Using Kgen to Generate Cross-Verified Apparent Equilibrium Constants (K^{*}) for Seawater Carbonate Chemistry

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Abstract

The state of acid-base equilibria in solutions is calculated using apparent equilibrium constants (K* 's). The accuracy of these K* 's is critical for calculations of environmental chemistry, for example the dissociation of dissolved CO2 in seawater. The K* 's required to calculate seawater carbonate chemistry are described by empirically determined functions, which are implemented within software packages used to calculate solution carbon speciation. Each of these software packages uses its own implementation of these K* calculations. This fragmented approach to K* calculation results in unintended and difficult to resolve discrepancies between outputs calculated by different pieces of software, particularly because of the empirical nature of the K* functions which are subject to revisions and improvements over time. We present 'Kgen', a collection of software to consistently calculate seawater K* 's in Python, R, and Matlab. Kgen provides a nearly identical interface for each language and, through use of a Continuous Integration/Continuous Delivery (CI/CD) pipeline, guarantees consistency between languages by automatically cross-checking results from all three implementations. Unifying the approach to K* calculation in this way provides an extensible platform for verifiable K* generation, which can be easily integrated into existing carbon speciation calculators to improve consistency of results.











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Key Points:

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8	•	Kgen is a software package to calculate apparent equilibrium constants for sea-
9		water carbonate system calculations
10	•	Kgen accounts for the influences of temperature, pressure, salinity, magnesium con-
11		centration, and calcium concentration
12	•	We provide a new implementation of the MyAMI model ('pymyami') to adjust ap-
13		parent equilibrium constants for changing seawater composition

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14 Abstract

The state of acid-base equilibria in solutions is calculated using apparent equilibrium con-15 stants (K*'s). The accuracy of these K*'s is critical for calculations of environmental chem-16 istry, for example the dissociation of dissolved CO_2 in seawater. The K^{*}'s required to 17 calculate seawater carbonate chemistry are described by empirically determined func-18 tions, which are implemented within software packages used to calculate solution car-19 bon speciation. Each of these software packages uses its own implementation of these 20 K^{*} calculations. This fragmented approach to K^{*} calculation results in unintended and 21 difficult to resolve discrepancies between outputs calculated by different pieces of soft-22 ware, particularly because of the empirical nature of the K^* functions which are subject 23 to revisions and improvements over time. We present 'Kgen', a collection of software to 24 consistently calculate seawater K^{*}'s in Python, R, and Matlab. Kgen provides a nearly 25 identical interface for each language and, through use of a Continuous Integration/Continuous 26 Delivery (CI/CD) pipeline, guarantees consistency between languages by automatically 27 cross-checking results from all three implementations. Unifying the approach to K^* cal-28 culation in this way provides an extensible platform for verifiable K^{*} generation, which 29 can be easily integrated into existing carbon speciation calculators to improve consis-30 tency of results. 31

32 Plain Language Summary

To calculate how various compounds in seawater behave (chemically speaking), it 33 is necessary to calculate what are known as 'apparent equilibrium constants' (denoted 34 by the symbol K^*). These K^* 's are used to determine proportions of compounds which 35 are present in seawater in a variety of forms. Before the work presented here, K^{*} cal-36 culation was done as a necessary aside within other software. This is an issue, because 37 different pieces of software can calculate them in subtly different ways, which leads to 38 differences in the outputs of these packages which are difficult to resolve. In this work, 39 we present a software package called Kgen that is exclusively devoted to generation of 40 K^{*}'s. The software is co-written in three programming languages (as all three are rou-41 tinely used for seawater calculations), and we integrate a testing procedure to ensure that 42 all three languages remain consistent. The result is a more coherent approach to K^{*} gen-43 eration that can be integrated into existing software. 44

45 **1** Introduction

Many compounds in seawater exist in acid-base equilibria, including fluorine, sul-46 phur, ammonia, nitrite, silica, phosphate, sulphate, boron, and (most importantly) car-47 bon. In solution, each of these compounds exists in an equilibrium state described by 48 one or more equilibrium constants (K's), which are used to quantify the relative activ-49 ity of the products and reactants of their dissociation reactions (see Millero (1974), and 50 for an example, see Equation 2). The relative activity of species in these dissociation re-51 actions is dependent on both thermodynamic conditions (temperature and pressure), and 52 the composition of the solution (salinity, concentration of other ions). Equilibrium con-53 stants may be derived from thermodynamic first principles for ideal solutions, but the 54 complexity of ion-ion interactions in seawater cause it to diverge from an ideal solution 55 (Huckel & Debye, 1923). To account for this non-ideal behaviour, equilibrium constants 56 in seawater are reframed as 'apparent' equilibrium constants (K^{*}'s), which are defined 57 in terms of the stoichiometric concentration of individual species (see Equation 3) - and 58 all terms related to activity are integrated into the K^* . We can empirically determine 59 K^{*}'s at specific solution chemistries and thermodynamic conditions, and K^{*}'s have been 60 empirically determined across a wide range of seawater salinities and thermodynamic con-61 ditions. To calculate these K^{*}'s between these specifically measured conditions, poly-62 nomials that capture the sensitivity of each K^* to temperature and salinity have been 63

created (Dickson & Goyet, 1994). Any attempt to calculate solution speciation in sea-64 water must begin by defining which equations to calculate K^* 's. Because the K^* equa-65 tions are empirical, they have been refined and developed over the years, and multiple 66 versions exist (Lewis & Wallace, 1998). Even though carefully-curated sets of 'Best Prac-67 tices' K^* 's exist (Dickson & Goyet, 1994), this makes it possible for different pieces of 68 software to be using different K^* equations, causing their outputs to deviate in complex 69 and difficult-to-identify ways. To resolve this, we present 'Kgen', a piece of software writ-70 ten synchronously across three programming languages (Python, Matlab, R) that guar-71 antees the provision of consistent K^{*} values in all three languages, thus removing this 72 source of uncertainty in seawater speciation calculators. 73

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1.1 Carbon in Seawater: The importance of K*'s

