# Deep Graph Neural Networks for Spatiotemporal Forecasting of Sub-Seasonal Sea Ice: A Case Study in Hudson Bay

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# Key Points:

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13	•	GraphSIFNet employs a sequence-to-sequence deep learning framework based on
14		the Graph Long-Short Term Memory (GCLSTM) framework for sub-seasonal sea
15		ice forecasting.
16	•	The model demonstrates superior performance over a statistical baseline in Hud-
17		son Bay, particularly in short- to medium-term predictions of sea ice concentra-
18		tion.
19	•	GraphSIFNet's graph-based approach provides a more natural representation of

sea ice dynamics, more closely resembling physically-based models than those based
 on two-dimensional kernel convolutions.

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#### 22 Abstract

This study introduces GraphSIFNet, a novel graph-based deep learning framework for 23 spatiotemporal sea ice forecasting. GraphSIFNet employs a specialized Graph Long-Short 24 Term Memory (GCLSTM) module within a sequence-to-sequence architecture to pre-25 dict daily sea ice concentration (SIC) and sea ice presence (SIP) in Hudson Bay over a 26 90-day time horizon. The use of graph networks allows the domain to be discretized into 27 arbitrarily specified meshes. This study demonstrates the model's ability to forecast over 28 an irregular mesh with higher spatial resolution near shorelines, and lower resolution oth-29 erwise. Utilizing atmospheric data from ERA5 and oceanographic data from GLORYS12, 30 the model is trained to model complex spatial relationships pertinent to sea ice dynam-31 ics. Results demonstrate the model's superior skill over a linear combination of persis-32 tence and climatology as a statistical baseline. The model showed skill particularly in 33 short- to medium-term (up to 35 days) SIC forecasts, with a noted reduction in root mean 34 squared error (RMSE) by up to 10% over the statistical baseline during the break-up sea-35 son, and up to 5% in the freeze-up season. Long-term (up to 90 days) SIP forecasts also 36 showed significant improvements over the baseline, with increases in accuracy of around 37 10% even at a lead time of 90 days. Variable importance analysis via feature ablation 38 was conducted which highlighted current sea ice concentration and thickness as critical 39 predictors. Thickness was shown to be important at longer lead times during the melt-40 ing season suggesting its importance as an indicator of ice longevity, while concentra-41 tion was shown to be more critical at shorter lead times which suggests it may act as an 42 indicator of immediate ice integrity. The study lays the groundwork for future exploration 43 into dynamic mesh-based forecasting, the use of more complex graph structures, and mesh-44 based forecasting of climate phenomena beyond sea ice. 45

# 46 1 Introduction

The drastic loss of Arctic sea ice volume is one of the most visible and immediate 47 impacts of climate change (J. Stroeve & Notz, 2018). The Arctic is the fastest-warming 48 region on Earth, and this warming is affecting the sea ice cover more than any other com-49 ponent of the climate system (Vihma, 2014; J. C. Stroeve et al., 2012; Cavalieri & Parkin-50 son, 2012). According to the National Snow and Ice Data Center (NSIDC), Arctic sea 51 ice extent (SIE)—the total area of the Arctic Ocean with at least 15% ice cover—is see-52 ing a steady decline. This is especially prominent in September when sea ice extent is 53 at its minimum (Serreze & Meier, 2019). Declining sea cover is connected to increasing 54 air temperatures, changes in atmospheric and oceanic circulation, the albedo feedback 55 loop, and the concentration of greenhouse gases in the atmosphere (J. C. Stroeve et al., 56 2012). The Arctic ice cover is of particular importance as it helps regulate the Earth's 57 climate, and the decline in sea ice and subsequent loss of reflectivity directly contribute 58 to the acceleration of climate change (Moon et al., 2019). Changes in Arctic sea ice cover 59 also disturb marine and terrestrial ecological dynamics (Post et al., 2013); create chal-60 lenges for Northern communities (Meier et al., 2014); and influence human activity as 61 new trade routes become available through the Arctic (Mudryk et al., 2021). Forecast-62 ing sea ice conditions is therefore becoming increasingly important as accurate knowl-63 edge of these changes would allow for more effective preparation. 64

In this study, we introduce a deep learning based sea ice forecasting model that em-65 ploys Graph Neural Networks (GNNs) integrated within a Long Short-Term Memory (LSTM) 66 module to predict daily sea ice concentration (SIC) and sea ice presence (SIP) in Hud-67 son Bay up to 90 days in advance. The choice of Hudson Bay as our study area is driven 68 by its important role as a shipping hub, the presence of communities living within the 69 region relying on maritime re-supply, and its unique characteristics as an in-land sea largely 70 isolated from the wider Arctic. The 90-day forecasting horizon addresses the needs for 71 planning and decision-making in industries such as shipping operations as well as the plan-72 ning requirements of local communities residing in the region. This time horizon cov-73

ers short-term (up to 7 days), medium-term (up to a month) and long-term (up to 3 months) 74 planning needs. The study highlights the effectiveness of GNNs in handling irregular spa-75 tial domains by dividing Hudson Bay into a spatially irregular mesh with a higher res-76 olution along shorelines. We evaluate the performance of two types of spatial graph con-77 volutions within the model: the basic Graph Convolutional Network (GCN) and an attention-78 based transformer convolution. The model was trained using sea ice and oceanographic 79 data from a coupled ice-ocean reanalysis product (GLORYS12 (Jean-Michel et al., 2021)), 80 as well as atmospheric data from the ECMWF Reanalysis v5 (ERA5 (Hersbach et al., 81 2020)). We validate the model's accuracy by comparing its predictions to a statistical 82 baseline and comparing forecasted and observed freeze-up and break-up dates at ports 83 on Hudson Bay. 84

85 2 Background

Sea ice forecasting is a spatiotemporal forecasting task which can be formulated as a next-frame prediction problem. Given a sequence of frames  $\mathbf{X} = (\mathbf{X}_{t-n}, ..., \mathbf{X}_{t-1}, \mathbf{X}_t)$ with  $\mathbf{X}_t \in \mathbb{R}^{w \times h \times c}$  where *n* is the number of frames in the sequence, *w* and *h* are the spatial dimensions of the frames, and *c* is the number of channels, the objective is to predict the next *T* frames in the sequence,  $X_{t+1}, ..., X_{t+T}$ .

While traditional time series modeling techniques such as ARIMA have been widely 91 used for forecasting, they are less effective for spatiotemporal forecasting due to their 92 inherent limitations in handling spatial dependencies and complex temporal dynamics. 93 ARIMA models, primarily designed for univariate time series, lack the capacity to ef-94 fectively model spatial relationships and multi-dimensional data structures, which are 95 critical in spatiotemporal forecasting. To address these limitations, methods like Vec-96 tor Autoregression (VAR) (Sims, 1980) and Spatial Autoregressive (SAR) (Anselin, 1988) 97 models were developed, offering improved handling of multivariate data and spatial de-98 pendencies, respectively. However, these models still struggled with dynamic spatial reqq lationships and non-linear interactions. Space-Time Autoregressive Integrated Moving 100 Average (STARIMA) models (Pfeifer & Deutsch, 1980) were introduced to better inte-101 grate spatial dependencies with temporal dynamics. Dynamic Linear Models (DLMs) 102 and State Space Models (Kalman, 1960) offered a framework for handling evolving tem-103 poral dynamics but were limited in their spatial modeling capabilities. 104

With the advent of deep learning, many neural network methods were developed for spatiotemporal problems, largely based on spatial convolutions with fixed-size twoor three-dimensional kernels (Oprea et al., 2022). These convolutional models are particularly well-suited for image data with a gridded structure such as images or video frames and allow for learning rich features that are present in real-world image sequences.

Graph Neural Networks (GNNs) offer a compelling alternative to Convolutional 110 Neural Networks (CNNs) for emulating models of physical processes, such as ice dynam-111 ics, for several reasons. One of the primary advantages of GNNs in this context is their 112 inherent ability to capture the spatial relationships between neighboring nodes through 113 graph edges, which can be arbitrarily specified. This is particularly crucial in applica-114 tions like sea ice dynamics, where the spatial relationships are fundamental in determin-115 ing heat and momentum exchanges, and other factors influencing ice processes. In GNNs, 116 both nodes and edges can encode information about the system, and graph convolutions 117 update these encodings by applying some non-linear function. This allows GNNs to ef-118 fectively model the exchange of physical quantities such as heat or ice volume at a given 119 location in space and time while accounting for the directionality of processes, which is 120 represented by directed edges. In contrast, CNNs operate on a fundamentally different 121 principle. They extract features such as edges or gradients from an input image by tun-122 ing kernel filters. This process involves convolving these filters over the input image to 123 identify patterns and features at various scales and orientations. While this approach 124



(a) CNN: Kernel filters (bottom figures) are learned to extract patterns in an image of a building.



