A climate model-informed nonstationary stochastic rainfall generator for design flood analyses in continental-scale river basins

Yuan Liu¹, Daniel Benjamin Wright¹, and David J Lorenz²

 1 University of Wisconsin-Madison $^{2}\mathrm{Center}$ for Climatic Research

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Abstract

Existing stochastic rainfall generators (SRGs) are typically limited to relatively small domains due to spatial stationarity assumptions, hindering their usefulness for flood studies in large basins. This study proposes StormLab, an SRG that simulates precipitation events at 6-hour and 0.03° resolution in the Mississippi River Basin (MRB). The model focuses on winter and spring storms caused by strong water vapor transport from the Gulf of Mexico—the key flood-generating storm type in the basin. The model generates anisotropic spatiotemporal noise fields that replicate local precipitation structures from observed data. The noise is transformed into precipitation through parametric distributions conditioned on large-scale atmospheric fields from a climate model, reflecting both spatial and temporal nonstationarity. StormLab can produce multiple realizations that reflect the uncertainty in fine-scale precipitation arising from a specific large-scale atmospheric environment. Model parameters were fitted for each month from December-May, based on storms identified from 1979-2021 ERA5 reanalysis data and AORC precipitation. Validation showed good consistency in key storm characteristics between StormLab simulations. Empirical return levels of simulated annual maxima agreed well with AORC data and displayed bounded tail behavior. To our knowledge, this is the first SRG simulating nonstationary, anisotropic high-resolution precipitation over continental-scale river basins, demonstrating the value of conditioning such stochastic models on large-scale atmospheric variables. The simulated events provide a wide range of extreme precipitation scenarios that can be further used for design floods in the MRB.

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A climate model-informed nonstationary stochastic rainfall generator for design flood analyses in continental-scale river basins

4 Yuan Liu¹, Daniel B. Wright¹, and David J. Lorenz²

¹Department of Civil and Environmental Engineering, University of Wisconsin-Madison,
 Madison, WI, USA.

7 ²Center for Climatic Research, University of Wisconsin-Madison, Madison, WI, USA.

8 Corresponding author: Yuan Liu (<u>yliu2232@wisc.edu</u>)

9 Key Points:

- A nonstationary stochastic rainfall generator driven by water vapor transport is proposed to simulate storms in the Mississippi Basin
- 1,000 synthetic years of precipitation events were simulated based on large-scale
 atmospheric variables from a global climate model
- Good agreement was found between distributions of simulated and reference annual
 precipitation maxima

17 Abstract

Existing stochastic rainfall generators (SRGs) are typically limited to relatively small domains 18 19 due to spatial stationarity assumptions, hindering their usefulness for flood studies in large 20 basins. This study proposes StormLab, an SRG that simulates precipitation events at 6-hour and 0.03° resolution in the Mississippi River Basin (MRB). The model focuses on winter and spring 21 22 storms caused by strong water vapor transport from the Gulf of Mexico-the key flood-23 generating storm type in the basin. The model generates anisotropic spatiotemporal noise fields 24 that replicate local precipitation structures from observed data. The noise is transformed into 25 precipitation through parametric distributions conditioned on large-scale atmospheric fields from 26 a climate model, reflecting both spatial and temporal nonstationarity. StormLab can produce 27 multiple realizations that reflect the uncertainty in fine-scale precipitation arising from a specific large-scale atmospheric environment. Model parameters were fitted for each month from 28 29 December-May, based on storms identified from 1979-2021 ERA5 reanalysis data and AORC 30 precipitation. Validation showed good consistency in key storm characteristics between StormLab simulations and AORC data. StormLab then generated 1,000 synthetic years of 31 32 precipitation events based on 10 CESM2 ensemble simulations. Empirical return levels of 33 simulated annual maxima agreed well with AORC data and displayed bounded tail behavior. To 34 our knowledge, this is the first SRG simulating nonstationary, anisotropic high-resolution 35 precipitation over continental-scale river basins, demonstrating the value of conditioning such stochastic models on large-scale atmospheric variables. The simulated events provide a wide 36 37 range of extreme precipitation scenarios that can be further used for design floods in the MRB.

38 **1 Introduction**

39 The Mississippi River Basin (MRB) has experienced multiple extreme floods throughout history, including the Spring 1927 Flood (J. A. Smith & Baeck, 2015), the Great Flood of 1993 40 41 (Dirmeyer & Brubaker, 1999; Dirmeyer & Kinter III, 2009), and the June 2008 Midwest Flood (Dirmeyer & Kinter III, 2009; Budikova et al., 2010). These floods were each caused by 42 43 sequences of severe large-scale rainstorms, all associated with strong water vapor transport from 44 the Gulf of Mexico (Moore et al., 2012; Lavers & Villarini, 2013; Su et al., 2023). The project 45 design flood was developed to estimate the maximum possible peak discharge and water levels 46 in the lower MRB, providing guidelines for flood protection infrastructure and management 47 (Gaines et al., 2019). Historical records of extreme rainstorms were placed in sequences, with 48 modest perturbations to their timing, location, and severity to create hypothetical worst-case 49 rainstorm scenarios (V. A. Myers, 1959). Of these, Hypo-Flood 58A, consisting of the rainstorm 50 in January 1937 followed shortly by rainstorms in January 1950 and then February 1938, 51 produced the largest discharge on the lower Mississippi River and has been used as the design 52 flood since 1955 (Lewis et al., 2019; McWilliams & Hayes, 2017). However, this approach is not 53 without limitations, including using a limited rainstorm record (i.e., pre-1950s) and providing 54 only one worst-case scenario, rather than a range of possible outcomes with occurrence 55 probabilities. With growing evidence that rainstorms are intensifying under future climate 56 change (e.g., Ban et al., 2015; Fischer & Knutti, 2016; Matte et al., 2021; Fowler et al., 2021), 57 historical design floods may fail to reflect the range of possible scenarios and may underestimate 58 future risks (Wright et al., 2014; Yu et al., 2020).

In principle, stochastic rainfall generators (SRGs) can generate synthetic precipitation data that extend short observation records and provide a wide range of extreme scenarios to support flood risk management (Wilks & Wilby, 1999; Ailliot et al., 2015; Benoit et al., 2018). SRGs be categorized into three main types based on their simulation dimensions: point or area-averaged models, multi-site models, and space-time models. We will focus on the last type, which generates spatiotemporal precipitation data over regular grids. Reviews of other model types can be found in, e.g., Waymire & Gupta (1981), Mehrotra et al. (2006), and Qin (2011).

66 Existing space-time SRGs can be further classified into three major categories: 1) cluster-based 67 models, 2) multifractal models, and 3) meta-Gaussian models. These categories are briefly 68 reviewed here. Cluster-based models conceptualize the precipitation system as a hierarchical organization in which groups of small rain "cells" (~10-50 km²) are embedded within large 69 rainband areas ($\sim 10^3 - 10^4$ km², e.g., Waymire et al., 1984). These models simulate rainband 70 centers using Poisson processes, with elliptical rain cells randomly generated around each 71 72 rainband following a clustered point process (e.g., Chen et al., 2021). Properties of these rain 73 cells, including intensity, lifetime, area, and orientation are modeled via distributions fitted to 74 observed data (Northrop, 1998). Total precipitation intensity at any location is calculated by 75 summing the contributions from all existing (and possibly overlapping) rain cells. Cluster-based 76 models were originally developed to simulate precipitation time series at single points (i.e., 77 Neyman-Scott and Bartlett-Lewis models, Rodriguez-Iturbe et al., 1997) and later expanded for 78 space-time simulations through work by Le Cam (1961), Waymire et al. (1984), Cowpertwait 79 (1995), Northrop (1998), and others. Recent improvements include randomizing model 80 parameters (Kaczmarska et al., 2014; Onof & Wang, 2020) and applying non-homogenous Poisson processes for rain cell arrivals (Burton et al., 2010). This modeling approach attempts to 81 82 mimic the physical organization of precipitation systems and enables analytical derivation for 83 key precipitation properties like moments and spatial correlation (Cowpertwait, 1995; Leonard et al., 2008). However, cluster-based models struggle to fully describe the precipitation statistical 84 85 structure across scales (Foufoula-Georgiou & Krajewski, 1995) and typically lack dependence modeling between precipitation properties, e.g., rain cell duration and intensity, with 86 87 implications for simulating extremes (Paschalis et al., 2013; Chen et al., 2021). The multi-level 88 model structure also introduces many parameters, increasing model calibration difficulty 89 (Jothityangkoon et al., 2000).