Carbon is present in seawater as four inorganic species: CO₂ (carbon dioxide), H₂CO₃ (carbonic acid), HCO_3^- (bicarbonate), and CO_3^{2-} (carbonate ion), which are related by the acid-base chain:

$$\operatorname{CO}_{2 \text{ (atm)}} \rightleftharpoons \operatorname{CO}_{2 \text{ (aq)}} + \operatorname{H}_2\operatorname{O} \rightleftharpoons \operatorname{H}_2\operatorname{CO}_3 \rightleftharpoons \operatorname{HCO}_3^- + \operatorname{H}^+ \rightleftharpoons \operatorname{CO}_3^{2-} + 2\operatorname{H}^+$$

75 At normal ocean conditions, carbonic acid has a vanishingly small concentration so typically it is incorporated into a single term with aqueous CO_2 as: 76 77

$$\mathrm{CO}_2^* = \mathrm{H}_2\mathrm{CO}_3 + \mathrm{CO}_2_{(aq)}$$

We can then express the acid-base chain as:

$$\operatorname{CO}_{2 \text{ (atm)}} \xleftarrow{K_0} \operatorname{CO}_{2 \text{ (aq)}}^* + \operatorname{H}_2 \operatorname{O} \xleftarrow{K_1} \operatorname{HCO}_3^- + \operatorname{H}^+ \xleftarrow{K_2} \operatorname{CO}_3^{2-} + 2\operatorname{H}^+$$

From which we may write an expression for the equilibrium constants $(K_0, K_1, and$ 78 K_2) and apparent equilibrium constants (K^*_0 , K^*_1 , and K^*_2) by taking the ratio of the 79 products and reactants. For example, the equilibrium constant K_1 is given in Equation 80 2, and the apparent equilibrium constant in Equation 3. 81

$$\operatorname{CO}_2 + \operatorname{H}_2\operatorname{O} \xleftarrow{K_1} \operatorname{HCO}_3^- + \operatorname{H}^+$$
 (1)

$$K_1 = \frac{\{\text{HCO}_3^-\}\{\text{H}^+\}}{\{\text{CO}_2\}\{\text{H}_2\text{O}\}}$$
(2)

$$K_1^* = \frac{[\text{HCO}_3^-][\text{H}^+]}{[\text{CO}_2][\text{H}_2\text{O}]}$$
(3)

where $\{X\}$ denotes the activity of species X, and [X] denotes the stoichiometric con-82 centration of species X in solution. 83

Because there are a number of acid-base chains in seawater, each sharing the com-84 mon ion H^+ , the relationship between the carbonate system and pH is affected by the 85 concentration of numerous other compounds that exist in weak acid-base forms in sea-86 water (such as boron and sulphate). It is therefore necessary to know the K^* 's for mul-87 tiple species, even if we only seek to determine the state of dissolved carbon in seawa-88 ter (Zeebe & Wolf-Gladrow, 2001). Carbon is of particular interest because it is the most 89 concentrated weak acid-base system in seawater, and because both ends of the carbon 90 acid-base chain have important environmental implications. The presence of atmospheric 91 CO_2 on the left means that determining the speciation of oceanic carbon is critical in 92 understanding our imminent and long-term climate trajectory, as well as understand-93 ing climates of the past. On the right-hand side, carbonate ion concentration is directly 94 proportional to the saturation state of calcium carbonate (Equation 4), which is a key 95

Language	Package	Reference	Palaeo-Seawater Method
Microsoft Excel	CO2SYS	Lewis and Wallace (1998)	-
Matlab	CO2SYS	Lewis and Wallace (1998)	-
Matlab	csys	Zeebe and Wolf-Gladrow (2001)	-
R	seacarb	Gattuso and Lavigne (2009)	-
R	seacarbx	Raitzsch et al. (2022)	MyAMI look-up-table
Python	PyCO2SYS	Humphreys et al. (2022)	-
Python	cbsyst	Branson et al. (2023)	Kgen & pymyami
Table 1	. Widely-used	software for calculating carbon specia	tion in seawater.

property for marine calcification (e.g. by foraminifera and corals), and long-term forma tion and dissolution of calcium carbonate that balances of inputs and outputs of carbon
 to the ocean.

$$\Omega = \frac{[\mathrm{Ca}^{2+}][\mathrm{CO}_3^{2-}]}{K_{sp\ calcite}^*} \tag{4}$$

Carbon is not the only acid-base sensitive element in seawater of geochemical interest. Boron is another of the key acid-base systems in seawater, partly due to its contribution to alkalinity (Zeebe & Wolf-Gladrow, 2001), but also because the isotopic ratio of borate ($\delta^{11}B_4$) is used as a proxy for palaeo pH of ancient seawater (Foster & Rae, 2016; Marschall & Foster, 2017). Calculation of pH from boron isotopes requires, among other factors, an estimate of K*_B (see Equation 5). Quantifying K*'s therefore has dual importance for the boron isotope community, as K*_B is required to translate $\delta^{11}B_4$ to pH (Equation 5), and all K*'s are needed to calculate palaeo CO₂ from palaeo pH. Partly as a result of this, the boron community is particularly invested in efforts to improve accuracy of K*'s in past seawater, where major ion chemistry is known to have varied (Dickson & Goyet, 1994).

$$\delta^{11}B_4 = \frac{\delta^{11}B_{sw} + (\delta^{11}B_{sw} - \epsilon) \cdot 10^{pK_B^* - pH}}{1 + \alpha \cdot 10^{pK_B^*} - pH}$$
(5)

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1.2 Calculating K*'s - Current Approach

