On the Energy Coupling from Magnetosonic Waves to High-Frequency Electromagnetic Ion Cyclotron Waves: Statistical Analysis

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December 27, 2023

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Key Points:

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9	• Energy coupling from MS to high-frequency EMIC waves through low-energy pro-
10	ton heating is investigated using correlation analysis
11	• High-frequency EMIC wave occurrence correlates well with large anisotropy of 10–
12	100 eV protons, required for the wave generation
13	• The correlation between low-energy protons and MS waves is rather poor, call-
14	ing for alternative explanation for the origin of these protons

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15 Abstract

In the inner magnetosphere, fast magnetosonic waves (MS waves) are known to resonantly 16 interact with ring current protons, causing these protons to gain energy preferentially 17 in the direction perpendicular to the background magnetic field. An anisotropic distri-18 bution of enhanced ring current protons is a necessary condition to excite electromag-19 netic ion cyclotron (EMIC) waves which are known to facilitate a rapid depletion of ultra-20 relativistic electrons in the outer radiation belt. So, when a simultaneous observation 21 of high-frequency EMIC (HFEMIC) waves, anisotropic low-energy protons, and MS waves 22 was first reported, a chain of energy flow from MS waves to HFEMIC waves through pro-23 ton heating was naturally proposed. In this study, we carry out a statistical analysis us-24 ing Van Allen Probes data to provide deeper insights into this energy pathway. Our re-25 sults show that the occurrence of HFEMIC waves exhibits good correlation with the en-26 hanced flux and anisotropy of low-energy protons, but the correlation between the low-27 energy protons and the concurrent MS waves is rather poor. The latter result is given 28 support by quasilinear diffusion analysis, indicating negligible momentum diffusion rates 29 at sub-keV energies, unless MS wave frequency gets very close to the proton cyclotron 30 frequency (which constitutes only a small number of the cases). The fact that the first 31 chain of the coupling is statistically inconclusive calls for an alternative explanation for 32 the major source of the low-energy anisotropic proton population in the inner magne-33 tosphere. 34

35 1 Introduction

Plasma waves are indispensable to the cross-energy and cross-species coupling in 36 space plasmas. Fast magnetosonic waves (MS waves) in the inner magnetosphere res-37 onantly interact with energetic ring current protons, causing these protons to gain en-38 ergy preferentially in the direction perpendicular to the background magnetic field (e.g., 39 Horne et al., 2000; Ma, Li, Yue, et al., 2019). An anisotropic distribution of enhanced 40 ring current protons is the necessary condition to excite electromagnetic ion cyclotron 41 (EMIC) waves (e.g., L. Chen, Thorne, Jordanova, Wang, et al., 2010) which are known 42 to facilitate a rapid depletion of ultra-relativistic electrons in the outer belt (Usanova 43 et al., 2014). So when a simultaneous observation of high-frequency EMIC (HFEMIC) 44 waves, anisotropic low-energy protons, and MS waves by Van Allen Probes (Mauk et al., 45 2013) was first reported by Teng et al. (2019), a chain of energy flow from MS waves to 46 HFEMIC waves through the heating of low-energy protons was naturally proposed (see 47 Asamura et al., 2021, Figure 4). The HFEMIC waves in this event were different from 48 typical ones in that the wave spectrum is narrow-banded ($\Delta f \lesssim 0.1 f_{cp}$, where f_{cp} is 49 the equatorial proton cyclotron frequency) and the peak frequency occurs at $\sim 0.95 f_{cp}$ (Teng 50 et al., 2019). According to linear theory (e.g., Kennel & Petschek, 1966; Teng et al., 2019), 51 such HFEMIC waves resonantly interact with sub-keV protons (as opposed to 10–100 52 keV protons associated with typical EMIC waves) and requires temperature anisotropy 53 $(A = T_{\perp}/T_{\parallel} - 1)$ well exceeding the value (~ 1) associated with the excitation of typ-54 ical EMIC waves (Yue et al., 2019; Jun et al., 2023). Indeed, the observation shows en-55 hanced 90°-peaked (in pitch angle space) proton fluxes at energy $\lesssim 100 \text{ eV}$, concurrent 56 with HFEMIC activity (see Teng et al., 2019, Figure 1). Shortly, Asamura et al. (2021) 57 reported a similar event detected by Arase (Miyoshi et al., 2018). Employing a technique 58 called wave-particle interaction analysis that enables calculation of the Joule heating rate 59 directly from wave and particle measurements, they presented compelling evidence for 60 the proposed chain of energy flow. 61

