Revisiting the Economic Value of Groundwater

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Revisiting the Economic Value of Groundwater

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Key Points:

- Revisited the theoretical framework of computing the economic value of groundwater in a dynamic context.
- Proved the existence of the dynamic reallocation value, which is generated by an intertemporal intake reallocation of groundwater users.
- Disregarding this new value can underestimates the value of groundwater as an essential instrument for climate adaptation.

Abstract

This paper revisits the theoretical framework of computing the economic value of groundwater in a dynamic context. Specifically, we prove that an additional type of economic value exists, that is, the dynamic reallocation value (DRV), which has been overlooked in existing studies, and we propose a new construction of the total economic value of groundwater with social implications for the role of groundwater in climate adaptation. We examine the existence of this new value and its underlying behavioural mechanism using a simple two-stage model, and then generalise the specification to a dynamic model with an arbitrary number of stages. We find that behind the positive values of DRV, users intentionally destabilize total water use by amplifying their reactions against surface water fluctuations and still realize a higher total expected benefit than in the case without uncertainty. We show that this behaviour is an intertemporal reallocation of groundwater intake against changes in intertemporal cost allocations caused by the users' stabilizing behaviours. Disregarding the DRV underestimates the economic value of groundwater as an essential instrument for climate adaptation.

1 Introduction

Over the past few decades, a considerable number of studies have attempted to quantify the economic value of groundwater in various locations worldwide and have explored improved groundwater management systems (e.g., Burt, 1964; Kim et al., 1989; Tsur, 1990; Tsur & Graham-Tomasi, 1991; Ramasamy, 1996; Amigues et al., 1997; National Research Council, 1997; Hernández-Mora et al., 2003; Pulido-Velázquez et al., 2004; Ranganathan & Palanisami, 2004; Syaukat & Fox, 2004; Kakumanu & Bauer, 2008; Diao et al., 2008; Palanisami et al., 2008; Marques et al., 2010; Ananthini & Palanisami 2010; Reichard et al., 2010; Nanthakumaran & Palanisami, 2011; Gomez & Rola, 2011; Palanisami et al., 2012; Kovacs et al., 2015; Rouhi Rad et al., 2017; Foster et al., 2017; MacEwan et al., 2017;Ashwell et al., 2018; Quintana-Ashwell & Gholson, 2022; Msangi & Hejazi, 2022). Most of these attempts are grounded in theoretical frameworks traced back to Tsur's seminal papers on the buffering role of groundwater (Tsur et al., 1989; Tsur, 1990; Tsur & Graham-Tomas, 1991; Gemma & Tsur, 2007).

The basic construction of such frameworks is as follows: the total economic value (TEV) of groundwater can be divided into the augmentation value (AV) and the stabilization value (SV). The AV is the value of being augmented by an increase in the average water intake through the exploitation of groundwater resources in addition to surface water. The SV is the value of mitigating the impact of surface water fluctuations by adjusting groundwater intake. Typically, groundwater extraction increases during periods of surface water shortage and decreases during periods of surface water abundance. Tsur presented a methodological framework for computing the values of these components.

The present paper revisits this framework in a dynamic context. Specifically, it proves the existence of an additional type of economic value, that is, the dynamic reallocation value (DRV), which has been overlooked in previous studies, including those conducted in a dynamic context. Furthermore, we propose a new construction of the total economic value of groundwater, with social implications for the role of groundwater in climate adaptation.

Similar to the SV, the DRV is derived from the adaptive behaviours of economic agents against surface-water variations under uncertain environments. However, they are conducted with different economic intentions and movements in opposite directions. They are optimizations against the changes in intertemporal cost allocations that occur as a reflection of stabilizing behaviours. Therefore, disregarding the DRV underestimates the economic value of groundwater as an essential instrument for climate adaptation.

Similar to most relevant studies (e.g., Peter et al., 2020; Monobina & Kurt, 2014; Abell et al., 2017; Cécile & Marine, 2019; Msangi & Hejazi, 2022), the present paper limit its attention to industrial and agricultural use of groundwater. We therefore do not deal with the economic benefits of nonconsumptive water use, such as landscapes, amenities, and tourism. In addition, we do not consider the environmental impacts of groundwater extraction, such as salt damage, land subsidence, and other externalities on human society and ecosystems. However, we discuss some policy implications of our findings regarding these issues in the discussion section.

The remainder of this paper is organised as follows: section 2 reviews the theoretical background of the economic value of groundwater. Section 3 describes our model formulation. Section 4 proves the existence of DRV, and discusses its underlying mechanism using a simple two-stage model. Section 5 generalises the findings to a model with an arbitrary number of stages, and presents some numerical illustrations of the DRV. Finally, Section 6 concludes the paper.

2. Theoretical Background

The basic idea of the Tsur's framework is the following. To compute the economic value of groundwater, we first use the difference between the expected net economic benefit of using both surface water and groundwater conjunctively and that of using only surface water, taking the latter as a baseline (Tsur, 1990; Reichard & Raucher, 2003; Sato, 2015). Specifically, we can calculate the economic value of groundwater V^u as follows:

$$V^u \triangleq E[F(w_u) - C(G_u) \cdot (w_u - S)] - E[F(S)], \# \tag{1}$$

where $F(\cdot)$ is a concave benefit function, w_u the benefit-maximizing total water use, $C(\cdot)$ a unit extraction cost that depends on the groundwater stock G_u, and S uncertain surface water whose known mean value is $\bar{S}$. In most groundwater literature, a unit cost function depends on the distance between the water table and ground surface. Although we implicitly incorporate the mathematical transformation from the stock amount to the above distance in the form of the function $C(\cdot)$ to simplify calculations, this doesn't have any effect on the essence of the solutions and conclusions below. We assume $C(\cdot)$ is strictly decreasing, that is, the smaller the groundwater stock, the higher the unit cost is. For simplicity, we assume that the user can utilize the surface water for free; therefore, the remaining $w_u - S$ represents the amount of groundwater used.

The difference obtained in (1) however contains both the AV and SV. To eliminate the AV and extract a pure SV, Tsur uses the difference in benefits when there is no uncertainty in S as another baseline. That is,

$$V^c \triangleq F(w_c) - C(G_c) \cdot (w_c - \bar{S}) - F(\bar{S}), \# \tag{2}$$

where w_c is the benefit-maximizing total water use in the case without uncertainty and G_c is the groundwater stock. The SV is then given by

$$SV \triangleq V^u - V^c. \# \tag{3}$$

In this simplified static problem, if the groundwater stocks are equal, that is, $G_u = G_c$, and so are the unit costs, the benefit-maximizing amount of water use are also the same, thereby indicating that $w_u = w_c$, and so are the expected pumping costs. This is because the benefit-maximizing amount of water used is determined at the level at which the marginal net benefit $F'(w)$ is equal to the marginal cost (unit cost) $C(G)$. Therefore, the user pumps an amount that can completely offset surface water fluctuations and stabilize the net benefit. Accordingly, the SV can eventually be computed as the difference in benefits with and without uncertainty when the user can only use surface water.

$$SV = V^u - V^c = F(\bar{S}) - E[F(S)]. \# \tag{4}$$

Thus, the SV can be expressed as a risk premium that the user is willing to pay to stabilize the surface water flow at the mean (Gemma & Tsur, 2007).

Using (4), the augmentation value can be computed as the remainder, $V^u - SV$:

$$AV \triangleq F(w_c) - C(G_c) \cdot (w_c - \bar{S}) - F(\bar{S}). \# \tag{5}$$

The total economic value of the groundwater is the sum $TEV \triangleq SV + AV$:

$$TEV = F(w_c) - C(G_c) \cdot (w_c - \bar{S}) - E[F(S)]. \# \tag{6}$$

Various studies have applied this approach to evaluate the economic value of groundwater in actual water environments (e.g., for cases in India, Ramasamy (1996), Ranganathan & Palanisami (2004), Gemma & Tsur (2007), Kakumanu & Bauer (2008), Palanisami et al. (2008), Ananthini & Palanisami (2010), Nanthakumaran and Palanisami (2011), and Palanisami et al. (2012); for cases in the United States, Tsur (1997), Kovacs et al. (2015), Kovacs & West, 2016; MacEwan et al. (2017), and Msangi & Hejazi (2022); for cases in Israeli, Tsur (1990)).

However, the transformation from (3) to (4) is not applicable to dynamic cases in general, even if the initial groundwater stocks were the same. Gemma and Tsur (2007) seem to be aware of this point. Hence, in an attempt to extend the Tsur (1990)'s framework to a dynamic environment, they avoided using a simple analogy of the risk premium in equation (4) but did not explore what exists in the gap between (3) and (4). The most recent attempt to apply Tsur's framework to a dynamic environment is Msangi and Hejazi (2022), which analyzes the impact of suboptimal behaviours and the physical constraints of extraction abilities on the economic value of groundwater. Through an empirical application to California, they showed that suboptimal behaviours diminish the AV while keeping the SV unaffected in the unconstrained case; however, the SV could be diminished in the constrained case. We will come back to this point in later sections.

On the other hand, the present paper argues that an additional type of economic value is hidden in the difference between V^u and V^c, that is,

$$V^u - V^c = SV + DRV. \quad (7)$$

Thus, the total economic value of the groundwater is composed of three components:

$$TEV = AV + SV + DRV. \quad (8)$$

3 Model Formulation

In each of the following analyses, we consider models with N users for the sake of generality, and denote the user set $\{1, \dots, N\}$ as $\mathcal{N}$. This enables us to examine the economic value of groundwater in both optimal and suboptimal environments. The former type of solution is described by a single decision-maker model, where the social planner distributes groundwater intake to each user during each time period to maximize the intertemporal sum of the aggregate net economic benefits of all users (henceforth, *single decision-maker regime*). The other type of solution is described by a multiple-user model in which each user plays a noncooperative dynamic game in choosing the amount of groundwater intake with the aim of maximizing its own intertemporal sum of net economic benefits (henceforth, *multiple-user regime*). Replacing $N = 1$ provides simpler scenarios for a single user.

The water environment in both regimes is governed by a stochastic dynamic process determined by two state variables: $G_{t-1} \in \mathcal{G}$, the groundwater stock, and $S_t \in \mathcal{S}$, the surface water flow, both available to users at the beginning of period t, where $\mathcal{G}$ and $\mathcal{S}$ represent sets of possible amount of the groundwater stock and surface water flow, respectively. The transition equation for the groundwater stock is as follows:

$$G_t = f(G_{t-1}, R_t, g_{1t}, \dots, g_{Nt}) \triangleq G_{t-1} + R_t - \sum\nolimits_{\mathcal{N}} g_{it}, \quad (9)$$

where g_{it} (≥ 0) is the groundwater intake by user i in period t and R_t (≥ 0) denotes the groundwater recharge in period t. Groundwater dynamics can be governed by a variety of interconnected hydrological processes driven by various climatic, topographic, and hydrogeological factors (Cuthbert et al., 2019). Therefore, more complex mechanisms, such as stochastic and spatially heterogeneous groundwater recharge, which are affected by local precipitation and surface water intake, can be introduced. However, for analytical simplicity, we don't touch on such complexities and use a fixed value, R, throughout all periods. However, such simplifications do not invalidate the essence of our argument on the existence of a new value, because the behavioural mechanism that generates it is the users' natural reactions to the underlying nature of the groundwater stock transition as argued below.

The surface flow S_t is given by:

$$S_t = \bar{S}_t + \xi_t, \quad \# \tag{10}$$

where $\bar{S}_t$ is the average flow amount that is expected in period t in normal years and ξ_t denotes the fluctuation from the average in period t, where $\xi_t > 0$ means a period of abundant water supply and $\xi_t < 0$ a period of water scarcity. For the analytic approach in the following section, we assume, like most groundwater literature (e.g., Burt, 1964; Tsur & Graham-Tomasi, 1991; Provencher & Burt, 1994; Knapp & Olson, 1995; Joodavi et al., 2015), that ξ_t is a stationary, temporally independent random variable of a known distribution with a zero mean and variance of σ^2.