The calculation of K^* 's for modern ocean conditions is contained incidentally within 100 multiple software packages designed to calculate carbon speciation in seawater (Table 101 1). From a user-facing perspective, these software packages all work in broadly the same 102 way. The user must provide two carbonate system parameters, and a variable number 103 of inputs to describe the thermodynamic and compositional properties of seawater - where 104 different to the archetypal standard $(25^{\circ}C, 0 \text{ bar, and } 35 \text{ psu})$. Some programs accept 105 additional arguments to describe concentrations of other ions in seawater, in particular 106 calcium and magnesium (again where different to the standard modern 10 mmol/kg cal-107 cium concentration and 53 mmol/kg magnesium concentration), or specifying which par-108 ticular expressions and coefficients to use for calculating apparent equilibrium constants. 109 However, the simplicity of these user-facing functions hide a number of complexities and 110 differences between calculation approaches, which have poorly constrained implications 111 for the resulting calculations, and remain opaque to most users. 112

113 1.3 Calculating K*'s - The Problem

The core problem underlying K^{*} calculation is that they can be calculated using any one of multiple empirical equations, and the choice of the 'best' K^{*} equations has evolved with time in a way that is not immediately apparent even to an expert user. For

modern seawater, it is currently standard practice to use the equations assembled in Dickson 117 and Goyet (1994), which have been implemented in all major carbon chemistry speci-118 ation calculators written across multiple platforms (Table 1). Most of these calculation 119 packages jointly implement both the calculation of K^{*}'s, and their application to calcu-120 late the state of the oceanic carbonate system. Throughout the history of these calcu-121 lators, both the software and our understanding of ocean chemistry have evolved, and 122 typographical errors in coefficients have been introduced and resolved, but not necessar-123 ily in concert across all the available packages. There is occasional indirect cross-validation 124 of K^{*} calculation between these packages by comparing their ability to calculate sim-125 ilar carbon speciation values (Orr et al., 2015), but no continuous checking to ensure that 126 all packages always produce comparable results, and little or no record of how and when 127 changes have been made to the calculation of K*'s in response to updates in our under-128 standing of the constants used to calculate them. If, for example, a K^* definition were 129 to be revised or modified today, each piece of calculation software would have to be in-130 dependently updated by a separate team of maintainers, leading to a period in which 131 each software package produces subtly different results, and it is not straightforward for 132 a user to determine which version has been used for a calculation, or indeed which ver-133 sion they are using themselves. This scenario of updating a K^{*} parameterisation has al-134 ready arisen, for instance recently Sulpis et al. (2020) published updated parameterisa-135 tions for K_{1}^{*} and K_{2}^{*} that improve performance at low temperatures, which are now used 136 as the default in PyCO2SYS (Humphreys et al., 2022), but have not been updated in 137 other calculation packages. This will cause deviation in the calculation of carbon param-138 eters between PyCO2SYS and other speciation calculation packages that a casual user 139 will not be aware of. 140

The challenge of evolving K^{*}'s is a particularly common occurrence in palaeosea-141 water calculations, where this lack of synchronisation and validation has caused issues 142 (CenCO2PIP Consortium, 2023). Palaeoseawater requires special consideration because 143 it is necessary to adjust K^{*}'s to account for secular changes in seawater composition (Millero 144 & Pierrot, 1998; Hain et al., 2015) through time, which affects the strength of ion-ion 145 interactions in the solution. Particular attention has been paid to calcium and magne-146 sium, which show the strongest interaction with the dissolved carbon species, and have 147 both changed over the last 100Myr (Tyrrell & Zeebe, 2004b; Hain et al., 2015; Zeebe & 148 Tyrrell, 2019). Sulphate also deserves special mention, as it features in the definition of 149 pH on the total and seawater scales (as described in Section 2.2.6 below), and is also known 150 to have changed concentration through geological time (Zeebe & Tyrrell, 2019; Weldeghe-151 briel et al., 2022). Our understanding of how to adjust K^* values to account for varia-152 tions in seawater composition is evolving relatively rapidly, with several proposed ap-153 proaches that are not uniformly implemented or used by the community. 154

In summary, the current norms for calculating K^{*} values create a number of pitfalls which are not always obvious to the end-user:

- The calculation of K^{*}'s is usually embedded within carbon calculation software. 157 and it is not always clear how they are being performed, or which coefficients are 158 being used. 159 • Over time, the equations used to calculate K*'s have evolved, or been refit to new 160 data. There is no standard way to ensure that different pieces of software are us-161 ing the same coefficients, or whether they adhere to current best practices. 162 • There are known ambiguities, typographical errors, and programmatic errors in 163 published constants and the implementation of the calculations, but no pipeline 164 presently exists to propagate improvements to all software packages simultaneously 165 to ensure they all produce consistent results. 166
- It is necessary to account for different pH scales (see Section 2.2.6).

The necessary complexity of K^{*} calculations, the fragmented nature of the imple-168 mentation of these calculations in numerous software packages, and the lack of routine 169 cross-checking between existing software packages create the ideal environment for difficult-170 to-detect errors to emerge, and for the resulting calculations of software packages to dif-171 fer. We seek to resolve this with a new tool, **Kgen**, which unifies the calculation of K^* 's 172 across the three major software languages - Python, Matlab and R. 173

2 Kgen: A unified framework for K^{*} Calculation 174

Kgen is designed to be a single, definitive source of K^{*} values in both modern and 175 palaeoseawater, which produces apparent equilibrium constants that are guaranteed to 176 be the same to within 0.01% across all three languages, thus removing a major source 177 of potential error between carbon speciation calculation methods. The overall goal of Kgen 178 is to provide a consistent approach to the calculation of K^* 's for seawater carbonate chem-179 istry calculations that can be used within other software that seeks to calculate ocean 180 carbon speciation. Specifically, we aim to: 181

- 1. Separate the generation of K^* 's from their use in carbonate chemistry calculation 182 programs. 183
- 2. Standardise and implement the same functions to calculate K^{*}'s in python, Mat-184 lab, and R.
- 3. Embed automated crosschecking of the K^* 's produced using each language at test 186 conditions to ensure ongoing consistency. 187
- 4. Ensure that other software which is required as a dependency of Kgen is portable 188 between languages (for example, to account for changes in seawater composition 189 the MyAMI software package is often used). 190

The use of Kgen across all three languages is comprehensively described in the on-191 line documentation, which should be the primary source for users seeking to install and 192 use Kgen. The overall conceptual approach that Kgen employs is described below. 193