(b) GNN: Non-linear functions are learned to model the relationship between neighboring nodes in a graph.

Figure 1: Conceptual comparison of the mechanisms of convolutional neural networks (CNN) and graph neural networks (GNN). (a) CNNs learn kernel filters which slide across the image to identify patterns in the image, such as edges or gradients. (b) GNNs learn a function to update a target node's state vector (A) by non-linearly combining the state vectors of its neighbours (B, C, D).

is highly effective for tasks like image recognition, where identifying and categorizing vi-125 sual patterns is key, it may not be as well-suited for learning the underlying physical laws 126 that govern interactions between points in space. CNNs typically lack the ability to ex-127 plicitly model directional relationships and complex dependencies between disparate points 128 in a spatial domain, which are critical in understanding and predicting physical phenom-129 ena like ice dynamics. A high-level visual representation of these two neural network types, 130 highlighting their structural and functional differences, is shown in Figure 1. CNNs lever-131 age spatial locality and translation invariance inherent in images through convolutional 132 layers with fixed-size filters that extract local features across the image. Techniques such 133 as the use of pooling operators, stride convolutions, or dilated filters can be used to cap-134 ture longer-range patterns and hierarchical information (K. He et al., 2016; Yu & Koltun, 135 2016). In contrast, message-passing GNNs can natively capture long-range patterns through 136 edge propagation, potentially reaching across the entire graph structure given a sufficiently 137 deep network. Although in most cases the underlying graphs are too large for informa-138 tion to be propagated globally, limited information propagation across can help mod-139 els gain a holistic view of the spatial domain and learn complex spatial patterns (Wu et 140 al., 2022). Additionally, most types of GNNs exhibit both translation and rotation in-141 variance as convolutions are applied indiscriminately to all nodes and the aggregation 142 operators are most often permutation invariant. Note that this is not always the case; 143 operators based on recurrent units such as the LSTM variant of GraphSAGE (Hamilton 144 et al., 2017) or sorting units such as the SortPooling aggregator (M. Zhang et al., 2018) 145 do not exhibit rotation invariance. Another noteworthy advantage of GNNs over CNNs 146 is their scalability due to the inherent parallelism in their architecture, allowing for ef-147 ficient processing of data over large regions or with fine resolution. This parallelism how-148 ever comes at the cost of higher memory usage which may become limiting, though this 149 can be circumvented by partitioning the graph and processing the subgraphs indepen-150 dently before combining the outputs. Overlapping subgraphs can be used to ensure no 151 spatial artifacts or discontinuities arise from the partitioning. 152

#### <sup>153</sup> **3 Related work**

Prior to the advent of deep learning techniques in sea ice forecasting, traditional 154 physics-based and statistical models were the mainstay for both short-term and long-155 term predictions. Dynamic models, often integrated within data assimilation systems, 156 such as the Pan-Arctic Ice-Ocean Modeling and Assimilation System (PIOMAS) (J. Zhang 157 & Rothrock, 2003), rely on solving physical equations to simulate the interactions be-158 tween sea ice, atmosphere, and ocean. These models are computationally intensive and 159 require extensive calibration, but are considered fairly reliable due to their capacity to 160 incorporate well-understood physical processes and parameters. On the other hand, sta-161 tistical models such as multiple linear regression (MLR) and autoregressive integrated 162 moving average (ARIMA) have been used for their simplicity and computational efficiency 163 relative to physical-based models (Petty et al., 2017). These models often utilize histor-164 ical sea ice concentration, temperature, and other meteorological variables to make short-165 term forecasts. However, they lack the ability to adequately capture the complex spa-166 tial and temporal patterns inherent in sea ice dynamics needed to forecast over longer 167 timeframes. 168

The application of deep learning techniques to sea ice forecasting has gained in-169 creasing attention in recent years due to their computational efficiency and generaliz-170 ability, particularly in the face of a changing climate and increased availability of large 171 training datasets. Early studies applying deep learning to sea ice forecasting were lim-172 ited to either spatial or temporal modelling. For instance, Chi and Kim (2017) used a 173 long-short term memory (LSTM) module to forecast sea ice on a per-pixel level but did 174 not consider spatial patterns. Kim et al. (2019) later used a deep neural network (DNN) 175 with two fully-connected layers to forecast sea ice concentration considering interactions 176 between pixels through dense layers but did not explicitly account for spatial autocor-177 relation. Later models based on the convolutional neural network (CNN) were able to 178 leverage spatial patterns. Andersson et al. (2021) used a U-net trained on both climate 179 simulation and observation data to forecast monthly sea ice concentration and was found 180 to out-perform the SEAS5 dynamical model, but did not explicitly model in the tem-181 poral dimensions. Spatiotemporal models were then proposed that unify spatial and tem-182 poral models. Liu et al. (2021) proposed a model based on the convolutional long-short 183 term memory (ConvLSTM) (X. Shi et al., 2015) to perform one-step ahead forecasting 184 of sea ice in the Barents sea which showed promise by outperforming statistical baselines. 185 Asadi et al. (2022) built on this work by proposing a sequence-to-sequence model based 186 on the ConvLSTM to forecast sea ice presence in Hudson Bay. The model generally out-187 performed the European Centre for Medium-Range Weather Forecasts's (ECMWF) subseasonal-188 to-seasonal (S2S) ensemble predictions (Vitart & Robertson, 2018). 189

GNN-based approaches have recently seen some attention in global climate mod-190 elling, motivated in part by successes in GNN-based physics simulation models such as 191 MeshGraphNets (Pfaff et al., 2020) or graph network simulators (Sanchez-Gonzalez et 192 al., 2020; Rubanova et al., 2022). Keisler (2022) first proposed a GNN for forecasting 193 the global climate using an autoregressive encoder-processor-decoder architecture. Grid-194 ded reanalysis data was encoded onto an icosohedron graph structure on which a message-195 passing neural network performed several steps of processing before being decoded back 196 onto the latitude-longitude grid. Results showed that the model is competitive in com-197 parison with state-of-the-art physical models when forecasting geopotential height and 198 temperature over a 6-day rollout with a 6-hour temporal step. Lam et al. (2022) built 199 upon this work with GraphCast, a similar model structure with the most notable dif-200 ference being the use of multiple icosahedron grids at varying spatial resolution. They 201 demonstrated greater skill than operational state-of-the-art physical models when fore-202 casting global temperature, precipitation, and wind patterns over a 10-day rollout at a 203 6-hour temporal step. 204

## <sup>205</sup> 4 Methodology

### 206 4.1 Data

In this study, ERA5 reanalysis data is used as atmospheric forcing data to train the models along with oceanographic variables from the GLORYS12 reanalysis product. Sea ice concentration estimates from GLORYS12 are used as the target variable and a proxy for the ground truth.

211 **4.1.1 ERA5** 

ERA5 (Hersbach et al., 2020) is a climate reanalysis dataset produced by ECMWF 212 that offers hourly estimates of climatic variables at a spatial resolution of  $0.25^{\circ}$  from 1979 213 to present. It is based on the IFS Cycle 41r2 4D-Var data assimilation system and in-214 cludes a wide range of climatic variables at different pressure levels of the atmosphere. 215 The IFS system assimilates observations from dozens of satellite missions and ground 216 stations to create a physically consistent best representation of atmospheric conditions. 217 Although the model does not have a coupled ocean-atmosphere component, it uses daily 218 passive microwave-derived sea ice concentration estimates from the Ocean and Sea Ice 219 Satellite Application Facilities (OSI-SAF) as boundary conditions (Hersbach et al., 2020). 220 In this study, we follow previous studies (Asadi et al., 2022; Andersson et al., 2021) and 221 use 2-meter temperature, 10-meter wind speeds, and surface sensible heat fluxes from 222 ERA5 as input features to our model (see Table 1) 223

# 224 **4.1.2** GLORYS12

GLORYS12 (Jean-Michel et al., 2021) is a global ocean and sea ice reanalysis data 225 product developed by the Copernicus Marine Environment Monitoring Service (CMEMS), 226 utilizing the LIM2 EVP NEMO 3.1 platform (Madec, n.d.) in the ORCA025 configura-227 tion designed by the DRAKKAR consortium. This configuration includes a global sea-228 ice model with a 1/4° Mercator grid. Atmospheric forcing for the ocean surface model 229 is provided by ECMWF's ERA-Interim (Dee et al., 2011) reanalysis data until 2019, and 230 ERA5 data thereafter. The spatial resolution of the ocean and ice models is  $1/12^{\circ}$ . The 231 data assimilation component of GLORYS12 includes in-situ temperature and salinity (T&S) 232 profiles, satellite sea surface temperature (SST), and along track sea-level anomalies de-233 rived from satellite altimetry. The assimilation of oceanic observations occurs using a 234 reduced-order Kalman filter, which is based on a singular evolutive extended Kalman 235 (SEEK) filter. The SEEK filter utilizes a three-dimensional multivariate background er-236 ror covariance matrix and operates on a 7-day assimilation cycle. The system also in-237 tegrates sea ice concentration observations from IFREMER/CERSAT. Historical records 238 are available from 1993 to present. This study uses GLORYS12 sea ice concentration, 239 thickness, velocities and sea surface temperatures. 240