Although it is often the case that the ring current proton populations accompany MS wave events (Ferradas et al., 2021; Wu et al., 2022) and several observational studies highlighted the ability of MS waves to energize them (Yuan et al., 2018; Ma, Li, Yue, et al., 2019; Hill et al., 2020), there is a growing body of work that questions the efficacy of MS wave-driven proton heating, particularly in the sub-keV range concerned here. In-

terestingly, Ferradas et al. (2021) showed that the majority of the H⁺ and He⁺ warm 67 ion flux enhancement events are not associated with direct observation of these waves, 68 although they did find that the flux enhancements and the pitch angle anisotropy in ab-69 sence of MS waves were weaker. Wu et al. (2022) presented a correlation analysis between 70 pancake pitch angle distributions of 10–300 eV protons and MS waves. Despite the con-71 clusion (drawn purely based on the concurrent observation statistics) that MS waves con-72 tributed to the formation of low-energy anisotropic proton distribution, they noted that 73 it is hard to justify this causal relationship from their diffusion analysis. Meanwhile, Min 74 et al. (2022) analyzed the event of Teng et al. (2019) in detail to test the proposed en-75 ergy coupling. They showed that while the observed 10-100 eV protons that exhibited 76 large anisotropy are the likely source of the concurrent HFEMIC waves, the MS wave-77 driven heating becomes ineffective in the energy range relevant to this event, as far as 78 the quasilinear process is concerned. On the other hand, Joseph et al. (2022) focused on 79 the relation between MS waves and pitch angle anisotropy of warm ($\lesssim 500 \text{ eV}$) protons 80 by a case study. From a comparative analysis involving two nearly identical cases of pitch 81 angle anisotropy of warm protons—one with concurrent MS waves and the other with-82 out them—and also from quasilinear theory, they concluded that MS waves are not re-83 sponsible for the primary heating of these warm protons. Alternatively, they proposed 84 that the recirculated polar wind plasma in the inner magnetosphere can cause the con-85 current appearance of heated protons and MS waves. 86

As for the low-energy proton-to-HFEMIC link, there is no statistical analysis to 87 draw a firm conclusion upon. Hence, in this study we carry out a statistical analysis us-88 ing Van Allen Probes data to provide further insights into the energy pathway proposed 89 by Teng et al. (2019) and Asamura et al. (2021). The aim of the study is (1) to help clear 90 up the role of MS waves in the low-energy proton heating and (2) to evaluate whether 91 the causal relation between HFEMIC waves and low-energy anisotropic protons is sta-92 tistically supported. Our results show that while the occurrence of HFEMIC waves ex-93 hibits a good correlation with the enhanced flux and anisotropy of low-energy protons, 94 the correlation between the key parameters of low-energy protons and concurrent MS 95 waves is rather poor. 96

The present paper is organized as follows: Section 2 outlines the data and event selection. In Section 3, we investigate the causal relationship between HFEMIC waves and low-energy protons. This is followed by an investigation of the coupling between lowenergy protons and concurrent MS waves in Section 4. Finally, Section 5 provides summary and discussion.

¹⁰² 2 Data and Event Selection

The Van Allen Probes provide comprehensive plasma wave and particle measure-103 ments in the inner magnetosphere (Mauk et al., 2013). Here, we utilize the data obtained 104 during the operation from 2013 to 2019. Observations of fields are from the Electric and 105 Magnetic Field Instrument Suite and Integrated Science (EMFISIS; Kletzing et al., 2013, 106 2023). Specifically, we utilize the data from the fluxgate magnetometer which records 107 the magnetic field at a maximum sampling rate of 64 Hz, and the waveform frequency 108 receiver (WFR) that provides wave magnetic power spectra from 10 Hz to 12 kHz. The 109 electric field data are provided by the electric fields and waves (EFW) instruments at 110 a maximum sampling rate of 32 Hz (Wygant et al., 2013; Breneman et al., 2022) and 111 used to identify low-harmonic MS waves. For low-energy protons, we utilize the data from 112 the Helium Oxygen Proton Electron (HOPE) instrument of the Energetic Particle Com-113 position and Thermal Plasma Suite which provides measurements of electrons and ions 114 over the 1 eV to 50 keV energy range with full pitch angle coverage (Funsten et al., 2013; 115 Spence et al., 2013). Finally, we use the background electron density data inferred from 116 the upper hybrid resonance frequency (Kurth et al., 2015). 117



Figure 1. A sample event detected by Probe A on August 27, 2017. (a) Electron density inferred from the upper hybrid frequency. (b) The parallel component of the magnetic field spectrogram from WFR in units of nT^2/Hz . The yellow dashed curves running across the panel are f_{ce} , $0.5f_{ce}$, $\sqrt{1836}f_{cp}$ (approximate lower hybrid frequency), and f_{cp} , where f_{ce} and f_{cp} are the equatorial electron and proton cyclotron frequencies, respectively. The white outline demarcates automatically identified MS waves (see Section 4 for details). (c) Proton differential flux at 90° pitch angle in units of $s^{-1}cm^{-2}sr^{-1}keV^{-1}$. The white trace running across the panel denotes the Alfvén energy, $E_A = m_p v_A^2/2$, where $v_A = B_{eq}/\sqrt{4\pi m_p n_e}$ is the Alfvén speed. (d) Pitch angle anisotropy parameter, A, given by Eq. (1). The white outline demarcates the region of enhanced A (see Section 4 for details). (e) Perpendicular component of the magnetic field spectrogram in the HFEMIC wave frequency range, given in units of nT^2/Hz . The start and end of HFEMIC wave activity are denoted by the magenta vertical lines in panels (a-d). (f) HFEMIC waves identified by the automatic algorithm. The magenta curves denote 0.89 and $1.01f_{cp}$, respectively.

