha=141\cdot \varphi+495\cdot \pi-66\cdot e+231$$

$$\mu-807\cdot \alpha=1205\cdot \pi-518\cdot \varphi-411\cdot e$$

The new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0$$

The new formula for the Avogadro's number N_A is:

$$N_A = \left(2e\alpha\sqrt{\alpha_G}\right)^{-1}$$

The mathematical formulas that connect dimensionless physical constants are:

$$\alpha_G(p)=\mu^2\cdot \alpha_G$$

$$\alpha=\mu\cdot N_1\cdot \alpha_G$$

$$\alpha\cdot \mu=N_1\cdot \alpha_G(p)$$

$$\alpha^2=N_1^2\cdot \alpha_G\cdot \alpha_G(p)$$

$$4\cdot e^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2=1$$

$$\mu^2=4\cdot e^2\cdot \alpha^2\cdot \alpha_G(p)\cdot N_A^2$$

$$\mu\cdot N_1=4\cdot e^2\cdot \alpha^3\cdot N_A^2$$

$$4\cdot e^2\cdot \alpha\cdot \mu\cdot \alpha_G^2\cdot N_A^2\cdot N_1=1$$

$$\mu^3=4\cdot e^2\cdot \alpha\cdot \alpha_G(p)^2\cdot N_A^2\cdot N_1$$

$$\mu^2=4\cdot e^2\cdot \alpha_G\cdot \alpha_G(p)^2\cdot N_A^2\cdot N_1^2$$

$$\mu=4\cdot e^2\cdot a\cdot a_G\cdot a_G(p)\cdot NA^2\cdot N_1$$

We reached the conclusion of the simple unification of the nuclear and the atomic physics:

$$10\cdot(e^{i\mu/a}+e^{-i\mu/a})^{1/2}=13\cdot i$$

We presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{Euler' number}{Gerford's constant} = \frac{e}{e^\pi} = e^{1-\pi} = 0,11748..$$

It presented the dimensionless unification of the fundamental interactions. We calculated the unity formulas that connect the coupling constants of the fundamental forces. From the most beautiful equation in mathematics Euler's identity it presented new beautiful equations of unification of the fundamental interactions. We calculated new unity formulas that connect the coupling constants of the fundamental forces.

We reached the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot a_s=10^7\cdot a_w$$

$$e^n\cdot a_s^2=10^7\cdot a_w$$

The imaginary dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$a_s^2=i^{2i}\cdot 10^7\cdot a_w$$

$$a_s^{2i}=i^2\cdot 10^{7i}\cdot a_w^i$$

We reached the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^n\cdot a_s\cdot \cos a^{-1}=1$$

The imaginary dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^n\cdot a_s\cdot (e^{i/a}+e^{-i/a})=2$$

$$a_s\cdot \cos a^{-1}=i^{2i}$$

$$a_s\cdot (e^{i/a}+e^{-i/a})=2\cdot i^{2i}$$

$$a_s^i+(\cos a^{-1})^{-i}=0$$

$$a_s^i+2^i\cdot (e^{i/a}+e^{-i/a})^{-i}=0$$

We reached the dimensionless unification of the weak nuclear and the electromagnetic interactions:

$$10^7\cdot e^n\cdot a_w\cdot \cos a^{-1}=e$$

The imaginary dimensionless unification of the weak nuclear and the electromagnetic interactions:

$$10^7\cdot e^n\cdot a_w\cdot (e^{i/a}+e^{-i/a})=2\cdot e$$

$$10^7\cdot a_w\cdot \cos a^{-1}=e\cdot i^{2i}$$

$$10^7\cdot a_w\cdot (e^{i/a}+e^{-i/a})=2\cdot e\cdot i^{2i}$$

$$10^{7i}\cdot a_w^i\cdot (\cos a^{-1})^i+e^i=0$$

We reached the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot a_w \cdot \cos a^{-1} = a_s$$

The imaginary dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot a_s$$

We reached the dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot N_A^2$$

The imaginary dimensionless unification of the gravitational and the electromagnetic interactions:

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2$$

$$4 \cdot a^2 \cdot a_G \cdot N_A^2 = i^2 \cdot e^{in-2}$$

We reached the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 1$$

$$2 \cdot e^{4n} \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot N_A^2 = 1$$