# 4.2 Meshing

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Meshes allow for greater flexibility in defining the model's spatial basis. Unlike two-242 dimensional convolutional approaches, which require defining a regular two-dimensional 243 grid of pixels over a region, meshes are comprised of cells of abitrary sizes, allowing the 244 modeler to control which areas are modelled in higher resolution (e.g., around ports or 245 passages of interest). Since cells are only defined in regions of interest we also avoid the 246 need to apply a land mask as a post-processing step, unlike in CNN-based approaches 247 which most often model over the whole region before applying a mask to exclude land 248 pixels from the output. 249

Figure 2 shows possible meshes for Hudson Bay using a 1/12 degree grid as the base resolution when trying to balance resolution and computational requirements. The mesh



Figure 2: Comparison of different mesh definitions for modeling Hudson Bay. (a) A high-resolution regular mesh with 32,856 cells, computationally intensive but highly detailed; (b) a four-times coarsened regular mesh with 2,425 cells lacking sufficient detail along land interfaces; (c) irregular mesh with 9,422 cells, a compromise for both computational efficiency and high resolution at land interfaces. This approach ensures no cell overlaps land while providing high-resolution data for critical regions like ports, passages, and areas of meteorological interest such as the Kivalliq latent heat polynya.

shown in (a) uses the base resolution as a regular mesh, which is computationally heav-252 ier with its 32,856 cells, while the mesh in (b) uses a regular four-times coarsened ver-253 sion of the same mesh with 2,425 cells, which may not have sufficient definition. At the 254 shoreline, this coarse mesh overlaps land but the model does not have the ability to ac-255 knowledge this overlapping. A  $4 \times 4$  cell with only one non-land pixel assigns the sea 256 ice concentration value to the entire cell, possibly undermining the model's ability to rea-257 son about volumetric continuity. As a compromise between resolution and computational 258 efficiency, an irregular mesh can be defined with the same four-times coarsened resolu-259 tion refined near shorelines such that no cell overlaps land. This is shown in (c). This 260 can be done by recursively splitting the cells of the base (coarsened) mesh in four equal 261 parts until no cell overlaps land. The result is a mesh with 9,422 cells. A secondary ad-262 vantage of this technique is that modelling around shorelines at a higher resolution may 263 be of interest to port operators or local communities. For shipping and freight purposes 264 in Hudson Bay, there is a keen interest in knowing the state of the ice near shipping ports 265 since some operations might required ice free conditions. However, large areas of nav-266 igable waters do not require the same high degree of spatial resolution since vessels have 267 the possibility to slightly change their routes, thus a coarser resolution is sufficient. 268

To convert gridded data from a grid representation  $X \in \mathbb{R}^{W \times H \times C}$  for data with *C* channels and  $W \times H$  spatial dimensions to a mesh representation  $G \in \mathbb{R}^{C \times N}$  with *N* cells, we first construct a sparse mapping tensor  $M \in \mathbb{R}^{N \times W * H}$  where entry (n, p)is assigned 1 if the  $p^{\text{th}}$  pixel of the flattened grid  $Y \in \mathbb{R}^{C \times W * H}$  should be mapped to cell *n*. We also construct a tensor  $P \in \mathbb{R}^N$  which stores the number of pixels which are mapped to each cell. Then, to convert a sample from a grid to a mesh representation, for each node we find the mean value of each of its constituent pixels with

$$G = Y M^T \oslash P \tag{1}$$



Figure 3: Input images are represented as graphs by relating each neighbouring pixel with edges. In this figure, a spatially irregular mesh is used to represent SIC in Hudson Bay, where red dots represent graph nodes and black lines represent edges.

where  $\oslash$  represents an element-wise or Hadamard division. *G* can be converted back to a grid representation by splitting the cells back into its constituent pixels as

$$\tilde{Y} = GM.$$
 (2)

Since Equation 1 takes the mean of the constituent pixels of each cell, it cannot be perfectly reverted, instead Equation 2 simply assigns the cell value to each of its constituent
pixels. Formulating these transformations as matrix multiplications allows for greater
GPU acceleration which is important if the input meshes are re-meshed dynamically during training, although this is not done in this study.

A graph can then be defined based on this mesh by assigning a node to each cell and placing edges between any two neighboring cell as in Figure 3. To preserve spatial awareness, the positions of each node and size of each cell are added as node features, and the length and angle of the edges are stored as edge features. The edges are therefore considered to be directed edges as the edge features are direction-dependent, that is, for two nodes  $x_i$  and  $x_j$ , the edge from  $x_i$  to  $x_j$   $(e_{ij})$  is not equivalent to the edge from  $x_j$  to  $x_i$   $(e_{ji})$ 

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# 4.3 Model Architecture

The proposed model uses graph convolutional long-short term memory (GCLSTM) modules within a sequence-to-sequence architecture. The GCLSTM module and the overall architecture are shown in Figure 4, and described in the subsections below.

 $4.3.1 \ GCLSTM$ 

The graph convolutional long-short term memory (GCLSTM) module used in this 295 work is a modified version of the model from Seo et al. (2018), which is in turn inspired 296 by the ConvLSTM first proposed by X. Shi et al. (2015). The module closely resembles 297 the peephole LSTM introduced by (Gers et al., 2002), with the only modification being 298 the addition of graph convolution operators over the hidden and input states at each of 299 the input, forget, cell and output gates in the place of weight matrices. This is repre-300 sented as the  $*\mathcal{G}$  block in Figure 4b. The graph convolution operators allow information 301 exchange between nodes through the directed edges. The model proposed by Seo et al. 302 (2018) uses a single Chebyshev graph convolution (M. He et al., n.d.) which has limited 303 spatial expressivity since a single convolution can only exchange information between 304



(a) Overall model architecture. The last hidden  $(h_t)$  and cell  $(c_t)$  states of the encoder act as the context vectors and are used as the initial states of the decoder. The encoder learns features from the *n* input timesteps, and the last hidden  $(h_t)$  and cell  $(c_t)$  states are retained as the context vector used to initiate the decoder, which unrolls over the fixed *m* desired output timesteps. The initial input to the decoder  $X_t$  is the ice channel of the last input timestep. GNN<sub>enc</sub> and GNN<sub>out</sub>, used to encode climatology at each output timestep  $(n_{t-o})$  and reduce the dimensionality of the output  $(o_{t-o})$ , respectively, are stacked spatial convolutions with leaky ReLU activations.



(b) Graph convolutional long-short term memory (GCLSTM) module. The module is based on the peephole LSTM (Gers et al., 2002), with the addition of K stacked graph convolutions applied to both the hidden states and input.

Figure 4: Model architecture showing (a) overall encoder-decoder architecture, and (b) a single graph convolutional long-short term memory (GConvLSTM) cell.  $\bigoplus$  represents element-wise addition, and  $\bigotimes$  represents element-wise multiplication.

immediate neighbors. Since the processes dominating ice formation and break-up are phys-305 ical processes occurring across space, we wish to increase the model's ability to recog-306 nize spatial patterns, and therefore use K stacked convolutions followed by leaky ReLU 307 activations, which provides information exchange over K hops. The peephole variant of 308 the LSTM is used here as it has been shown to outperform the vanilla LSTM (Joshi et 309 al., 2022), particularly for video understanding (Srivastava et al., 2015). The convolu-310 tion operator taking the place of GraphConv in Figure 4b can be arbitrarily selected from 311 the myriad graph convolution operators that have been proposed. In this work, we eval-312 uate both the graph transformer convolution from Y. Shi et al. (2021), and the more ba-313 sic Graph Convolutional Network (GCN) first proposed by Kipf and Welling (Kipf & Welling, 314 2017). 315

In the graph transformer convolution, the feature vector of a given node  $i, x_i$ , is updated by aggregating information from its neighbors  $j \in \mathcal{N}(i)$ , and the node itself, using edge features from i to  $j, e_{ij}$ . The governing equation for the graph transformer convolution is

$$x'_{i} = W_{1}x_{i} + \sum_{j \in \mathcal{N}(i) \cup i} \alpha_{ij}(W_{2}x_{j} + W_{3}e_{ij})$$
(3)

where  $\mathcal{N}(i)$  denotes the neighbors of node *i*, *W* are weight matrices that project the inputs to their latent representation where the attention coefficients  $\alpha_{ij}$  are given by

$$\alpha_{ij} = softmax \left( \frac{(W_4 x_i)^T (W_4 x_j + W_3 e_{ij})}{\sqrt{d}} \right) \tag{4}$$

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The attention weights allow the model to selectively attend to a given node's neighbors based on their node and edge feature vectors. The inclusion of edge features and an edge specific weight matrix allows the model to learn to relate the edge features to better reflect anisotropic evolution of the model state.