Because our main focus is the energy channel that gives rise to HFEMIC waves, 118 we work with the events that specifically contain them. Figure 1 displays a sample event 119 on August 27, 2017 where enhanced MS waves (Figure 1b) and low-energy ($\leq 1 \text{ keV}$) 120 protons (Figure 1c) occurred concurrently with HFEMIC waves (Figure 1e). Evident from 121 the density profile of Figure 1a, not only HFEMIC waves (denoted by two vertical dashed 122 lines) but also MS waves and low-energy proton enhancement were all found outside the 123 plasmapause which is demarcated by the sudden drops in the density (one near 2200 UT 124 on August 27 and another at 0430 UT on the following day). In addition to the flux en-125 hancement, low-energy protons during this period also exhibited strong pitch-angle anisotropy, 126 which will be described in detail in Section 3. 127

Considering that the number of events are small (Teng et al., 2019), we narrowed 128 down candidate events first by visually inspecting magnetic field spectrograms. Although 129 laborious, it was fairly straightforward to identify them visually because of the distinct 130 characters of HFEMIC waves. While doing so, we also excluded the period where HFEMIC 131 and typical EMIC waves appear simultaneously, and counted as one when two spacecraft 132 with a small separation saw the same HFEMIC waves. We then generate boolean masks 133 based on the criteria: (1) the sum of all three components of magnetic spectral power 134 greater than 0.002 nT²/Hz and (2) frequency interval $0.89 < f/f_{cp} < 1.01$. We apply 135 to each mask array a five-pixel Gaussian filter and label the values greater than 0.4 as 136 HFEMIC waves. As an example, Figure 1f displays the identified HFEMIC wave signa-137 tures. Since there can be multiple patches of wave activity in one orbit, as the final step, 138 we require that the longest blob in Figure 1f be at least five minutes long. 139

In the end, we found a total of 26 events. (The full list is tabulated in Supporting 140 141 Information Table S3.) In comparison, a somewhat larger number of events (38 events) were found in Teng et al. (2019), who examined data from 2012 to 2018 based on a dif-142 ferent set of criteria. Since the statistical properties of HFEMIC waves we have found 143 (reproduced in Supporting Information S1 and S4) are consistent with those of Teng et 144 al. (2019), our events can be regarded as a subset of theirs. We note that almost all HFEMIC 145 events were found within 5° magnetic latitude and are associated with the electron plasma 146 to cyclotron frequency ratio $f_{pe}/f_{ce} \lesssim 10$ which is the typical condition outside the plas-147 masphere. We also note that the increased sample size by reducing the minimum dura-148 tion criterion did not change the fundamental conclusions of the present study due to 149 the low occurrence of HFEMIC wave events. 150

¹⁵¹ 3 HFEMIC Activity vs. Low-energy Protons

¹⁵² **3.1 Correlation Analysis**

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To understand the source of HFEMIC waves, here we statistically examine low-energy protons during HFEMIC activity. In addition to elevated proton fluxes, pitch-angle anisotropy is an important parameter for HFEMIC wave growth. In fact, the case event examined by Teng et al. (2019, Figure 2b) exhibits a very anisotropic distribution in the sense that $T_{\perp} \gg T_{\parallel}$. To systematically measure the degree of anisotropy of low-energy proton distribution, we calculate the pitch angle anisotropy parameter (M. W. Chen et al., 1999; Li et al., 2009)

$$A(E_i) = \frac{\int_0^{\pi} j(E_i, \alpha) \sin^3 \alpha d\alpha}{2 \int_0^{\pi} j(E_i, \alpha) \cos^2 \alpha \sin \alpha d\alpha} - 1$$
(1)

at every energy channel E_i , where *j* stands for the particle flux. Figure 1d displays this parameter for the sample event. Evidently, the enhancement of *A* is concurrent with the flux enhancement in the same energy range, which is markedly pronounced during the HFEMIC activity. Although not shown here, the corresponding pitch angle distribution in this energy range is sharply peaked at $\alpha = 90^{\circ}$, similar to Teng et al. (2019, Figures 1e and 1f).



Figure 2. Low-energy (10-500 eV) proton anisotropy statistics during HFEMIC activity. (a) A versus L shell scatter plot. The dots with identical color belong in the same event. The 90th percentile of the fluxes measured at different energies is shown. (b) Histogram of A. The red solid, blue dashed, and gray dot-dashed lines correspond to the histograms of 90th, 75th, and 50th percentile values. (c) A statistical dependence of A as a function of energy. The solid black curve and the shaded region denote the median and the inter-quartile range (IQR) of A, respectively.

This is not specific to this sample case—indeed, protons in the 10–500 eV range exhibited a flux enhancement and elevated anisotropy (like Figure 1c) for all HFEMIC events. Since the pitch angle anisotropy is one of the important parameters, we extract A in this energy range and examine the correlation with the concurrent HFEMIC waves. For this, the proton flux data were averaged over a two-minute period with one-minute overlap. In the end, we obtained a two-dimensional array of A, one in time and another in energy.