The imaginary dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interaction

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i}$$

$$e^{4n} \cdot a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = 1$$

$$2 \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i}$$

$$4 \cdot e^{2-i} \cdot a^2 \cdot a_G \cdot a_s^i \cdot N_A^2 = i^2$$

$$2 \cdot e^{n(1-i)} \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = i^2$$

$$2 \cdot e^{n-i} \cdot a_s^{1-i} \cdot a \cdot a_G^{1/2} \cdot N_A = i^2$$

$$a_s^i \cdot a^{i-2} \cdot a_G^{(1/2i)-1} \cdot N_A^{i-2} = i^2 \cdot 2^{2-i} \cdot e^2$$

We reached the dimensionless unification of the weak nuclear,the gravitational and electromagnetic interactions:

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = e^2$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot N_A^2 = 1$$

The imaginary dimensionless unification of the weak nuclear,the gravitational and electromagnetic interactions:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$4 \cdot 10^{14} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot N_A^2 = i^{8i}$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot N_A^2 = i^{8i}$$

$$10^{14} \cdot e^{4n} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot N_A^2 = i^{8i}$$

We reached the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2$$

$$\alpha_s\cdot \cos\alpha^{-1}=4\cdot 10^7\cdot N_A^2\cdot \alpha_w\cdot \alpha^2\cdot \alpha_G$$

The imaginary dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$8\cdot 10^7\cdot N_A^2\cdot \alpha_w\cdot \alpha^2\cdot \alpha_G=\alpha_s\cdot (e^{i/\alpha}+e^{-i/\alpha})$$

$$\alpha_s^{2i}=2\cdot i^2\cdot e^n\cdot 10^{7i}\cdot \alpha_s\cdot \alpha\cdot \alpha_G^{1/2}\cdot \alpha_w^i\cdot N_A$$

$$\alpha_s^{i+2}=4\cdot i^2\cdot e^i\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2$$

$$\alpha_s^{2(i+1)}=4\cdot i^2\cdot 10^{14+7i}\cdot \alpha_w^{2+i}\cdot \alpha^2\cdot \alpha_G\cdot N_A^2$$

$$\alpha_s^{2i-1}=2\cdot i^2\cdot e^n\cdot 10^{7i}\cdot \alpha\cdot \alpha_G^{1/2}\cdot \alpha_w^i\cdot N_A$$

From these expressions resulting the unity formulas that connects the strong coupling constant α_s ,the weak coupling constant α_w ,the proton to electron mass ratio μ ,the fine-structure constant α ,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A ,the gravitational coupling constant α_G of the electron,the gravitational coupling constant of the proton $\alpha_G(p)$,the strong coupling constant α_s and the weak coupling constant α_w :

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2$$

$$\mu^2\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G(p)\cdot N_A^2$$

$$\mu\cdot N_1\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^3\cdot N_A^2$$

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \mu\cdot \alpha_G^2\cdot N_A^2\cdot N_1$$

$$\mu^3\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \alpha_G(p)^2\cdot N_A^2\cdot N_1$$

$$\mu\cdot \alpha_s=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha_G\cdot G(p)^2\cdot N_A^2\cdot N_1^2$$

$$\mu\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \alpha_G\cdot \alpha_G(p)\cdot N_A^2\cdot N_1$$

We found the formula for the Gravitational constant:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i}(2\alpha_s\alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i}e^2(2\cdot 10^7\alpha_w\alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = \alpha_s^2(2\cdot 10^7\alpha_w\alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

It presented the theoretical value of the Gravitational constant $G=6.67448\times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$. This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a

research group announced new measurements based on torsion balances. They ended up measuring $6.674184 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ and $6.674484 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ -of-swinging and angular acceleration methods, respectively. We calculated the expression that connects the gravitational fine structure constant with the four coupling constants:

$$\alpha_g^2 = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. It presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$|p|^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6}$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot a_G^3 \cdot a_w^6$$

$$a_s^{12} \cdot a^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot a_G^3 \cdot a_w^6$$

For the cosmological constant equals:

$$\Lambda = \left(2e a^2 N_A \right)^{-6} \frac{c^3}{G \hbar}$$

$$\Lambda = i^{12i} \left(2 a_s a^2 N_A \right)^{-6} \frac{c^3}{G \hbar}$$

$$\Lambda = i^{12i} e^6 \left(2 \cdot 10^7 a_w a^3 N_A \right)^{-6} \frac{c^3}{G \hbar}$$

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \frac{c^3}{G \hbar}$$

$$\Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{a^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar}$$

The Equation of the Universe is:

$$\frac{\Lambda G \hbar}{c^3} = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{a^2 \alpha_s^4} \right)^3$$

The following expressions connect the gravitational fine-structure constant α_g with the four coupling constants. Perhaps the gravitational fine structure constant α_g is the coupling constant for the fifth force.