We compare the transformer convolution with the Graph Convolutional Network (GCN) proposed by Kipf and Welling (2017), as it is a commonly used and simpler convolution operator. The GCN operator is defined by the equation

$$x_i' = W^T \sum_{j \in \mathcal{N}(i) \cup i} \frac{e_{ij}}{\sqrt{\hat{d}_j \hat{d}_i}} x_j \tag{5}$$

where X is a weight matrix,  $\hat{d}_i = 1 + \sum_{j \in \mathcal{N}(i)} e_{ij}$  and  $e_{ij}$  are the edge weights from ito j. Since  $e_{ij}$  must be a scalar, here we use the normalized distance between nodes as the edge weights. Note that this limits the spatial awareness of the model as it does not receive information about the nodes' relative positions, unlike the transformer convolution.

#### 335

#### 4.3.2 Sequence-to-Sequence Architecture

The GCLSTM module is used within a sequence-to-sequence encoder-decoder struc-336 ture to learn features from the inputs and evolve the sea ice state forward in time. The 337 overall architecture is shown in Figure 4a. Since navigation and offshore operations are 338 affected at various degree by the presence and concentration of sea ice, our model fore-339 casts both SIC and SIP as a multi-task learning approach. Although sea ice presence—defined 340 as any pixel where SIC is greater than 15%—can be derived from the forecasted SIC val-341 ues, a model trained without the secondary SIP forecasts would not be optimized for this 342 15% threshold. It was also found through experimentation that including SIP as a sec-343 ondary task improved SIC forecasts in the break-up and freeze-up seasons. 344

The encoder is responsible for learning rich spatiotemporal features from the in-345 put sequence while the decoder is responsible for evolving the state forward in time from 346 these learned features. The encoder therefore acts as an information bottleneck, mean-347 ing it is crucial that the encoder is sophisticated enough to distill the inputs into a con-348 text vector with sufficient information for the decoder to use in the unrolling process. 349 Given a sufficiently rich context vector, the decoder does not necessarily need to learn 350 additional spatial features within the context vector, nor during the unrolling process. 351 Therefore, in this work we use a spatiotemporal GCLSTM module in the encoder block, 352 and a simple LSTM in the decoder block. Although the decoder block also contains graph 353 convolutions (e.g., in  $GNN_{out}$ ), the distinction between the two is that the GCLSTM 354 in the encoder block integrates graph convolutions within the temporal model allowing 355 for simultaneous spatial and temporal modelling, while the decoder block models tem-356 poral and spatial dynamics separately, with  $GNN_{out}$  being used mainly for dimension-357 ality reduction. Using an LSTM rather than a GCLSTM module in the decoder block 358 also greatly reduces training time in the case where there are fewer input timesteps than 359 output timesteps. Note that experiments with a GCLSTM in the decoder were also run 360 but showed no improvements over using an LSTM. 361

The encoder processes each input timestep sequentially, updating the hidden and 362 cell states at each timestep with layer normalization (Ba et al., 2016) applied to the hid-363 den and cell states after each timestep to increase model stability. The final hidden and 364 cell states are the high-dimensional vectors that are taken as the context vectors that 365 contain the learned features from the input and are used to initialize the hidden and cell 366 state of the decoder. The last input ice state is used as the initial input to the decoder 367 (or start token) since we wish to evolve the state forward from this initial state. The de-368 coder is run recurrently for the desired number of output timesteps in a similar fashion 369 to the encoder but using the last step's prediction  $(y_{t-1})$  as the input for the current step 370  $(y_t).$ 371

Since sea ice is highly seasonal, the model is susceptible to a form of modal collapse 372 wherein the model converges to a local minimum, predicting only the average sea ice con-373 ditions for a particular day of the year. These daily averages are known as the climate 374 normals or climatology. For long-term forecasting of climatological variables, climatol-375 ogy can perform reasonably well compared to dynamic or statistical models due to strong 376 seasonality. Since we wish to outperform climatology and expect the model to learn to 377 use it as a heuristic, we choose to include it as an input such that model can focus on 378 learning departures from normal conditions. This was shown to be beneficial for sea ice 379 forecasting in a previous study (Asadi et al., 2022). Climate normals are calculated as 380 the mean ice concentration values for each day of the year over the entire training set 381 and are encoded into latent space using a shallow multi-layer GNN before being com-382 bined with the decoder output by element-wise addition. The result is then fed through 383 a multi-layer GNN with leaky ReLU activations to reduce the dimensionality to two, and 384 finally through a hyperbolic tangent activation to map the values between -1 and 1. This 385 output represents the change in sea ice conditions and is added to the last timestep's pre-386 diction. Since both SIC and SIP should be bound between 0 and 1, the output is passed 387 through a sigmoid layer that produces the final predictions. 388

389

390

# 4.4 Experimental set-up

# 4.4.1 Mesh Definition

To illustrate the advantage of using graph networks, experiments were designed to demonstrate the ability to produce forecasts over an irregular mesh. To this end, experiments were run on an irregular mesh as well as the coarsened regular mesh described in Section 4.2 and shown in Figure 2b and Figure 2c. The irregular mesh is refined to a higher resolution at the land edges by splitting the base 1/3° mesh if a cell intersects



Figure 5: Monthly sea ice concentration anomalies in Hudson Bay from 1993-2020. Highlights periods of higher and lower-than-average sea ice concentrations.

a one-cell buffer around land. This buffer is used since near-shore dynamics can be par-396 ticularly complex. By extending high-resolution meshing slightly beyond the immedi-397 ate land-water interface, the model may be better equipped to capture these complex 398 dynamics occurring in these more critical regions. The resulting irregular mesh contains 399  $1/12^{\circ}$ ,  $1/6^{\circ}$  and  $1/3^{\circ}$  sized cells. To show that the complexities introduced by this ir-400 regular mesh is not a detriment to the model, a separate experiment is conducted by train-401 ing the same model over the regular  $1/3^{\circ}$  mesh. This should be an easier task than the irregular mesh, therefore showing similar performance over either meshes is sufficient to 403 demonstrate that the model is resolution-agnostic. 404

#### 405

#### 4.4.2 Data Partitioning

The Hudson Bay region, including Hudson Strait, James Bay and Foxe Basin, un-406 dergoes a cyclical transformation in its ice cover characterized by complete freezing dur-407 ing the winter months and total melt in the summer, with some multi-year ice possible 408 in Foxe Basin. This seasonal cycle is subject to considerable inter-annual variability, both 409 in terms of the rate at which these processes occur and the timing of these transitions. 410 Figure 5 illustrates this variability by showing monthly SIC anomalies between 1993 and 411 2020. These anomalies are computed as the mean differences between observed SIC and 412 the long-term average concentration for each corresponding month. The data reveals dis-413 tinct periods of anomalous behavior in SIC. Specifically, the years 1993 to 1997 were marked 414 by higher-than-average SIC, indicating that during these years, Hudson Bay experienced 415 an earlier freeze-up and a delayed break-up season. In contrast, the period from 2010 416 to 2012 exhibited anomalously low SIC, characterized by a late onset of freeze-up and 417 an earlier melting season. Including data from both these anomalous periods along with 418 years that exhibit more typical ice conditions is critical for enhancing model robustness 419 in the face of varying environmental conditions. This is particularly important in the con-420 text of climate change, where shifts in temperature and weather patterns could further 421 exacerbate the variability in sea ice conditions. The data is therefore partitioned into 422 a sequential 20-year, 3-year, 3-year split, wherein data from 1993-2013 is used for train-423 ing, 2013-2016 is used for validation, and 2016-2019 is used for testing. Note however 424 that the test period only includes years with normal or lower-than-usual ice conditions. 425 Although this bias may not be optimal, lower-than-normal ice conditions may be more 426 representative of future ice conditions in the Hudson region (J. Stroeve & Notz, 2018) 427