Users make decisions on groundwater intake after observing the realization of surface water flows during the current period. Let $s_{it} = \varepsilon_i S_t$ denote the amount of surface water utilized by user i in period t, where ε_i is the share of user i and $\sum_{\mathcal{N}} \varepsilon_i = 1$. For simplicity, we assume that users can use surface water within this range at no additional cost. Let w_{it} be the total amount of water used by user i in period t; thus, $w_{it} = g_{it} + s_{it}$.

$F_i(w_{it})$ represents the instantaneous benefit accruing to user i in period t, which is assumed to be quadratic for acquiring analytical solutions:

$$F_i(w_{it}) \triangleq a_i w_{it} - b w_{it}^2, \#$$

where a_i and b are positive constants. This represents diminishing returns to production, which accords with most production practices as reported in many groundwater literature (e.g., Gisser & Sánchez, 1980; Provencher & Burt, 1994; Gardner et al., 1997; Msangi & Hejazi, 2022; Quintana-Ashwell & Gholson, 2022). Based on this, we introduce user heterogeneity by differentiating parameter a_is. Although we do not differentiate parameter b to obtain analytical solutions for the dynamic game, this differentiation allows us to cover a broad range of heterogeneity in terms of production scale and technology.

Let $C_i(G_t)$ denote the unit cost of user i for pumping groundwater to the surface, which depends on the groundwater stock.

$$C_i(G_t) \triangleq c_i - dG_t, \#$$

where c_i and d are positive constants. Therefore, the cost is inversely proportional to the total inventory. This is consistent with the assumptions of most groundwater studies such as those of Gisser and Sánchez (1980) and Gardner et al. (1997). Moreover, although we do not differentiate the parameter d to obtain analytical solutions, the differentiation of c_i enables us to represent a considerable amount of heterogeneity in pumping facilities and the spatial diversity of an aquifer. Again, the specifications of these parameters do not invalidate our arguments on the new value.

The instantaneous net benefit, including the pumping cost, for user i in period t is given by:

$$\pi_i(g_{it}, G_{t-1}, S_t) \triangleq F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1}) g_{it}. \#$$

The period set $\{1, \dots, T\}$ is denoted by $\mathcal{T}$, and let $\Pi_i\colon (\mathcal{G} \times \mathcal{S} \times \mathrm{U}_{i1} \times \mathrm{U}_{-i1}) \times \dots \times (\mathcal{G} \times \mathcal{S} \times \mathrm{U}_{iT} \times \mathrm{U}_{-iT}) \to \mathbb{R}_{\geq 0}$ denote the discounted intertemporal sum of user i's expected net benefits:

$$\Pi_i(G_0, S_1, g_{i1}, g_{-i1}, \dots, G_{T-1}, S_T, g_{iT}, g_{-iT}) \triangleq E\left[\sum_{t \in \mathcal{T}} \beta^{t-1} [F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1}) g_{it}]\right], \quad (11)$$

where U_{it} is the set of admissible actions of user i in period t, and $\beta \in [0, 1]$ is a discount factor. Symbols with the subscript $-i$ indicate that they are a variable or set for the users excluding user i. The social planner maximizes the discounted intertemporal sum of the aggregate expected net benefits $\Pi\colon (\mathcal{G} \times \mathcal{S} \times \mathrm{U}_{11} \times \dots \times \mathrm{U}_{N1}) \times \dots \times (\mathcal{G} \times \mathcal{S} \times \mathrm{U}_{1T} \times \dots \times \mathrm{U}_{NT}) \to \mathbb{R}_{\geq 0}$:

$$\begin{aligned} \Pi(G_0, S_1, g_{11}, \dots, g_{N1}, \dots, G_{T-1}, S_T, g_{1T}, \dots, g_{NT}) &\triangleq \sum_{i \in \mathcal{N}} \Pi_i(G_0, S_1, g_{i1}, g_{-i1}, \dots, G_{T-1}, S_T, g_{iT}, g_{-iT}) \\ &= E\left[\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} \beta^{t-1} [F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1}) g_{it}]\right], \end{aligned}$$

subject to equations (9) and (10), and the initial stock level G_0. One of the possible requirements for admissible actions is, $U_{it} \coloneqq [0, G_{t-1}]$, that is, users can exploit the aquifer to its whole stock level. In the following, we assume that the total groundwater intake does not exceed the current groundwater stock within a single period. We come back to a drawback of this simplification in the discussion section.

In the multiple-user regime, user i maximizes the discounted intertemporal sum of the expected net benefits (11) subject to (10), the initial stock level G_0, and the transition equations of the groundwater stock:

$$G_t = f_i(G_{t-1}, g_{it}, g_{-it}) \triangleq G_{t-1} + R - g_{it} - \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} g_{jt}\,, \qquad t \in \mathcal{T}.$$

Let γ_{it} denote an admissible strategy of user i for $S_t \in \mathcal{S}, G_{t-1} \in \mathcal{G}, t \in \mathcal{T}$, and let Γ_{it} denote the set of admissible strategies. We can then describe the dynamic process as an N-user T-stage discrete-time stochastic dynamic noncooperative game defined by $\{\mathcal{N}, \mathcal{T}, \mathcal{G}, \mathcal{S}, \{U_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{f_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Gamma_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Pi_i\}_{i\in\mathcal{T}}\}$.

4 Two-stage Model

We start by demonstrating the existence of a new value using a simple two-stage model and examine the underlying economic mechanisms.

4.1 Existence of the DRV

For the two-stage model, by solving backwards from the second stage, we obtain unique solutions for each regime and for cases with and without uncertainty (See SI1 in the Supporting Information for solutions and derivation). In the following discussion, we use the notations in Table 1 for the variables derived from these solutions:

Table 1. Notations for the variables derived from the solutions.

(a) Single decision-maker regime

Notation	Description
$\pi_u^{\text{single}} = \pi_{u1}^{\text{single}} + \pi_{u2}^{\text{single}}$	aggregate expected net benefit (and its temporal decomposition) in the uncertain case
$\pi_c^{\text{single}} = \pi_{c1}^{\text{single}} + \pi_{c2}^{\text{single}}$	aggregate expected net benefit (and its temporal decomposition) in the certain case
$w_{u1}^{\text{single}}(S_1)$	aggregate water use at the first stage after observing S_1 in the uncertain case
$w_{c1}^{\text{single}}(\bar{S})$	aggregate water use at the first stage after observing $\bar{S}$ in the certain case
$g_{u1}^{\text{single}}(S_1)$	aggregate groundwater intake at the first stage after observing S_1 in the uncertain case
$g_{c1}^{\text{single}}(\bar{S})$	aggregate groundwater intake at the first stage after observing $\bar{S}$ in the certain case

(b) Multiple-user regime

Notation	Description
$\pi_u^{\text{multi}} = \pi_{u1}^{\text{multi}} + \pi_{u2}^{\text{multi}}$	aggregate expected net benefit (and its temporal decomposition) in the uncertain case
$\pi_c^{\text{multi}} = \pi_{c1}^{\text{multi}} + \pi_{c2}^{\text{multi}}$	aggregate expected net benefit (and its temporal decomposition) in the certain case
$w_{u1}^{\text{multi}}(S_1)$	aggregate water use at the first stage after observing S_1 in

	the uncertain case
$w_{c1}^{\mathrm{multi}}(\bar{S})$	aggregate water use at the first stage after observing $\bar{S}$ in the certain case
$g_{u1}^{\mathrm{multi}}(S_1)$	aggregate groundwater intake at the first stage after observing S_1 in the uncertain case
$g_{c1}^{\mathrm{multi}}(\bar{S})$	aggregate groundwater intake at the first stage after observing $\bar{S}$ in the certain case

Note that the expected net benefits in the table are the expected values evaluated before the realization of surface water in the first period, whereas water use and groundwater intake are the values that users determine after observing it. In addition, we don't use the discount factor to evaluate the expected net benefits, although the solutions used here, that is, $g_{u1}^{\mathrm{single}}(S_1)$, $g_{c1}^{\mathrm{single}}(\bar{S})$, $g_{u1}^{\mathrm{multi}}(S_1)$ and $g_{c1}^{\mathrm{multi}}(\bar{S})$, are the results of users' decisions with discounting. Therefore, we evaluate the economic value of each period equally. Summing up the discounted net benefits is another option for evaluating the economic value of groundwater in a dynamic context and may sometimes be more appropriate for resource management practices. However, as researchers, we take a different approach for our analytical purpose to evaluate users' behaviours equally throughout the period.

Although we explain the reason behind the name later, we define the dynamic reallocation value (DRV) as follows:

Definition 1. The dynamic reallocation value (DRV*)* is the difference in the intertemporal sum of the aggregate expected net benefit in cases with and without uncertainty in surface water:

$$\begin{gathered} DRV_{\mathrm{single}} \triangleq \pi_u^{\mathrm{single}} - \pi_c^{\mathrm{single}}, \\ DRV_{\mathrm{multi}} \triangleq \pi_u^{\mathrm{multi}} - \pi_c^{\mathrm{multi}}. \# \end{gathered} \tag{12}$$

We can easily derive the following from the solutions of the two-stage model:

Proposition 1. The dynamic reallocation value (DRV) is positive in both the single decision-maker and multiple-user regimes. That is,

$$\begin{gathered} DRV_{\mathrm{single}} = \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}\sigma^2 > 0, \\ DRV_{\mathrm{multi}} = \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}\sigma^2 > 0. \# \end{gathered} \tag{13}$$

For the full proof, see SI2 in the Supporting Information.

This requires significant reconsideration of the specifications of the economic value of groundwater used in the literature, which indicates that the above differences are zero. First, the transformation from (3) into (4) is incorrect in dynamic environments. Second, we argue that the specification of the SV in (3) is not appropriate, because the difference $V^u - V^c$ contains a different type of economic value. That is,

$$\begin{aligned} V^u_{\text{single}} - V^c_{\text{single}} &= SV_{\text{single}} + DRV_{\text{single}}, \\ V^u_{\text{multi}} - V^c_{\text{multi}} &= SV_{\text{multi}} + DRV_{\text{multi}}, \# \end{aligned} \tag{3'}$$

where for the computation of SV_{single} and SV_{multi}, we use the specification in (4). In the two-stage model:

$$SV_{\text{single}} = SV_{\text{multi}} = \sum_{t=1}^{2} \sum_{i \in \mathcal{N}} \{F_i(\varepsilon_i \bar{S}) - E[F_i(\varepsilon_i S_t)]\} = 2b \left(\sum_{i \in \mathcal{N}} \varepsilon_i^2 \right) \sigma^2. \# \tag{4'}$$

In the latter half of this section, we explain why the dynamic reallocation value should not be considered part of the SV.

Third, the above considerations redefine the composition of the total economic value of groundwater. Based on (5), the augmentation values can be derived as follows:

$$\begin{aligned} AV_{\text{single}} &= \pi_c^{\text{single}} - \sum_{t=1}^{2} \sum_{i \in \mathcal{N}} F_i(\varepsilon_i \bar{S}), \\ AV_{\text{multi}} &= \pi_c^{\text{multi}} - \sum_{t=1}^{2} \sum_{i \in \mathcal{N}} F_i(\varepsilon_i \bar{S}). \# \end{aligned} \tag{5'}$$

We can therefore derive a new composition:

$$\begin{aligned} TEV_{\text{single}} &\triangleq \pi_u^{\text{single}} - \sum_{t=1}^{2} \sum_{i \in \mathcal{N}} E[F_i(\varepsilon_i S_t)] = AV_{\text{single}} + SV_{\text{single}} + DRV_{\text{single}}, \\ TEV_{\text{multi}} &\triangleq \pi_u^{\text{multi}} - \sum_{t=1}^{2} \sum_{i \in \mathcal{N}} E[F_i(\varepsilon_i S_t)] = AV_{\text{multi}} + SV_{\text{multi}} + DRV_{\text{multi}}. \# \end{aligned} \tag{14}$$

Studies that measure the economic value of groundwater using the specification of SV in (3) most likely overestimate the magnitude of SV, and those that use the specification of (4) overestimate the magnitude of AV.