90 Multifractal models were developed based on empirical evidence that precipitation systems 91 exhibit similar statistical properties across different temporal and spatial scales (i.e., scale 92 invariance, see Schertzer & Lovejoy, 1987; Gupta & Waymire, 1993; Menabde et al., 1997). A common modeling approach, known as the discrete multiplicative random cascade (MRC), 93 94 involves recursively subdividing the simulation domain into small squares and multiplying the 95 precipitation intensity in each square by a random weight drawn from a fitted distribution 96 (Sharma et al., 2007; Serinaldi, 2010; Rupp et al., 2012). A similar process is applied along the 97 time dimension using a different dividing factor to account for anisotropy between space and 98 time(Marsan et al., 1996; Veneziano et al., 2006). Other model variations include continuous 99 MRC (Marsan et al., 1996), temporal Markov chain coupled with spatial MRC (Over & Gupta, 100 1996; Jothityangkoon et al., 2000), and rainfall cascade decomposition (Seed et al., 2013; Raut et 101 al., 2019). Multifractal models aim to reflect precipitation physical structures and require fewer 102 parameters than cluster-based models (Raut et al., 2018). Nevertheless, these models assume that 103 precipitation fields are spatially isotropic and only represent limited types of space-time scaling 104 properties. These assumptions can introduce bias when precipitation does not exhibit perfect 105 invariance over different temporal and spatial scales (Serinaldi, 2010).

106 Meta-Gaussian models represent precipitation evolution by simulating an underlying 2D 107 Gaussian noise field that attempts to mimic the spatiotemporal correlation structure of actual 108 precipitation (Baxevani & Lennartsson, 2015). This noise field is then transformed into 109 precipitation amounts using a non-linear function, often a parametric precipitation distribution, 110 generating simulations that are statistically consistent with observed precipitation (Mascaro et 111 al., 2023). Meta-Gaussian models have become increasingly popular in recent years (Vischel et 112 al., 2011; Sigrist et al., 2012; Papalexiou et al., 2021), with major differences in how they 113 represent precipitation spatial structures. Approaches include parametric covariance functions 114 (Paschalis et al., 2013; Papalexiou, 2018), turning band methods (Leblois & Creutin, 2013), fast 115 Fourier transforms (FFTs, Nerini et al., 2017), stochastic partial differential equations (Fuglstad et al., 2015), and kernel convolutions (Fouedjio et al., 2016). A key advantage of meta-Gaussian 116 117 models is their flexibility in modeling precipitation distributions and spatiotemporal dependence 118 structures, which can yield improved extreme value simulations compared to other SRGs. 119 However, they often involve many parameters and high computational demand for large-scale 120 simulations (Paschalis et al., 2013; Papalexiou & Serinaldi, 2020). Modeling temporal 121 autocorrelation also remains challenging due to its dependence on storm movement and on 122 spatial correlation (Storvik et al., 2002). Beyond the three major categories described, there also 123 exist some other space-time rainfall generators such as variogram-based simulations (Schleiss et 124 al., 2014) and deep learning models (e.g., Zhang et al., 2020; Leinonen et al., 2021; Rampal et 125 al., 2022).

126 Most existing SRGs assume the precipitation field is second-order stationary, i.e., the mean of 127 precipitation is constant across the region, with covariance between two grid cells depending 128 only on their separation distance rather than their absolute locations (Fuglstad et al., 2015; Benoit 129 et al., 2018; Hristopulos, 2020). This limits simulation domains to relatively small areas where 130 precipitation can be considered homogeneous (Vischel et al., 2011; Schleiss et al., 2014; 131 Papalexiou et al., 2021). Such an assumption precludes the usage of such SRGs for simulating 132 extreme precipitation over continental-scale basins like the MRB. Extreme rainstorms in large 133 basins are likely to arise from different types of weather systems (e.g., tropical cyclones, 134 extratropical cyclones, mesoscale convective systems) exhibiting distinct precipitation 135 distributions and correlation structures (Barlow et al., 2019; B. Liu et al., 2020; Schumacher & 136 Rasmussen, 2020). Precipitation distributions also vary spatially, with regions near moisture 137 sources or subject to orographic lifting showing higher means and variances (Hitchens et al., 2013; Marra et al., 2022). Nonstationarity in spatial correlation structures is clearly evident in 138 139 high-resolution precipitation observations, with patterns varying significantly within different 140 parts of large weather systems like extratropical cyclones (Niemi et al., 2014; Nerini et al., 141 2017). Moreover, observed increases in extreme precipitation due to anthropogenic climate 142 change suggest temporal nonstationarity in precipitation properties, challenging the usage of 143 temporally stationary distributions of precipitation amount (Pan et al., 2016; Gori et al., 2022). 144 Consequently, stationary SRGs will struggle in several key respects to represent the strong 145 spatiotemporal nonstationarity of extreme precipitation within a large basin. Several efforts have 146 been made to model nonstationary precipitation fields (e.g., Fuglstad et al., 2015; Fouedjio et al., 147 2016; Nerini et al., 2017; Fouedjio, 2017), but to our knowledge, no SRG has been developed to simulate nonstationary precipitation fields for a basin as large as the MRB. 148

149 In this study, we propose the Space-Time nOnstationary Rainfall Model for Large Area Basins 150 (StormLab), an SRG that generates 6-hour, 0.03° resolution rainstorms over the MRB. Following 151 the meta-Gaussian framework, the model generates spatiotemporally correlated noise fields and 152 converts them to actual precipitation patterns via precipitation distribution functions. Unlike some other SRGs that simulate precipitation without conditioning on additional atmospheric 153 154 variables, StormLab performs conditional simulations where precipitation distributions vary 155 across space and time, dependent on large-scale atmospheric variables from global climate 156 models (GCMs). Such conditioning incorporates spatial and temporal nonstationarity of 157 precipitation and realistically represents how local-scale precipitation is embedded within and 158 driven by the larger-scale atmospheric environment. Nonstationarity in the spatial correlation of 159 precipitation is further addressed by replicating local correlation structures from precipitation 160 observations using a "local window" FFT approach. StormLab was used to generate 1,000 161 synthetic years of winter and spring rainstorms conditioned on GCM outputs. StormLab can 162 alternatively be understood as a stochastic downscaling approach, i.e., establishing statistical 163 relationships between coarse GCM data and local precipitation observations. In this study, 164 "rainstorm" is used to denote a heavy precipitation event, though it may contain snow or mixed-165 phase precipitation. All input data and stochastic simulations consider liquid water equivalent, 166 without distinction of precipitation phase. The remainder of the paper is organized as follows: 167 Section 2 describes the study basin and datasets. Section 3 details the proposed methods. Results 168 are shown in Section 4, followed by discussion in Section 5. A summary and conclusions are 169 provided in Section 6.

170

171 2 Study Site and Data Processing

172 The Mississippi River Basin, the largest watershed in North America, has a drainage area of 3.2 million km² and consists of five major subbasins: the Arkansas-Red, Missouri, Upper 173 174 Mississippi, Ohio-Tennessee, and Lower Mississippi (Figure 1a). Major flood events occur 175 during winter and spring (December-May), resulting in significant socioeconomic impacts across 176 the basin (Myers & White, 1993; Mutel, 2010; A. B. Smith & Katz, 2013). Due to the basin's 177 large and complex drainage network, the locations and spatiotemporal patterns of heavy 178 precipitation greatly influence flood responses and damages throughout the basin (Su et al., 179 2023).