Figure 2a displays a scatter plot of A versus L shell. Since A is also a function of 174 175 energy, we choose the 90th percentile of the fluxes measured at different energies at each time point. The dots with identical color belong in the same event. Similar to the sam-176 ple event in Figure 1, the pitch angle anisotropy is consistently large during HFEMIC 177 activity: $A \gtrsim 2$ for all events and $A \geq 5$ for 71% of all. Notwithstanding the small 178 number of samples, it appears that A is not strongly related to L, which appears con-179 sistent with the L dependence of the HFEMIC wave amplitude and the electron plasma-180 to-cyclotron frequency ratio (shown in Figures S4c and S4f). Another way to look at this 181 may be that A has an upper bound at ~ 10 independent of L, which may be interpreted 182 as a result of the self-regulating process by generating HFEMIC waves, as shown for typ-183 ical EMIC waves (e.g., Gary & Lee, 1994; Denton et al., 1994; Yue et al., 2019). Figure 2b 184 shows histograms of A. Clearly, the 75th percentile curve does not deviate too far from 185 the 90th percentile curve and the majority of the median A values are greater than 2. 186 In Figure 2c, we find that A peaks at around 100 eV, which is substantially lower than 187 the energy $(\geq 1 \text{ keV})$ associated with the typical EMIC wave excitation (L. Chen, Thorne, 188 Jordanova, Wang, et al., 2010). 189

From this result, we can conclude that HFEMIC waves are strongly associated with enhanced fluxes of low-energy (10–500 eV) protons with markedly elevated pitch angle anisotropy.

3.2 HFEMIC Instability Analysis

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To gain further insights into the free energy source of HFEMIC waves, we carry out linear instability analysis using the low-energy proton data. Following the formulation of L. Chen et al. (2013), one can write the approximate growth rate in parallel propagation in a more data-agnostic way

$$\gamma = \frac{2\pi^2 \omega_{p0}^2}{\partial D_r / \partial \omega_r} \int_{E_{\parallel}}^{\infty} \frac{dE}{2E} \left[-\frac{\omega_r}{k_{\parallel}c} \mathcal{J}_h - \sqrt{\frac{E - E_{\parallel}}{2m_p c^2}} \frac{\partial \mathcal{J}_h}{\partial \alpha} \right] \Big|_{E_{\parallel} = E_{\rm res}},\tag{2}$$

where ω_r is the real part of the angular wave frequency ω ; k_{\parallel} is the parallel wave num-199 ber; D_r is the real part of the dispersion relation, $D(\omega, k_{\parallel}) = 0$; $\omega_{p0} = \sqrt{4\pi n_0 e^2/m_p}$ 200 is the proton plasma frequency; $\mathcal{J}_h = m_p c^2 j_h / (n_0 c)$ is the normalized hot proton flux; 201 and $E_{\rm res}$ is the parallel resonant energy. We approximate the energy integral and pitch 202 angle derivative in the right side from $\mathcal{J}_h(E_i,\alpha_i)$ given in discrete energy and pitch an-203 gle space. The real part of wave frequency ω_r is obtained from the cold plasma disper-204 sion relation of a proton-electron plasma. The fact that protons make up of the entire 205 ion species is not an unreasonable assumption in the regime where the wave frequency 206 approaches f_{cp} , but the cold plasma assumption is generally considered to be invalid in 207 this regime where $E_{\rm res}$ becomes small enough that thermal protons start to resonantly 208 interact with the waves. Nevertheless, since an elaborate fitting of model distributions 209 like in Teng et al. (2019) is not practical for all events we have found, we use this ap-210 proximate formula to get general idea of how the instability behaves qualitatively and 211 then pick one case to carry out a more appropriate analysis. 212

For growth rate calculation, we average the data over the 5-minute period centered at the longest HFEMIC wave blob (see Figure 1f) and include protons only in the energy range 10–1000 eV. The number density accounted for by this population is less than



Figure 3. Summary of HFEMIC instability analysis. (a) Superposition of linear growth rates for all events calculated by Eq. (2). The event denoted with red color (detected by Probe A on August 19, 2017) is examined in detail in panels (b-e). (b) Linear growth rate from full kinetic theory at parallel propagation for the chosen event. The solid curve is the result for a model distribution fit to the data, and the dashed curve is the result for a model distribution with slightly enhanced anisotropy. (c) Proton flux as a function of energy and pitch angle from the particle data. A pair of dashed curves indicate the resonant energy corresponding to $f/f_{cp} = 0.95$. (d) Energy-pitch angle distribution of proton flux of a model proton distribution with slightly enhanced anisotropy.

²¹⁶ 25% (on average 15%) of the total electron density (meaning that protons of < 10 eV ²¹⁷ make up the majority) and the average temperature is ~ 50 eV. In Figure 3a, the ap-²¹⁸ proximate growth rates from Eq. (2) are superimposed for all HFEMIC events. The zigzag ²¹⁹ pattern in all curves is owing to \mathcal{J}_h given in discrete energy and pitch angle space with ²²⁰ coarse resolution. No event exhibits pronounced wave growth at $f/f_{cp} \gtrsim 0.9$.

