A Physics-Enhanced Neural Network for Estimating Longitudinal Dispersion Coefficient and Average Solute Transport Velocity in Porous Media

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Abstract

Dispersion coefficients and the average solute transport velocity are pivotal for groundwater solute transport modeling. Accurately and efficiently determining these parameters is challenging due to difficulties in directly correlating them with pore-space structure. To address this issue, we introduced the Physics-enhanced Convolutional Neural Network-Transformer (PhysenCT-Net), an innovative model designed to concurrently estimate the longitudinal dispersion coefficient and average solute transport velocity in three-dimensional porous media. PhysenCT-Net exhibited excellent predictive performance on unseen testing datasets and significantly reduced computational demands. Comprehensive evaluations confirmed its robust generalization across various flow conditions and pore structures. Notably, the longitudinal dispersion coefficient predictions closely align with established empirical relationships involving flow velocity, affirming the model's physical interpretability and potential to aid in simulating transport phenomena in porous media.

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- 2 A Physics-Enhanced Neural Network for Estimating Longitudinal Dispersion
- 3 Coefficient and Average Solute Transport Velocity in Porous Media

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8 Key Points:

- Accurately predicting key parameters of the advection-dispersion equation for solute
 transport in porous media
- Developing direct mapping relationships between three-dimensional porous media
 images and parameters by the physics-enhanced neural network
- Network demonstrates exceptional prediction accuracy and generalization, providing parameters that align with an empirical formula
- 15
- 16

17 Abstract

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- 19 solute transport modeling. Accurately and efficiently determining these parameters is
- 20 challenging due to difficulties in directly correlating them with pore-space structure. To address
- 21 this issue, we introduced the Physics-enhanced Convolutional Neural Network-Transformer
- 22 (PhysenCT-Net), an innovative model designed to concurrently estimate the longitudinal
- 23 dispersion coefficient and average solute transport velocity in three-dimensional porous media.
- 24 PhysenCT-Net exhibited excellent predictive performance on unseen testing datasets and
- 25 significantly reduced computational demands. Comprehensive evaluations confirmed its robust
- 26 generalization across various flow conditions and pore structures. Notably, the longitudinal 27 dispersion coefficient predictions closely align with established empirical relationships involving
- flow velocity, affirming the model's physical interpretability and potential to aid in simulating
- 29 transport phenomena in porous media.

30 Plain Language Summary

31 Advection-dispersion equation serves as the mathematical model for solute transport in aquifers

- 32 and other porous media, with dispersion coefficients and average solute transport velocity
- 33 (advective velocity) being key parameters. Typically, these are derived from optimization
- 34 inversion of experimental data. Traditional pore-scale direct numerical simulation methods,
- 35 while precise, demand considerable computational resources and time. To overcome these
- 36 limitations, we developed PhysenCT-Net, a physics-enhanced neural network, aimed at
- 37 simultaneously predicting the longitudinal dispersion coefficient and average solute transport
- velocity. The network integrates physical parameters such as molecular diffusion coefficient,
- 39 Darcy velocity, and porosity into three-dimensional images of porous media through a
- 40 convolutional neural network. The Transformer network then processes the high-dimensional
- 41 data, with fully connected layers finalizing the concurrent prediction of both parameters. Direct
- simulations combined with an inversion technique calculate these parameters to establish the
 ground truth of the dataset. The trained PhysenCT-Net demonstrated outstanding predictive
- 44 performance across various scenarios, proving its robustness. Transfer learning techniques
- further enhanced its generalization across different flow conditions and pore-space structures.
- 46 Moreover, PhysenCT-Net represents a major advancement in physical interpretability for
- 47 parameter prediction through deep learning methods.