One model is trained for each month of the year, each denoted as a 'monthly model'.
Each monthly model was trained using data from the respective month with a 15-day
buffer before and after the beginning and end of the month respectively. For example,
the April model is trained with input data for each day between March 16 and May 15

over all training years. A longer buffer of one month was tested but did not lead to sig-432 nificant improvements in model performance. In inference mode, each model is used only 433 to produce a forecast with inputs from its respective month. For example, to generate 434 90 day forecasts for April, a 90 day forecast is launched for each day between April 1 and 435 April 30. Training separate model for each month of the year was done since we expect 436 the dynamics that must be learned for one time of the year to be sufficiently different 437 from other times of the year such that each model will have greater accuracy by concen-438 trating efforts in learning specific ice dynamics (Asadi et al., 2022). As a secondary ben-439 efit, this also allows training to be carried out more efficiently as each monthly model 440 can be trained in parallel. 441

442 4.4.3 Input Features

Sea ice concentration data from GLORYS12 serve as the target variable, while at-443 mospheric variables from ERA5, combined with oceanographic variables from GLORYS12, 444 are used as input features. Sea ice dynamics are primarily influenced by factors such as 445 air and sea temperature (Wang et al., 2019), wind (Stammerjohn et al., 2003), heat fluxes 446 (Ivanov et al., 2012), and ocean salinity (Yao et al., 2000), thus we include these vari-447 ables as input features. The 10 chosen input variables are listed in Table 1, along with 448 the rationale for their selection. It should be noted that ERA5 hourly variables are re-449 gridded from their original  $0.25^{\circ}$  grid to match the GLORYS12  $1/12^{\circ}$  grid, and resam-450 pled to match the GLORYS12 daily temporal resolution. This is achieved through spa-451 tial linear interpolation and aggregation from an hourly to a daily resolution using a sim-452 ple mean. The input sequence length is 10 days and the spatial domain as a grid is  $229 \times$ 453 361. Since the model operates over the mesh domain rather than the grid domain, the 454 dimensionality of the inputs to the encoder as (input steps, number of nodes, input fea-455 tures) is  $10 \times 9,422 \times 10$  for the irregular mesh and  $10 \times 2,425 \times 10$  for the regular 456 mesh. The input to the decoder is the context vectors provided by the encoder as well 457 as the climatology for each forecast day. The output dimensionality is  $90 \times 9,422 \times 2$ 458 for the irregular mesh, and  $90 \times 2,425 \times 2$  for the regular mesh. 459

4.4.4 Baseline Model

As a baseline model with which to compare the model, we use a combination of two 461 common statistical baselines: persistence and climatology. Persistence refers to persisting the most recent sea ice conditions and tends to perform well at very short forecast 463 lengths particularly outside of the freezing and melting seasons. Climatology refers to 464 the pixel-wise average SIC for each day of the year where the average is taken over the 465 historical period of interest. Climatology tends to perform best relative to forecast mod-466 els at longer lead times. For forecasts produced over a seasonal scale, a stronger base-467 line than either persistence and climatology can be derived by combining the two using 468 a weighted average with the relative weights varying by lead time, where more weight 469 is given to persistence than climatology at short lead times and more weight is given to 470 climatology than persistence at long lead times. The form chosen for the baseline model 471 is 472

$$F = (1 - \gamma)P + \gamma C, \tag{6}$$

473 where

460

$$\gamma(t) = \gamma_0 \times e^{-\lambda t}.\tag{7}$$

 $\gamma_0$  is set to 1 since we know persistence to be a strong predictor at short lead times, and  $\lambda_{75}$   $\lambda$  is optimized by minimizing the mean squared error over the training dataset for each month. The resulting weights are shown as a heatmap in Figure 6.

Short	Full Name	Source	Rationale for Inclusion
sic	Sea ice concentration	GLORYS12	Direct measure of what is being forecasted; crucial for temporal dynamics and initial conditions.
sit	Sea ice thickness	GLORYS12	Provides insights into the resiliency and robustness of the ice, affecting its likelihood to melt or deform.
siuv	Sea ice velocities	GLORYS12	Indicates the direction and speed of sea ice movement.
SO	Sea water salinity	GLORYS12	Salinity affects the freezing point of sea water and is crucial in the dy- namics of ice formation and melt.
sst	Sea surface temperature	GLORYS12	The temperature of surrounding sea water directly affects ice melt and formation rates.
t2m	2-meter temperature	ERA5	Air temperature can provide ad- ditional context for the thermal conditions affecting the sea ice sur- face.
u10/v10	10-meter wind velocity	ERA5	Influences the motion and deforma- tion of sea ice.
sshf	Surface sensible heat flux	ERA5	Surface sensible heat flux is an indicator of the heat exchange between the atmosphere and the sea surface, affecting ice melt and formation.
х	x-position of each node		Provides the latitudinal spatial context for each data point.
У	y-position of each node		Provides the longitudinal spatial context for each data point.
doy	Day of the year		Provides temporal context.
csize	Cell size		Provides the relative size of the area covered by each cell for addi- tional spatial context.

Table 1: Selected input variables to the encoder, data source and rationale for inclusion.

477

#### 4.4.5 Model Hyperparameter Configurations and Implementation

This study evaluates three distinct models, listed in Table 2. Our primary focus 478 is the GraphSIFNet-Att model, which incorporates three TransformerConv spatial con-479 volutions in the GCLSTM block and is trained on the irregular mesh described in Sec-480 tion 4.2 for 35 epochs. That is, in Figure 4b,  $*\mathcal{G}$  uses the TransformerConv as the Graph-481 Conv block with K = 3. For comparison, we examine the GraphSIFNet-Att-Reg model 482 which is identical in architecture but trained on the coarsened regular mesh from Sec-483 tion 4.2 for 35 epochs. Additionally, we compare with the GraphSIFNet-GCN model, 484 which employs six GCN convolutions within the GCLSTM module, that is, the Graph-485 Conv block is the GCN with K = 6. GraphSIFNet-GCN is trained over the irregular 486 mesh for 45 epochs. Each of these models have the same number of parameters (approx-487 imately 123,000). 488



Figure 6: Gamma values for the baseline model (Equation 6) showing the balance between persistence and climatology by the month of the launch date and lead time. Gamma values near 0 favor persistence while values near 1 favor climatology. Less variable ice seasons such as January/February and August/September rely more on persistence for longer lead times.

Each model uses a 10-day input sequence to predict the subsequent 90 days. A hid-489 den dimension size of 32 is used for each of the hidden state and cell state of the encoder 490 and decoder LSTMs, as well as in all graph convolutional layers. The GNN used to en-491 code climatology (GNN<sub>enc</sub>) is comprised of a single graph convolution layer, and the out-492 put GNN (GNN<sub>out</sub>) is comprised of 3 stacked convolution layers with leaky ReLU ac-493 tivations. The hidden size, number of spatial convolutions and number of GCLSTM/LSTM 494 layers were chosen based on small-scale experiments which aimed to keep the model sim-495 ple yet effective. The optimizer is the Adam optimizer with an initial learning rate of 496 0.001 reducing by 10% every 5 epochs. An L2 regularization value of 0.01 is applied to 497 the weights reduce the risk of overfitting, and gradient clipping with a value of 1.0 is ap-498 plied to mitigate the risk of gradient explosion due to the extended forecast length. Early 499 stopping was used if no improvement in the validation loss was observed for 10 epochs. 500 Since the model produces two outputs, a custom loss function was used that combines 501 a mean square error (MSE) loss from the continuous SIC prediction and binary cross-502 entropy (BCE) loss from the probabilistic SIP prediction. The BCE loss is scaled by a 503 factor of 0.1 and added to the MSE loss before back-propagation. Since losses are cal-504 culated over a mesh with cells of varying physical sizes, the losses are also scaled by the 505 size of each cell. This prevents the model from over-valuing correct predictions in areas 506 of higher spatial resolution. The models are implemented in Pytorch using the pytorch-507 geometric (Fey & Lenssen, 2019) package and trained on a single Tesla V100 GPU hosted 508 by the Digital Research Alliance of Canada. A summary of models tested and training 509 times is given in Table 2. 510

# 511 5 Results

In this section, the GraphSIFNet-Att model is evaluated by comparing its performance with the statistical baseline and contrasting with the two other configurations: GraphSIFNet-Att-Reg and GraphSIFNet-GCN. Using GraphSIFNet-Att, insights from the attention weights, the results of a variable importance experiment, and an evaluation of its ability to predict break-up and freeze-up dates are also presented. Table 2: Summary of developed model configurations. The models differ in their spatial convolutions and their underlying meshes, with the aim of contrasting the attention-based transformer convolution with the graph convolutional network, as well as demonstrating the model's ability to model over an irregular mesh.