180 The Analysis of Record for Calibration (AORC) dataset provides reference precipitation data at 6-hour, 0.03° resolution over the MRB from 1979-2021. The dataset was created by combining 181 182 hourly NLDAS-2 and Stage IV precipitation data (Fall et al., 2023), which we resampled to 6-183 hour intervals in this study. ERA5 reanalysis, produced by the European Centre for Medium-184 Range Weather Forecast (ECMWF), provides hourly, 0.25° fields of large-scale atmospheric 185 variable fields (Hersbach et al., 2018); we used precipitation, precipitable water, integrated water 186 transport (IVT), and 850 hPa winds over the study region from 1979-2021. ERA5 fields were 187 resampled to 6-hour intervals and spatially upscaled to match the resolution of the CESM2 dataset (described next), using the Python package "xESMF" (Zhuang et al., 2020). The CESM2 188 189 Large-Ensemble Project provides 100 ensemble members of historical (1850-2014) and future 190 projection (2015-2100; SSP3-7.0 scenario) simulations using the CESM2 model (Danabasoglu et 191 al., 2020; Rodgers et al., 2021). We used ten ensemble members from 1951-2050, equivalent to

- 192 1,000 years of atmospheric simulations, from which precipitation, precipitable water, IVT, and
- 193 850 hPa winds were retrieved at approximately 1-degree resolution and 6-hour intervals. The
 194 CESM2 large-scale atmospheric variables were bias-corrected against ERA5 data for each
- 195 season using the CDF-t method (M. Vrac et al., 2012; see Supporting Information S1).

196 **3 Methods**

197 **3.1 Rainstorm Identification**

198 Rainstorms associated with strong water vapor transport were identified over the MRB in ERA5 199 and CESM2. First, we applied a storm tracking method STARCH (Y. Liu & Wright, 2022b) to 200 identify periods of intense water vapor transport over the tracking domain (29-50° N, 79-113° W, Figure 1a). STARCH applies a high threshold (500 kg $m^{-1}s^{-1}$) to the IVT fields to identify 201 regions of intense water vapor transport at each time step. These regions were then expanded to a 202 lower threshold (250 kg m⁻¹s⁻¹) to capture more IVT areas. The identified IVT objects were 203 tracked across time by computing the overlap ratio between objects at consecutive time steps. If 204 205 the overlap ratio exceeds 0.2, the objects were considered the same IVT event and tracked 206 through time (Figure 1a, see Supporting Information S2 for more details). These IVT events 207 were considered as indicators for individual rainstorms and associated with concurrent 208 precipitation, precipitable water, and wind speed fields from ERA5/CESM2 data (e.g., Figure 209 1b). To focus on extreme events that potentially lead to large-scale flooding, we only selected 210 rainstorms with precipitation covering >10% of the basin for more than 24 hours. The starting 211 and ending periods of a rainstorm were also cropped if the precipitation coverage was <10% of 212 the basin. For ERA5 data, we identified 1,088 rainstorms from December-May during 1979-213 2021 and associated AORC precipitation (Figure 1c). For CESM2, we identified 26,166 214 rainstorms over the 1,000 years.



Figure 1. STARCH rainstorm tracking algorithm: (a) IVT object identification and tracking; (b)
Attached concurrent ERA5 precipitation and (c) AORC precipitation to identified IVT event.
The top-left panel also shows the Mississippi River Basin, including the (1) Lower Mississippi,
(2) Arkansas-Red, (3) Missouri, (4) Upper Mississippi, and (5) Ohio-Tennessee subbasins.

220

221 **3.2 Stochastic Rainfall Generator**

The Space-Time nOnstationary Rainfall Model for Large Area Basins (StormLab) was 222 developed based on ideas from Notaro et al. (2014) under a meta-Gaussian framework. For a 223 224 given ERA5/CESM2 rainstorm, 2D Gaussian noise was generated to represent precipitation 225 spatiotemporal structures and converted to precipitation amounts through parametric 226 distributions conditioned on large-scale atmospheric variables. Multiple noise realizations were 227 simulated to produce an ensemble of possible precipitation scenarios for each rainstorm event. 228 The model consists of three key components, described further below: 1) time-varying 229 precipitation distributions; 2) spatiotemporal noise generation; and 3) precipitation ensemble 230 simulation.

231 3.2.1 Time-Varying Precipitation Distributions

Parametric precipitation distributions conditioned on large-scale atmospheric variables were
 modeled at each 0.03° grid cell. The distributions consist of two components:

1) A precipitation occurrence model that predicts the probability of precipitation occurrence P_{wet} in a grid cell at time *t* using logistic regression (Sperandei, 2014):

236
$$P_{wet}(t) = \frac{1}{1 + \exp(-[\alpha_0 + \alpha_1 P R_m(t) + \alpha_2 P W_m(t)])}$$
(1)

where PR_m and PW_m are ERA5/CESM2 precipitation and precipitable water linearly interpolated to the grid, (i.e., the conditioned large-scale atmospheric variables). For model fitting and validation, PR_m and PW_m were obtained from ERA5 rainstorms (see Section 3.3.1). For model simulation, PR_m and PW_m were obtained from CESM2 rainstorms, under the assumption that the statistical relationships derived from ERA5 and AORC hold for CESM2 (see Section 3.3.2). α_0 , α_1 , and α_2 are fitted parameters.

243 2) A precipitation magnitude model that predicts the distribution of precipitation amount 244 in a "wet" grid cell using a transformed nonstationary gamma distribution (TNGD). Let *x* 245 represent the precipitation amount at a single grid point. We assumed *x* follows a generalized 246 gamma distribution with shape parameters a > 0, c > 0, and scale b > 0 (Stacy, 1962):

247
$$f_X(x,a,c,b) = \frac{c}{b\Gamma(a)} \left(\frac{x}{b}\right)^{ca-1} \exp(-\left(\frac{x}{b}\right)^c)$$
(2)

Applying a transformation $y = x^c$ gives $y \sim Gamma(a',b')$, where a' = a and $b' = b^c$ (see Supporting Information S3 for derivations):

250
$$f_{Y}(y,a',b') = \frac{1}{b'\Gamma(a')} (\frac{y}{b'})^{a'-1} \exp(-\frac{y}{b'})$$
(3)

251 The gamma distribution has mean $\mu = a'b'$ and variance $\sigma^2 = a'(b')^2$. To incorporate 252 nonstationarity, the mean μ was modeled as a function of large-scale atmospheric variables 253 (Scheuerer & Hamill, 2015):

254
$$\mu(t) = \frac{\mu_c}{\beta_0} \log \left\{ 1 + (\exp(\beta_0) - 1) \left[\beta_1 + \beta_2 \frac{PR_m(t)}{\overline{PR}} + \beta_3 \frac{PW_m(t)}{\overline{PW}} \right] \right\}$$
(4)

where μ_c is the mean of a stationary gamma distribution fitted to all the ERA5 rainstorms' AORC precipitation data at the grid, \overline{PR} and \overline{PW} are the climatological mean of ERA5 precipitation and precipitable water, $\beta_0, \beta_1, \beta_2$ and β_3 are fitted parameters. The variance σ^2 and shape parameter *c* were kept constant over time. The parameter *c* in the TNGD controls the tail heaviness of the distribution, allowing better fitting of extreme precipitation values compared to a basic gamma distribution. The transformation to variable *Y* also creates a simple relationship between the mean and distribution parameters that simplifies parameter estimation.

262 3.2.2 Spatiotemporal Noise Generation

263 We followed the STREAM (Hartke et al., 2022) approach, which itself is based heavily on 264 Pysteps (Nerini et al., 2017; Pulkkinen et al., 2019), to simulate spatiotemporally correlated 265 Gaussian noise that replicates the spatial and temporal dependency structure of the actual precipitation. Given an AORC precipitation field at the first time step t_0 of a rainstorm, we 266 divided the field into overlapping spatial windows and computed the 2D Fourier amplitude 267 268 spectrum within each window (Figure 2a). The amplitude spectra capture the spatial correlation 269 of precipitation within each window while omitting phase information that determines the exact 270 location of precipitation. For windows with less than 10% precipitation coverage, the global 271 amplitude spectrum of the entire field was used.