We reached the dimensionless unification of the five fundamental interactions:

$$\alpha_g^2 \cdot a_s^{12} \cdot a^6 \cdot e^{6\pi} = 10^{42} \cdot a_w^6 \cdot a_G^3$$

The imaginary dimensionless unification of the five fundamental interactions:

$$\alpha_g^{2i} \cdot \alpha_s^{12i} \cdot \alpha^{6i} = 10^{7i} \cdot i^2 \cdot \alpha_w^i \cdot \alpha_G^{i/2}$$

$$\alpha_g^{-i/3} \cdot \alpha_s^{2-2i} \cdot \alpha^{-i-2} \cdot \alpha_w^{i-2} \cdot \alpha_G^{(i-2)/2} = 4 \cdot i^2 \cdot 10^{14-7i} \cdot N_A^2$$

$$\alpha_g^{1/3i} \cdot \alpha_s^{2i-1} \cdot \alpha^{i-1} \cdot \alpha_w^{-i} \cdot \alpha_G^{-(1+i)/2i} = 2 \cdot i^2 \cdot e^n \cdot 10^{7i} \cdot N_A$$

We presented the law of the gravitational fine-structure constant α_g followed by ratios of maximum and minimum theoretical values for natural quantities. Length l , time t , speed v and temperature T have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}}$$

Energy E , mass M , action A , momentum P and entropy S have another min/max ratio, which is the square of α_g :

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}}$$

Force F has min/max ratio which is α_g^4 :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}}$$

Mass density has min/max ratio which is α_g^5 :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}}$$

Perhaps for the minimum distance l_{min} apply:

$$l_{min} = 2 \cdot e \cdot |p|$$

The maximum distance l_{max} is:

$$l_{max} = L_H = c \cdot H_0^{-1} = \alpha_g^{-1} \cdot l_{min}$$

For the minimum mass M_{min} apply:

$$M_{min} = \frac{m_{pl}^2}{M_{max}} = \alpha_g m_{pl} = \frac{\alpha_G}{\alpha^3} m_e = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e$$

From the dimensionless unification of the fundamental interactions we discover a new simple Large Number Hypothesis which calculates the Mass, the Age and the Radius of the universe. The expressions for the mass of the observable universe are:

$$M_U = \alpha^{-1} \cdot \alpha_g^{-4/3} \cdot m_e = \alpha^3 \cdot \alpha_G^{-2} \cdot m_e = (2 \cdot e \cdot \alpha^2 \cdot N_A)^2 \cdot N_1 \cdot m_p = \mu \cdot \alpha \cdot N_1^2 \cdot m_p = 1.153482 \times 10^{53} \text{ kg}$$

The expressions who calculate the number of protons in the observable universe are:

$$N_{Edd} = \frac{M_U}{m_p} = \mu \alpha N_1^2 = \frac{N_1}{\alpha_g^{2/3}} = \left(2e\alpha^2 N_A \right)^2 N_1 = \left(\frac{r_e}{l_{pl}} \right)^2 N_1 = 6.9 \times 10^{79}$$

The diameter of the observable universe will be calculated to be equal to the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot R_U = N_1 \cdot \lambda_c$$

The expressions for the radius of the observable universe are:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 = \frac{N_1}{2\alpha} r_e = \frac{1}{2\mu \alpha_G} r_e = \frac{m_{pl}^2 r_e}{2m_e m_p} = \frac{\hbar c r_e}{2G m_e m_p} = \frac{\alpha \hbar}{2G m_e^2 m_p}$$