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2.1 Equilibrium Constants in Modern and Ancient Seawater

Kgen builds on current state-of-the-art methods for calculating K^{*}'s in modern and 195 palaeoseawater. Here, we outline the approach we use to deal with the influence of tem-196 perature, salinity, pressure, and seawater composition on calculated K^{*} values. 197

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2.1.1 Temperature and Salinity

Functions for surface ocean K^{*}'s in terms of temperature and salinity have been 199 calibrated over a number of years and in a number of publications, before being defini-200 tively compiled by Dickson and Goyet (1994). Alternative formulations exist for other 201 environments, and there are a number of published alternatives for seawater that use sub-202 tly different coefficients, however here we focus on the recommended best practices for 203 seawater in Dickson and Goyet (1994). The same K^{*}'s were restated in Hain et al. (2015) 204 but with small edits to the order of coefficients. Here we use the following equations for 205 K*'s: 206

$$\begin{split} & K_{0}^{*} = e^{\left(a_{0} + \frac{(100 \cdot a_{1})}{T} + a_{2} \cdot \log\left(\frac{T}{100}\right) + S \cdot \left(a_{3} + \frac{a_{4} \cdot T}{100} + a_{5} \cdot \left(\frac{T}{100}\right)^{2}\right)\right)} & (6) \\ & K_{1}^{*} = 10^{\left(a_{0} + \frac{a_{1}}{T} + a_{2} \cdot \log\left(T\right) + a_{3} \cdot S + a_{4} \cdot S^{2}\right)} & (7) \\ & K_{2}^{*} = 10^{\left(a_{0} + \frac{a_{1}}{T} + a_{2} \cdot \log\left(T\right) + a_{3} \cdot S + a_{4} \cdot S^{2}\right)} & (8) \\ & K_{B}^{*} = e^{\left(a_{0} + \frac{a_{1}}{T} + a_{2} \cdot \log\left(T\right) + a_{3} \cdot S + a_{4} \cdot \sqrt{S} + a_{5} \cdot S + a_{6} \cdot S^{1.5} + a_{7} \cdot S^{2}\right) + \log\left(T\right) \cdot \left(a_{8} + a_{3} \cdot \sqrt{S} + a_{10} \cdot S\right) + a_{11} \cdot T \cdot \sqrt{S}} & (9) \\ & K_{W}^{*} = e^{\left(a_{0} + \frac{a_{1}}{T} + a_{2} \cdot \log\left(T\right) + \sqrt{S} \cdot \left(\frac{a_{3}}{T} + a_{4} + a_{5} \cdot \log\left(T\right)\right) + I \cdot \left(\frac{a_{6}}{T} + a_{7} + a_{8} \cdot \log\left(T\right)\right) + \frac{\left(a_{0} \cdot I \cdot 1.5\right)}{T} + \frac{a_{10} \cdot I \cdot 2}{T} + \log\left(1 - 0.001005 * S\right)} & (11) \\ & K_{F}^{*} = e^{\left(\frac{a_{0}}{T} + a_{1} + a_{2} \cdot \log\left(T\right) + \sqrt{I} \cdot \left(\frac{a_{3}}{T} + a_{4} + a_{5} \cdot \log\left(T\right)\right) + I \cdot \left(\frac{a_{6}}{T} + a_{7} + a_{8} \cdot \log\left(T\right)\right) + \frac{\left(a_{0} \cdot I \cdot 1.5\right)}{T} + \frac{a_{10} \cdot I \cdot 2}{T} + \log\left(1 - 0.001005 * S\right)} & (11) \\ & K_{F}^{*} = e^{\left(\frac{a_{0}}{T} + a_{1} + a_{2} \cdot \log\left(T\right) + \sqrt{J} \cdot \left(\frac{a_{3}}{T} + a_{4} + a_{5} \cdot \log\left(T\right)\right) + I \cdot \sqrt{S} + \left(a_{4} + a_{5} \cdot T + \frac{a_{6}}{T}\right) + a_{7} \cdot S + a_{8} \cdot S^{1.5}} & (13) \\ & K_{sp,c}^{*} = 10^{\left(a_{0} + a_{1} \cdot T + \frac{a_{2}}{T} + a_{3} \cdot \log_{10}\left(T\right) + \sqrt{S} \cdot \left(a_{4} + a_{5} \cdot T + \frac{a_{6}}{T}\right) + a_{7} \cdot S + a_{8} \cdot S^{1.5}} & (14) \\ & K_{P1}^{*} = e^{\left(\frac{a_{0}}{T} + a_{1} + a_{2} \cdot \log\left(T\right) + \sqrt{S} \cdot \left(\frac{a_{1}}{T} + a_{4}\right) + S \cdot \left(\frac{a_{5}}{T} + a_{6}\right)\right)} & (15) \\ & K_{P2}^{*} = e^{\left(\frac{a_{0}}{T} + a_{1} + a_{2} \cdot \log\left(T\right) + \sqrt{S} \cdot \left(\frac{a_{1}}{T} + a_{4}\right) + S \cdot \left(\frac{a_{5}}{T} + a_{6}\right)} & (16) \\ & K_{P3}^{*} = e^{\left(\frac{a_{0}}{T} + a_{1} + a_{2} \cdot \log\left(T\right) + \sqrt{S} \cdot \left(\frac{a_{1}}{T} + a_{3}\right) + S \cdot \left(\frac{a_{5}}{T} + a_{6}\right)} & (17) \\ & K_{Si}^{*} = e^{\left(\frac{a_{1}}{T} + a_{2} + a_{3} \cdot \log\left(T\right) + \sqrt{J} \cdot \left(\frac{a_{1}}{T} + a_{5}\right) + I \cdot \left(\frac{a_{6}}{T} + a_{7}\right) + I^{2} \cdot \left(\frac{a_{8}}{T} + a_{9}\right) + \log\left(1 - 0.001005 * s\right)} & (18) \\ \end{array}$$

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Where T is the temperature in degrees Celsius, S is the salinity, and I is the ionic strength as defined in Dickson and Goyet (1994) $\left(I = \frac{19.924S}{1-1.005S}\right)$. Each a_n is a coefficient (Table 2), determined empirically for modern seawater, which will provide a K* at the specified temperature and salinity at sea surface pressure and modern ocean composition. If using the MyAMI approach in Hain et al. (2015), these a_n coefficients are adjusted to account for changes in palaeoseawater composition.