Name	Convolution (# stacked layers)	Mesh	Approximate training time
GraphSIFNet-Att	TransformerConv (3)	Irregular (1/12° - 1/3°)	10h (30 epochs)
GraphSIFNet-Att-Reg	TransformerConv (3)	Regular $(1/3^{\circ})$	8h (30 epochs)
GraphSIFNet-GCN	GCN(6)	Irregular $(1/12^{\circ} - 1/3^{\circ})$	10h~(45  epochs)
Baseline	N/A	N/A	N/A

#### 5.1 Baseline Performance

517

The performance of the baseline statistical model defined by Equation 6 for both 518 the SIC and SIP forecasting task is shown in Figure 7a and Figure 7b, respectively. These 519 heatmaps are generated by calculating the spatial average of the root mean squared er-520 ror (RMSE) over the domain using only the test years (2016-2019). The errors are grouped 521 by the month of the launch dates and lead times. For instance, the value in the top right 522 corner of the error heatmaps (January, 90-day lead time) indicates the mean RMSE for 523 all 90-day forecasts launched in January, that is, forecasts for dates spanning April 1st 524 to May 1st. The two clearly visible bands of higher RMSE values correspond to the break-525 up and freeze-up seasons, the former normally spanning from the beginning of May to 526 mid-July and the latter normally spanning from the beginning of November to the end 527 of December. These seasons are the most difficult to forecast as the timing and pattern 528 of the break-up and freeze-up vary between years. Conversely, August to beginning of 529 October are largely ice-free, thus the errors are near zero. In the winter months, that 530 is, mid-December to the beginning of April, ice is present throughout the Hudson Bay 531 system though some open water can sporadically be found around shorelines, for exam-532 ple due to offshore winds, thus SIC RMSE values during the winter months are small 533 but not zero. 534



Figure 7: Performance of the baseline statistical model on SIC (a) and SIP (b) over the test years aggregated by the month of the launch date and lead time.

#### 5.2 GraphSIFNet-Att Performance



Figure 8: RMSE heatmaps for the SIC forecasting task by month and lead time for the GraphSIFNet-Att model (a), and the RMSE differences between GraphSIFNet-Att and the baseline (b) where negative values (blue) indicate a reduction in model error relative to the baseline.



Figure 9: Accuracy heatmaps for the SIP forecasting task by month and lead time for the GraphSIFNet-Att model (a), and the difference between GraphSIFNet-Att and the baseline (b) where positive values (red) in the difference plots indicate an increase in model accuracy relative to the baseline.

The performance of GraphSIFNet-Att model and the difference in performance relative to the baseline model is shown in Figure 8 and Figure 9 for SIC and SIP forecasts, respectively. Since persistence and climatology are usually used as baselines seperately, the difference in performance relative to both are shown in Section Appendix A. Models are evaluated against GLORYS12 SIC and SIP on the full-resolution 1/12° GLORYS12 grid.

For the majority of the months and lead times, the GraphSIFNet-Att model exhibits improvements in SIC forecasts over the baseline, with minor exceptions. The model

Task	Model	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Avg.
SIC	GraphSIFNet-GCN	0.29	0.19	0.02	0.11	0.67	-0.33	0.18	0.22	0.00	0.11	-0.37	0.40	0.12
	GraphSIFNet-Att-Reg	0.43	0.12	-0.16	0.30	-0.19	-0.71	0.22	0.03	-0.08	1.19	-0.79	0.51	0.07
SIP	GraphSIFNet-GCN	0.00	-0.01	0.05	0.14	-0.54	0.08	-0.06	-0.07	0.37	0.13	-0.38	-0.03	-0.03
	GraphSIFNet-Att-Reg	0.00	-0.03	80.0	-0.22	-0.10	0.25	-0.10	0.02	0.18	-0.21	0.83	-0.15	0.05

Figure 10: Difference in monthly SIC RMSE [%SIC] and SIP [%]accuracy between GraphSIFNet-Att-Reg and GraphSIFNet-GCN relative to GraphSIFNet-Att averaged over all 90 forecast days. Negative RMSE differences and positive accuracy differences indicate better performance on the part of GraphSIFNet-Att relative to the other models.

exhibits the largest improvements over the baseline in its short- to medium-term fore-544 casts of the break-up season (lead times 5 to 45 launched in May to July). These show 545 up to a 10% improvement over the baseline. At longer timesteps, the improvements over 546 the baseline during the break-up period (launched in March and April) are less pronounced, 547 hovering around 2-3%. However at these long lead times even small improvements demon-548 strate forecast skill and can provide value to users of the system. During the winter months 549 when the region is almost entirely frozen, the model still exhibits a 2-3% improvement 550 over the baseline at all lead times. This suggests that the model may be able to better 551 capture the effects of off-shore winds mechanically creating open water regions along the 552 shoreline. During freeze-up, the model only shows skill over the baseline at short lead 553 times from 0 to 25 days. Longer forecasts beyond 25 days perform on par with the base-554 line or only marginally better. Forecasts launched in November with a 30 to 55 day lead 555 time perform worse than the baseline, indicating difficulty in capturing the final stages 556 of ice formation. 557

The SIP accuracy heatmaps in Figure 9 show similar patterns, with increases in accuracy of up to 20% from the GraphSIFNet-Att model over the baseline during the break-up process, and more modest increases during the freeze-up process. Notably, however, GraphSIFNet-Att outperforms quite significantly (> 10%) even at long lead times. This indicates that although the model may struggle to forecast the precise SIC at these lead times, it still has skill in forecasting the point at which the ice will completely melt or break up.

565

# 5.3 Comparison Between Model Configurations

Differences in both SIC RMSE and SIP accuracy between the GraphSIFNet model 566 configurations, averaged for all timesteps for each month, are shown in Figure 10. GraphSIFNet-567 GCN and GraphSIFNet-Att-Reg demonstrate comparable performance relative to GraphSIFNet-568 Att, with differences being largely insignificant when aggregated across the entire region. 569 To better understand the differences in their capabilities, spatial monthly SIC RMSE 570 maps for the 15-, 30-, and 60-day lead times for forecasts launched in May and Novem-571 ber are presented in Figure 11. These correspond to parts of the break-up and freeze-572 up periods, respectively. Panels a) and c) show the impact of the convolution operator, 573 while panels b) and d) show the impact of the mesh resolution. 574

Early (15-day) forecasts in the Northwest region of Hudson Bay, launched in May, are best captured by GraphSIFNet-Att-Reg. This region is characterized by a latent heat polynya, suggesting that the coarser uniform resolution mesh may aid the model in forecasting the formation and behavior of the polynya. Using a finer resolution mesh in this region might cause the model to overemphasize local variations in sea ice concentration and thickness, potentially obscuring the broader spatial patterns crucial for accurate polynya forecasting. Both GraphSIFNet-GCN and GraphSIFNet-Att-Reg outperform GraphSIFNetAtt in the 15- and 30-day forecasts launched in November in Hudson Strait. The freezeup in Hudson Strait is characterized by rapid changes in ice formation and movement influenced by strong ocean currents. These conditions create a highly dynamic and challenging environment for sea ice prediction. Since all three models exhibit similar performance, the additional interpretability granted by the attention weights in GraphSIFNet-Att motivates the use of GraphSIFNet-Att over the others.

#### 5.4 Attention Maps

588

The use of transformer convolution in the model enhances its interpretability. By 589 examining the attention weights in the encoder's first layer of graph convolutions, insights 590 can be gleaned into how the model encodes the input data. According to Equation 3 and 591 Equation 4, each node is assigned attention weights for its neighboring nodes based on 592 learned weight matrices in each transformer layer. The softmax function ensures that 593 the sum of all attention weights for a given node's neighbors equals 1. Consequently, the 594 node is updated using a weighted average of its neighbors' features, which are projected 595 into a latent space. Due to the large number of edges, visualizing these weights on a sim-596 ple map is challenging. A simpler approach for visualization involves calculating the pri-597 mary direction from which each node is updated. This can be done by summing the at-598 tention weights as vectors ( $\alpha$  values in Equation 3 with the direction of their respective 599 edges) for each node. These can be represented by arrows, the magnitude of which is pro-600 portional to the difference in weights. For example, a node with evenly distributed at-601 tention weights among eight neighbors would be represented as a single dot, whereas a 602 node with a dominant westward neighbor would have a large arrow pointing westward. 603 These arrows can be interpreted as indicating the direction of information flow through 604 the graph as the model processes the input maps. 605