48 **1 Introduction**

- 49 Precise modeling of solute dispersion within porous media holds significant relevance across
- various scientific and engineering disciplines, such as carbon dioxide sequestration, oil recovery,
- seawater intrusion into aquifers, and groundwater hydrology (Bear, 2013; Kamrava, Sahimi, et
- al., 2021; Sahimi, 2011; Xiong et al., 2020; Xiong et al., 2023). The transport of a conservative
- solute in porous media is governed by the advection-dispersion equation (ADE), wherein
- 54 dispersion coefficients and average solute transport velocity (advective velocity in ADE) are
- pivotal parameters (Bear, 2013; Dentz et al., 2018; Lee et al., 2018). Dispersion coefficients
- cannot be directly measured and require be inferred or calibrated based on hypotheses related to
- 57 observable characteristics (Abderrezzak et al., 2015; Ahsan, 2008). Previous studies have
- 58 empirically or experimentally determined the longitudinal dispersion coefficient as a function of
- 59 hydraulic and geometric parameters, such as Péclet number (Afshari et al., 2018; De Arcangelis

- et al., 1986; Sahimi et al., 1986). Typically, the average advective velocity is assumed to be
- equivalent to the average fluid velocity; however, column experiments and pore-scale
- 62 simulations have revealed discrepancies. This is because the solute distribution within pore
- 63 spaces of porous media is not uniform (Darland & Inskeep, 1997; Rovey & Niemann, 2005;
- 64 Zhang & Lv, 2009). Therefore, precise determination of the average advective velocity is
- essential for accurately simulating solute transport in porous media.
- 66 Pore-scale computational simulation methods, like pore network modelling (PNM), have been
- employed to investigate dispersion (Kamrava, Im, et al., 2021; Sahimi & Imdakm, 1988).
- 68 However, PNM simplifies the pore space structure through inherent network approximations and
- 69 treats the pore space as if it were composed of well-mixed entities, resulting in dispersive
- concentration profiles (Sadeghi et al., 2020; Yang et al., 2016). Consequently, other numerical
- 71 simulation methods that directly model images of pore space, such as lattice Boltzmann method
- (LBM) or particle tracking, have increasingly been adopted to study dispersion in fluid flow
 (Bijeljic et al., 2011; Blunt et al., 2013; Hasan et al., 2020). Direct simulation methods can
- accurately compute macroscopic parameters of porous media, yet their substantial time and
- resource demands are significant limitations (Bedrunka et al., 2021; Kamrava, Im, et al., 2021).
- Therefore, a novel method that strikes a balance between efficiency and accuracy in parameter
- estimation is essential for effectively predicting solute transport processes.
- In this letter, we introduce a physics-enhanced neural network, specifically designed to estimate
- ⁷⁹ longitudinal dispersion coefficient (D_L) and average solute transport velocity (u). The network
- 80 leverages LBM and particle tracking simulations, combined with a parameter inversion approach,
- to provide a more efficient yet accurate solution for simulating transport phenomena in porous
- 82 media.
- 83 Deep learning, or neural networks, have evolved as a superior alternative to traditional machine
- learning for analyzing complex system parameters (LeCun et al., 2015; LeCun et al., 1989;
- 85 Vaswani et al., 2017). Notably, convolutional neural networks (CNNs) and Transformer
- 86 networks have become leading architectures in computer vision and natural language processing,
- 87 respectively, driving significant progress and forming the cornerstone of various deep learning 88 applications (Daylin et al. 2018; He et al. 2017; He et al. 2016) CNNs excel in detecting leavel
- applications (Devlin et al., 2018; He et al., 2017; He et al., 2016). CNNs excel in detecting local
 patterns and spatial information in images, making them ideal for linking the geometric features
- of porous media with their transport and physical properties, such as permeability and diffusivity
- (Elmorsy et al., 2022; Kamrava, Im, et al., 2021; Kamrava, Sahimi, et al., 2021; Rabbani et al.,
- 2020; Tang et al., 2022). On the other hand, Transformer networks, with their self-attention
- mechanism, proficient in capturing long-range dependencies, enhancing feature extraction and
- generalization when combined with CNNs (Bai & Tahmasebi, 2022; Vaswani et al., 2017). This
- 95 integrated CNN-Transformer approach achieves high accuracy with fewer parameters, proving
- advantageous in computational resource-limited settings and facilitating research on large-sized
- porous media (Meng et al., 2023). Moreover, the infusion of physical information into the
- network—permitting the direct assimilation of porous media's physical parameters during
- 99 training—markedly elevates the model's predictive precision and its generalization capacity
- 100 (Kamrava, Im, et al., 2021; Meng et al., 2023; Tang et al., 2022). In this letter, we propose the
- 101 Physics-enhanced Convolutional Neural Network-Transformer (PhysenCT-Net), designed for the
- 102 prediction of parameters D_L and u in large-sized ($200 \times 200 \times 1,000$ cubic voxels) porous media
- 103 flows.