4.2 Behavioural mechanisms of the DRV

However, what is DRV and why should it be distinguished from SV and AV? To answer this, the behavioural mechanisms of the users that generate this value need to be comprehensively understood. From the solutions shown in SI1 in the Supporting Information, we can easily demonstrate how the users' groundwater intake reacts to surface water fluctuations.

Proposition 2. When the surface water in the first period, S_1, deviates from its mean value by $S_1 - \bar{S}$, the aggregate groundwater intake responds to it by more than $S_1 - \bar{S}$ in both the single decision-maker and multiple-user regimes. That is,

$$\begin{aligned} g_{u1}^{\text{single}}(S_1) - g_{u1}^{\text{single}}(\bar{S}) = g_{u1}^{\text{single}}(S_1) - g_{c1}^{\text{single}}(\bar{S}) &= -\frac{4b^2}{4b^2 - N^2 d^2 \beta}(S_1 - \bar{S}), \\ g_{u1}^{\text{multi}}(S_1) - g_{u1}^{\text{multi}}(\bar{S}) = g_{u1}^{\text{multi}}(S_1) - g_{c1}^{\text{multi}}(\bar{S}) &= -\frac{4b^2}{4b^2 - N d^2 \beta}(S_1 - \bar{S}). \# \end{aligned} \tag{15}$$

This is significantly different from the stabilizing behaviour implied by previous studies in the specification of Equation (4), where the groundwater intake responds to the surface water fluctuation on a one-to-one basis to ensure that the former movement perfectly offsets the latter change. If the surface-water content increases by $S_1 - \bar{S}$, the groundwater intake declines by $S_1 - \bar{S}$. If the surface water decreases by $S_1 - \bar{S}$, the groundwater intake increases by $S_1 - \bar{S}$. However, Proposition 2 suggests that groundwater intake not only stabilizes the fluctuation but also destabilizes the total water use. From Equation (15), we can easily derive the following:

$$\begin{aligned} w_{u1}^{\text{single}}(S_1) - w_{u1}^{\text{single}}(\bar{S}) = w_{u1}^{\text{single}}(S_1) - w_{c1}^{\text{single}}(\bar{S}) &= -\frac{N^2 d^2 \beta}{4b^2 - N^2 d^2 \beta}(S_1 - \bar{S}), \\ w_{u1}^{\text{multi}}(S_1) - w_{u1}^{\text{multi}}(\bar{S}) = w_{u1}^{\text{multi}}(S_1) - w_{c1}^{\text{multi}}(\bar{S}) &= -\frac{N d^2 \beta}{4b^2 - N d^2 \beta}(S_1 - \bar{S}). \# \end{aligned} \tag{16}$$

In specification (4), the surface water fluctuation has no effect on the total water use because it is perfectly absorbed by the offsetting movement of the groundwater intake; however, Equation (16) reveals that it has an effect. When the amount of surface water increases, the total water declines and as the surface-water decreases, the total water increases.

This intended destabilization decreases the expected benefit of the first period, but it is more than covered in the second period, as shown in the next proposition, which can easily be calculated from the results shown in SI1 in the Supporting Information. This leads to the intertemporal sum of the expected benefit being greater than that in a certain case, as shown in Proposition 1.

Proposition 3. The aggregate expected net benefit in the first period in the case with uncertainty is less than that in the case without uncertainty, whereas the aggregate expected net benefit in the second period in the case with uncertainty is greater than that in the case without uncertainty. That is,

$$\begin{aligned}
\pi_{u1}^{\text{single}} - \pi_{c1}^{\text{single}} &= -\frac{N^3 b d^4 \beta^2}{(4b^2 - N^2 d^2 \beta)^2}\sigma^2 < 0, \\
\pi_{u2}^{\text{single}} - \pi_{c2}^{\text{single}} &= \frac{4Nb^3 d^2}{(4b^2 - N^2 d^2 \beta)^2}\sigma^2 > 0, \\
\pi_{u1}^{\text{multi}} - \pi_{c1}^{\text{multi}} &= -\frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}\sigma^2 < 0, \\
\pi_{u2}^{\text{multi}} - \pi_{c2}^{\text{multi}} &= \frac{4Nb^3 d^2}{(4b^2 - Nd^2\beta)^2}\sigma^2 > 0. \#
\end{aligned} \tag{17}$$

From these results, we can expect that there is another consideration in users' intake decisions that differs from the stabilizing behaviour. Therefore, we aim to elucidate the reason behind users' intentionally destabilizing water use and why such behaviours generate higher total benefit than that in cases without uncertainty.

To examine these points graphically, we further simplify the model in four respects: first, we consider a single user model with the instantaneous benefit function $F(w_t) = aw_t - bw_t^2$; second, we consider that the surface water takes between two values S_L ($= 0$ for simplicity) and S_H with a probability of $1/2$ for each and with the mean value $\bar{S}$ ($= S_H/2$); third, there is no groundwater recharge ($R = 0$); and fourth, the discount factor $\beta = 1$. These simplifications are only for graphical illustration, and the argument below holds for the more general specifications discussed thus far.

In the first stage, after observing surface water S_1, the user faces the following problem:

$$\max_{g_1} F(S_1 + g_1) - C(G_0)g_1 + E_1\left[F\big(S_2 + g_2(S_2, g_1)\big) - C(G_0 - g_1)g_2(S_2, g_1)\right], \#$$

where $g_2(S_2, g_1)$ is the solution in the second period with stock level $G_0 - g_1$ and the observation of S_2:

$$g_2(S_2, g_1) = \frac{1}{2b}\big(a - c + d(G_0 - g_1)\big) - S_2. \# \tag{18}$$

As discussed in the previous section, we excluded cases in which the user exploits the entire stock in a single period. The first-order condition then provides the benefit-maximizing intake g_1^*:

$$F'(S_1 + g_1^*) = C(G_0) + E_1[-C'(G_0 - g_1^*)g_2(S_2, g_1^*)].\# \quad (19)$$

The benefit-maximizing groundwater intake is therefore ensured when the marginal benefit is equal to the sum of the unit cost of the first period (the first term on the right side) and the marginal user cost (the second term). The latter is the future pumping cost that would have been saved by decreasing a marginal unit of groundwater intake in the first period. In other words, this is the opportunity cost of the current extraction.

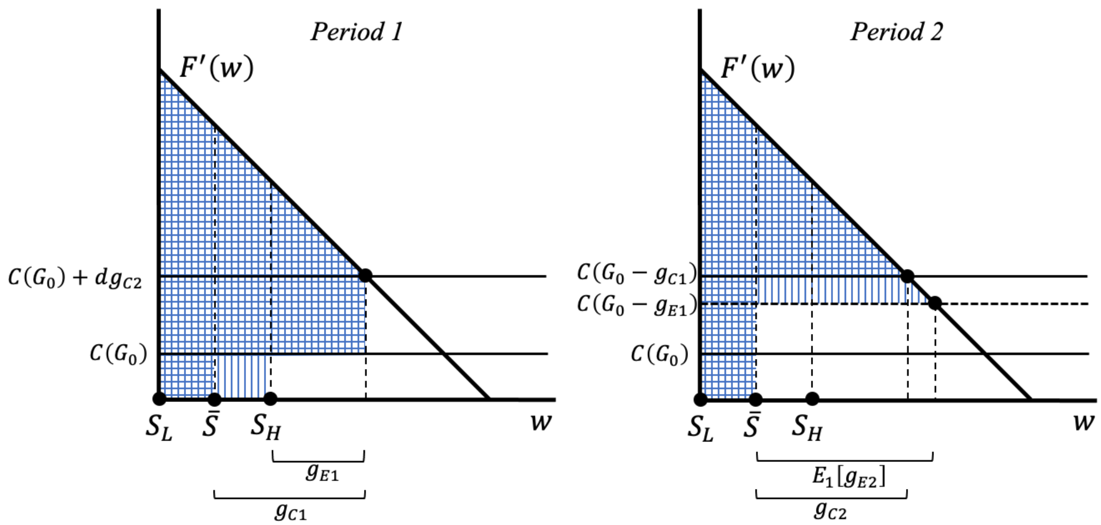
We examine this mechanism in two steps. First, we introduce a policy in which the user absorbs the surface water fluctuation perfectly in the first period and keeps the total water use for that period constant (at the mean value). This is not the optimal behaviour but provides a very good case for understanding the behavioural mechanism of dynamic reallocation. We call this *Policy E* (where *E* represents *exact stabilization*) and denote it by g_{Et}. Next, we introduce the optimal policy described in Proposition 2. In this policy, the user amplifies its reaction against surface water fluctuation to generate an artificial destabilization but can achieve a full dynamic reallocation value. We call this *Policy R* (where *R* represented *reallocation*) and denote it by g_{Rt}. In addition, we call a reference policy that the user would take when there is no uncertainty *Policy C* (where *C* represents *certainty*) and denote it by g_{Ct}. In the following figures, we describe the user's intake decisions and the corresponding benefits and costs after observing (a) S_H and (b) S_L during the first period.

Policy E

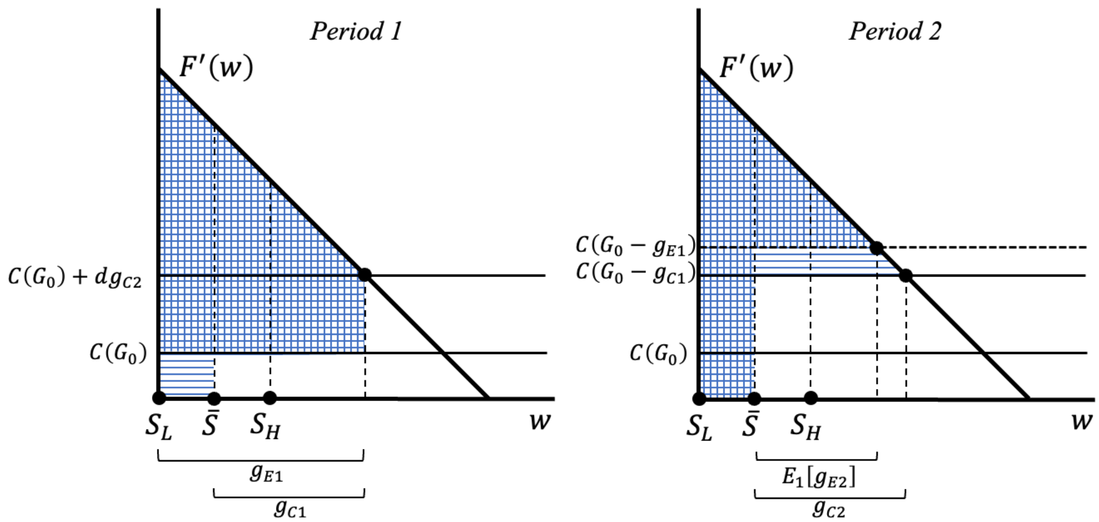
Figure 1 shows a comparison of Policies E and C. In Policy C, the total water use in the first period is determined at the intersection of the marginal benefit curve $F'(w)$ and the sum of the unit cost and marginal user cost $C(G_0) + dg_{C2}$. Policy E also maintains this amount by changing the groundwater intake g_{E1} to offset the surface water fluctuation in an exact manner. The expected net benefits evaluated in period 0 are the same for both policies. This is exactly the same situation as that captured by the simplification of Equation (4). Therefore, the SV in period 1 is evaluated purely by the risk premium in (4).