Figure 12 provides examples of attention weights of the input gate for a single in-606 put image during both freeze-up (Figure 13a) and melting (Figure 13b) seasons. Although 607 the attention mechanism is applied to the hidden and input tensors at each of the LSTM 608 gates, it is most informative to visualize the weights that are applied to the inputs since 609 the inputs are physically interpretable. Note that attention weights at land interfaces 610 are omitted for visual clarity, as they are numerous and the lack of nodes on land means 611 the dominant direction is always away from the shore. In the freeze-up condition, the 612 model directs information flow generally from the southeast to the northwest. This sug-613 gests that the model learns the importance of understanding the sea ice and atmospheric 614 conditions of nodes to the northwest, aligning with the direction of freezing. It it is log-615 ical that a node that contains water should know the condition of its 3-hop neighbor to 616 the northwest, as if this neighbor is frozen, it is likely that this node will freeze in the 617 near future. Conversely, during the melting season, arrows point towards open water, 618 indicating that nodes with icy conditions but with water-containing neighbors should 619 consider these neighbors important as they indicate the node is likely to melt soon. No-620 tably, the magnitude of the arrows is larger at the ice edge and nearly zero in the con-621 solidated ice region, which could reflect the localized nature of the break-up process com-622 pared to the more gradual freeze-up. That is, the break-up process is largely confined 623 to the ice edge, while freeze-up gradually occurs across the region, as seen by changes 624 in sea ice concentration. Nodes in the open water region during the melting season are 625 less likely to change and, therefore, do not require attention to specific neighbors. Note 626 that although the weights are visualized on the sea ice concentration inputs, they ap-627 ply indiscriminately to all input features. Interestingly, the model appears to prioritize 628 sea ice thickness over concentration, evidenced by the larger attention weights where thick-629 630 ness drops more dramatically than concentration in Figure 13b. This is logical given the importance of thickness in determining the rate at which the ice will melt or break up. 631 Additionally, the attention weights in the open-water region during the freeze-up con-632 dition appear to be influenced by surface sensible heat flux, suggesting its significance 633 as an input feature. 634







(b) May —  $\Delta$ (GraphSIFNet-Att, GraphSIFNet-Att-Reg)



(c) November —  $\Delta$ (GraphSIFNet-Att, GraphSIFNet-GCN)



(d) November —  $\Delta$ (GraphSIFNet-Att, GraphSIFNet-Att-Reg)

Figure 11: Comparison of SIC RMSE for GraphSIFNet-Att, GraphSIFNet-Att-Reg, and GraphSIFNet-GCN models at 15-, 30-, and 60-day forecast lead times, initiated in May and November. The figure shows the difference in RMSE between GraphSIFNet-Att and both GraphSIFNet-Att-Reg and GraphSIFNet-GCN. Negative values indicate a reduction in error in the GraphSIFNet-Att relative to the other indicated model.



(a) December 1, 2014

(b) July 1, 2014

Figure 12: Visualization of attention weights of the input gate applied to the input tensors during the freeze-up (a) and melting (b) seasons overlaid on the sea ice concentration input. Arrows indicate the primary direction and magnitude of information flow based on the learned attention weights. Attention weights at the land interfaces are omitted for clarity. The attention weights appear to be largely influenced by sea ice concentration, but other input variables also influence the weights, for example surface sensible heat flux in (a), and sea ice thickness in (b).

#### 5.5 Variable Importance

635

The models are trained with a number of input variables (refer to Table 1), which 636 we anticipated the model might utilize to make its predictions. However, these variables 637 may not contribute equally to the resulting predictions. In this section, we explore the 638 significance of each feature by feature ablation through omission (Fong & Vedaldi, 2017). 639 Specifically, we produce forecasts using the trained GraphSIFNet-Att model by substi-640 tuting each input variable, one at a time, with white Gaussian noise generated using the 641 mean and standard deviation of the real inputs. Figure 13 shows the resulting difference 642 in RMSE when re-generating predictions on the test years using the June and Decem-643 ber models when each variable is replaced with noise. 644

During the break-up process (June model), the model largely relies on the input sea ice concentration and sea ice thickness to make its predictions, but also considers the ice velocities, sea surface temperature and sea salinity to a smaller degree. Other variables do not significantly affect the resulting predictions. The model appears to use sea ice concentrations to inform near-term forecasts (days 0 through 20), and sea ice thickness to inform its medium-term forecasts (days 0 through 35). This makes intuitive sense as thickness is an indicator of the ice cover's longevity making it relevant at longer fore-



Figure 13: Feature ablation with noise injection for the June and November GraphSIFNet-Att models. Positive values indicate an increase in RMSE when each respective variable is replaced with noise.

cast steps, while sea ice concentration is more important for immediate predictions since
lower ice concentrations are normally associated with ice parcels that are already breaking up. Note that at forecast steps larger than 35 days, forecasts launched in June are
largely forecasting periods where Hudson Bay is fully open water, thus none of the input features contribute to the resulting forecasts.

Similarly, during the freeze-up process, the model relies on sea ice thickness, sea 657 ice concentration, sea ice velocity and sea surface temperature to make its predictions. 658 Again, the model largely considers sea ice concentration to make its shorter term fore-659 casts (days 10 through 25), while considering ice velocity and thickness for medium-term 660 forecasts (days 15 through 40). Ice velocity may be indicating areas where ice migrates, 661 thereby creating space for new ice formation. The difference between the vertical and 662 horizontal ice velocity component (usi and vsi) may indicate that they offer redundant 663 information, thus it is sufficient for the model to consider one of the components. Again, 664 November forecasts at larger than 40 days are largely forecasting periods of full ice cover, 665 therefore omitting input features does not impact the scores. It is also worth noting that 666 in both cases, the model does not appear to consider the variables originating from ERA5. 667 This could point to a mismatch between ERA5 and GLORYS12, which would be unsur-668 prising as GLORYS12 uses ERA-Interim as model forcing at the surface. Since the tar-669 get variables are derived from GLORYS12, the models therefore prioritize input features 670 originating from GLORYS12. 671

To illustrate the impact of these variables on the resulting predictions, a sample 672 GraphSIFNet-Att forecast is shown in Figure 14, along with the same forecast when re-673 placing sea ice concentration and sea ice thickness (SIT) with noise as described above. 674 Replacing either SIC or SIT with noise does not significantly affect the 1-day forecast, 675 suggesting the model uses persistence as a heuristic at very short lead times. Beyond the 676 10-day forecast, predictions are affected by the noise injections, with the model forecast-677 ing a quicker melt when sea ice thickness is replaced with noise, consistent with the the-678 ory that thickness is used as a signal of ice longevity. When SIC is replaced with noise, 679 the model persists more of the ice in the 20-day forecast, suggesting that SIC is also im-680 portant for ice integrity. 681

Although this technique offers some insight into feature importance, it should be noted that since the models are not re-trained, the observed changes in performance due to feature omission may not perfectly reflect the true importance of each feature. This is because the model has been optimized to make predictions based on the full set of features, therefore the omission of any one feature changes the input space in a way that the model was not specifically trained to handle. Moreover, the interdependencies be-



Figure 14: Sample 1-, 10-, 20-, and 30-day forecasts from GraphSIFNet-Att launched on June 15, 2014. The climatology for each forecast day is shown for reference, and the results of running inference after replacing sea ice concentration (SIC) and sea ice thickness (SIT) with noise is shown.

tween features are not accounted for in this single-feature ablation approach. Variables in the dataset may interact in complex, non-linear ways that are not captured by examining each variable in isolation. Despite these limitations, this feature ablation technique provides useful insights into the relative importance of the different input features used in these particular trained models (Fong & Vedaldi, 2017). Since we know which features the models are using, we know which input variables should be more closely monitored.

694

# 5.6 Estimating Break-up and Freeze-up Dates

A potential use-case for sea ice prediction in Hudson Bay is the estimation of break-695 up and freeze-up dates in key locations, as these dates have significant implications for 696 maritime navigation and local communities. We evaluate the GraphSIFNet-Att model's 697 performance in estimating the freeze-up date at three key ports in Hudson Bay: the ports 698 of Churchill, Quaqtaq and Inukjuak. The port of Churchill is mostly used to export grain 699 while the ports of Quaqtaq and Inukjuak are regularly used for community resupply. These 700 three ports were chosen as their locations are representative of the varying sea ice con-701 ditions found in the Hudson Bay region. In this study, the validation and test year (2014 702 to 2019) serve as the period for assessing the predicted break-up and freeze-up dates. These 703 dates are determined using the same criteria as the previous study, which follows the def-704 inition given by the Canadian Ice Service (CIS). That is, the freeze-up date at a given 705 site is defined as the initial day when open water persists for 15 consecutive days, with 706 open water being defined as a SIC of less than 15%. Conversely, the break-up date is de-707 fined as first day at which SIC exceeds 15% for 15 consecutive days. The 30-day and 60-708 day predicted break-up and freeze-up dates are determined using the same criteria, but 709 with open water and ice conditions being defined as a sea ice presence probability less 710 than and greater than 50%, respectively. For each port, we take the mean pixel value 711 of a  $3 \times 3$  window around the nearest pixel to the port locations. 712