272 Next, we filtered Gaussian white noise locally using the amplitude spectrum from each window:

273
$$n_{p_1,p_2} = FFT^{-1} \left\{ |R_{p_1,p_2}| \circ \mathbf{N} \right\}$$
(5)

where n_{p_1,p_2} is the spatially correlated noise field, $|R_{p_1,p_2}|$ is the local amplitude spectrum for the 274 window at the position (p_1, p_2) , N is the Fourier Transform of uncorrelated Gaussian white 275 276 noise, and (o) is pointwise multiplication. This generates noise with local anisotropy and spatial 277 dependence matching that of the original AORC precipitation within the window. The final 278 nonstationary noise field is obtained by summing the local noise from all windows. We chose an 279 empirical window size of (128, 128) grid cells to balance capturing localized precipitation 280 structures and maintaining sufficient samples within each window. A 0.3 overlap ratio between 281 windows was chosen to improve the smoothness of the final noise field. For any overlapping 282 regions between windows, the noise values were averaged.

The correlated noise field at the next time step t+1 was modeled by a lag-1 autoregressive (AR(1)) model (Figure 2b):

285
$$n(t+1, i, j) = \rho n(t, i-v_t, j-u_t) + \sqrt{1-\rho^2} n(t+1, i, j)$$
(6)

where n(t+1, i, j) is the noise value to be calculated at time t+1 and grid cell position (i, j), $n(t, i-v_t, j-u_t)$ is the noise value at time t, which means the noise is advected by the 850hPa north-south and east-west wind vector (v_t, u_t) from position $(i-v_t, j-u_t)$ at time t to position (i, j) at time t+1, n(t+1, i, j) is the new spatially correlated noise generated at t+1, and ρ is the correlation coefficient between AORC precipitation at t and t+1.



291

Figure 2. Process of spatiotemporal precipitation generation. (a) Generating spatially correlated noise using local window FFT based on high-resolution AORC precipitation; (b) Generating temporally correlated noise using the AR(1) model. ρ represents the correlation coefficient between precipitation fields at *t* and *t*+1; (c) Transforming noise fields to precipitation fields through CDFs.

Generating space-time correlated noise requires amplitude spectra from high-resolution AORC precipitation. For CESM2 rainstorms, corresponding high-resolution precipitation fields are not available. To obtain plausible spatial precipitation structures, we used a precipitation field matching method to find surrogate high-resolution precipitation fields from historical AORC

data (Figure 3). This matching assumes rainstorms with similar large-scale atmospheric
 conditions will exhibit similar precipitation patterns and spatial correlations. For each CESM2
 time step, we computed the average Euclidean distance between the CESM2 atmospheric
 variable fields (precipitation, precipitable water, and IVT) and those from ERA5 rainstorms
 during 1979-2021:

306
$$d = \sqrt{\sum_{i=1}^{N} (PR_{ERA5,i} - PR_{CESM2,i})^{2}} + \sqrt{\sum_{i=1}^{N} (PW_{ERA5,i} - PW_{CESM2,i})^{2}} + \sqrt{\sum_{i=1}^{N} (IVT_{ERA5,i} - IVT_{CESM2,i})^{2}}$$
(7)where N is the

307 closest matching ERA5 time steps based on minimum Euclidean distance to the CESM2 time 308 step. One of those five time steps was randomly sampled with a probability proportional to the 309 inverse of the Euclidean distance. This ensures that ERA5 time steps with more similar 310 atmospheric variables to the CESM2 time step have a higher probability of being sampled. The 311 sampled time step provides an AORC precipitation field serving as the surrogate high-resolution 312 precipitation, whose amplitude spectra are then used to generate spatially correlated noise. This 313 matching is performed independently at each time step. In effect, this finds historical 314 precipitation fields that resemble the spatial structure of a given CESM2 rainstorm. It only uses local and global amplitude spectra and discards the phase information, thus reducing effects from 315 316 location mismatch between AORC and CESM2 precipitation. In rare cases when the local 317 window with CESM2 precipitation contained no AORC data, the global amplitude spectrum of 318 surrogate precipitation field was used as an approximation. Since the sampled AORC fields may lack temporal continuity, the correlation coefficient ρ in Equation 6 was computed based on 319 320 CESM2 precipitation when simulating CESM2 rainstorms.



Figure 3. Process of precipitation field matching method. (1) Randomly selecting an ERA5 time step with the closest atmospheric variable fields to those at the target CESM2 time step; (2) Using AORC precipitation at the sampled ERA5 time step to provide local (high-resolution) amplitude spectra for the precipitation simulation at the target CESM2 time step.

326 3.2.3 Precipitation Ensemble Simulation

The actual precipitation field was obtained by transforming the noise field through the timevarying parametric precipitation distributions described in Section 3.2.1 (Figure 2c). The correlated noise field was first converted to a probability field using the CDF of the standard Gaussian distribution. If the transformed probability P(t) at one grid is lower than the current dry probability $P_{dry}(t) = 1 - P_{wet}(t)$, the simulated precipitation was set to zero. If P(t) exceeded

332 $P_{dry}(t)$, the excess was rescaled to 0-1 as $P'(t) = \frac{P(t) - P_{dry}(t)}{1 - P_{dry}(t)}$. This P'(t) was then converted to

333 y(t) using the inverse CDF of the fitted nonstationary Gamma distribution (Equation 3). The 334 final precipitation x(t) was calculated as $x(t) = y(t)^{1/c}$. Multiple correlated noise realizations for 335 a rainstorm can be transformed to generate an ensemble of precipitation realizations. Each 336 realization represents a possible precipitation scenario consistent with the current large-scale 337 atmospheric state.

338

339 3.3 Experimental Design

An overview of the model experiments is as follows: The SRG was first fitted based on AORC precipitation and ERA5 large-scale atmospheric variables and then cross-validated (Section 3.3.1). Then, the SRG was used to simulate precipitation fields based on large-scale atmospheric variables from CESM2 rainstorms (Section 3.3.2). This assumes that the ERA5 and biascorrected CESM2 rainstorms share similar characteristics, such that the established ERA5-AORC relationships can be applied to downscale CESM2 storms.

346 3.3.1 Model Fitting and Evaluation

347 The precipitation occurrence and magnitude models were fitted for each month from December-348 May using ERA5 rainstorms for 1979-2021. Fitting was performed at each 0.03° grid in the 349 study domain (29-50°N, 79-113°W), consisting of 1024×630 grids. 6-hour AORC precipitation 350 data and associated large-scale atmospheric variables (ERA5 precipitation and precipitable 351 water) were extracted from the identified rainstorms. For logistic regression, AORC data were 352 converted to 0 (dry) or 1 (wet) using a 0.2 mm threshold. The Python Scikit-learn package 353 (Pedregosa et al., 2011) was used to estimate the logistic regression parameters using maximum 354 likelihood. For TNGD, samples with AORC precipitation > 0.2 mm were first fitted to a 355 stationary generalized gamma model using Scipy's method of moments (Virtanen et al., 2020). 356 The estimated shape parameter c transformed the precipitation data as $y = x^{c}$. The new variable 357 y was then fitted to the nonstationary gamma distribution (Equation 3). The distribution

parameters were estimated by minimizing the mean continuous ranked probability score (CRPS,
 units in mm, see Hersbach, 2000) using the optimization function in Scipy:

$$\frac{1}{n}\sum_{t=1}^{n} CRPS(F_t, y_t)$$
(8)

361
$$CRPS(F_t, y_t) = \int_{-\infty}^{+\infty} \left[F_t(z) - \mathbf{1} \left(y_t \le z \right) \right]^2 dz$$
(9)

where F_t is the CDF of nonstationary gamma distribution at time t, y_t is the transformed AORC precipitation, n is the sample size, and $\mathbf{1}(y_t \le z)$ is a Heaviside step function, which is 1 if $y_t \le z$ and 0 otherwise. CRPS measures the distance between CDFs, so minimizing the mean CRPS minimizes the difference between the parametric and empirical CDFs of observations at the grid.