But the truth is that the impact of the fluctuation does not disappear at all. It is transferred to period 2 through the corresponding change in the groundwater stock and unit cost, which is represented by the differences between the solid and dotted horizontal lines on the right side of Figure 1(a) and (b). Note that the intake of Policy E in period 2 (g_{E2}) is shown as the expected amount evaluated before the realization of surface water in this period.

Surprisingly, even in Policy E, which replicates the standard stabilizing behaviour, if we stand at period 0 (the moment before observing S in period 1), the expected net benefit is larger than that of Policy C. Why does the case with uncertainty achieve a higher expected net benefit than that of the case without uncertainty, even with a concave benefit function (i.e. a risk-averse agent)? Figure 2 shows the increments and decrements in benefits and costs over the values of Policy C. When considering the benefit side only, policy E obtains a lower expected value by the amount corresponding to the area of the triangle in the grey shaded area on the left. This is normal for risk-averse agents. However, on the cost side, it achieves a higher expected reduction by the amount corresponding to the shaded square in the middle. Consequently, the expected net benefit of Policy E is higher than that of Policy C, as indicated by the area of the shaded triangle on the right. Therefore, the source of the higher net benefit is the cost side. Why, however, does Policy E achieve a larger cost reduction? In period 1, the user increases the intake when it observes S_H and decreases it when S_L to stabilize the benefit in the period. These behaviours can simultaneously be seen as an intertemporal reallocation of the groundwater intake, which in turn affects the intertemporal allocation of groundwater stock and thereby that of unit pumping cost. In the case of our two-stage model, the increase (decrease) in intake in period 1 increases (reduces) the unit pumping cost in period 2. This makes the relative price of groundwater in period 2 to period 1 higher (lower) than that of Policy C. Thus, transferring the intake from period 2 to period 1 or from period 1 to period 2 reduces the pumping cost in period 2. In other words, the intertemporal reallocation of groundwater intake, which occurs as a result of the stabilizing behaviour in period 1, generates a higher expected net benefit in Policy E than in Policy C through a cost reduction realized by the corresponding intertemporal reallocation of the unit pumping cost.

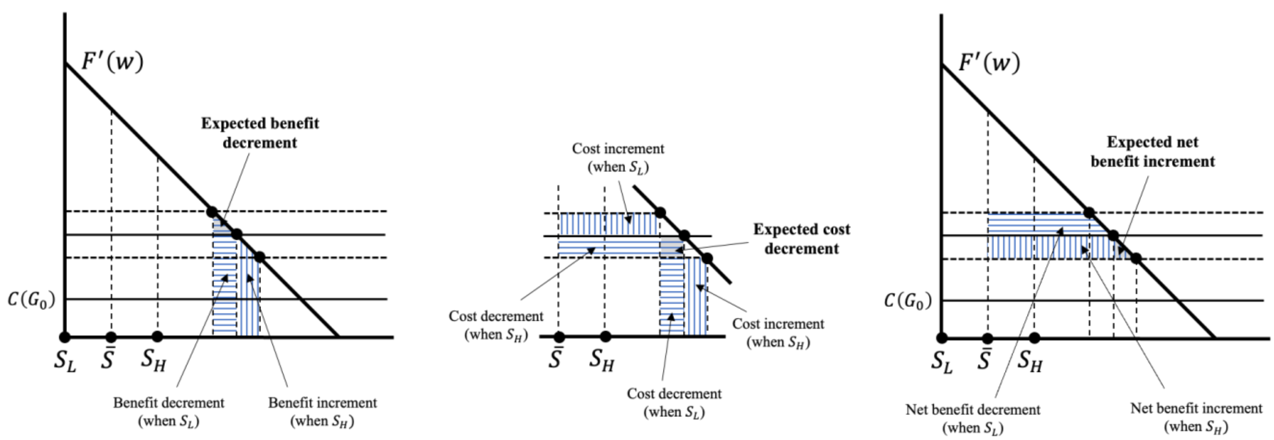


(a) S_H in period 1



(b) S_L in period 1

Figure 1. User's intake decisions and corresponding net benefits for Policies E and C. The line segment that is declining to the right is the marginal benefit curve $F'(w)$. The horizontal lines represent the unit cost or the sum of the unit cost and marginal user cost. The areas in the vertical stripes represent the net benefits achieved by Policy E and the horizontal stripes represent those achieved by Policy C.

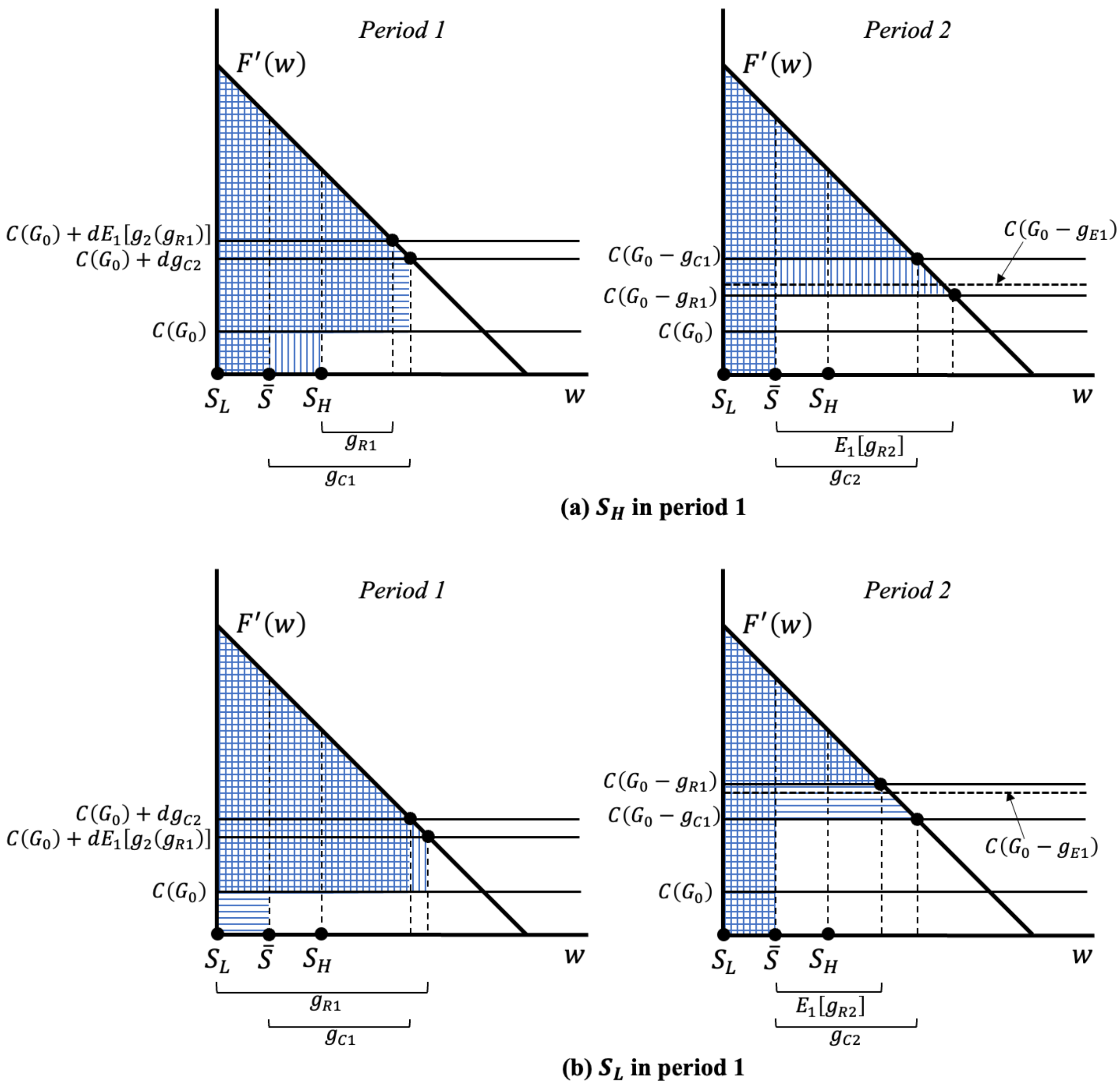


(a) Expected benefit **(b) Expected cost** **(c) Expected net benefit**

Figure 2. Increments and decrements in expected benefit and cost of Policy E over Policy C**.** The areas in the vertical stripes represent the increments and the horizontal stripes represent the decrements in (a) benefit, (b) cost, and (c) net benefit. The areas of the shaded triangles or squares represent the increments or decrements in the expected amount evaluated in period 0.

Policy R

Policy E is not optimal because the intake in period 1 is a simple reaction to the surface water fluctuation of the period and not the benefit-maximizing intake derived from equation (19). In Policy R, the user determines the intake to equate the marginal benefit with the sum of the unit cost and marginal user cost, which reflects the relative price of groundwater in period 2 over period 1. Figure 3 illustrates these behaviours. In period 1, the user increases the intake to more than that of Policy E when it observes S_H and decreases the intake to more than that of Policy E when it observes S_L. This destabilizes the benefit in period 1 and lowers the expected net benefit of the period. However, it achieves a much larger cost reduction in period 2 than that of Policy E and generates a higher total expected net benefit. This is why the artificial destabilization described in Proposition 2 decreases the expected net benefit in the first period but increases it in the second period, as stated in Proposition 3, and finally results in an increased total expected net benefit, as stated in Proposition 1.



(a) S_H in period 1

(b) S_L in period 1

Figure 3. User's intake decisions and corresponding net benefits for Policies R and C**.** The areas in the vertical stripes represent the net benefits achieved by policy R and the horizontal stripes represent those achieved by policy C. The line segment that is declining to the right is the

marginal benefit curve $F'(w)$. The horizontal lines represent the unit cost or the sum of the unit cost and marginal user cost.

In summary, the DRV is derived from users' optimization to the changes in intertemporal cost allocations that occur as a reflection of their stabilizing behaviours. Users actively reallocate their groundwater intake intertemporally to save their pumping costs throughout the periods, thereby achieving a higher total benefit even in the case with uncertainty than in the case without uncertainty. We, therefore, call this value the "dynamic reallocation value."

5 Dynamic Model with An Arbitrary Number of Stages

5.1 Generalization of the DRV

We first generalise the formulation of the DRV in Equation (13) to models with an arbitrary number of stages T. Subsequently, we examine how the generalized DRV reacts to changes in major parameters using some numerical illustrations.

Proposition 4. In the single decision-maker regime, the dynamic reallocation value (DRV) in a dynamic problem of maximizing $\Pi: (\mathcal{G} \times \mathcal{S} \times \mathrm{U}_{11} \times \ldots \times \mathrm{U}_{N1}) \times \ldots \times (\mathcal{G} \times \mathcal{S} \times \mathrm{U}_{1T} \times \ldots \times \mathrm{U}_{NT}) \to \mathbb{R}_{\geq 0}$ subject to (9), (10), and the initial stock level G_0 is given by $DRV_{\text{single}} = \sum_{\mathcal{T}} \Xi_t$, where

$$
\begin{aligned}
\Xi_t \triangleq & -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2 \\
& + \frac{\Phi(t)\big(Nd - b\Phi(t)\big)}{N}\Bigg\{[\Psi(t-1)]^2 + \big[\Psi(t-2)\big(1-\Phi(t-1)\big)\big]^2 + \cdots \\
& + \left[\Psi(1)\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]^2\Bigg\}\sigma^2, \quad 4 \leq t \leq T,
\end{aligned}
$$

$$
\begin{aligned}
\Xi_3 \triangleq & -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2 \\
& + \frac{\Phi(t)\big(Nd - b\Phi(t)\big)}{N}\Big\{[\Psi(t-1)]^2 + \big[\Psi(t-2)\big(1-\Phi(t-1)\big)\big]^2\Big\}\sigma^2, \\
& t = 3,
\end{aligned}
$$

$$
\Xi_2 \triangleq -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2 + \frac{\Phi(t)\big(Nd - b\Phi(t)\big)}{N}[\Psi(t-1)]^2\sigma^2, \qquad t = 2,
$$

$$
\Xi_1 \triangleq -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2, \qquad t = 1.
$$

For the definition of the functions Ψ and Φ and the proof, see SI3 in the Supporting Information.