We investigate in detail the case highlighted in red which shows a local bump in the growth rate at around $f/f_{cp} = 0.93$. The corresponding energy-pitch angle distribution of proton flux is displayed in Figure 3c. Typical for all events, the flux exhibits a sharp enhancement in the vicinity of $\alpha = 90^{\circ}$ (which is resolved by only *three* pixels!). For simplicity, we assume a model of two bi-Maxwellian distributions to fit the data

$$f_j = \frac{n_j}{\pi^{3/2} \theta_{\parallel j} \theta_{\perp j}^2} e^{-v_{\parallel}^2/\theta_{\parallel j}^2} e^{-v_{\perp}^2/\theta_{\perp j}^2}.$$
(3)

The fitting parameters are: $n_1 = 0.2n_0$, $\theta_{\parallel 1} = 0.02v_A$, and $T_{\perp 1}/T_{\parallel 1} = 10$ for the first component, and $n_2 = 0.0045n_0$, $\theta_{\parallel 2} = 0.065v_A$, and $T_{\perp 2}/T_{\parallel 2} = 5$ for the second. The charge-neutralizing background population is assumed to have $\theta_{\parallel 3} = \theta_{\perp 3} = 0.01v_A$. (Here, $v_A = B_{eq}/\sqrt{4\pi m_p n_e}$ is the Alfvén velocity.) Figure 3d shows the model distribution which reasonably compares to the actual data. We solve the full kinetic dispersion relation at parallel propagation (e.g., L. Chen et al., 2013). The result shown in Figure 3b (blue curve) indicates no noticeable wave growth.

It is not surprising to see that the observed proton distribution is in a marginally 234 stable state. Waves and particles self-consistently evolve and the previous analysis (Teng 235 et al., 2019; Min et al., 2022) showed that the instability is rather weak. So, it is likely 236 that the observed distributions have already been relaxed substantially. In fact, previ-237 ous studies of EMIC waves show that almost all events fall under the instability thresh-238 old curve in anisotropy-parallel beta space (e.g., Gary & Lee, 1994; Denton et al., 1994; 239 Noh et al., 2018; Yue et al., 2019; Jun et al., 2023). To our knowledge the instability thresh-240 old analysis in the high-frequency EMIC regime has not been done, so such a theory-241 observation comparison will be valuable to understand the low-energy proton to HFEMIC 242 wave energy coupling chain. Another point worth mentioning is that the analysis here 243 had to contend with the coarse pitch angle resolution of the particle measurement—clearly, 244 the three-pixel coverage is not enough to resolve the sharp flux enhancement in the im-245 mediate vicinity of $\alpha = 90^{\circ}$ shown in Figure 3c. A pair of dashed curves in Figure 3c 246 denotes the resonant energy, $E_{\rm res}$, corresponding to $f/f_{cp} = 0.95$ (a typical peak fre-247 quency of the observed HFEMIC waves). Since Eq. (2) involves the pitch angle gradi-248 ent of proton fluxes evaluated at $E_{\rm res}$, the high-resolution data near $\alpha = 90^{\circ}$ is crucial 249 for accurate HFEMIC instability calculation. As a demonstration of this point, if we in-250 crease the anisotropy of the first component in Figure 3d slightly (to $T_{\perp 1}/T_{\parallel 1} = 15$), 251 HFEMIC waves can grow at $f/f_{cp} \approx 0.91$ (red dashed curve in Figure 3b), meaning 252 that a slight increase of anisotropy renders the model distribution unstable. The corre-253 sponding energy-pitch angle distribution is shown in Figure 3e. It will be difficult to dis-254 tinguish between the model distributions in Figures 3d and 3e by coarse sampling in pitch 255 angle as in Figure 3c. Therefore, it is reasonable to conjecture that the actual pitch an-256 gle anisotropy (and its gradient at E_{res}) is greater than what is estimated in Figure 2, 257 which of course favors the scenario that the anisotropic low-energy protons are the free 258 energy source of HFEMIC waves. 259

4 MS Waves vs. Low-energy Protons

4.1 Correlation Analysis

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Having shown a positive correlation between HFEMIC occurrence and the enhancement of low-energy proton anisotropy, we now turn to the correlation analysis between the low-energy proton enhancement and MS waves. Since the MS wave-driven heating occurs preferentially in the direction perpendicular to the magnetic field, we once again
utilize the anisotropy parameter of Eq. (1) of protons. For MS waves, the key parameters are the wave amplitude and harmonic number (assuming that wave normal angles are quasi-perpendicular).

For statistical analysis, we identify MS waves from the WFR data based on the cri-269 teria: wave normal angle greater than 70° , ellipticity within ± 0.25 (i.e., linear polariza-270 tion), and harmonic frequency $f \leq 42 f_{cp}$. The white contour in Figure 1b demarcates 271 the identified MS waves based on these criteria. Even though no minimum harmonic fre-272 quency is imposed, all but one event show MS waves at $f \gtrsim f_{cp}$. It should be noted 273 that the WFR data can miss very low-harmonic MS waves due to the low sensitivity and 274 coarse frequency resolution in the low-frequency regime. Ma, Li, Bortnik, et al. (2019) 275 performed a survey using both fluxgate and search coil magnetometers of Van Allen Probes 276 and found that low-harmonic MS waves can have high power at L > 4 outside the plasma-277 pause. Furthermore, Teng et al. (2021) showed that even the fluxgate magnetometer on 278 board Van Allen Probes can miss some low-harmonic MS waves with weak magnetic field 279 intensity because of relatively high measurement thresholds. After checking the fluxgate 280 magnetic field and EFW data, we found three events of low-harmonic MS waves which 281 show up only in the EFW data. (Due to the sampling limit of EFW, only the first five 282 harmonic modes can be examined.) Their low occurrence rate (and the fact that these 283 waves are absent from the fluxgate data) suggests that the WFR data alone should be sufficient for the statistical analysis below. 285