к*	^a 0	^a 1	a_2	a ₃	a_4	a_5	^a 6	a_7	^a 8	a_9	a_{10}	^a 11
K*0	-60.2409	93.4517	23.3585	0.023517	-0.023656	0.0047036						
K*1	61.2172	-3633.86	-9.67770	0.011555	-0.0001152							
K*2	-25.9290	-471.78	3.16967	0.01781	-0.0001122							
K [*] w	148.9652	-13847.26	-23.6521	118.67	-5.977	1.0495	-0.01615					
K [*] B	148.0248	137.1942	1.62142	-8966.90	-2890.53	-77.942	1.728	-0.0996	-24.4344	-25.085	-0.2474	0.053105
K [*] S	141.328	-4276.1	-23.093	-13856	324.57	-47.986	35474	-771.54	114.723	-2698	1776	
K [*] _{SDA}	-171.945	-0.077993	2903.293	71.595	-0.068393	0.0017276	88.135	-0.10018	0.0059415			
K [*] _{spC}	-171.9065	-0.077993	2839.319	71.595	-0.77712	0.0028426	178.34	-0.07711	0.0041249			
K* _{P1}	-4576.752	115.525	-18.453	-106.736	0.69171	-0.65643	-0.01844					
K*P2	-8814.715	172.0883	-27.927	-160.340	1.3566	0.37335	-0.05778					
K*P3	-3070.75	-18.141	17.27039	2.81197	-44.99486	-0.09984						
K [*] Si	-8904.2	117.385	-19.334	-458.79	3.5913	188.74	-1.5998	-12.1652	0.07871			
K [*] F	874	-9.68	0.111									

Table 2. Coefficients used for calculation of K^* 's at standard conditions for modern seawater composition from Dickson and Goyet (1994), and available online.

2.1.2 Pressure

Accounting for changes in pressure is achieved by the following formula (Zeebe & Wolf-Gladrow, 2001).

$$\frac{K_{deep}^*}{K_{surface}^*} = e^{-\left(\frac{\Delta V}{R(T-273.15)}\right)P + \left(0.5\frac{\Delta\kappa}{R(T-273.15)}\right)P^2}$$
(19)

Where:

$$\Delta V = a_0 + a_1 T + a_2 T^2$$
$$\Delta \kappa = b_0 + b_1 T$$

Unfortunately, Equations 6 to 18, and coefficients used in Equation 19 have a history of typographical errors. While we can not preclude the potential of further typographical or other errors here, by using the same coefficients in all three languages we ensure that all three implementations are comparable, and any improvements are simultaneously propagated to all software which uses Kgen. Here we use coefficients from Hain et al. (2015) (Table 3).

	$ a_0$	a_1	a_2	b_0	b_1
$\overline{\mathrm{K}^*_0}$	0	0	0	0	0
K_{1}^{*}	-25.50	0.1271	0	-0.00308	0.0000877
K_{2}^{*}	-15.82	-0.0219	0	0.00113	-0.0001475
${\rm K^*}_{\rm B}$	-29.48	0.1622	-0.002608	-0.00284	0
$\mathrm{K*}_{\mathrm{W}}$	-20.02	0.1119	-0.001409	-0.00513	0.0000794
${\rm K}^{*}{}_{\rm S}$	-18.03	0.0466	0.000316	-0.00453	0.00009
K_{F}^{*}	-9.78	-0.0090	-0.000942	-0.00391	0.000054
$\mathrm{K^*}_{\mathrm{spC}}$	-48.76	0.5304	0	-0.01176	0.0003692
K^*_{spA}	-45.96	0.5304	0	-0.01176	0.0003692
K^*_{P1}	-14.51	0.1211	-0.000321	-0.00267	0.0000427
K^*_{P2}	-23.12	0.1758	-0.002647	-0.00515	0.00009
$\mathrm{K*}_{\mathrm{P3}}$	-26.57	0.2020	-0.0030420	-0.00408	0.0000714
${\rm K*}_{\rm Si}$	0	0	0	0	0

Table 3. Coefficients used in the pressure correction of K^* 's from Hain et al. (2015) and available online.

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2.1.3 Seawater Composition

Palaeoceanographic studies of seawater chemistry need to account for the influence 221 of long-term secular evolution in seawater composition. Solution modelling programs such 222 as PHREEQC (Parkhurst & Appelo, 2013) are not sufficiently accurate in seawater, and 223 a range of other tools and methods have been developed to tackle this problem. Broadly 224 speaking, there are two approaches: use simple sensitivity parameters (Tyrrell & Zeebe, 225 2004b; Zeebe & Tyrrell, 2019) that modify K*'s as a linear function of seawater com-226 position, or use a more complex Pitzer speciation model (Hain et al., 2015). The Pitzer 227 ion interaction model adjusts each of the K*'s using empirical coefficients that describe 228 ion-ion interactions in the solution, and the impact of these interactions on the activ-229 ities of relevant ions in seawater. All currently available carbonate chemistry speciation 230 programs that provide the facility to account for changing seawater composition have 231 opted to use the MyAMI Pitzer ion interaction from Hain et al. (2015), which is writ-232 ten in Python. MyAMI is designed to account for non-standard seawater calcium and 233 magnesium concentrations by adjusting the coefficients in Equations 6-18. MyAMI can 234 used in three ways: by directly calling functions, using the command line interface, or 235 using a look-up table of parameters. Both the command line interface and the look-up 236 table approach have been used in previous publications such as Henehan et al. (2019); 237 Rae et al. (2021). 238