367 We used five-fold cross-validation to evaluate out-of-sample model performance. The original 43 years of data were split into five folds. Each fold used 80% of the data for fitting and 20% for 368 369 simulation and validation. To simulate an ERA5-based rainstorm, we first generated correlated 370 noise fields based on AORC precipitation. The noise fields were converted to precipitation 371 through the precipitation distributions conditioned on ERA5 large-scale atmospheric variables. 372 We simulated 20 realizations per rainstorm, representing possible precipitation patterns other 373 than the AORC precipitation. Looping through the five folds generated out-of-sample 374 simulations for all ERA5 rainstorms. Key rainstorm characteristics, including total precipitation, 375 rainstorm area, and space-time autocorrelation, were compared between simulations and AORC 376 data. The Brier Score (BS) was used to assess the accuracy of predicted precipitation occurrence 377 (wet or dry) at each grid (Benedetti, 2010):

378
$$BS(x_t, x_t) = \frac{1}{K} \sum_{i=1}^{K} \left[\mathbf{1}(x_{t,k} > 0) - \mathbf{1}(x_t > 0) \right]^2$$
(10)

379 where K is the number of realizations, $x_{t,k}$ is the kth simulated precipitation at time t, and x_t is the AORC precipitation. BS ranges from 0 to 1, with 0 for perfect accuracy and 1 for highly 380 381 inaccurate predictions. We also computed the Reduction CRPS as 382 $RCRPS(F_t, x_t) = CRPS(F_t, x_t) / \sigma_0$ to assess the accuracy of simulated precipitation distribution at each grid cell (Trinh et al., 2013), where F_t is the empirical CDF of the 20 precipitation 383 384 realizations at time t and σ_0 is the climatological standard deviation of the AORC precipitation. Compared to the original CRPS, RCRPS is normalized and thus independent of the precipitation 385 386 magnitude at each grid, allowing spatial comparison of model performance across grids and 387 subbasins. A lower value of RCRPS indicates a smaller difference between the CDFs of 388 simulated and AORC precipitation.

390 3.3.2 CESM2-based Rainstorm Simulations

391 Random precipitation fields were generated for storms identified from ten CESM2 ensemble 392 members. For a given event, surrogate high-resolution precipitation fields were sampled using 393 the matching method described above (Section 3.2.2). Correlated noise was then generated and 394 transformed to precipitation based on the precipitation distributions conditioning on CESM2 395 large-scale atmospheric variables. To reduce computation, we simulated a single realization for 396 each rainstorm in the ten 100-year members. The resulting storm total precipitation distributions 397 were then compared against ERA5 rainstorm total precipitation accumulations. To estimate 398 extreme precipitation frequency, we selected simulated annual maxima rainstorms and generated 399 20 additional realizations for each event. Bootstrap resampling was implemented to estimate 400 return levels of annual maxima: 1) In each bootstrap sample, we randomly resampled 1,000 years 401 with replacement and selected one of the 20 annual maxima realizations for each year. 2) The 402 empirical CDF of annual maximum precipitation was obtained from this bootstrap sample. 3) 403 Steps 1 and 2 were repeated 5,000 times to estimate mean return levels and their uncertainties.

404

405 **4 Results**

406 4.1 ERA5 and CESM2 Rainstorm Identification

407 The STARCH method identified intense IVT events and associated rainstorms over the basin 408 (e.g., Figures 1, 4a-c, and 11a-c). The tracked IVT objects exhibited an elongated, narrow 409 structure, with IVT intensities gradually decreasing from the center toward the edge. In most 410 cases, only one IVT object was present at a time, moving steadily from southwest to northeast 411 across the basin. Extensive precipitation associated with fronts or extratropical cyclones was 412 found near the high IVT regions, showing a clear link between strong water vapor transport and 413 heavy precipitation in the basin. Good agreement was found between ERA5 precipitation 414 patterns and AORC data, suggesting that ERA5 simulates realistic large-scale atmospheric 415 environments in which the rainstorms occurred. However, precipitation locations and spatial distributions still differed at local scales, suggesting that local precipitation is partly determined 416 417 by mesoscale or finer-scale processes that are not fully resolved by ERA5 or CESM2.

418 Similar rainstorm characteristics were found between the 1,088 rainstorms identified from the 419 43-year ERA5 data and the 26,166 rainstorms from the 1,000-year bias-corrected CESM2 data 420 (Figure 5). The average annual number of rainstorms from December-May was 25 in ERA5 and 421 26 in CESM2 (Figure 5a). Average storm duration was around 50 hours for both datasets, with 422 maximum durations reaching 240 hours in ERA5 and 300 hours in CESM2 (Figure 5b). The 423 average precipitation area over the basin was about 1.3 million km², approximately 40% of the 424 basin area (Figure 5c). The average precipitation rate of the CESM2 rainstorms was 0.8% lower 425 than that of the ERA5 rainstorms, while average precipitable water and IVT were 4.3% and 3.5% 426 higher (Figures 5d-f). However, CESM2 rainstorms had a larger standard deviation in average 427 precipitation rate (18% higher) and lower standard deviations in average precipitable water and 428 IVT (5.2% and 5.9% lower). CESM2 rainstorms also exhibited more extreme values, with the 429 largest average precipitation rate of 10.6 mm/6-hour compared to 7 mm/6-hour in ERA5 430 rainstorms, likely due to its larger sample size and the inclusion of future climate projections.

431 Overall, bias-corrected CESM2 data produced rainstorms generally consistent with those from

the ERA5 data, allowing them to be used for precipitation simulations. However, more complex

433 biases may still exist in the CESM2 data after correction which can introduce some uncertainty

434 into the simulated results.



435

Figure 4. Spatiotemporal patterns of an ERA5-based rainstorm starting at 00:00 UTC on 29
December 2019. (a) ERA5 IVT; (b) ERA5 precipitation; (c) ERA5 precipitable water; (d) The
first realization of noise; (e) The first realization of simulated precipitation; (f) The second

439 realization of simulated precipitation; (g) Reference AORC precipitation.

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Figure 5. Distributions of rainstorm characteristics from ERA5 (blue) and CESM2 (red). (a)
Annual storm number from Dec-May; (b) Storm duration; (c) Average precipitation area; (d)
Average precipitation rate; (e) Average precipitable water; (f) Average integrated water vapor
flux. *n* is the total number of identified rainstorms.

445 4.2 ERA5-based Rainstorm Simulation

440

446 This section evaluated the model performance by examining ERA5-based simulated rainstorms. 447 Figure 4 shows a rainstorm in December 2019, with typical atmospheric conditions conducive to 448 heavy precipitation: an extratropical cyclone system with intense IVT and high precipitable 449 water covering the basin (Figures 4a-c). The Gaussian noise field captured local spatial structures 450 of the AORC precipitation (Figure 4d). For example, the upper-left noise was smooth and 451 continuous in the first time step, while the lower-right was fragmented, consistent with the 452 AORC precipitation pattern (Figure 4g). The simulated spatiotemporal precipitation patterns 453 were similar to the AORC data, exhibiting a comma-like structure (Figure 4e-g). The model 454 simulated heavy precipitation in the regions of high ERA5 precipitation and precipitable water, 455 showing positive relationships between the mean of precipitation distributions and large-scale atmospheric variables. Notably, however, the local patterns and hotspots of simulated 456 457 precipitation differed from the AORC and varied across realizations, reflecting the stochasticity 458 in small-scale precipitation processes that could occur under the same large-scale atmospheric 459 conditions.

The simulated and AORC total precipitation fields for three extreme rainstorms preceding the 1997, 2011, and 2018 Mississippi Floods are shown in Figure 6. The rainstorms exhibited similar spatial orientations and coverage, with intense precipitation concentrated over the Lower Mississippi and Ohio-Tennessee subbasins. The simulated precipitation patterns and amounts generally agreed with the AORC data. For instance, total precipitation realizations for the 2018

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465 event ranged from 47-57 mm with a mean of 51 mm, close to the 50 mm amount from AORC. 466 Intense precipitation locations and shapes, however, differed from the AORC precipitation since 467 the simulations were intended to represent multiple possible extreme scenarios. The AORC 468 precipitation displayed stronger anisotropy and long-range spatial dependence (e.g., the more 469 elongated arc of intense precipitation in the 1997 event, Figure 6b). Also, the AORC 470 precipitation exhibited more continuous spatial variations, while the simulations contained more 471 fragmented, noise-like textures. This suggests some discrepancies remained between the 472 simulated and AORC precipitation structures that were not fully captured by the rainfall 473 generator.