Proposition 5. In the multiple-user regime, the dynamic reallocation value (DRV) in a N-user discrete-time stochastic infinite dynamic noncooperative game of a finite horizon, $\{\mathcal{N}, \mathcal{T}, \mathcal{G}, \mathcal{S}, \{U_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{f_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Gamma_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Pi_i\}_{i\in\mathcal{T}}\}$, is given by $DRV_{\text{multi}} = \sum_{\mathcal{T}} \tilde{\Xi}_t$, where

$$\begin{aligned}\tilde{\Xi}_t \triangleq &-\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2 \\ &+\frac{\tilde{\Phi}(t)\left(Nd-b\tilde{\Phi}(t)\right)}{N}\left\{\left[\tilde{\Psi}(t-1)\right]^2+\left[\tilde{\Psi}(t-2)\left(1-\tilde{\Phi}(t-1)\right)\right]^2+\cdots\right. \\ &\left.+\left[\tilde{\Psi}(1)\prod_{\tau=2}^{t-1}\left(1-\tilde{\Phi}(\tau)\right)\right]^2\right\}\sigma^2, \quad 4\leq t\leq T,\end{aligned}$$

$$\begin{aligned}\tilde{\Xi}_3 \triangleq &-\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2 \\ &+\frac{\tilde{\Phi}(t)\left(Nd-b\tilde{\Phi}(t)\right)}{N}\left\{\left[\tilde{\Psi}(t-1)\right]^2+\left[\tilde{\Psi}(t-2)\left(1-\tilde{\Phi}(t-1)\right)\right]^2\right\}\sigma^2, \\ &t=3,\end{aligned}$$

$$\tilde{\Xi}_2 \triangleq -\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2+\frac{\tilde{\Phi}(t)\left(Nd-b\tilde{\Phi}(t)\right)}{N}\left[\tilde{\Psi}(t-1)\right]^2\sigma^2, \qquad t=2,$$

$$\tilde{\Xi}_1 \triangleq -\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2, \qquad t=1.$$

For the definition of the functions $\tilde{\Psi}$ and $\tilde{\Phi}$ and the proof, see SI4 in the Supporting Information.

5.2 Numerical illustrations

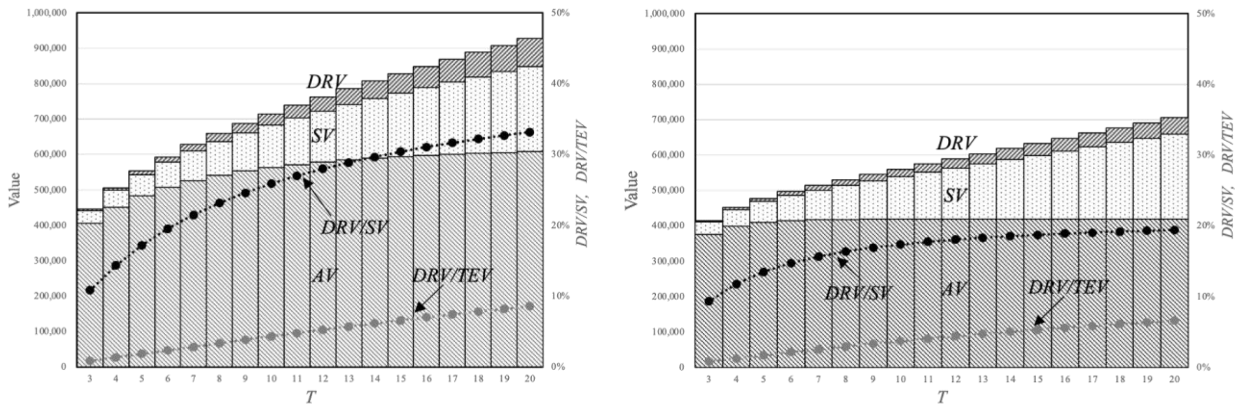
To analyze how the dynamic reallocation value reacts to changes in major parameters, such as the number of stages or the variance of surface water fluctuation, and how such reactions differ between the single-decision-maker regime and the multiple-user regime, this subsection provides some numerical illustrations of each type of economic value by applying a set of sample parameter values to the analytical results of the previous section and subsection (especially, Propositions 4 and 5). The values used are listed in Table 2. Note that the purpose of this subsection is not to simulate the concrete values of the DRV using actual water data. Rather, we aim to examine the basic responses of the DRV to changes in major parameters in a theoretical setting. So the values in the table are arbitrarily chosen to allow clearer graphical demonstrations in the figures below, and they do not have concrete physical and monetary units.

Table 2. Parameter Values Used in Numerical Illustration

Parameter	Description	Value
a_i	First-order coefficient of instantaneous benefit function	12,200
b	Second-order coefficient of instantaneous benefit function	300
c_i	Pumping cost intercept	21,000
d	Pumping cost slope	[15, 20]
G_0	Initial groundwater stock	1,000
$\bar{S}$	Average surface water supply	100
σ^2	Variance of surface water supply	[0, 600]
R	Natural groundwater recharge	0.1
N	Number of users	10
ε_i	Share of water right	$1/N$
β	Discount factor	0.98
T	Number of stages	{3, 4, …, 20}

Figure 4 shows the composition of the three value types at a different number of stages ($T = 3, 4, \ldots, 20,\ d = 20,\ \sigma^2 = 400$). First, we note that all values, including the DRV, increase as the number of stages T increases, but in different manners. The increment in the AV over Ts decreases as T increases. This is because of the users' intertemporal levelling behaviour of groundwater use within a given stock amount. The SV increases linearly; the increment in the SV over Ts is constant. This is natural if we consider the SV specification in Equation (4). However, the increment in the DRV increases as T increases. This is because, as was revealed in the previous section, the source of the DRV is the intertemporal reallocation of groundwater intake, and it is transferred to the following stages through the corresponding change in stock and cost. Every intake at each stage impacts the following stages; hence, the DRV increases with increasing increments as the time horizon is prolonged. As a result, the share of the DRV in the total economic value of groundwater increases as T increases, and the ratio of the DRV to the SV also increases as T increases.

Second, the multiple-user regime exhibits lower values than the single-decision-maker regime exhibits, except for the SV, which is the same between the two regimes. In addition, the share of the DRV in the total economic value or to the SV is lower in the multiple-user regime than in the single-decision-maker regime. The results for the AV and SV are consistent with the findings of previous studies (e.g., Gemma & Tsur, 2007). A new finding is about the DRV. If we compare the equation of (15) between the two regimes, the users respond to the surface water fluctuations by more than the amount of fluctuation, but the extent is weaker in the multiple-user regime. Overexploitation of groundwater in a suboptimal environment hinders users from fully utilizing reallocation opportunities.



(a) Single decision-maker regime

(b) Multiple-user regime

Figure 4. Composition of economic value in various numbers of stages. The bar charts represent the values of AV, SV, and DRV, and the line graphs represent the DRV/SV and DRV/TEV ratios.

Figure 5 shows how the SV and DRV change as the variance of the surface water fluctuation increases. As can be predicted from the formulas, both SV and DRV respond linearly to the variance increase; however, the figure indicates that the slope of the DRV is smaller than that of the SV. It is not easy to show the reason for the smaller slope analytically, but an intuitive explanation may be, as we discussed in the previous section, the DRV can be seen as a by-product of the users' stabilizing behaviour. Therefore, the DRV utilizes surface water fluctuations to a lesser extent than the SV does. Again, the slope of the DRV is smaller in the multiple-user regime than in the single-decision-maker regime.

Figure 5 also shows how DRV responds to different levels of the pumping slope parameter d, which is the marginal unit cost with respect to stock level G. The larger the parameter value, the more the unit cost responds to a marginal change in stock level. As shown in the figure, the slope of the DRV curve increases as d increases.

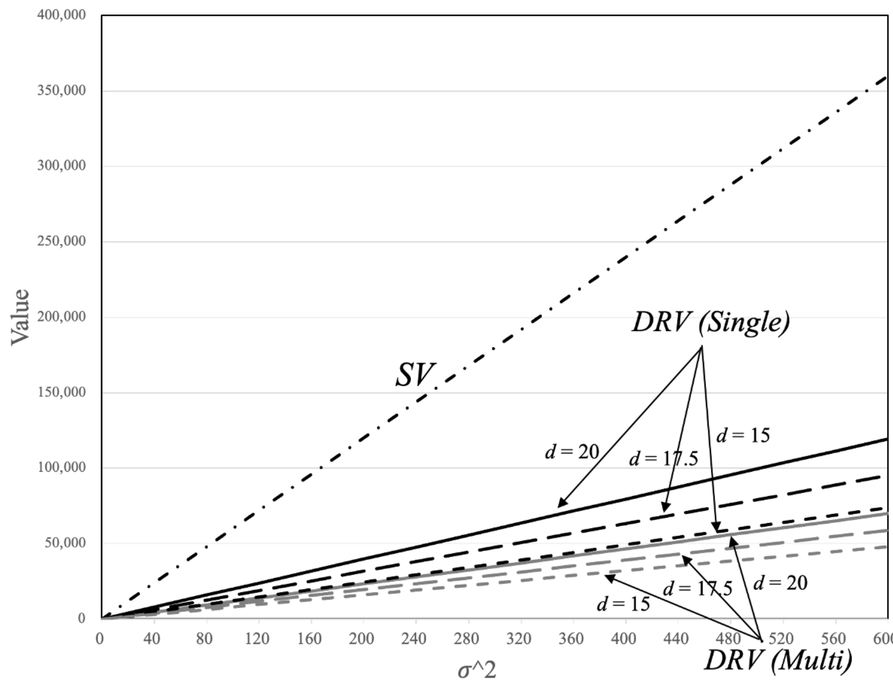


Figure 5. SV and DRV for different levels of variance in surface water fluctuation.

6 Discussion and Conclusions

In this study, we revisited the total economic value of groundwater. Specifically, we proved the existence of a dynamic reallocation value and proposed a new construction of the total economic value of groundwater comprising three components: augmentation value (AV), stabilization value (SV), and dynamic reallocation value (DRV).

Furthermore, we showed the economic mechanisms underlying the DRVs using simple analytical models. Similar to the SV, the DRV is derived from the adaptive behaviours of economic agents against surface-water variations under uncertain environments. However, they are conducted with different economic intentions and movement in opposite directions. Our model results showed that users intentionally destabilize their water use by increasing or decreasing their groundwater intake by more than the amount of surface water fluctuations. Such seemingly irrational behaviours arise from their optimization against changes in intertemporal cost allocations that occur as a reflection of stabilizing behaviours. That is, the stabilization behaviour of one period can simultaneously be seen as an intertemporal reallocation of groundwater intake from or to the following periods. Such reallocations change the unit pumping cost, and thereby, the relative price of groundwater in the future. Users actively take advantage of this to save their pumping costs throughout the period and achieve a higher total benefit, even in cases with uncertainty than in cases without uncertainty.

In addition, we analyzed how the DRV reacts to changes in parameters such as the number of stages or the variance of surface water fluctuation using numerical illustrations. First, we found that the share of DRV in the total economic value of groundwater increases as the time horizon increases. Second, DRV diminishes in a suboptimal environment with multiple users because the overexploitation of groundwater hinders users from fully utilizing reallocation opportunities.

Unfortunately, the DRV has been overlooked in all existing studies, including those conducted in dynamic contexts. Typically, studies using the simplified specification of the SV are likely to include the DRV in the AV unconsciously, and thereby overestimate the AV. Therefore, they estimate the value of groundwater to adapt to climate instability only in terms of its stabilization function. However, as shown in this study, users can derive additional value from groundwater than simply offsetting surface water fluctuations. In other words, even if the TEV itself is not affected, disregarding DRV can underestimate the value of groundwater as an essential instrument for climate adaptation. Although the present paper did not apply our results to actual water data, it is preferable that the economic valuations of existing empirical studies be re-examined using our new framework incorporating DRV.