Similarly, we apply a set of criteria to systematically select the enhanced anisotropy 286 of low-energy protons: Guided by Figure 2a, we choose an anisotropy threshold A > 3287 in the 10–500 eV range. In the end, two out of 26 events did not meet this minimum re-288 quirement. The white contour in Figure 1d indicates the identified region of enhanced 289 A. Although visually the region of enhanced anisotropy extends nearly to the end of the 290 plot, the later half of the region is not selected because of the anisotropy being lower than 291 the threshold. In fact, the two events that did not meet the threshold still exhibit a clear 292 90°-peaked pitch angle distribution. In that sense, the anisotropy threshold A = 3 is 293 a conservative choice. (A threshold value of A = 4 does not change the fundamental 294 result here, only reducing the number of data points.) 295

Figure 4a shows a relation between MS wave amplitude and harmonic number. De-296 spite the data scatter, there is a noticeable inverse relationship: The smaller the harmonic 297 number is, the larger the wave amplitude tends to get (e.g., Ma, Li, Bortnik, et al., 2019). 298 Also, there are a lot more samples at low harmonic frequencies, which can be understood 299 by the fact that MS waves with larger amplitude are more easily detectable. Figure 4b 300 plots MS wave amplitude against magnetic latitude. The wave occurrence is clearly con-301 fined to within $\pm 5^{\circ}$ latitude and the amplitude maximizes at the equator (e.g., Board-302 sen et al., 2016). Certainly, the inverse relationship and latitudinal confinement of the 303 MS wave occurrence in Figure 4 are the typical features expected from MS waves. 304

Figure 5a displays for each event the fraction of MS wave occurrence over the duration of enhanced proton anisotropy. A 100% means that MS waves occurred for the entire duration of enhanced anisotropy. So, for 15 out of 24 events, MS waves lasted as long as (and perhaps longer than) the anisotropy enhancement did. Even for those below 100%, the coverage is greater than 50% (with one exception having a 25% coverage).

However, the good MS wave coverage does not necessarily mean the causal relationship. Figure 5b shows a relation between 90th percentile A values $(A_{90th}$ calculated in the same way as in Figure 2a) and MS wave amplitude. Since there are events with fractional MS wave coverage, there are points with no corresponding MS wave power. These are denoted by the cross symbols in the left side and make up of 11% of data points in the figure. (Despite no concurrent MS waves, they still have large A_{90th} values (~5) associated with them.) For those that do have finite MS wave power associated them,



Figure 4. (a) MS wave amplitude versus power-weighted average frequency, f_{avg} , normalized by f_{cp} . The dots and vertical bars denote the mean and one standard deviation, respectively. (b) MS wave amplitude versus magnetic latitude. The dots with the same color belong in the same event.

the correlation between A_{90th} and MS wave amplitude is not so clear. If the low-energy 317 proton heating (preferentially in the perpendicular direction) is driven by the concur-318 rent MS waves, one would expect to see a positive correlation between A_{90th} and MS wave 319 amplitude. Although the mean value (denoted by open circles) does seem to show a mild 320 increase with MS wave amplitude in the weak-amplitude region, the trend flattens out 321 at the large amplitude region (where we expect to see an efficient acceleration by MS waves 322 and thus a more anisotropic distribution). In general, it is quite difficult to make out a 323 clear trend because of the large data scatter. Similarly, Figure 5c shows a relation be-324 tween A_{90th} and power-averaged MS wave harmonic number (f_{avg}/f_{cp}) . No particular 325 dependence stands out in this case, either. Considering that the harmonic number is in-326 versely related to MS wave amplitude in Figure 4a, a decreasing trend should be expected 327 here. In that regard, the increasing trend shown in the weak-amplitude region in Fig-328 ure 5b may not be related to the MS wave-driven heating at all. Figures 5d and 5e show 329 correlations of the average proton energy normalized by the Alfvén energy (E_{avg}/E_A) 330 with the MS wave amplitude and harmonic number, respectively. We use the normal-331 ized energy because in linear theory the energy of protons in resonance with MS waves 332 are scaled by E_A (see, e.g., Horne et al., 2000). Similar to the previous two plots, it is 333 hard to glean any meaningful statistical correlations due to the large data scatter. In-334 terestingly, the trend of E_{avg}/E_A in Figure 5d appears to be quite similar to the trend 335 shown in Figure 5b. 336

In summary, despite the decent coverage by MS waves during the period of enhanced anisotropy (and fluxes) of low-energy protons, the lack of correlations between the key parameters that are relevant to MS wave-driven heating suggests that statistically the low-energy proton enhancement driven by concurrent MS waves is inconclusive.