- 104 The structure of the letter is as follows: Section 2 introduces the numerical simulation methods
- of LBM and particle tracking, describes the data processing for training the PhysenCT-Net, and

106 presents the network framework. In Section 3, we detail the research methodology of the

- 107 PhysenCT-Net model, discussing the impact of varying data volumes, Péclet numbers,
- cementation and differences in particle size on the model's predictive and generalization
- capabilities. Section 4 summarizes and concludes the findings of the letter.





Figure 1. Schematic of the PhysenCT-Net

112 2 The Methodology

113 We first elucidate the methods for calculating the longitudinal dispersion coefficient D_L and

114 average solute transport velocity u of the porous media, as well as the process of preparing and

preprocessing data for PhysenCT-Net. Subsequently, we describe the framework of PhysenCT-

116 Net.

117 2.1 Porous Media Generation and Parameter Calculation

118 We utilized the sedimentation-producing method (Batchelor, 1972; Pilotti, 1998) to fabricate

three-dimensional (3D) porous media, subsequently modeling the flow field and solute transport

by LBM and particle tracking method (Jiang & Wu, 2021; Mostaghimi et al., 2012).

121 The process of creating porous media via the sedimentation-producing method involves

- sequentially depositing spherical grains into a column. Each spherical grain starts at the top and
- descends freely until it reaches the column's bottom or lands on another grain. If the descending
- grain encounters another, it slides along the latter's surface until it finds a stable position, either
- 125 at the bottom or in a state of minimum potential energy, thus preserving the porous medium's
- topological characteristics. This iterative process continues with subsequent grains. The spherical

grains used in this method have diameters ranging from 0.4 to 1.2 mm, forming a porous

medium with dimensions of $4 \text{ mm} \times 4 \text{ mm} \times 20 \text{ mm}$. Water flow is oriented along the column's long axis, with the porous medium's lateral boundaries defined as periodic to simulate an infinite

130 system.

131 Generating 10,000 flow field samples for the dataset is computationally demanding. To

efficiently manage the intricate boundaries within porous media, we employed the bounce-back

- scheme in LBM, specifically adopting the D3Q19 model to simulate pore-level flows. This
- 134 model is formulated as:

$$f_i(x+e_i\Delta t,t+\Delta t) = f_i(x,t) + \frac{\Delta t}{\tau} \left[f_i^{eq}(x,t) - f_i(x,t) \right]$$
(1)

- 135 where $f_i(x, t)$ represents the density distribution function in the *i*th direction at discrete lattice
- point x and $f_i^{eq}(x,t)$ denotes the equilibrium distribution function, determined by the updated
- 137 fluid velocities (Chen & Doolen, 1998).
- 138 For simulating solute molecule movement in porous media, we utilized the particle tracking
- 139 method(Srinivasan et al., 2010). The solute's breakthrough curve at the porous media exit was
- 140 calculated and then fitted by the ADE to determine the D_L and u. Solute transport within the
- porous media entails two dynamics: advective movement with water flow and random Brownian
- 142 motion, which can be mathematically described as:

$$r(t + \Delta t) = r(t) + u(r(t))\Delta t + 2\sqrt{D_m\Delta t}\xi$$
⁽²⁾

- 143 where $u(\mathbf{r})$ indicates the flow velocity at the position r, D_m denotes molecular diffusion
- 144 coefficient, and ξ represents a random variable conforming to a standard normal distribution.
- 145 The migration of conservative solutes in the porous media can be described by the ADE with the
- 146 initial and boundary conditions (Van Genuchten, 1981):