474 Consistency also existed in the temporal evolution of total precipitation, with the SRG ensemble 475 spread (i.e., min to max) containing most of the AORC time series (Figure 7). Notably, the SRG 476 ensemble mean only represents the general trend of simulations and does not necessarily match 477 the AORC data. This is because the AORC precipitation is also a single realization of an 478 underlying random process, and therefore may not always locate at the mean at every time point. 479 The spatial autocorrelation functions (ACF) of simulated rainstorms were generally consistent 480 with the AORC data, though slightly lower from 0-400 km (Figures 8a-b). The rainstorms 481 exhibited short-term dependence, with temporal ACF dropping below 0.2 after the 12-hour lag 482 (Figure 8c), supporting the AR(1) temporal noise model. The lag-1 ACF coefficients matched 483 the AORC, except being lower for the 2018 event. The spatiotemporal patterns, precipitation 484 time series, and ACFs were also examined for the remaining ERA5 rainstorms, yielding similar agreement. 485



488 Figure 6. Total precipitation patterns from simulations (single realization) and AORC for 489 rainstorms in (a-b) February 1997, (c-d) April 2011, and (e-f) February 2018.



491 Figure 7. Time series of basin-average total precipitation from simulations (blue) and AORC
492 (grey) for rainstorms in (a) February 1997, (b) April 2011, and (c) February 2018.

493

494 Basin-average total precipitation and rainstorm areas were compared between SRG ensemble 495 mean and AORC for each rainstorm. Despite the model generating variant spatiotemporal 496 patterns across realizations, the average total precipitation was highly correlated with the AORC 497 data (correlation $\rho = 0.97$, Figure 9a). This consistency is partly because the time-varying 498 precipitation distributions were conditioned on ERA5 large-scale atmospheric variables, which 499 indirectly limits the range of possible precipitation values. Another explanation stems from the 500 atmospheric water balance: the net water vapor flux entering the river basin—governed by large-501 scale atmospheric variables—should balance the total precipitation during an event, thus 502 constraining possible precipitation amounts. There was also agreement in total and intense (>20 503 mm) precipitation areas between simulations and reference ($\rho = 0.90$ and 0.96, Figures 9b-c). For smaller rainstorms under 1.5 million km^2 , however, the model tended to overestimate total 504 505 precipitation area. This overestimation may result from the coarse ERA5 data not fully resolving 506 the AORC precipitation patterns and overpredicting precipitation occurrence in some cases. 507 Figure 1b at 18 hours, for example, illustrated an example where ERA5 generated greater 508 precipitation coverage than the AORC data.

509 Model performance was also evaluated at each grid cell using the Brier Score and RCRPS. The 510 average BS over the basin was 0.11, suggesting good accuracy in simulating precipitation 511 occurrence (Figure 10a). Comparatively higher BS was seen in the Ohio-Tennessee (an average 512 of 0.14), Lower Mississippi (0.13), and Upper Mississippi (0.12) subbasins (Figure 10b). The 513 average RCRPS was 0.18 over the basin, indicating satisfactory agreement between simulated 514 and AORC precipitation distributions (Figure 10c). The Ohio-Tennessee subbasin had the 515 highest average RCRPS of 0.25, followed by 0.22 in the Lower Mississippi and 0.2 in the Upper 516 Mississippi subbasins (Figure 10d). The BS and RCRPS patterns suggest higher SRG 517 uncertainties in regions experiencing more extreme and frequent rainstorms. A potential 518 explanation is that these regions exhibit more complex precipitation processes, e.g., localized 519 convection or orographic lift, leading to more uncertain relationships between large-scale 520 atmospheric environments and local precipitation.



523 Figure 8. Comparison of spatiotemporal ACFs between simulations (red) and AORC data (black)

for rainstorms in February 1997, April 2011, and February 2018. (a1-3) Average spatial ACF in

525 East-West direction; (b1-3) Average spatial ACF in North-South direction; (c1-3) temporal ACF

526 of basin-average total precipitation.



528 Figure 9. Comparison of rainstorm characteristics between SRG ensemble mean and AORC for 529 each ERA5 rainstorm. (a) Basin-average total precipitation; (b) Total precipitation area; (c) Total 530 intense precipitation area (>20 mm).



Figure 10. (a) Brier Scores across the simulation domain; (b) Boxplots of Brier Scores in the
Mississippi River Basin and the five subbasins; (c) Reduction CRPSs across the simulation
domain; (d) Boxplots of Reduction CRPSs in the Mississippi River Basin and the five subbasins.
The five subbasins are also shown in (a), including (1) Lower Mississippi, (2) Arkansas-Red, (3)
Missouri, (4) Upper Mississippi, and (5) Ohio-Tennessee subbasins.

538 4.3 CESM2-based Rainstorm Simulation

539 Spatiotemporal patterns of a CESM2 rainstorm in March 2022 are shown in Figure 11. The 540 matched AORC precipitation, despite being sampled from different time steps, exhibited similar locations and shapes to the CESM2 precipitation, providing reasonable approximations of 541 precipitation spatial structures. The simulated precipitation fields more closely resembled the 542 543 CESM2 patterns rather than the sampled AORC fields. This is because precipitation occurrence and magnitude are mainly determined by precipitation distributions conditioned on CESM2 544 large-scale atmospheric variables, while only the spatial correlation was drawn from the AORC 545 546 data. The SRG realizations had very similar precipitation coverage and shape, with differences 547 primarily in intense precipitation locations. This indicates the SRG ensemble members are not fully independent due to their shared reliance on the same large-scale atmospheric conditions. 548



Figure 11. Spatiotemporal patterns of a CESM2-based rainstorm starting at 12:00 UTC on 25 March 2022. (a) CESM2 IVT; (b) CSEM2 precipitation; (c) CSEM2 precipitable water; (d) Matched AORC historical precipitation at 2020-03-29 00:00, 1983-04-14 06:00, and 2019-03-15 00:00, respectively; (e) The first realization of simulated precipitation; (f) The second realization of simulated precipitation. The date does not mean this CESM2 rainstorm happened historically, as it is generated from CESM2 simulations representing various atmospheric states under historical forcing.

549

557 One realization of simulated total precipitation and associated atmospheric variables for three 558 extreme rainstorms are shown in Figure 12. The events produced basin-average precipitation of 559 67, 75, and 91 mm, roughly equivalent to 50-year, 100-year, and 1,000-year events based on the 560 GEV distribution fitted to the reference data (GEV fitting shown in Figure 15). All three 561 rainstorms occurred in early spring and were characterized by long-lived (>160 hours) strong 562 water vapor transport from the Gulf of Mexico. Consistent with ERA5 rainstorms, the simulated 563 precipitation patterns exhibited southwest-northeast orientations, with intense precipitation near 564 high CESM2 precipitation and precipitable water. The 1,000-year event displayed the greatest area of heavy precipitation (>300 mm), covering nearly the entire Lower Mississippi and parts of 565 adjacent subbasins. Multiple peaks occurred in the simulated precipitation series (Figure 13), 566 567 which is also an evident trait in ERA5 rainstorms (Figure 7). Despite showing similar trends to the original CESM2 precipitation, SRG ensemble members showed greater variability in 568 569 temporal precipitation series. This enhanced variability in the SRG ensembles is more evident in 570 the total precipitation amounts. For example, the total precipitation produced by the SRG ranged 571 from 79-99 mm for the April 1962 event, whereas the bias-corrected CESM2 data showed 78 572 mm (Figure 13c). This suggests that the SRG introduces variability not only in the 573 spatiotemporal precipitation patterns but also in the total precipitation amount of a rainstorm

- 574 event. This randomness could potentially cover more extreme scenarios and lead to important
- 575 variability in flood responses.

576



577

- 578 Figure 12. Total CESM2 precipitation (a, d, g), average CESM2 precipitable water (b, e, h), and
- 579 total simulated precipitation (c, f, i) of CESM2 rainstorms in March 2022 (162 hours), April

580 2029 (162 hours), and April 1962 (264 hours).



Figure 13. Time series of basin-average total precipitation of CESM2 rainstorms in (a) March 2022, (b) April 2029, and (c) April 1962. The dark blue line represents the rainstorm realization shown in Figure 12, and the grey lines represent the 20 SRG ensemble members. The light blue line represents the bias-corrected CESM2 data.