Unfortunately, there are a number of issues with this approach that are not immediately evident to the end user. First, there is a known issue with the published lookup table affecting, in particular, the K*_B values (CenCO2PIP Consortium, 2023). K*_B values generated from the look-up table diverge from K*_B calculated using MyAMI by up to 66%. This look-up table has since be regenerated and can be found in compressed R data published in (Raitzsch et al., 2022), but is formatted slightly differently to the

original. We provide a drag-and-drop replacement for the original table here. While we 245 maintain that Kgen is an overall more effective long-term solution for repeatable K^{*} gen-246 eration, this corrected look-up table provides a convenient solution to remove the most 247 egregious issues with K^{*}'s in pre-existing scripts that use this method. Second, there is 248 an undocumented difference in the order of parameters returned by MyAMI when us-249 ing the command line and function-call interfaces, which can lead an unwary user to ap-250 ply dramatically different correction factors at a given magnesium and calcium concen-251 tration. Finally, and more fundamentally, to provide coefficients for input into Equations 6-252 18, MyAMI runs the underlying Pitzer model across a grid of temperature and salinity 253 conditions at the specified ionic composition of seawater, then fits the relevant K^{*} equa-254 tion to these gridded data. This is convenient, in that it presents the user with famil-255 iar coefficients for ease of use, but is problematic because it assumes that the empirical 256 K^{*} formulation can accurately describe the response to temperature and salinity in mod-257 ified seawater chemistry. This is fragile because there is no guarantee of linearity or smooth-258 ness in the response of these coefficients to calcium and magnesium concentrations. The 259 polynomials used to describe K^* 's may also be non-unique, in that different combina-260 tions of coefficients may resolve to the same result within a particular domain, but have 261 vastly different predictions outside that domain. Polynomials can also be sensitive, with 262 apparently small changes in the coefficients resulting in a large change in the predicted 263 response. As the look-up table to find coefficients for these polynomials is provided at 264 0.1 mmol/kg resolution, it is almost always necessary to interpolate the coefficients to 265 the exact magnesium and calcium concentrations of interest, and there is no guarantee that these interpolated coefficients are good predictors of the K^{*}'s. Ultimately, the fact 267 that there are multiple ways to interact with MyAMI that produce significantly differ-268 ent results make it difficult to determine the impact of these complications on the broader 269 literature, in particular because historically code was rarely archived alongside studies 270 so information on exactly how the scripts were used has been lost. 271

We resolve these issues by providing a new version of the MyAMI model in the pymyami package, written in Python. pymyami is a re-implementation of the MyAMI Specific Ion Interaction Model (Hain et al., 2015) which benefits from:

- Increased speed by 2+ orders of magnitude due to vectorisation the direct calculation of correction factors (rather than re-fitting the K* functions to gridded parameters).
- Improved transparency, by working with the original parameter tables from Millero and Pierrot (1998) rather than embedding parameter values within the code, which identified and resolved several typographic errors within MyAMI. The correction of these typographic errors result in differences of between 0-3.8% in the correction factors calculated by pymyami compared to MyAMI, with no difference in K*_W, and the largest difference in K*₂.
- Improved flexibility, allowing an expert user to change the concentrations of a greater number of ions (Na⁺, K⁺, Mg²⁺, Ca²⁺, Sr²⁺, Cl⁻, B(OH)₄⁻, HCO₃⁻, CO₃²⁻ and SO₄²⁻).

In addition to these improvements, pymyami also changes the output structure from MyAMI. Instead of calculating new coefficients for input into Equations 6-18 at a specified calcium and magnesium concentration, pymyami calculates a single multiplicative correction factor $(f_{(T,S,Mg,Ca}))$ for each K^{*} as a direct function of the specified ocean composition:

$$K_{x (T,S,Mg,Ca)} = K_{x (T,S)} f_{x (T,S,Mg,Ca)}$$
(20)

This removes the assumption that the functional form of Equations 6-18 remains con-

stant with changes in seawater chemistry, and simplifies the implementation and inte-gration into Kgen.

To make the seawater composition corrections available outside of python, we pro-290 vide two approaches: a polynomial approximation of the correction factors, and a mech-291 anism to call the full python implementation of pymyami (which can be imported in the 292 case of Matlab, or used via the reticulate platform in R). The approximation method uses 293 a high-dimensional polynomial to estimate a correction factor as a function of temper-294 ature, salinity, and magnesium and calcium concentration $(f_{(T,S,Mg,Ca}))$ natively in each 295 language. This is useful in cases where speed is paramount (e.g. uncertainty propagation using random sampling methods such as Monte Carlo), but these approximations 297 are only accurate to within 0.2% of the directly calculated values. This inaccuracy is small 298 given the relative scale of other uncertainties in palaeo-oceanographic reconstructions, 299 so the approximation method is likely to be sufficient for most purposes. However, we 300 would caution users to avoid the approximation in cases where a very high level of pre-301 cision is required. 302

303

2.2 Practicalities: Usability and Automated Consistency Checking

As well as providing state-of-the-art K^{*} calculation, Kgen brings a number of key considerations which are designed to make the process of K^{*} calculation as painless and reliable as possible for the end-user.

307

2.2.1 Consistent Interface

Implementations of Kgen in python, R, and Matlab provide user-facing functions with the same naming scheme and input arguments, allowing straightforward migration between languages. Kgen provides two main functions for calculating a single K^{*}:

- 311 calc_K(name,temp_c,sal,p_bar,magnesium,calcium)
- or multiple K*'s
- calc_Ks(names,temp_c,sal,p_bar,magnesium,calcium)

These combine lower-level functions to calculate the K^{*} for modern seawater, apply a pressure correction factor created with the function

316 calculate_pressure_correction(name,temp_c,p_bar)

followed by, if necessary, a correction for seawater composition calculated by pymymami

319 calculate_seawater_correction(name,temp_c,sal,magnesium,calcium)

The primary source of documentation for code is online, however here we list these functions here to demonstrate the benefit of having consistently named functions and function arguments.

2.2.2 Automated Consistency Checking

Each implementation of Kgen performs three sequential actions (as shown in Figure 1). First, required apparent equilibrium constants are determined for surface ocean seawater at the prescribed temperature and salinity. Second, a multiplicative pressure correction is determined. Third, a multiplicative seawater composition correction is determined. The externally available functions of Kgen reflect these three actions.