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - u \frac{\partial C}{\partial x}$$
(3)

$$C(x,t)|_{t=0} = 0 (4)$$

....

$$\frac{\partial C}{\partial x}\Big|_{x \to \infty} = (\text{finite}) \ (t \ge 0) \tag{5}$$

$$\left(-D_L \frac{\partial C}{\partial x} + uC\right)\Big|_{x=0} = uC_0 \tag{6}$$

- 147 where C is the solute concentration, C_0 the concentration of the injected solute, x the solute
- 148 migration distance, u the average advective velocity, and D_L the longitudinal dispersion
- 149 coefficient.
- 150 This mathematical model has an exact solution, which can be formulated as:

$$C(x,t) = C_0 \left\{ \frac{1}{2} \operatorname{erfc}\left(\frac{x-ut}{2\sqrt{D_L t}}\right) + \sqrt{\frac{u^2 t}{\pi D_L}} \exp\left[-\frac{(x-ut)^2}{4D_L t}\right] - \frac{1}{2} \left[1 + \frac{ux}{D_L} + \frac{u^2 t}{D_L}\right] \exp\left(\frac{ux}{D_L}\right) \operatorname{erfc}\left(\frac{x+ut}{2\sqrt{D_L t}}\right) \right\}$$
(7)

- 151 where the symbols are defined as above, *erfc* is the complementary error function.
- 152 For each sample of porous media, we designated two Darcy flow velocities and four molecular
- diffusion coefficients, resulting in four breakthrough curves C(t) at the outlet. The D_L and u
- were precisely fitted via solute concentration analytical solutions (Eq. 7) and *SciPy* optimization
- inversion algorithm (Virtanen et al., 2020), respectively. The process minimizes the sum of
- squared residuals to align predicted concentration values $\hat{C}(t)$ closely with observed ones C(t),
- thus accurately identifying optimal values for D_L and u.
- 158 2.2 Dataset Preparation
- 159 To enhance training of the neural network, we augmented the image dataset through vertical,
- 160 horizontal, and diagonal flipping, effectively tripling the number of porous medium images

- 161 (Elmorsy et al., 2022). This strategy mitigated the issue of a single 3D image correlating with
- 162 multiple D_L or u. We then constructed a dataset comprising 10,000 samples, with porosity (ε) of
- 163 the 3D porous media ranging from 0.42 to 0.46. The Péclet number (Pe), defined as Pe =
- 164 $vd/(\varepsilon D_m)$, where v represents the Darcy velocity, d the average grain diameter, varied between 12
- and 12,000, with an average value of 806 and a standard deviation of 1227. The dimensions of
- 166 the porous media images were $200\Delta x \times 200\Delta y \times 1,000\Delta z$, with $\Delta x = \Delta y = \Delta z = 20 \ \mu m$.

167 We constructed three-channel porous medium images infused with physical information entails

- the following steps: For each two-dimensional (2D) single-channel image within a sequence
- (depicting slices of a 3D porous medium), separate 2D parameter matrices for D_m and the slice
- pore velocity ($v_{sli} = v/\varepsilon_{sli}$), where ε_{sli} denotes the porosity of the 2D slice, respectively, are developed. The numerical values of these matrices are unified as D_m or v_{sli} and are then resized to
- align with the dimensions of the slice images. By integrating these two matrices with the original
- image along the channel axis, a 2D three-channel image with physical information is created,
- subsequently enhancing the entire sequence with physics.
- 175 2.3 The Neural Network Framework