586 The simulated total precipitation distributions of all CESM2 rainstorms (26,166 events) generally 587 agreed with the AORC data for each month (Figure 14). An exception was May, where the 588 model tended to simulate more events with 5-10 mm total precipitation. The L-kurtosis, a 589 measure of tail heaviness based on L-moments (T. J. Smith et al., 2023), was computed for each 590 month using 1,000 bootstraps (Table 1). The average L-kurtosis was 0.161 for the simulations 591 and 0.167 for the AORC, suggesting similar tail behavior. However, the simulations exhibited 592 significantly lighter tails (lower L-kurtosis) in December and heavier tails (higher L-kurtosis) in 593 May, likely resulting from remaining biases in CESM2 data.

Month	Dec	Jan	Feb	Mar	Apr	May	Avg
CESM2 rainstorms	0.147	0.156	0.154	0.159	0.153	0.196	0.161
AORC	0.218	0.138	0.175	0.155	0.166	0.151	0.167

594 **Table 1.** L-kurtosis of total precipitation distributions.

595 The return levels of annual maximum total precipitation from the CESM2-based simulations 596 agreed well with the AORC data (Figure 15). In contrast, the original bias-corrected CESM2 data 597 significantly underestimated the extreme precipitation, indicating it may not be suitable for 598 directly representing extreme scenarios. For comparison, a GEV distribution was also fitted to 599 the reference annual maxima using L-moments (T. J. Smith et al., 2023). The simulation-based 600 estimates were slightly higher than the GEV for 10-500 year recurrence intervals, staying closer 601 to the AORC precipitation points. However, beyond 500 years, the estimates increased slower 602 than the GEV, yielding a lower 1,000-year return level of 89 mm. This likely results from the 603 GEV exhibiting a heavy, unbounded tail (shape parameter = 0.1), while the SRG simulations 604 seem to be bounded. The bounded tail implies a maximum possible event in the basin, which is 605 physically reasonable because the precipitation was constrained by large-scale atmospheric 606 variables and space-time correlation structures. Also, the GEV tail behavior can be highly 607 uncertain due to fitting difficulty given the limited sample size. The tail heaviness of extreme 608 precipitation has been investigated and discussed by many studies (e.g., National Research 609 Council, 1994; Koutsoyiannis, 2004; Papalexiou & Koutsoyiannis, 2013; Francesco Serinaldi & 610 Kilsby, 2014; Marani & Ignaccolo, 2015). It is widely recognized that commonly-used extreme value distributions (including GEV) may not adequately represent the underlying precipitation 611 612 processes, causing high certainties at the tails. To address this, the SRG may help investigate tail 613 behavior based on large-sample simulations. Our results suggest the extreme precipitation driven 614 by strong IVT is bounded in the Mississippi River Basin.



616 Figure 14. Distributions of basin-average total precipitation between simulated CESM2 617 rainstorms (red) and AORC (black) from December-May. *n* is the total number of rainstorms.





619

620 Figure 15. Comparison of estimated return levels of annual maximum precipitation based on

- 621 stochastic simulations conditioned on CESM2 rainstorms (blue line) and the GEV distribution
- 622 (red line). Dashed gray line represents the original bias-corrected CESM2 precipitation.

623 **5. Discussion**

624 **5.1 Contribution to Design Flood in the lower Mississippi River Basin**

The simulations from our SRG potentially contribute to the design flood in MRB by extending 625 the choice of historical rainstorms to over 1,000 synthetic years of events based on CESM2 data. 626 627 The design flood study of V. A. Myers (1959) revealed that it is a critical sequence of extreme 628 rainstorms—as opposed to a single precipitation event—that is likely to cause the most extreme 629 flood in the lower MRB (Su et al., 2023). In other words, the flood magnitude depends on storm 630 arrival orders in the MRB and time of concentration-flood discharge due to downstream 631 precipitation can be amplified by peak discharges coming from upstream rainstorms. While 632 developing new hypothetical storm sequences is beyond the scope of this study, we highlight that 633 rainstorm sequences can be determined directly based on their arrivals in CESM2 simulations, providing a large number of more physically reasonable sequences compared to "manual 634 635 creation" of one (as in Hypo-Flood 58A) or several sequences. Note that a rainstorm sequence 636 with 1,000-year total precipitation does not necessarily result in a flood with 1,000-year peak 637 discharge-because flood responses are determined by many other factors, including watershed antecedent conditions, spatiotemporal precipitation patterns, and arrival locations of rainstorms. 638 639 Therefore, future work will be to integrate the precipitation simulations with large-scale 640 hydrologic models to analyze peak discharge outputs and identify rainstorm sequences that result 641 in floods at certain average recurrence intervals.

642 **5.2 Model Uncertainty**

Multiple sources of uncertainties exist in the model that can influence the simulation results. This
 section discusses three major sources of uncertainty: 1) uncertainty from data sources; 2)
 uncertainty from precipitation distributions; and 3) uncertainty from precipitation spatiotemporal
 structures.

647 5.2.1 Uncertainty from Data Sources

648 Biases in datasets contributed to model uncertainties in different ways. The AORC data, created 649 by downscaling daily NLDAS-2 and Stage IV precipitation, may not fully capture the extreme 650 precipitation values at hourly scales, introducing bias when fitting precipitation distributions and 651 extracting precipitation spatial correlation structures. Meanwhile, although ERA5 is corrected by extensive observations via data assimilation, biases in large-scale atmospheric variables can be 652 653 higher in earlier periods (e.g., before 2000) due to fewer available observational data. A key 654 source of uncertainty arises from the way in which ERA5 and CESM2 data are used-while the 655 rainfall generator was fitted based on AORC precipitation and ERA5 large-scale atmospheric 656 variables, CESM2 data were used as covariates to simulate rainstorms, assuming the fitted 657 statistical relationships can be transferred to CESM2. Though bias correction of CESM2 was 658 used to reduce its biases relative to ERA5, this could not eliminate discrepancies in spatial and 659 temporal correlation structures. For instance, CESM2 tends to generate smoother precipitation

patterns and larger areas of intense IVT and precipitable water than ERA5. To reduce this
uncertainty, more advanced bias correction methods could be applied to correct temporal and
spatial structures of CESM2 data (e.g., Cannon, 2018; Mathieu Vrac & Thao, 2020; Robin &
Vrac, 2021). However, one must be careful, as such multi-aspect bias corrections may damage or
lose information in the original CESM2 data, such as climate change signals.

665 5.2.2 Uncertainty from Precipitation Distributions

666 The logistic regression model achieved good prediction accuracy in precipitation occurrence (see 667 Brier Scores in Figure 10), although spatial variations existed across sub-basins. Greater uncertainties originated from fitting the TNGD due to its more complicated structure and larger 668 number of parameters. Compared to simply fitting a Gamma distribution, TNGD improved tail 669 670 fitting with the transformation parameter c. However, some grids still exhibited under- or overestimation of extreme precipitation. The transformed variable y was assumed to have a 671 672 distribution mean being a non-linear function of large-scale atmospheric variables and a constant 673 standard deviation, which simplified climate change influences as precipitation variance may 674 increase in the future. Also, while maximum precipitable water scales at approximately 7%/K 675 following the Clausius-Clapeyron equation (Held & Soden, 2006), extreme precipitation 676 generally does not follow the same scaling, depending on factors such as precipitation type, 677 duration, and location (Lochbihler et al., 2017; X. Zhang et al., 2017; Fowler et al., 2021). This suggests that precipitable water, when used as the covariate, may increase faster than actual 678 679 precipitation in future projections, resulting in overestimation of the distribution mean. A 680 possible modification is to use an alternative covariate suggested by O'Gorman & Schneider 681 (2009) that incorporates vertical velocity and moist-adiabatic lapse rate, which scales better with 682 extreme precipitation under temperature changes.