Figure 15 displays the predicted dates of freeze-up/breakup at the three ports with 713 30 and 60 days of lead time compared to the actual observed dates for the validation and 714 test years along with the mean absolute error. Predicted dates falling within 7 days of 715 the observed dates are considered correct, visualized by the pink shaded area. This def-716 inition of a correct forecast is in line with a previous study (Asadi et al., 2022). The 30-717 day forecasted break-up and freeze-up dates for Churchill are noticeably inferior to the 718 other two ports, likely due to challenges presented by the latent heat polynya in the North-719 west of Hudson Bay. The uniform forecasts of freeze-up dates at Churchill can be inter-720 preted as an admission that the model does not have skill here and resorts to forecast-721 ing the same mean day every year. Break-up predictions at Inukjuak also pose a chal-722 lenge for the model, likely due to freshwater inflows from the James Bay area affecting 723 the timing and rate of melt. Quaqtaq sees the most successful predictions, with all freeze-724 up dates falling within 7 days of the observed date. 725

In Figure 16, the break-up and freeze-up accuracies are shown spatially for the en-726 tire region. These accuracies are calculated as the proportion of years with predicted break-727 up or freeze-up dates within 7 days of the observed date. These are compared to pre-728 dictions made using the climate normals. The model performs equally or better than cli-729 matology for most of the region in predicting break-up dates at both 30-days and 60-730 days of lead time. However, there is a strong pattern in the freeze-up maps where the 731 model performs worse than climatology in the western half of the bay but still outper-732 forms climatology in the eastern half and in Hudson Strait. This is unsurprising as Hud-733 son Bay begins its freeze-up process in the northwest corner of the bay, thus the onset 734 of that initial freezing is difficult to predict. Once the bay has begun freezing over, the 735 model can better predict the timing of the rest of the bay. Although we might expect 736 the model to use atmospheric conditions such as temperature to predict the onset of freeze-737 up, the model only has access those atmospheric conditions 30 or 60 days prior to the 738



(b) Freeze-up date estimates

Figure 15: Break-up and freeze-up dates predicted by GraphSIFNet-Att at Churchill, Inukjuak, and Quaqtaq ports for lead times of 30 and 60 days for the years 2014 to 2019 compared to the observed dates from GLORYS12. The pink shaded area represents a 7-day buffer around a perfect forecast. Samples which fall within this buffer are deemed correct forecasts. The annotated numbers in parentheses are the error for each year.

forecast date. There may not be a strong enough signal in those initial conditions to allow the model to accurately predict how quickly the temperatures will drop.

# 741 6 Conclusion

The study presented in this paper demonstrated the effectiveness of using a GNN-742 based spatiotemporal forecasting model for predicting daily sea ice concentration and 743 sea ice presence in Hudson Bay over a 90-day time horizon. To demonstrate the ability 744 of GNNs to handle spatially irregular meshes, models were trained on both a uniform 745 regular mesh and an irregular mesh with higher resolution near shorelines. The proposed 746 model uses an attention-based transformer spatial convolution to learn spatial features 747 from the input, which was shown to have similar performance compared to the more ba-748 sic graph convolutional network. The attention-based convolution however has the ad-749 ditional benefit of increasing the model's interpretability, motivating its use. 750

Results from this study highlighted the model's skill in predicting sea ice dynam-751 ics, with particular success noted in short- to medium-term forecasts during the break-752 up season when compared to a linear combination of persistence and climatology as a 753 statistical baseline. The model performed as well or better on the irregular mesh as on 754 the regular mesh, with the exception of some difficulty capturing the initial freeze-up in 755 the Northwest region of Hudson Bay as well as the polynya formation at longer lead times. 756 This suggests that improvements could be made in refining the model's sensitivity to com-757 plex spatial features associated with irregular meshes, particularly in areas where ice dy-758 namics are highly variable. This could involve more sophisticated positional and spatial 759 encoding, perhaps by projecting the positional, cell size, distance and angle encodings 760 into higher dimensional latent space. The model showed similar overall performance be-761 tween the model using the transformer convolution and the GCN within the GCLSTM 762 module, with some differences in performance in certain regions such as Hudson Strait. 763 This suggested potential overfitting in the model using the spatial transformer convo-764 lution. 765

The attention mechanism within the transformer convolution offered interpretabil-766 ity by highlighting the primary direction and magnitude of information flow in the en-767 coder, which aligned with known physical processes such as the direction of freezing and 768 melting. A feature ablation experiment indicated the trained model's reliance on sea ice 769 concentration, thickness and velocities to inform its predictions. Other variables did not 770 contribute significantly to the resulting forecasts, which could explain the model's poor 771 performance in forecasting the Kivalliq latent heat polynya. A evaluation of the model's 772 ability to predict freeze-up and break-up dates was conducted, revealing the model's lim-773 ited ability to forecast the onset of freeze-up in Hudson Bay, as well as the onset of break-774 up in the Northwest region which is influenced by the polynya. The model however still 775 showed skill over the statistical baseline in these tasks. 776

Several potential avenues for future work exist. In a GNN, each node is processed 777 as a separate sample by the network. This has two major implications. First, one input 778 image  $X \in \mathbb{R}^{W \times \hat{H} \times C}$  does not necessarily need to be processed fully at once, instead, 779 nodes could also be sampled in batches sequentially until the full sample has been pro-780 cessed. This would be helpful in the case where the region is large and modelling it in 781 its entirety would be infeasible due to memory constraints. Second, since each node has 782 its own hidden and cell states, cells can be combined by averaging the states or split by 783 duplicating the states. This means that the underlying mesh could be dynamic in time, 784 evolving as the underlying data changes (e.g. as the ice conditions evolves). For exam-785 ple, one could define a dynamic mesh which has a higher resolution at the ice edge where 786 the ice conditions are known to be more dynamic. As the ice conditions evolve, so too 787 would the underlying mesh. The advantages are two-fold. First, it allows for a reduc-788 tion in data volume with minimal information loss, contrary to the static mesh used in 789



(a) 30-day break-up date estimate map



(b) 60-day break-up date estimate map



(c) 30-day freeze-up date estimate map



(d) 60-day freeze-up date estimate map

Figure 16: Break-up and freeze-up date estimate maps from the climatological baseline (a), GraphSIFNet-Att model predictions (b), and the difference between the two (c). Positive values in the difference plots indicate an increase in accuracy from the model relative to the baseline, where accuracy is defined as the proportion of predictions falling within 7 days of the observed date for the years 2014 to 2019.

this work which has information loss where the data has high spatial variance. Second, 790 the dynamic mesh could help the model learn more sophisticated dynamics and is more 791 consistent with physical simulation software. This idea was explored in (Pfaff et al., 2020). 792 Another avenue for future work could be a deeper investigation of the adjacency matrix. 793 In this study, edges were placed between any two directly spatially adjacent cells. How-794 ever, edges could also be placed between distant cells thereby widening the receptive field 795 without adding convolutions. This could be investigated by transforming the adjacency 796 matrix into a learnable matrix optimized during training. Furthermore, node sampling 797 strategies could also be used to reduce training time. Specifically, adaptive sampling tech-798 niques could be employed where nodes in dynamic regions, such as the ice edges known 799 for their fluctuating conditions, are sampled with higher frequency compared to the more 800 static areas. Incorporating long-term weather forecasts from third party sources such as 801 the Canadian Global Ice Ocean Prediction System (GIOPS) could also be beneficial, par-802 ticularly in forecasting freeze-up. Lastly, multi-resolution modelling either through an 803 ensemble of models operating over meshes of different resolution or using multiple meshes 804 of varying resolutions within a single model could be explored. This may help the model 805 better capture both large-scale and small-scale phenomena. 806

# <sup>807</sup> Appendix A Additional RMSE Heatmaps



Figure A1: RMSE heatmaps for the SIC forecasting task by month and lead time for the GraphSIFNet-Att model (a), and the RMSE differences between GraphSIFNet-Att and persistence (b) and climatology (c) where negative values (blue) indicate a reduction in model error relative to the baseline.

# <sup>808</sup> Data Availability Statement

ERA5 atmospheric reanalysis data (Hersbach et al., 2020) are available at https:// doi.org/10.24381/cds.adbb2d47, and GLORYS12 ocean reanalysis data (Jean-Michel et al., 2021) are available at https://doi.org/10.48670/moi-00021.

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Figure A2: RMSE heatmaps for the SIC forecasting task by month and lead time for the GraphSIFNet-Att-Reg model (a), and the RMSE differences between GraphSIFNet-Att-Reg and persistence (b) and climatology (c) where negative values (blue) indicate a reduction in model error relative to the baseline.



Figure A3: RMSE heatmaps for the SIC forecasting task by month and lead time for the GraphSIFNet-Att-Reg model (a), and the RMSE differences between GraphSIFNet-GCN and persistence (b) and climatology (c) where negative values (blue) indicate a reduction in model error relative to the baseline.

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