176 The effectiveness of deep learning methods relies on the correct neural architecture and a

- sufficiently large dataset. In this letter, PhysenCT-Net primarily processes images of porous
- media infused with physical information through the CNN-Transformer architecture. A
- schematic of the PhysenCT-Net is depicted in Figure 1. The CNN comprises four convolutional
- layers, with the first two layers featuring 4×4 2D convolution kernels, a stride of 2, and padding of 1, and the number of channels in feature maps increasing sequentially from 12 to 24. The
- kernel size of the subsequent two convolutional layers is adjusted to 3×3 , while maintaining a
- 183 stride of 2 and padding of 1, with the channels in the feature maps escalating sequentially from
- 48 to 96. Following each convolutional operation, Batch Normalization (Ioffe & Szegedy, 2015)
- and the activation function of rectified linear unit (ReLU) (Nair & Hinton, 2010) are applied to
- 186 enhance the network's non-linear representation capability. By setting up four convolutional
- 187 layers, the three-channel (physics-enhanced) images of porous medium slices are transformed
- into 96-channel feature maps to represent their high-dimensional deep information. Subsequently,
 a fully connected layer compresses the feature map of each slice image into a feature vector with
- a fully connected layer compresses the feature map of each slice image into a feature vector with a dimensionality of 200. This same process is applied to each slice image of the porous medium,
- 190 a undersionality of 200. This same process is applied to each since image of the porous medium, 191 transforming the original 3D image into a sequence represented by a series of feature vectors.
- 192 This sequence is then fed into the Transformer network as input.

The Transformer network, a deep learning model based on the multi-head self-attention 193 mechanism, utilizes the mechanism to capture and learn the global dependencies within a 194 sequence, thereby acquiring a long-term representation of the sequence (see Vaswani et al. 195 196 (2017), for a comprehensive review of the terminology of the Transformer). We adopted the vanilla model proposed by Vaswani et al. (2017), which consists of six identical encoder layers, 197 each containing two sublayers. The first sublayer, a multi-head attention layer, captures the 198 relationships between elements (representing the compressed information of porous medium 199 slice images) within the sequence. The second sublayer, a feed-forward fully connected layer, is 200 employed for further feature extraction. Residual connections (He et al., 2016; Mo et al., 2019) 201 202 and regularization operations between sublayers enhance the network's generalization ability and training stability. The Transformer network incorporates positional encoding into its input to 203

204 capture the relative positional information of elements within the sequence; we employed

sinusoidal encoding (McAulay & Quatieri, 1995) for the purpose. After processing the input

sequence, the Transformer network averages the outputs along the sequence length dimension,

207 compressing them into a fixed-size vector. This vector then serves as the input to the fully 208 connected layers following the Transformer network, with the output of the layers being the D_L

209 and *u*.

210 **3 The Computational Procedure and Results**

Prior to training the neural network, standardizing parameter values is crucial to achieve consistency in data scale and distribution. Given the wide magnitude ranges for *v* and *u* (from 10^{-5} to 10^{-3} m/s), D_m (from 10^{-10} to 10^{-9} m²/s), and D_L (from 10^{-9} to 10^{-7} m²/s), we applied a logarithmic transformation to these parameters. Specifically, for *v* and *u*, a scaling factor of 10^{5} is utilized preceding the logarithmic transformation to adjust the values to a more practical range. Similarly, for D_m and D_L , a scaling factor of 10^{10} is employed. This preprocessing step ensures the parameter values are uniformly scaled, facilitating the neural network's learning process.

218 During the training process of the neural network, we initially divided the dataset into a training

set (80% of the data, totaling 8,000 samples) and a testing set (20% of the data, totaling 2,000

samples). The training set was further divided into a training subset and a validation subset at a

ratio of 8:2. We employed the Adam optimizer (Kingma & Ba, 2014) with training epochs set to

30 and batch size to 32. The initial learning rate was set at 1e-4, with decay to 10%, 5%, and 2.5%

of the initial rate at epochs 10, 20, and 25, respectively. The root mean square error (Chai &
Draxler, 2014) was selected as the network's loss function. We ran the model five times and