We Found the value of the radius of the universe $R_U=4.38 \times 10^{26}$ m. The expressions for the radius of the observable universe are:

$$T_U = \frac{R_U}{c} = \frac{N_1 r_e}{2\alpha c} = \frac{r_e}{2\mu\alpha_G c} = \frac{\alpha N_1 \alpha_0}{2c} = \frac{\alpha \hbar}{2c G m_e^2 m_p} = \frac{\hbar r_e}{2G m_e m_p}$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2 \varphi^{-5}}{3^3 \pi \varphi^5} \times 10^{-120}$$

Perhaps for the minimum time t_{min} apply:

$$t_{min}=2 \cdot e \cdot t_{pl}$$

We proved the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions propose a possible solution for the density parameters of baryonic matter, dark matter and dark energy:

$$\Omega_B = e^{-n} = i^{2i} = 0.0432 = 4.32\%$$

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.7357 = 73.57\%$$

$$\Omega_D = 2 \cdot e^{1-n} = 2 \cdot e \cdot i^{2i} = 0.2349 = 23.49\%$$

The beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda = e^{i/a} + e^{-i/a}$$

The sum of the contributions to the total density parameter at the current time is $\Omega_0 = 1.0139$. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold, such as the Poincaré dodecahedral space, all of which are quotients of the 3-sphere. These results prove that the weather space is finite. The state equation w has value:

$$w = -24 \cdot e^{-n} = -24 \cdot i^{2i} = -1.037134$$

For as much as $w < -1$, the density actually increases with time.

The Equations of the Universe are:

$$e^{6\pi} \frac{\Lambda G \hbar}{c^3} = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

$$6e^{5\pi} \frac{G \hbar H_0^2}{c^5} = 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{11}}$$

$$e^{7\pi} \frac{G \hbar \Lambda^2}{c H_0^2} = 6 \cdot 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{13}}$$

All these equations are applicable for all energy scales.

References

- [1] Pellis, Stergios, Unification Archimedes constant π , golden ratio ϕ , Euler's number e and imaginary number i (October 10, 2021)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3975869>
- [2] Heyrovská R., "The Golden ratio, ionic and atomic radii and bond lengths", *Molecular Physics*, 103, pp.877-882 (2005)
- [3] Pellis, Stergios, Exact formula for the Fine-Structure Constant α in Terms of the Golden Ratio ϕ (October 13, 2021)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4160769>
- [4] Pellis, Stergios, Fine-Structure Constant from the Golden Angle, the Relativity Factor and the Fifth Power of the Golden Mean (September 5, 2022)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4247343>
- [5] Pellis, Stergios, Exact expressions of the fine-structure constant (October 20, 2021)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3975861>
- [6] Pellis, Stergios, Fine-structure constant from the Archimedes constant (October 11, 2022)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4245208>
- [7] Pellis, Stergios, Fine-Structure Constant from the Madelung Constant (July 27, 2022)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4174644>
- [8] Pellis, Stergios, Exact mathematical expressions of the proton to electron mass ratio (October 10, 2021)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3967998>
- [9] Pellis, Stergios, Unity formula that connect the fine-structure constant and the proton to electron mass ratio (November 8, 2021)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3963425>
- [10] Pellis, Stergios, Exact mathematical formula that connect 6 dimensionless physical constants (October 17, 2021)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3963427>
- [11] Jeff Yee The Relationship of the Mole and Charge (2019)
<https://vixra.org/pdf/1904.0445v1.pdf>
- [12] European organization for nuclear research (CERN) (2021)
<https://arxiv.org/pdf/2111.10431.pdf>
- [13] Pellis, Stergios, Theoretical value for the strong coupling constant (January 1, 2022)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3998175>
- [14] Pellis, S. (2023) Unity Formulas for the Coupling Constants and the Dimensionless Physical Constants. *Journal of High Energy Physics, Gravitation and Cosmology*, 9, 245-294.
<https://doi.org/10.4236/jhepgc.2023.91021>
- [15] Pellis, Stergios, Dimensionless Unification of the Fundamental Interactions (August 27, 2022)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4201780>
- [16] Pellis, Stergios, Unification of the fundamental interactions (2022)
DOI: 10.13140/RG.2.2.12296.70405
- [17] Pellis, Stergios, Unification of the Fundamental Forces (2022)
DOI: 10.13140/RG.2.2.33651.60967
- [18] Department of Physics and Astronomy Georgia State University
<http://hyperphysics.phy-astr.gsu.edu/hbase/Particles/weastr.html>
- [19] Sánchez, J. (2018) Calculation of the Fine-Structure Constant. *Journal of High Energy Physics, Gravitation and Cosmology*, 4, 510-518.
<https://doi.org/10.4236/jhepgc.2018.43029>
- [20] Pellis, Stergios, Theoretical Value of the Gravitational Constant (May 7, 2023).
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4442074>
- [21] Pellis, Stergios, Formula for the Gravitational constant (January 2023).
DOI: 10.13140/RG.2.2.19656.60166
- [22] Pellis, Stergios, Dimensionless Solution for the Cosmological Constant (September 14, 2022)
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4219292>
- [23] Pellis, Stergios, Unification of atomic physics and cosmology (2022)
DOI: 10.13140/RG.2.2.11493.88804
- [24] L. Nottale, Mach's principle, Dirac's large numbers and the cosmological constant problem (1993)