The Kgen project is managed through a git repository, augmented with GitHub 329 actions to automate cross-comparison between the three languages. There are multiple 330 comparisons made to ensure consistency. The first validates the output from Kgen against 331 check values given in Zeebe and Wolf-Gladrow (2001). The second runs Kgen in all three 332 languages to calculate K^{*}'s for a wide variety of oceanic conditions, then quantifies the 333 difference between them - with an automatic failure if the difference is greater than 0.01%. 334 Updated values for parameters or functions can not be merged into the repository with-335 out first passing these tests, ensuring ongoing consistency. 336

Versioning uses the semantic naming scheme (X.Y.Z), with consistency between all three languages guaranteed at the X.Y level, and bug fixes for individual languages incrementing the Z version number. Each increase in version number triggers an automated GitHub action which stores an indicative value how much each K* has changed between versions. This framework has been implemented for python, R, and Matlab, but is extensible, and open to future implementations of Kgen in other languages.

343

2.2.3 Traceable Version Numbering

The semantic version numbering scheme of Kgen, e.g. '1.0.0', is in the form of [ma-344 jor release].[K value update].[minor patch] across all three software languages. We guar-345 antee that all versions of the software that share the same [major release].[K value up-346 date] numbers will produce quantitatively identical K values, regardless of the language. 347 Whenever there is a change in the underlying code that causes a change in K values, these 348 will be recorded and described in the online manual. Thus, as long as a user states which 349 version of Kgen was used in a calculation, it will be possible to exactly reproduce that 350 calculation by installing a legacy version of Kgen. 351

352

2.2.4 Integrating Kgen Into Carbonate Chemistry Solvers

We recommend assimilating Kgen into carbonate chemistry software by treating 353 it as a dependency. With Python and R this can be managed using your preferred pack-354 age manager to specify versions of Kgen compatible with the carbonate chemistry soft-355 ware. For Matlab, Kgen can be directly incorporated with the carbonate chemistry pack-356 age using the Matlab file exchange. Kgen can run alongside other methodologies of cal-357 culating K^{*}'s (for instance where apparent equilibrium constants appropriate for fresh-358 water and seawater are required within the same analysis). Kgen can be integrated into existing carbonate chemistry software packages in such a way that does not break back-360 wards compatibility and K^* 's from a variety of methods can be easily compared. 361

362

2.2.5 Simplified Calculation Sequence

The current procedure to calculate K*'s proceeds along one of two paths, depend-363 ing on whether the target is modern or ancient seawater. These two pathways are illus-364 trated in Figure 1. If the target is modern seawater, we proceed along the left hand path 365 - using Equations 6-18 (or analogous equations) to calculate modern, surface seawater 366 K^{*}'s. A pressure correction is applied to adjust K^{*}'s to be appropriate for depth. If the 367 target is ancient seawater, then the calculation is more involved. Coefficients for Equa-368 tions 6-18 must first be estimated for the palaeo seawater composition, then the calcu-369 lation proceeds as before. The pressure correction step remains the same. 370

371 2.2.6 pH Scales

The primary contribution of Kgen is in providing a central, cross-checked funcitonality for calculating consistent K* values. The collection of these calculations in one place does not, however, address some of the outstanding issues surrounding K* calculation.



Figure 1. The pathways for calculation of K^* 's for carbonate chemistry software. The current, two step paradigm is shown on the left, and is branched depending on whether K^* 's are required for modern or palaeo seawater. Kgen linearises this workflow into three separate steps.

pH can be expressed on a variety of scales: the NBS scale, the free scale, the to-376 tal scale, or the seawater scale. The free scale is the simplest, as it accounts only for free 377 hydrogen ions, while the total scale accounts for hydrogen sulphate, and the seawater 378 scale accounts for both hydrogen sulphate and hydrogen fluoride. The motivation be-379 hind these scales is practical in nature, in that, while we might typically be interested 380 in free hydrogen ion concentration, pH measurement apparatus may respond more closely 381 to the combined influence of free hydrogen ions, hydrogen sulphide, and hydrogen flu-382 oride. 383

When calculating pH of a palaeo ocean, it would seem sensible to use the free scale - as there is no pristine preserved seawater on which modern analytical equipment can be used. However, as carbonate system programs (and the equations which govern K^{*}'s) were originally targeted at modern seawater, most equations are given on the total pH scale - or are converted to be on an approximate total scale. We adopt this convention here, reporting almost all K^{*}'s on the total scale.

There are some important exceptions though. $K^*_{sulphate}$ and $K^*_{fluoride}$ are used in the definition of the total and seawater pH scales. They are therefore given on the free scale to avoid producing a self referential scale. The other major exception is in the calculation of the pressure correction for K^* 's, which understood to have been given on the seawater scale (Rae, 2011). To perform this pressure correction, we convert all K^* 's to the seawater scale, apply the correction, the convert them all back to the total scale (in line with CO2SYS (Lewis & Wallace, 1998) and PyCO2SYS (Humphreys et al., 2022)).

³⁹⁷ **3 Results: Kgen Performance**

The primary result of this paper is the Kgen software library itself. We take this opportunity to analyse the output of Kgen at a variety of conditions, both to validate our approach, and to illustrate various properties of the K*'s.

401 3.1 Comparison

We have validated the K*'s produced by Kgen against K*'s from MyAMI and the sensitivity parameter approach of Zeebe and Tyrrell (2019) (see Figure 2). We find that MyAMI and Kgen produce identical K*'s across profiles for temperature and salinity. MyAMI, Kgen (using pymyami), and the sensitivity parameter approach illustrated in Zeebe and Tyrrell (2019) show broad agreement, with the most notable discrepancies being in K*₁, K*₂, K*_{spC}, and K*_A. The differences are mostly minor, with the exception of K*_a which belows differently (as precised parts doin (Using et al. 2018))





Figure 2. The sensitive of K^* 's to temperature, salinity, magnesium concentration and calcium concentration using three different methods. Each K^* is shown in a row and each control in a column. There is generally good agreement between the three methods. The largest offsets are between the sensitivity parameter approach (Zeebe & Tyrrell, 2019) and the Pitzer model approach in particular for K^*_1 . pymyami and MyAMI diverge only slightly, as a result of correction of typographical errors.