4.2 Quasilinear Diffusion

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According to Min et al. (2022), quasilinear theory does not seem to favor efficient heating of low-energy protons, either. Considering near-equatorially mirroring protons interacting with MS waves at quasi-perpendicular propagation, the momentum diffusion



Figure 5. Correlation analysis between MS waves and low-energy protons. (a) Fraction of MS wave occurrence over the duration of enhanced proton anisotropy. (b) 90th percentile $A(A_{90th})$ versus MS wave amplitude. The data points with no MS wave power (for those events having less than a 100% coverage) are shown with the cross symbols on the left side. The number of data points with finite MS wave power is about 3100 and the number with no MS wave power is about 400. (c) A_{90th} versus f_{avg}/f_{cp} , where f_{avg} is the power-weighted average MS wave frequency. (d) Average proton energy normalized by the Alfvén energy (E_{avg}/E_A) versus MS wave amplitude. (e) E_{avg}/E_A versus f_{avg}/f_{cp} . In panels (b-e), the open circles and vertical bars correspond to the mean and one standard deviation, respectively.

coefficient can be written as (Min & Liu, 2021)

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$$D_{\perp\perp} = \pi \Omega_p^2 \sum_n \frac{\omega^2}{k_\perp^2} \frac{W_B(\omega)}{B_{\rm eq}^2} \frac{n^2 \Omega_p^2}{k_\perp^2 v_\perp^2} J_n^2 \left(\frac{k_\perp v_\perp}{\Omega_p}\right),\tag{4}$$

where $\Omega_p = 2\pi f_{cp}, k_{\perp}$ is the perpendicular wave number, v_{\perp} is the perpendicular com-347 ponent of proton velocity, n is the resonance order, $J_n(x)$ is the Bessel function of the 348 first kind, and $W_B(\omega)$ is the magnetic field power spectral density. Figure 6a plots $D_{\perp\perp}$ 349 as a function of E/E_A , corresponding to n = 2, 3, 5, and 10. For all curves, the momen-350 tum diffusion coefficient peaks at $E \gtrsim E_A$ and monotonically decreases with decreas-351 ing energy. In addition, the smaller the harmonic number gets, the slower the decreas-352 ing rate becomes. Therefore, it is with the small harmonic MS waves that lead to a max-353 imal scattering rate in the low-energy regime. As we will see, $E_A \gtrsim 1$ keV in our events. 354 So, it is only the first few harmonic modes that will be most effective in the low-energy 355 $(\sim 10-100 \text{ eV})$ proton heating. 356

For qualitative analysis, we calculate the momentum diffusion coefficient of equa-357 torially mirroring protons using the MS wave power spectra identified in the previous 358 subsection, assuming that MS waves propagate strictly perpendicular to the background 359 magnetic field. Although the latter assumption is not valid in general, Min et al. (2022) 360 showed that Eq. (4) can qualitatively represent the overall trend of the bounce-averaged 361 diffusion rate of near-equatorially mirroring protons (see also Supporting Information 362 S2 and S5). In Figure 6b, we show the ratio of $D_{\perp\perp}$ at E = 10 and 100 eV (black and 363 red dots, respectively) to the maximum of $D_{\perp\perp}$ at each time bin, plotted against f_{avg}/f_{cp} . 364 The horizontal dashed line is drawn at 10^{-3} , meaning that the diffusion rate is three or-365 ders of magnitude smaller than the maximum. The majority of points are below the 10^{-3} 366 mark. Furthermore, there is an inverse relationship between the ratio and the harmonic 367 number, and the diffusion rate gets larger for more energetic protons (red versus black 368 dots). In Figure 6c, we show the energy at which $D_{\perp\perp}$ maximizes versus the Alfvén en-369 ergy. The peak energy is typically greater than E_A (and within a factor of two). This 370 is essentially controlled by the Bessel function term in Eq. (4) and the dispersion rela-371 tion approximately given by $\omega \sim v_A k_{\perp}$ (L. Chen, Thorne, Jordanova, & Horne, 2010). 372 We emphasize that even though WFR can miss very low-harmonic MS waves, the num-373 ber of such events identified from the fluxgate and EFW data is actually small. 374

375

5 Summary and Discussion

We carried out a comprehensive statistical analysis using the Van Allen Probes data to provide deeper insights into the energy coupling from MS waves to HFEMIC waves through the heating of low-energy protons. We identified 26 HFEMIC wave events from both spacecraft for the entire mission period and performed correlation analyses among the key parameters relevant to diagnose the suggested chain of energy flow. Our findings can be summarized as follows:

1. For all events, HFEMIC waves are strongly associated with enhanced fluxes and 382 elevated pitch angle anisotropy of low-energy (10-500 eV) protons. The pitch an-383 gle anisotropy during HFEMIC activity is much larger than the threshold value 384 $(A \sim 1)$ needed to excite typical EMIC waves and statistically peaks at energy 385 ~ 100 eV. The linear instability calculation indicated that the observed low-energy 386 protons are marginally stable to HFEMIC waves. However, part of the reason has 387 to do with the low pitch angle resolution of the proton flux data, which can smooth 388 out the rapid variation of proton flux in the vicinity of 90° pitch angle and thus 389 underestimate the actual pitch angle anisotropy and its gradient at such a low res-390 onant energy. 391

2. Although MS waves and enhanced low-energy protons occurred semi-concurrently, the lack of correlations between the key parameters that are relevant to MS wave-



Figure 6. (a) Momentum diffusion coefficient, $D_{\perp\perp}$, of Eq. (4) corresponding to the harmonic modes, n = 2, 3, 5, and 10. The energy on the horizontal axis is normalized by the Alfvén energy, E_A . (b) Ratio of $D_{\perp\perp}$ at 10 (black) and 100 (red) eV to the maximum of $D_{\perp\perp}$, calculated using the observed MS wave power spectra. (c) Energy at the maximum diffusion coefficient, E_{\max} , versus Alfvén energy, E_A .