<http://www.luth.obspm.fr/%7Eluthier/nottale/arlambda.pdf>

[25] Pellis, Stergios, The Equation of the Universe (2023).

DOI: 10.13140/RG.2.2.24768.40960

[26] Pellis, Stergios, Maximum and Minimum Values for Natural Quantities (December 10, 2022)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4306280>

[27] Pellis, Stergios, The coupling constant for the fifth force (April 2023).

DOI: 10.13140/RG.2.2.32481.99686

[28] Pellis, Stergios, Gravitational fine-structure constant (December 2022).

DOI: 10.13140/RG.2.2.34770.43206/2

[29] Pellis, Stergios, Unification of the Microcosm and the Macrocosm (April 24, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4429653>

[30] Pellis, Stergios, Dimensionless theory of everything (June 5, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4469668>

[31] Pellis, Stergios, The Dimensionless Equations of the Universe (June 23, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4488688>

[32] Pellis, Stergios, The Dimensionless Fractal Universe (August 2023)

DOI: 10.13140/RG.2.2.25324.54407/1

[33] Pellis, Stergios, The Equations of the Unified Physics (September 22, 2023).

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4731914>

[34] Pellis, Stergios, New Large Number Hypothesis of the universe (August 1, 2023).

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4529088>

[35] Forsythe, C. J. & Valev, D. T. (2014). Extended mass relation for seven fundamental masses and new evidence of large number hypothesis. *Physics International*, 5(2), 152-158. <https://doi.org/10.3844/pisp.2014.152.158>

[36] Pellis, Stergios, The mass scale law of the Universe (August 31, 2023).

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4606510>

[37] R. Adler Comment on the cosmological constant and a gravitational alpha (2011)

<https://arxiv.org/pdf/1110.3358.pdf>

[38] S. Roy S. Bhattacharya B V Sreekantan Dirac's Large Number Hypothesis: Is it Related to 40 Hz Gamma Oscillation or Consciousness? (2015)

https://www.researchgate.net/publication/282278875_Dirac's_Large_Number_Hypothesis_Is_it_Related_to_40_Hz_Gamma_Oscillation_or_Consciousness?

[39] Pellis, Stergios, Theoretical value for the Hubble Constant (August 21, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4547576>

[40] Pellis, Stergios, Poincaré Dodecahedral Space Solution of The Shape of The Universe (December 31, 2022).

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4316617>

[41] Pellis, Stergios, The Shape of The Universe (March 2023).

doi: 10.13140/RG.2.2.16034.09922

[42] J.-P. Luminet A cosmic hall of mirrors (2005)

<https://arxiv.org/abs/physics/0509171>

[43] <https://astronomy.swin.edu.au/cosmos/>

[44] Pellis, Stergios, Solution for the Density Parameter of Dark Energy (December 25, 2022).

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4406279>

[45] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq, J.-P. Uzan Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background (2003)

<https://arxiv.org/abs/astro-ph/0310253>

[46] Pellis, Stergios, Solution for the cosmological parameters (December 2022).

DOI: 10.13140/RG.2.2.23917.67047/2

[47] Pellis, Stergios, Equation of state in cosmology (December 2022).

DOI: 10.13140/RG.2.2.17952.25609/1

[48] Pellis, Stergios, Euler's identity in unification of the fundamental interactions (January 21, 2024).

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4702100>