The dominant factor driving differences between K*'s is the chosen approach (Pitzer ion model vs sensitivity parameters). The sensitivity parameter style approach of Zeebe and Tyrrell (2019) gives noticeably different results to the pitzer ion model approach demon-

strated in Hain et al. (2015) and here. Differences between MyAMI and pymyami are

⁴¹³ much smaller, and are driven by small typographical corrections to MyAMI.

3.2 K^{*} Properties



Figure 3. The sensitivity of pH and CO_2 to each individual factor (temperature, salinity, pressure, magnesium concentration, and calcium concentration) is shown above at a reference condition of DIC=2000 μ mol/kg and alkalinity=2300 μ mol/kg, and standard modern seawater conditions.

To evaluate the sensitivity of pH and CO_2 to temperature, salinity, pressure, mag-415 nesium concentration, and calcium concentration, we prescribe a standard set of condi-416 tions then vary each parameter independently. The estimates of $\frac{\partial pH}{\partial T}$, $\frac{\partial pH}{\partial S}$, $\frac{\partial CO_2}{\partial T}$ etc., 417 are shown in Figure 3, alongside the second derivative of each parameter (e.g. the sen-418 sitivity of $\frac{\partial pH}{\partial T}$ to temperature). From this we find that pressure is the strongest driv-419 ing factor of pH and CO₂ across the range found in the ocean (only one tenth of which 420 is shown in Figure 3), followed by temperature, while the influence of other factors is more 421 muted. Pressure is the only parameter which has a negative correlation to CO_2 . The sen-422 sitivity of pH and CO_2 to salinity, pressure, magnesium concentration, and calcium con-423 centration is almost constant across their range, whereas the influence of temperature 424 is much more nonlinear, with a much greater sensitivity of CO_2 to temperature at higher 425 temperatures. The lower subplots in Figure 3 allow us to express the sensitivity of pH 426 and CO_2 to each individual factor quantitatively and, for modern ocean conditions, get 427 some sense of the uncertainty stemming from each parameter - though it is important 428 to note that (as described above) these are partial derivatives, and the combinative ef-429 fects of parameters may be different to treating each factor independently. In particu-430

431 lar, we know that seawater calcium and magnesium concentrations have inversely var-

⁴³² ied across the Cenozoic (Weldeghebriel et al., 2022).



Figure 4. Each subplot shows a single K^* , normalised to the modern day K^* across a range of seawater calcium and magnesium concentrations. Lighter colours indicate that the K^* at these conditions are similar to the modern ocean, warmer colours indicate higher K^* , and cooler colours indicate lower K^* . The evolution of calcium and magnesium concentration over the Cenozoic is plotted in black, with keypoints 1. 100Ma, 2. Eocene (56Ma), 3. Miocene (14Ma), and the large point representing modern seawater.

To evaluate the likely magnitude of the combined effect of magnesium and calcium, we can calculate the impact of various combinations of calcium and magnesium concentration on each K^{*} (as depicted in Figure 4). Most K^{*}'s show a diagonal sensitivity pattern (meaning the influence of magnesium and calcium are compensatory to one another) - the exception is K^{*}_W, which is more strongly dependent on magnesium than calcium. We find the influence of magnesium and calcium concentration is strongest on K^{*}₂, K^{*}_W, K^{*}_{spC}, K^{*}_{spA}, and weakest on K^{*}₀ and K^{*}₁.

The trajectory of magnesium and calcium concentration across the Cenozoic (Zeebe 440 & Tyrrell, 2019) is shown on top of the sensitivity in Figure 4 in a black line. As mag-441 nesium concentration has generally increased over the Cenozoic while calcium concen-442 tration has generally fallen, their compensatory effect means the K^{*}'s are very similar 443 to their value during the earliest Cenozoic. It is somewhat inherent in the nature of mag-444 nesium and calcium to be inversely correlated (Tyrrell & Zeebe, 2004a), which is ben-445 eficial to us as it means that palaeo K^* 's are unlikely to be far away from their modern 446 day value by changes in seawater composition. That said, transiently during the Eocene, 447 Oligocene, and Miocene, K^{*}'s were generally higher than present day values. 448

449 4 Summary

We provide Kgen, a multilanguage approach to calculation of apparent equilibrium constants (K^{*}'s). Kgen resolves discrepancies between previous software packages which calculate K^{*}'s, and provides a mechanism for ongoing consistency through automated crosschecking of generated K^{*}'s across a wide range of conditions.

In addition to the direct benefits Kgen brings, we also believe the Kgen repository provides a convenient programming language comparison as applied to geochemistry. Kgen attempts to improve clarity in the equations which underlie the calculation of apparent equilibrium constants and how these influence chemical speciation, in particular as related to atmospheric CO₂ calculations.

Kgen is already in use within cbsyst (Branson et al., 2023), a carbonate chemistry
library written in python which is applicable to both modern and palaeo seawater compositions, alongside a number of convenience functions for calculating the carbon system from boron speciation and isotopes. We are currently exploring the best way of integrating Kgen into existing carbonate chemistry software packages (seacarb/seacarbx
for R, and CO2SYS for Matlab).

465 **5 Open Research**

The software generated during this project is managed through our GitHub repository, which each release of Kgen archived using Zenodo. Two alpha release versions have been archived already, Kgen v0.2.0 (Whiteford et al., 2023a), and Kgen v0.3.0 (Whiteford et al., 2023b). Kgen v1.0.0 will stabilise alongside the publication of this manuscript and similarly be available through GitHub and archived on Zenodo.

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Figure 1.



Figure 2.



Figure 3a.



Figure 3b.



Figure 3c.



Figure 3d.



Figure 5.