224 Draxier, 2014) was selected as the network's loss function. We fail the model rive times and 225 calculated the average coefficient of determination (\mathbb{R}^2) values for D_L and u on the validation

subset and testing set. The average R² for D_L on the validation subset was 0.9914, and for *u* was

227 0.9964; for the testing set, the average R^2 for D_L was 0.9913, and for *u* was 0.9962. The

computational time for training dataset by LBM and particle tracking amounted to 10 million

229 CPU seconds. While training the neural network required approximately 100,000 GPU seconds

on four NVIDIA GeForce RTX 4090 GPUs. The direct prediction of D_L and u for the testing set

by the trained PhysenCT-Net model required merely 1,600 GPU seconds. This represents a

substantial improvement in computational efficiency, marked by several orders of magnitude,

when compared to direct simulation approaches.







showcase the optimal prediction results and porous medium structures in the dataset; (c)

237 illustrates four box plots for each sample group, representing predictions of D_L and u for both

validation and testing sets. Each box plot delineates the mean (triangle symbol), median

(horizontal line within the box), outliers (dots), and the 95% confidence interval (shaded area)

according to the *t*-distribution.

To examine the effect of training sample size on the predictive accuracy and generalization

capability of the model, we modified the training dataset proportions to 10%, 20%, 40%, and 60%

of the total data, while maintaining a constant testing set size. Each model variant, defined by its

training sample size, underwent five rounds of training. Subsequently, we computed and

illustrated the model's predictive accuracy (\mathbb{R}^2) and its 95% confidence intervals employing the *t*-

distribution methodology (Lange et al., 1989; Reich & Barai, 1999). As depicted in Figure 2,
 PhysenCT-Net exhibited remarkable precision and resilience. The model was capable of

247 In sentence. The model was capable of 248 delivering precise predictions for unseen testing data with a minimal quantity of training samples

(including instances where the training sample size was less than the testing sample size), with

an observable enhancement in accuracy correlating with increased training data volume.

251 Bijeljic and Blunt (2006) employed the PNM method to investigate the relationship between

hydrodynamic dispersion coefficients and Pe, while Mostaghimi et al. (2012) determined D_L as a

function of the *Pe* through direct simulation. Instead of directly establishing a numerical

relationship between Pe and D_L , we explored and evaluated the PhysenCT-Net model's

255 capabilities to predict D_L and u with varying *Pe* conditions.

The PhysenCT-Net was set up with Pe < 1,000 (incorporating 7,552 samples) for training,

partitioning the dataset into training and validation sets at an 8:2 ratio. The dataset for Pe > 1,000

conditions was designated as the testing set with 2,432 samples. The training approach was kept

consistent, ensuring the dataset size strictly aligned with multiples of the batch size. The findings

reveal that PhysenCT-Net (referenced in Figures 3a and 3b), with training conditions Pe < 1,000,

261 yielded promising results on the validation set (totaling 1,504 samples), achieving R² values of

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- 262 0.985 for D_L and 0.997 for u. However, its generalization capability on the unseen testing set (Pe >263 1,000) was compromised, evidenced by R² values of 0.881 for D_L and 0.928 for u. To improve 264 the model's generalization capacity, transfer learning methods were employed (Devlin et al.,
- 2018; Yosinski et al., 2014). We refined the overall parameters of the trained models by utilizing
- 10% of the testing set samples, then proceeded to predict the remaining 90% of the testing set
- samples. During this phase, we adjusted the model's batch size to 16 and lowered the initial
- learning rate to 1e-5 to fit the smaller data sample size. The inset graphs at the lower right
- 269 corners of Figures 3a and 3b display the model's predictions and \mathbb{R}^2 values for 90% of the testing
- set, illustrating that transfer learning improved the model's generalization ability.