The major methodological limitations of this paper are as follows. First, similar to almost all existing groundwater studies (e.g., Gisser & Sánchez, 1980; Provencher & Burt, 1994; Gardner et al., 1997; Msangi & Hejazi, 2022; Quintana-Ashwell & Gholson, 2022), we used a quadratic form for the benefit function (production function), which enabled us to derive simple analytical and even reduced-form solutions. Although we believe that our conclusions are not affected by function types, as long as they allow for diminishing marginal benefits, an assumption that accords with most production practices, we can numerically examine other types of benefit functions in future studies. Second, we used a stationary, temporally independent random variable for surface water fluctuations. This is because the typical situations that the current study addresses are those in which industrial or agricultural users tackle fluctuations in a relatively short period of time, for example, monthly. However, we can examine our findings in broadened environments, such as Markovian disturbances (e.g., Srikanthan & McMahon, 1985, 2001) or even in cases in which distributions are completely unknown, through numerical simulations using reinforcement learning. Third, the present study used a relatively simple setting for hydrological processes, such as deterministic recharge; however, we can examine our framework under more complex interactions between precipitation, surface water flow, and groundwater recharge both natural and artificial (e.g., Barlow et al., 2003; Vedula et al., 2005; Hantush, 2005; Fleckenstein et al., 2006; Pulido-Velázquez et al., 2006; Pulido-Velázquez et al., 2007; Marques et al., 2010; Reznik et al., 2022). Finally, we excluded cases in which the entire stock is exploited or should be kept above a threshold level, or cases in which groundwater supply is physically limited or reduced by its depletion. These cases have been extensively studied in some literature (e.g., Gisser & Sánchez, 1980; Gisser & Allen, 1984; Zeitouni, 2004; Msangi & Hejazi, 2022; Rouhi Rad et al., 2017; Foster et al., 2017). Although excluding these allows us to focus on a simple analytical demonstration of the DRV, it can take away the possibilities of considering different types of responses to intertemporal reallocation of intake that can generate the DRV. For example, it is known that, when the stock is binding, the user cost comprises not only the depth cost but also the stock cost (Provencher and Burt, 1993). It is therefore very likely that the DRV increases when user consider the latter type of user cost. We leave the evaluation of DRV in such cases for future study.

Finally, let us discuss some policy implications that we can derive from the study findings. First, the existence of the DRV augments the importance of sustainable groundwater management, particularly in areas threatened by surface-water fluctuations under climate change. Groundwater can provide those areas with larger economic benefits beyond its stabilizing effects. Second, overexploitation can reduce these benefits under insufficient regulation. Proper regulations are essential not only for avoiding the exhaustion of resources but also for fully utilizing the SV and DRV of groundwater. Third, although this paper did not directly address issues related to non-consumptive water use and externalities of groundwater extraction, the discovery of the new value indirectly contributes to addressing such issues because, as discussed above, the DRV provides users of groundwater with stronger incentives for its sustainable management. Finally, a growing body of literature have simulated optimizated conjunctive management of surface water and groundwater using machine learning models including genetic algorithm (e.g., Safavi et al., 2010; Safavi & Esmikhani, 2013 & 2016; Safavi & Falsafioun, 2016; Rezaei et al., 2017; Sepahvand et al., 2019). Although, most of these literatures have not captured dynamic reallocation behaviours presented in this paper explicitly, it is valuable to separate them from other types of optimization using these models and quantify the economic benefit of such behaviours.

Acknowledgments

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Conflicts of interest

The authors declare no conflicts of interest relevant to this study.

Data availability statement

The present paper is supplemented by the Supporting Information. The data used for the numerical illustrations in Figure 4 and 5 are available at Zenodo via https://doi.org/10.5281/zenodo.10887433.

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Water Resources Research

Supporting Information for

Revisiting the Economic Value of Groundwater

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Contents of this file

SI1, SI2, SI3, and SI4.

Introduction

We supplement the solution process of the two-stage model and the proof of Propositions 1, 4, and 5 in this supporting material.

SI1.

1. Single decision-maker regime

Since the intertemporal net benefit function is additively separable with respect to the instantaneous sum of the users' net benefits, we can solve the problem in two steps: the determination of the total water intake g_t for each period, where $g_t = \sum_{i\in\mathcal{N}} g_{it}$, and the allocation of water pumping to each user within period t taking the total water intake g_t as given.

Consider the problem of the second step first:

$$\max_{g_{1,2},\dots,g_{N,2}} \sum_{i\in\mathcal{N}} [F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1) g_{i2}],$$

subject to $\sum_{i\in\mathcal{N}} g_{it} = g_t$. By solving the problem, we get

$$\begin{aligned} g_{it} &= \frac{\tilde{a}_i}{2b} + \frac{S_t}{N} + \frac{g_t}{N} - \varepsilon_i S_t \\ w_{it} &= \frac{\tilde{a}_i}{2b} + \frac{S_t}{N} + \frac{g_t}{N}, \end{aligned} \tag{A.1}$$

for all $i \in \mathcal{N}$ and for all $t \in \{1, 2\}$, where $\tilde{a}_i \triangleq a_i - a/N$ and $a \triangleq \sum_{i\in\mathcal{N}} a_i$. Therefore, the maximized instantaneous aggregate net benefit for given S_t, G_{t-1}, and g_t is given by

$$\pi_t(g_t, S_t, G_{t-1}) \triangleq W(S_t) + (H(S_t) + dG_{t-1}) g_t - \frac{b}{N} g_t^2, \tag{A.2}$$

where

$$W(S_t) \triangleq \sum_{i=1}^{N}\left[a_i\left(\frac{\tilde{a}_i}{2b}+\frac{S_t}{N}\right)\right] - b\sum_{i=1}^{N}\left(\frac{\tilde{a}_i}{2b}+\frac{S_t}{N}\right)^2,$$
$$H(S_t) \triangleq \frac{a - Nc - 2bS_t}{N}.$$

Next, we consider the problem of determining the total water intake g_t for each period. By solving backword from period 2, we obtain the following solution:

$$g_2(G_1, S_2) = \frac{a - Nc + NdG_1}{2b} - S_2. \tag{A.3}$$

The problem of the first period in the uncertain case is then given by:

$$\max_{g_1} W(S_1) + (H(S_1) + dG_0)g_1 - \frac{b}{N}g_1^2 + \beta E_1[\pi_2(g_2(G_1, S_2), S_2, G_0)|S_1].$$

Subsequently, we get

$$g_{u1}^{\text{single}}(S_1) = \frac{1}{4b^2 - N^2d^2\beta}[(2b - Nd\beta)X - 2b(2bS_1 - Nd\beta\bar{S}) - N^2d^2\beta R], \tag{A.4}$$

where $X \triangleq a - Nc + NdG_0$ and $w_{u1}^{\text{single}}(S_1)$ is given by $w_{u1}^{\text{single}}(S_1) = g_{u1}^{\text{single}}(S_1) + S_1$.

For the above solution to satisfy the necessary and sufficient conditions, we further require the following from the second-order condition:

$$4b^2 - N^2d^2\beta > 0. \tag{A.5}$$

Using (A.1), (A.3), and (A.4), we get:

$$\begin{aligned}\pi_{u1}^{\text{single}} = &\left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 - \frac{Nd^2\beta^2(2b - Nd)^2}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2\\ &- \frac{N^3bd^4\beta^2}{(4b^2 - N^2d^2\beta)^2}R^2 - \frac{N^2d^3\beta^2(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R - \frac{N^3bd^4\beta^2}{(4b^2 - N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A.6}$$

$$\begin{aligned}\pi_{u2}^{\text{single}} = &(c - dG_0)\bar{S} + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 - \frac{d(2b - Nd\beta)(8b^2 - N^2d^2\beta - 2Nbd)}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2\\ &+ \frac{4Nb^3d^2}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R + \frac{4Nb^3d^2}{(4b^2 - N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A.7}$$

$$\begin{aligned}\pi_{u}^{\text{single}} = &\left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2\\ &- \frac{d[Nd\beta^2(2b^2 + N^2d^2 - 2Nbd) + 2b^2(4b - Nd - 2Nd\beta)]}{2b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2\\ &+ \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{d(2b - Nd)(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R\\ &+ \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A.8}$$

where $X_i \triangleq a_i - c + dG_0$.

Similarly, the problem of the first period in the certain case is given by:

$$\max_{g_1} W(\bar{S}) + (H(\bar{S}) + dG_0)g_1 - \frac{b}{N}g_1^2 + \beta\pi_2(g_2(G_1,\bar{S}),\bar{S},G_0).$$

Subsequently, we get:

$$g_{c1}^{\text{single}}(\bar{S}) = \frac{1}{4b^2 - N^2d^2\beta}[(2b - Nd\beta)(X - 2b\bar{S}) - N^2d^2\beta R]. \tag{A.9}$$

In addition, $w_{c1}^{\text{single}}(\bar{S})$ is given by $w_{c1}^{\text{single}}(\bar{S}) = g_{c1}^{\text{single}}(\bar{S}) + \bar{S}$. Using solutions (A.1), (A.3), and (A.9), we obtain:

$$\begin{aligned}\pi_{c1}^{\text{single}} &= \left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &- \frac{Nd^2\beta^2(2b-Nd)^2}{4b(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})^2 - \frac{N^3bd^4\beta^2}{(4b^2-N^2d^2\beta)^2}R^2 - \frac{N^2d^3\beta^2(2b-Nd)}{(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})R,\end{aligned} \tag{A.10}$$

$$\begin{aligned}\pi_{c2}^{\text{single}} &= (c - dG_0)\bar{S} + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 - \frac{d(2b-Nd\beta)(8b^2-N^2d^2\beta-2Nbd)}{4b(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})^2 \\ &+ \frac{4Nb^3d^2}{(4b^2-N^2d^2\beta)^2}R^2 + \frac{4b^2d(2b-Nd)}{(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})R,\end{aligned} \tag{A.11}$$

$$\begin{aligned}\pi_{c}^{\text{single}} &= \left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 \\ &- \frac{d[Nd\beta^2(2b^2+N^2d^2-2Nbd)+2b^2(4b-Nd-2Nd\beta)]}{2b(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})^2 \\ &+ \frac{Nbd^2(4b^2-N^2d^2\beta^2)}{(4b^2-N^2d^2\beta)^2}R^2 + \frac{d(2b-Nd)(4b^2-N^2d^2\beta^2)}{(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})R.\end{aligned} \tag{A.12}$$

2. Multiple-user regime

User i's problem of the second period for given S_2 and G_1 is:

$$\max_{g_{i2}} F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1)g_{i2}.$$

Hence, the solution for this is:

$$g_{i2}(G_1, S_2) = \frac{a_i - c + dG_1}{2b} - \varepsilon_i S_2. \tag{A.13}$$

User i's problem of the first period in the uncertain case is given by

$$\max_{g_{i1}} F_i(g_{i1} + \varepsilon_i S_1) - C_i(G_0)g_{i1} + \beta E_1[\pi_i(g_{i2}(G_1,S_2),G_1,S_2)|S_1].$$

Subsequently, we obtain:

$$g_{u1}^{\text{multi}}(S_1) = \frac{1}{4b^2 - Nd^2\beta}[2b(X - 2bS_1) - d\beta(X - 2b\bar{S}) - Nd^2\beta R], \tag{A.14}$$

and $w_{u1}^{\text{multi}}(S_1)$ is given by $w_{u1}^{\text{multi}}(S_1) = g_{u1}^{\text{multi}}(S_1) + S_1$.