driven heating suggests that statistically the role of MS waves as the driver of anisotropic 394 low-energy protons is questionable. This result is given support by the quasilin-395 ear analysis where the momentum diffusion rate maximizes at energy slightly larger 396 than the Alfvén energy (which is $\gtrsim 1 \text{ keV}$) and the scattering efficiency drops pre-397 cipitously with a decreasing energy. Although the resonant interactions with low-398 harmonic MS waves can elevate the scattering efficiency and we indeed found sev-399 eral events of low-harmonic MS waves, for most of the cases the diffusion rate at 400 10–100 eV is several orders of magnitude lower compared to the maximum rate. 401

All things considered, it is not unreasonable to believe that the low-energy protons with 402 enhanced anisotropy are the free energy source of HFEMIC waves, but it is compara-403 tively hard to justify the resonant interactions with MS waves as the (primary) source 404 of the enhanced low-energy protons. It is not to say that the results of Asamura et al. 405 (2021), which is based on a quantitative analysis, are erroneous, but it makes more sense, 406 in general, to view the semi-concurrent MS waves and enhanced low-energy protons as 407 having a common driver, rather than being causally related. Having said that, we can-408 not rule out any non-resonant, nonlinear effect we have neglected here. Theoretical and 409 particle-in-cell simulation studies (Artemyev et al., 2017; Sun et al., 2017; Min et al., 2022) 410 highlighted that such an effect may be important. However, at this point more quan-411 titative theories need to be materialized and even then they must reconcile the obser-412 vational results presented in Figure 5. 413

Other possibilities not considered here may include the spatial effect: The low-energy 414 protons could have been energized at earlier local time where there were strong MS wave 415 activity and then drifted to where the measurement was made. In this way, the weak cor-416 relation between MS waves and low-energy protons could be explained if the measure-417 ment was made far from the MS wave source region. According to Ma, Li, Bortnik, et 418 al. (2019, Figure 4), the occurrence of low-harmonic MS waves appears to peak slightly 419 ahead of our HFEMIC events (Figure S4a), when AE^{*} (defined as the maximum geo-420 magnetic auroral electrojet (AE) index value in the previous 3 hr) is larger than 500 nT 421 (i.e., moderate substorm activity). However, although we did not examine the geomag-422 netic conditions, Jun et al. (2021, 2023) reported that H-band EMIC waves with frequen-423 cies from 0.23 to $0.95 f_{cp}$ tend to occur during relatively quiet geomagnetic conditions, 424 which is not in favor of this scenario. 425

On the other hand, MS waves and low-energy protons need not be causally related 426 to each other just because they appear concurrently. Considering how frequently pan-427 cake distributions of low-energy protons are found with MS waves shown in a recent study (Wu 428 et al., 2022) (although one should be careful in the interpretation of Wu et al. (2022, Fig-429 ure 4) because of the different normalization), it is possible that they have a common 430 driver. Joseph et al. (2022) recently proposed an alternative explanation for the origin 431 of low-energy, anisotropic protons (see Figure 6 therein). In this scenario, the polar wind 432 outflow is intensified under geomagnetically disturbed conditions. Depending on the strength 433 of the southward interplanetary magnetic field, the entry point of the polar wind can be 434 closer to, or far away from, the Earth. As the polar wind plasma particles get injected 435 towards the Earth, they gain energy adiabatically, preferentially in the direction perpen-436 dicular to the background magnetic field. The plasma particles whose entry point is far 437 away from the Earth can attain ring current energies, and those that have entered closer 438 to the Earth becomes the warm plasma cloak with a high anisotropy. Thus, the former 439 population can be the source for MS waves and the latter becomes the source of HFEMIC 440 waves. In that regard, this scenario may be able to explain the semi-concurrent MS waves 441 and low-energy anisotropic proton population. On the other hand, not all HFEMIC waves 442 seem to occur during geomagnetically disturbed times, as reported by Jun et al. (2021, 443 2023). Nevertheless, this is an interesting idea that warrants further investigation. 444

Enhanced solar wind dynamic pressure is also known to cause proton temperature anisotropy on the dayside of the magnetosphere as a result of adiabatic heating (Anderson & Hamilton, 1993; McCollough et al., 2010). Interestingly, this is also the region where most of our HFEMIC events were found. However, this mechanism is unlikely to explain our low-energy proton observations because we would have seen an enhancement in anisotropy in all energies consistently and an elevated anisotropy alone is not sufficient to excite MS waves.

The warm plasma population is indispensable to the dynamics in the magnetosphere. Therefore, revealing the processes involved in the perpendicular acceleration of low-energy protons is important for quantifying the cross-scale and cross-energy coupling.

455 6 Open Research

The authors acknowledge the EMFISIS data obtained from https://emfisis.physics
.uiowa.edu/Flight/, the HOPE data obtained from http://www.rbsp-ect.lanl.gov/
data_pub/, and the EFW data from http://www.space.umn.edu/data/rbsp/.

459 Acknowledgments

K.M. graciously acknowledges the support from the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2020R1C1C100999612).
Q.M. would like to acknowledge the NASA grant 80NSSC20K0196, NSF grant AGS-2225445,
and the support from NSF Geospace Environment Modeling focus group "Self-Consistent
Inner Magnetospheric Modeling."

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