 $\begin{array}{c} 271 \\ 272 \\ 727 \\$

- graphs of each subfigure, illustrating direct predictions of the model on Pe < 1,000 (validation) 273 and Pe > 1,000 (testing) datasets, respectively, while the inset graphs depict predictions for 90% 274 of the testing datasets after transfer learning. (c)-(h) demonstrate model performance for 275 predicting conditions with average grain diameters of 0.8 < d < 1.2 mm, 1.2 < d < 1.6 mm, and 276 1.6 < d < 2 mm. In the main graphs, blue dots represent direct predictions by the trained model 277 on the dataset of 600 samples, while red dots indicate predictions for the remaining 90% by the 278 retrained model after transfer learning. The inset graphs show the pore structure of the porous 279 280 media in the corresponding datasets.
- 281
- The primary dataset comprised 10,000 porous media with no cementation and grain sizes ranging
- from 0.4 to 1.2 mm. To evaluate the prediction performance of the trained models on cemented
- porous media with grain sizes between 0.8 and 2 mm, we developed three additional datasets.
- Each dataset contains 600 samples, featuring porosities spanning 0.37 to 0.39, 0.39 to 0.43, and
- 0.41 to 0.45, respectively. The model with the best generalization performance, as shown in Figures 2a and 2b, was adopted to directly predict D_I and u for each of these groups. The
- outcomes, represented by the blue scatter points in Figures 3c to 3h, confirmed the model's
- effectiveness in precisely estimating parameters for these unencountered porous media samples.
- Moreover, the application of transfer learning, represented by the red scatter points in the same
- figures, significantly enhanced the model's generalization ability. This finding highlights that

- transfer learning, which involves fine-tuning model parameters with limited sample sizes,
- substantially strengthens the model's generalization capabilities, crucial for broader applicability.
- 294 Leveraging the physics-enhanced neural network and transfer learning strategies discussed in
- this letter, we anticipate that the pretrained models will be adept at accurately estimating physical
- 296 parameters across diverse porous media and under varied flow conditions.

During direct simulations, we assigned two Darcy flow velocities to each porous medium to calculate parameters D_L and u. In the training process of PhysenCT-Net, we applied three image

- transformations to each porous medium sample to prevent redundancy of input-output pairs in
- the deep learning dataset. Here, we created a unique dataset by assigning 256 different Darcy
- flow velocities to the same porous medium image samples while maintaining constant D_m . Six
- porous medium samples were selected for analysis. The trained PhysenCT-Net model reliably predicted an increment in the D_L as average fluid velocity increased. This pattern complies with
- the $D_L = \alpha_L \cdot u_{avg}^n + D_m$ empirical formula proposed by Cherry and Freeze (1979), where α_L
- denotes the longitudinal dispersivity and u_{avg} the average fluid velocity ($u_{avg} = v/\varepsilon$), validating
- 306 the model's competency in providing parameter predictions consistent with physical principles.
- 307 Such alignment emphasizes the model's proficiency in accurately capturing the fundamental
- 308 physics governing solute transport in porous media.



309

Figure 4. Predicted longitudinal dispersion coefficient versus average fluid velocity by the trained PhysenCT-Net model.

312 5 Conclusions

In this letter, we introduce the development of PhysenCT-Net, a significant advancement in utilizing neural networks to predict crucial solute transport parameters in porous media, such as

- the longitudinal dispersion coefficient and average solute transport velocity. The PhysenCT-Net
- methodology combines the precision of numerical simulation methods with the computational
- efficiency of deep learning technologies. It adeptly integrates three-dimensional images of
- porous media with pertinent physical parameters governing the dispersion process, showcasing
- robust performance and adherence to physical principles. Our study establishes the basis for
- future research that synergizes deep learning with direct pore-scale simulations, aiming for the
- rapid and precise determination of critical parameters to enhance the modeling of transport

- 322 phenomena in porous media and align computational predictions closely with real-world
- 323 behaviors.

324 Data Availability Statement

- 325 Part of the dataset, codes, and trained models are accessible at
- 326 <u>https://doi.org/10.5281/zenodo.10858994</u> and can also be obtained from the corresponding
- 327 author for detail upon reasonable request.

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- 334 335

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