For the above solution to satisfy the necessary and sufficient conditions, we require the following from the second-order condition:

$$4b^2 - d^2\beta > 0 \tag{A.15}$$

Using the solutions (A.13) and (A.14), we get:

$$\begin{aligned}\pi_{u1}^{\text{multi}} &= \left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &+ \frac{d^3\beta^2(2b - d\beta)(8\beta^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 - \frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}R^2 \\ &- \frac{d^3\beta^2(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R - \frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}\sigma^2,\end{aligned} \tag{A.16}$$

$$\begin{aligned}\pi_{u2}^{\text{multi}} &= (c - dG_0)\bar{S} - \frac{d(2b - d\beta)(8b^2 - Nd^2\beta - 2Nbd)}{4b(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2 \\ &+ \frac{4Nb^3d^2}{(4b^2 - Nd^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R + \frac{4Nb^3d^2}{(4b^2 - Nd^2\beta)^2}\sigma^2,\end{aligned} \tag{A.17}$$

$$\begin{aligned}\pi_{u}^{\text{multi}} &= \left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2 \\ &- \frac{d(2b - d\beta)(4b^2 - d^2\beta^2)(8b^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}R^2 \\ &+ \frac{d(4b^2 - d^2\beta^2)(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R + \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}\sigma^2.\end{aligned} \tag{A.18}$$

Similarly, the problem of the first period in the certain case is given by:

$$\max_{g_{i1}} F_i(g_{i1} + \varepsilon_i\bar{S}) - C_i(G_0)g_{i1} + \beta\pi_i(g_{i2}(G_1, \bar{S}), G_1, \bar{S}).$$

Subsequently, we get:

$$g_{c1}^{\text{multi}}(\bar{S}) = \frac{1}{4b^2 - Nd^2\beta}[(2b - d\beta)(X - 2b\bar{S}) - Nd^2\beta R^g]. \tag{A.19}$$

In addition, $w_{c1}^{\text{multi}}(\bar{S})$ is given by $w_{c1}^{\text{multi}}(\bar{S}) = g_{c1}^{\text{multi}}(\bar{S}) + \bar{S}$. Using solutions (A.13) and (A.19), we obtain:

$$\begin{aligned}\pi_{c1}^{\text{multi}} &= \left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &+ \frac{d^3\beta^2(2b - d\beta)(8\beta^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 - \frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}R^2 \\ &- \frac{d^3\square\square^2(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.20}$$

$$\begin{aligned}\pi_{c2}^{\text{multi}} &= (c - dG_0)\bar{S} - \frac{d(2b - d\beta)(8b^2 - Nd^2\beta - 2Nbd)}{4b(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2 \\ &+ \frac{4Nb^3d^2}{(4b^2 - Nd^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.21}$$

$$\pi_c^{\text{multi}} = \left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2$$
$$- \frac{d(2b - d\beta)(4b^2 - d^2\beta^2)(8b^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}R^2 \qquad \text{(A. 22)}$$
$$+ \frac{d(4b^2 - d^2\beta^2)(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R.$$

SI2.

Proof of Proposition 1

From (A.8) and (A.12), we obtain:

$$DRV_{\text{single}} = \pi_u^{\text{single}} - \pi_c^{\text{single}} = \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}\sigma^2.$$

From (A.5), we can demonstrate $DRV_{\text{single}} > 0$.

Similarly, from (A.18) and (A.22), we obtain:

$$DRV_{\text{multi}} = \pi_u^{\text{multi}} - \pi_c^{\text{multi}} = \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}\sigma^2.$$

From (A.15), we can demonstrate $DRV_{\text{multi}} > 0$.

SI3.

Proof of Proposition 4

First we find a general solution for groundwater intake in the case of an arbitrary number of stages. Let $\gamma_t = (\gamma_{1t}, \ldots, \gamma_{Nt}) \in \Gamma_t = \Gamma_{1t} \times \ldots \times \Gamma_{Nt}$ denote an admissible action rule of the social planner, where Γ_{it} is the set of admissible action rules concerning user i in period t. Let $V(t, G_{t-1}, S_t)$ denote the optimal value function in period $t \in \mathrm{T}$ given the current groundwater stock G_{t-1} and the realization of surface flow S_t,

$$V(t, G_{t-1}, S_t) \triangleq \max_{\gamma_t\in\Gamma_t,\ldots,\gamma_T\in\Gamma_T} E_t\left[\sum_{t\in\mathrm{T}}\sum_{i\in\mathrm{N}}\beta^{t-1}\left[F_i(\gamma_{i\tau} + \varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}\right]\right]. \qquad \text{(C. 1)}$$

The recursive structure of the returns leads to the following Bellman optimality equation (Bellman 1952; Basar, 2012):

$$V(t, G_{t-1}, S_t) = \max_{\gamma_t\in\Gamma_t}\sum_{i\in\mathrm{N}}[F_i(\gamma_{i\tau} + \varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}] + \beta E_{t+1}[V(t+1, G_t, S_{t+1})],$$
$$V(T+1, G_T, S_{T+1}) = 0. \qquad \text{(C. 2)}$$

Now we prove the following action rules constitute a unique solution for groundwater intake.

$$\gamma_{iT}^*(S_T, G_{T-1}) = \frac{1}{2b}[\Theta_i(S_T) - Nd^2\beta G_{T-1}],$$
$$\gamma_{it}^*(S_t, G_{t-1}) = \frac{1}{v_t}\Big[\frac{v_t}{2b}\Theta_i(S_t) + \frac{Nd^2\beta\rho_{t+1}}{2b}\Theta(S_t) - d\beta\rho_{t+1}\Theta(\bar{S}) - Nd^2\beta\eta_t R$$
$$+ d(v_{t+1} - Nd\beta\rho_{t+1})G_{t-1}\Big], \qquad t \le T-1, \qquad \text{(C. 3)}$$

where

$$\Theta_i(S_t) \triangleq a_i - c_i - 2b\varepsilon_i S_t, \qquad \Theta(S_t) \triangleq \sum\nolimits_{i\in\mathcal{N}} (a_i - c_i) - 2bS_t,$$
$$\rho_t \triangleq \begin{cases} 1, & t = T \\ \upsilon_{t+1} - 2\beta\rho_{t+1}(Nd - b), & t \le T-1 \end{cases}$$
$$\upsilon_t \triangleq \begin{cases} 2b, & t = T \\ 2b\upsilon_{t+1} - N^2d^2\beta\rho_{t+1}, & t \le T-1 \end{cases}$$
$$\eta_t \triangleq \begin{cases} 0, & t = T \\ \beta\eta_{t+1}(2b - Nd) + \rho_{t+1}, & t \le T-1. \end{cases}$$

Also consider

$$E_{t-1}\left[\frac{\partial V(t, G_{t-1}, S_t)}{\partial G_{t-1}}\right] = \frac{d}{\upsilon_t}[\rho_t\Theta(\bar{S}) + Nd\beta\eta_t(2b - Nd)R + Nd\rho_t G_{t-1}]. \tag{C.4}$$

For $t = T$ and $T - 1$, solving backward from T, we can easily show (C.3) and (C.4) are true. Assume that they also hold for some $t = k + 1$ $(1 \le k \le T - 2)$:

$$\begin{aligned} \gamma^*_{ik+1}(S_{k+1}, G_k) = \frac{1}{\upsilon_{k+1}}\bigg[&\frac{\upsilon_{k+1}}{2b}\Theta_i(S_{k+1}) + \frac{Nd^2\beta\rho_{k+2}}{2b}\Theta(S_{k+1}) - d\beta\rho_{k+2}\Theta(\bar{S}) \\ &-Nd^2\beta\eta_{k+1}R + d(\upsilon_{k+2} - Nd\beta\rho_{k+2})G_k\bigg], \\ E_k\left[\frac{\partial V(k+1, G_k, S_{k+1})}{\partial G_k}\right] = &\frac{d}{\upsilon_{k+1}}[\rho_{k+1}\Theta(\bar{S}) + Nd\beta\eta_{k+1}(2b - Nd)R + Nd\rho_{k+1}G_k]. \end{aligned} \tag{C.5}$$

Consider the problem for $t = k$:

$$\max_{g_{1,k},\cdots,g_{N,k}} \Omega(S_k) + \sum\nolimits_{i\in\mathcal{N}} [\Theta_i(S_k) + dG_{k-1}]g_{ik} - b\sum\nolimits_{i\in\mathcal{N}} g_{ik}^2 + \beta E_k[V(k+1, G_k, S_{k+1})|S_k],$$

where $\Omega(S_t) \triangleq (\sum_{\mathcal{N}} a_i\varepsilon_i)S_t - b(\sum_{\mathcal{N}} \varepsilon_i^2)S_t^2$. By using (C.5), we obtain the following solution:

$$\begin{aligned} g_{ik} = \gamma^*_{ik}(S_t, G_{t-1}) = \frac{1}{\upsilon_k}\bigg[&\frac{\upsilon_k}{2b}\Theta_i(S_k) + \frac{Nd^2\beta\rho_{k+1}}{2b}\Theta(S_k) - d\beta\rho_{k+1}\Theta(\bar{S}) \\ &-Nd^2\beta\eta_k R + d(\upsilon_{k+1} - Nd\beta\rho_{k+1})G_{k-1}\bigg]. \end{aligned} \tag{C.6}$$

By using (C.6), we can demonstrate the following:

$$E_{k-1}\left[\frac{\partial V(k, G_{k-1}, S_k)}{\partial G_{k-1}}\right] = \frac{d}{\upsilon_k}[\rho_k\Theta(\bar{S}) + Nd\beta\eta_k(2b - Nd)R + Nd\rho_k G_{k-1}]. \tag{C.7}$$

From equation (C.6) and (C.7), equation (C.3) and (C.5) also holds for $t = k$. By mathematical induction, they are true for all $t \le T - 1$.

Subsequently, we find the DRV. The aggregate groundwater intake is given by:

$$\begin{aligned} g_T &= \frac{1}{\upsilon_T}[\Theta(S_T) - N^2d^2\beta\eta_T R + NdG_{T-1}], \\ g_t &= \frac{1}{\upsilon_t}[\upsilon_{t+1}\Theta(S_t) - Nd\beta\rho_{t+1}\Theta(\bar{S}) - N^2d^2\beta\eta_t R + Nd(\upsilon_{t+1} - Nd\beta\rho_{t+1})G_{t-1}], \qquad t \le T-1. \end{aligned} \tag{C.8}$$

We rewrite (C.8) as:

$$g_t = \Lambda(t) + \Phi(t)G_{t-1} + \Psi(t)S_t, \tag{C.9}$$

where

$$\Lambda(t) \triangleq \begin{cases} \dfrac{1}{\upsilon_T}\left[\sum_{i\in\mathcal{N}}(a_i - c_i) - N^2 d^2 \beta \eta_T R\right], & t = T \\ \dfrac{1}{\upsilon_t}\left[\upsilon_{t+1}\sum_{i\in\mathcal{N}}(a_i - c_i) - Nd\beta\rho_{t+1}\Theta(\bar{S}) - N^2 d^2 \beta \eta_t R\right], & t \le T-1 \end{cases}$$

$$\Phi(t) \triangleq \begin{cases} \dfrac{Nd}{\upsilon_T}, & t = T \\ \dfrac{Nd(\upsilon_{t+1} - Nd\beta\rho_{t+1})}{\upsilon_t}, & t \le T-1 \end{cases}$$

$$\Psi(t) \triangleq \begin{cases} -\dfrac{2b}{\upsilon_T}, & t = T \\ -\dfrac{2b\upsilon_{t+1}}{\upsilon_t}, & t \le T-1. \end{cases}$$

Using this, the groundwater stock G_{t-1} can be transformed into:

$$\begin{aligned} G_{t-1} &= \left[\prod_{\tau=1}^{t-1}\big(1-\Phi(\tau)\big)\right]G_0 \\ &- \left\{\Psi(t-1)S_{t-1} + \Psi(t-2)\big(1-\Phi(t-1)\big)S_{t-2} + \cdots + \left[\Psi(1)\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]S_1\right\} \\ &+ \left\{1 + \big(1-\Phi(t-1)\big) + \cdots + \left[\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]\right\}R \\ &- \left\{\Lambda(t-1) + \big(1-\Phi(t-1)\big)\Lambda(t-2) + \cdots + \left[\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]\Lambda(1)\right\}. \end{aligned} \tag{C.10}$$

In addition, the solutions (C.3) can be transformed into:

$$g_{it} = \gamma_{it}^{*}(S_t, G_{t-1}) = \frac{\hat{a}_i}{2b} + \frac{\Lambda(t)}{N} + \frac{\Phi(t)}{N}G_{t-1} + \frac{1}{N}\big(1 - N\varepsilon_i + \Psi(t)\big)S_t, \tag{C.11}$$

where $\hat{a}_i \triangleq a_i - c_i - \frac{1}{N}\sum_{i=1}^{N}(a_i - c_i)$. Substitute (C.11) into the aggregate instantaneous net benefit

$$\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t) \triangleq \sum_{i\in\mathcal{N}}[F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1)g_{i2}]. \tag{C.12}$$

Extracting only the terms with $S_1^2, \dots, S_T^2$ from $\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)$ by using (C.10), we obtain

$$\begin{aligned} -\frac{b}{N}\big(1+\Psi(t)\big)^2 S_t^2 + \frac{\Phi(t)\big(Nd - b\Phi(t)\big)}{N}\Big[\Psi(t-1)^2 S_{t-1}^2 + \Psi(t-2)^2\big(1-\Phi(t-1)\big)^2 S_{t-2}^2 \\ + \cdots + \Psi(1)^2\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)^2 S_1^2\Big]. \end{aligned} \tag{C.13}$$

If we take the expected value of $E_0[\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)]$, the terms with σ^2 are generated by replacing $S_1^2, \dots, S_T^2$ in (C.13) with σ^2. They give $\Xi_1, \dots, \Xi_T$ in Proposition 4.

SI4.

Proof of Proposition 5

The procedure is the same as in the proof of Proposition 4 (SI3). We first prove the following strategy constitutes a unique feedback Nash equilibrium solution for $\{\mathcal{N}, \mathcal{T}, \mathcal{G}, \mathcal{S}, \{U_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{f_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Gamma_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Pi_i\}_{i\in\mathcal{T}}\}$.

$$\begin{gathered}\gamma_{iT}^{**}(G_T, S_T) = \frac{1}{2b}[\Theta_i(S_T) + dG_{T-1}],\\ \gamma_{it}^{**}(G_{t-1}, S_t) = \frac{1}{\tilde{\upsilon}_t}\left[\frac{\tilde{\upsilon}_t}{2b}\Theta_i(S_t) - \frac{d\beta\tilde{\upsilon}_t(\tilde{\rho}_{t+1} + N\tilde{\varphi}_{t+1})}{2b\tilde{\upsilon}_{t+1}}\Theta_i(\bar{S}) + \frac{d^2\beta\tilde{\rho}_{t+1}}{2b}\Theta(S_t)\right.\\ \left.- \frac{d\beta(d^2\beta\tilde{\rho}_{t+1}^2 - \tilde{\upsilon}_t\tilde{\varphi}_{t+1})}{2b\tilde{\upsilon}_{t+1}}\Theta(\bar{S}) - d^2\beta\tilde{\eta}_t R + d(\tilde{\upsilon}_{t+1} - d\beta\tilde{\rho}_{t+1})G_{t-1}\right],\\ t \le T-1,\end{gathered} \tag{D.1}$$

where

$$\tilde{\rho}_t \triangleq \begin{cases} 1, & t = T \\ \tilde{\upsilon}_{t+1} - \beta\tilde{\rho}_{t+1}(Nd + d - 2b) - \dfrac{(N-1)d\beta\tilde{\rho}_{t+1}(2b - Nd)(\tilde{\upsilon}_{t+1} - d\gamma\tilde{\rho}_{t+1})}{\tilde{\upsilon}_t}, & t \le T-1 \end{cases}$$

$$\tilde{\upsilon}_t \triangleq \begin{cases} 2b, & t = T \\ 2b\tilde{\upsilon}_{t+1} - Nd^2\beta\tilde{\rho}_{t+1}, & t \le T-1 \end{cases}$$

$$\tilde{\eta}_t \triangleq \begin{cases} 0, & t = T \\ \beta\tilde{\eta}_{t+1}\tilde{\mu}_{t+1}(2b - Nd) + \tilde{\rho}_{t+1}, & t \le T-1 \end{cases}$$

$$\tilde{\mu}_t \triangleq \begin{cases} 1, & t = T \\ \dfrac{2b\tilde{\upsilon}_{t+1} - d^2\beta\tilde{\rho}_{t+1}}{\tilde{\upsilon}_t}, & t \le T-1 \end{cases}$$

$$\tilde{\varphi}_t \triangleq \begin{cases} 0, & t = T \\ \dfrac{\beta\tilde{\mu}_t(2b - Nd)[d\tilde{\rho}_{t+1}(\tilde{\upsilon}_{t+1} - d\beta\tilde{\rho}_{t+1}) + \tilde{\upsilon}_t\tilde{\varphi}_{t+1}]}{2b\tilde{\upsilon}_{t+1}}, & t \le T-1. \end{cases}$$

Moreover, consider

$$\begin{gathered}E_{t-1}\left[\frac{\partial V^i(t, G_{t-1}, S_t)}{\partial G_{t-1}}\right] = \frac{d}{\tilde{\upsilon}_t}[(\tilde{\rho}_t + N\tilde{\varphi}_t)\Theta_i(\bar{S}) - \tilde{\varphi}_t\Theta(\bar{S})\\ + d\beta\tilde{\eta}_t\tilde{\mu}_t(2b - Nd)R + d\tilde{\rho}_t G_{t-1}].\end{gathered} \tag{D.2}$$

For $t = T$ and $T - 1$, solving backward from T, we can show that (D.1) and (D.2) are true. Assume they hold for $t = k + 1$ ($1 \le k \le T - 2$), and we can prove they are also true for all $t \le T - 1$ in the same way as SI3.

The aggregate groundwater intake is given by:

$$\begin{gathered}g_T = \frac{1}{\tilde{\upsilon}_T}[\Theta(S_T) - Nd^2\beta\tilde{\eta}_T R + NdG_{T-1}],\\ g_t = \frac{1}{\tilde{\upsilon}_t}[\tilde{\upsilon}_{t+1}\Theta(S_t) - d\beta\tilde{\rho}_{t+1}\Theta(\bar{S}) - Nd^2\beta\tilde{\eta}_t R + Nd(\tilde{\upsilon}_{t+1} - d\beta\tilde{\rho}_{t+1})G_{t-1}], \qquad t \le T-1.\end{gathered} \tag{D.3}$$

Hence, we rewrite (D.3) as:

$$g_t = \widetilde{\Lambda}(t) + \widetilde{\Phi}(t)G_{t-1} + \widetilde{\Psi}(t)S_t, \tag{D.4}$$

where

$$\widetilde{\Lambda}(t) \triangleq \begin{cases} \dfrac{1}{\tilde{v}_T}\left[\displaystyle\sum_{i=1}^{N}(a_i - c_i) - Nd^2\beta\tilde{\eta}_T R\right], & t = T \\ \dfrac{1}{\tilde{v}_t}\left[\tilde{v}_{t+1}\displaystyle\sum_{i=1}^{N}(a_i - c_i) - d\beta\tilde{\rho}_{t+1}\Theta(\bar{S}) - Nd^2\beta\tilde{\eta}_t R\right], & t \leq T-1 \end{cases}$$

$$\widetilde{\Phi}(t) \triangleq \begin{cases} \dfrac{Nd}{\tilde{v}_T}, & t = T \\ \dfrac{Nd(\tilde{v}_{t+1} - d\beta\tilde{\rho}_{t+1})}{\tilde{v}_t}, & t \leq T-1 \end{cases}$$

$$\widetilde{\Psi}(t) \triangleq \begin{cases} -\dfrac{2b}{\tilde{v}_T}, & t = T \\ -\dfrac{2b\tilde{v}_{t+1}}{\tilde{v}_t}, & t \leq T-1. \end{cases}$$

Using this, the groundwater stock G_{t-1} can be transformed into:

$$\begin{aligned} G_{t-1} = &\left[\prod_{\tau=1}^{t-1}\left(1 - \widetilde{\Phi}(\tau)\right)\right]G_0 \\ &-\left\{\widetilde{\Psi}(t-1)S_{t-1} + \widetilde{\Psi}(t-2)\left(1 - \widetilde{\Phi}(t-1)\right)S_{t-2} + \cdots + \left[\widetilde{\Psi}(1)\prod_{\tau=2}^{t-1}\left(1 - \widetilde{\Phi}(\tau)\right)\right]S_1\right\} \\ &+\left\{1 + \left(1 - \widetilde{\Phi}(t-1)\right) + \cdots + \left[\prod_{\tau=2}^{t-1}\left(1 - \widetilde{\Phi}(\tau)\right)\right]\right\}R \\ &-\left\{\widetilde{\Lambda}(t-1) + \left(1 - \widetilde{\Phi}(t-1)\right)\widetilde{\Lambda}(t-2) + \cdots + \left[\prod_{\tau=2}^{t-1}\left(1 - \widetilde{\Phi}(\tau)\right)\right]\widetilde{\Lambda}(1)\right\}. \end{aligned} \tag{D.5}$$

In addition, (D.1) can be transformed into:

$$g_{it} = \gamma_{it}^{**}(S_t, G_{t-1}) = \frac{\hat{a}_i}{2b} + \frac{\widetilde{\Lambda}(t)}{N} + \frac{\widetilde{\Phi}(t)}{N}G_{t-1} - Z_{it} + \frac{1}{N}\left(1 - N\varepsilon_i + \widetilde{\Psi}(t)\right)S_t, \tag{D.6}$$

where

$$Z_{it} = \begin{cases} 0, & t = T \\ \dfrac{d\beta}{2b}\left[\dfrac{\tilde{\rho}_{t+1} + N\tilde{\varphi}_{t+1}}{\tilde{v}_{t+1}}\Theta_i(\bar{S}) + \dfrac{Nd^2\beta\tilde{\rho}_{t+1}^2 - N\tilde{v}_t\tilde{\varphi}_{t+1} - 2b\tilde{v}_{t+1}\tilde{\rho}_{t+1}}{N\tilde{v}_t\tilde{v}_{t+1}}\Theta(\bar{S})\right], & t \leq T-2. \end{cases}$$

Substituting (D.6) into the aggregate instantaneous net benefit $\pi(g_{1t}, \ldots, g_{Nt}, G_{t-1}, S_t)$ and extracting only the terms with $S_1^2, \ldots, S_T^2$ by using (D.5), we get:

$$\begin{aligned} -\frac{b}{N}\left(1 + \widetilde{\Psi}(t)\right)^2 S_t^2 + \frac{\widetilde{\Phi}(t)\left(Nd - b\widetilde{\Phi}(t)\right)}{N}&\left[\widetilde{\Psi}(t-1)^2 S_{t-1}^2 + \widetilde{\Psi}(t-2)^2\left(1 - \widetilde{\Phi}(t-1)\right)^2 S_{t-2}^2\right. \\ &\left. + \cdots + \widetilde{\Psi}(1)^2\prod_{\tau=2}^{t-1}\left(1 - \widetilde{\Phi}(\tau)\right)^2 S_1^2\right]. \end{aligned} \tag{D.7}$$

If we take the expected value of $E_0[\pi(g_{1t}, \ldots, g_{Nt}, G_{t-1}, S_t)]$, the terms with σ^2 are generated by replacing $S_1^2, \ldots, S_T^2$ in (D.7) with σ^2. They give $\tilde{\Xi}_1, \ldots, \tilde{\Xi}_T$ in Proposition 5.