683 5.2.3 Uncertainty from Precipitation Spatiotemporal Structures

684 While the SRG generated overall consistent rainstorm patterns and shapes as compared to the 685 AORC precipitation, approximating precipitation structures using spatiotemporal Gaussian noise can introduce uncertainties in precipitation simulations. Although the local-window FFT 686 improved representation of local precipitation spatial structures, it reduced samples in each 687 688 window and decreased the accuracy of the frequency spectrum that is related to spatial 689 correlation. Precipitation fields in the local windows were assumed to be stationary, which can 690 neglect some highly localized, anisotropic precipitation processes like local convection and 691 orographic effects. Cropping the precipitation field by window also lost precipitation long-range 692 structural information, weakening long-range dependence in simulated precipitation (see Section 693 4.2). Non-linear transformation via precipitation distributions can reduce correlation strength 694 between Gaussian noise (Papalexiou, 2022), which may explain the reduced spatial ACF of 695 simulated rainstorms (see Section 4.2 and Figure 8). One possible improvement is applying a 696 residual spectrum to the original amplitude spectrum as compensation for correlation loss during 697 transformation. For the time dimension, the AR(1) model omitted precipitation temporal 698 dependence beyond lag 2 and can also be influenced by the quality and resolution of 850 hPa 699 wind data, introducing additional uncertainties.

Another key source of uncertainty came from the precipitation field matching method for
 CESM2 rainstorms. The sampled AORC precipitation—although sharing similar large-scale

702 atmospheric variable patterns-can have different spatial coverage and local precipitation 703 structures to the unknown precipitation fields of CESM2 rainstorms. Also, independent sampling 704 at each time step reduced temporal coherence of spatial structures, compared to ERA5 rainstorm 705 simulations based on consecutive AORC fields. Since AORC data only covered 43 years, there 706 might be no suitable AORC precipitation pattern for some CESM2 rainstorms. These 707 uncertainties arose from the rainfall generator's reliance on high-resolution precipitation for 708 spatial correlation information. To address this limitation, one potential solution is to use 709 parametric spatial correlation models for each local window, with coefficients estimated based 710 on current large-scale atmospheric variables. Another way is leveraging deep learning models 711 like generative adversarial networks (e.g., pix2pix, Isola et al., 2017) or diffusion models (e.g., 712 SR3, Saharia et al., 2022) to predict high-resolution precipitation or spectrum fields.

713 **6 Summary and Conclusions**

714 In the study, a stochastic rainfall generator StormLab was proposed to simulate 6-hour, 0.03° 715 rainstorm fields driven by strong water vapor flux in the Mississippi River Basin. The model 716 used a local-window FFT-based approach to simulate nonstationary Gaussian noise fields that 717 capture local spatial structures and temporal evolution of precipitation. The noise fields were 718 transformed into actual precipitation amounts through precipitation distributions at each grid cell, 719 where precipitation occurrence was modeled with logistic regression, and precipitation 720 magnitude was represented by a transformed nonstationary Gamma distribution (TNGD). The 721 precipitation probability and mean of TNGD were modeled as functions of large-scale 722 atmospheric variables (i.e., ERA5/CESM2 precipitation and precipitable water) to incorporate 723 spatial and temporal nonstationarity. Given the large-scale atmospheric variable fields of a 724 rainstorm, the model could simulate multiple precipitation realizations representing possible 725 precipitation scenarios.

726 Spatiotemporal rainstorms driven by strong IVT were identified from 1979-2021 based on ERA5 727 reanalysis and high-resolution AORC precipitation using a rainstorm tracking method STARCH. 728 Precipitation distributions were fitted monthly at each grid cell for December-May based on the 729 identified rainstorms. To evaluate the model performance, key rainstorm characteristics were 730 compared between StormLab simulations and reference data, including spatiotemporal patterns, 731 total precipitation, precipitation area, and spatiotemporal ACFs, showing good consistency. The 732 average Brier Score and Reduction CRPS were 0.11 and 0.18 over the basin, suggesting high 733 precipitation occurrence accuracy and good agreement between simulated and AORC 734 precipitation distributions. However, these metrics were higher in the Ohio-Tennessee, Lower, 735 and Upper Mississippi River Basins, indicating greater model uncertainty in regions with more 736 extreme precipitation.

The model generated 1,000 synthetic years of rainstorms based on events from 10 bias-corrected CESM2 ensemble simulations from 1950-2050. Since CESM2 lacks high-resolution precipitation data to extract spatial correlation structures, a matching algorithm was implemented to sample historical AORC fields for each CESM2 event. Empirical return levels of annual maximum precipitation from the simulations showed better agreement with the AORC data than the GEV distribution. In future work, the model simulations will provide extreme rainstorm samples to update the current design storm sequence in the lower Mississippi River Basin. 744 The proposed model overcomes limitations of existing SRGs by enabling nonstationary 745 simulations for continental-scale river basins. By focusing on the key flood-generating 746 mechanism (i.e., strong IVT), the model avoids the difficulty of representing multiple complex 747 weather systems and precipitation processes. Notably, while this study focused on simulating 748 winter and spring extreme rainstorms, the model can be extended for continuous, long-term 749 simulations. Another advantage is that the model predicts precipitation distributions—rather than 750 specific precipitation values—conditioned on large-scale atmospheric variables, followed by 751 generating multiple noise and precipitation realizations. This reflects the concept that there are 752 multiple possible local precipitation processes consistent with a given large-scale atmospheric 753 state. Therefore, the model can also be viewed not only as a step forward in stochastic rainfall 754 generation, but also as a novel stochastic downscaling approach to disaggregate coarse GCM 755 outputs. Compared to the original CESM2 precipitation, the model is able to introduce variability 756 not only in the spatiotemporal precipitation patterns but also in the total precipitation amounts. 757 This suggests that conditioning on large-scale atmospheric variables can provide additional information to improve SGR simulations of storm arrivals, spatiotemporal structures, and 758 759 precipitation distributions, particularly under a changing climate.

760 The model is not without limitations, however. One major shortcoming is its reliance on 761 sufficient gridded precipitation and atmospheric variable data for fitting distributions and 762 extracting precipitation spatial structures. Biases in the atmospheric variables will propagate 763 through the fitted precipitation distributions and influence simulations. There are also 764 uncertainties when using spatial correlation structures from historical AORC precipitation for 765 CESM2 rainstorms. One way to improve the model is to use parametric spatial covariance models for local precipitation structures, with parameters inferred from large-scale atmospheric 766 767 variable fields. Future work will integrate the model simulations with hydrologic models to study 768 influences of rainstorm sequences on basin flooding. Another direction is applying the model to 769 other weather systems like tropical cyclones that contribute to coastal floods and other extreme 770 hazards.

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777 Data Availability Statement

778 AORC precipitation data can be downloaded from the National Oceanic Atmospheric 779 Administration data storage website (https://hydrology.nws.noaa.gov/aorc-historic/). ERA5 780 Reanalysis is available at the **ECMWF** Climate Store data Data 781 (https://doi.org/10.24381/cds.adbb2d47). CESM2 Large-Ensemble Project data is available from 782 Climate Data Gateway at the National Center Atmospheric for Research

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- 783 (https://www.earthsystemgrid.org/dataset/ucar.cgd.cesm2le.atm.proc.6hourly_ave.html).
- 784 StormLab codes are available on GitHub (<u>https://github.com/lorenliu13/StormLab</u>). STREAM
- and STARCH codes are available on Zenodo (http://doi.org/10.5281/zenodo.6564982, Hartke,
- 786 2022, and https://doi.org/10.5281/zenodo.7091017, Y. Liu & Wright, 2022b). Other codes and
- 787 data from this study are available from the authors upon request.
- 788

789 **Conflict of Interest**

- 790 The authors declare no conflicts of interest relevant to this study.
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Water Resources Research

Supporting Information for

A climate model-informed nonstationary stochastic rainfall generator for design flood analyses in continental-scale river basins

Yuan Liu¹, Daniel B. Wright¹, and David J. Lorenz²

¹Department of Civil and Environmental Engineering, University of Wisconsin-Madison, Madison, WI, USA.

²Center for Climatic Research, University of Wisconsin-Madison, Madison, WI, USA.

Corresponding author: Yuan Liu (yliu2232@wisc.edu)

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Text S1 to S3 Tables S1

Additional Supporting Information (Files uploaded separately)

No additional supporting information.

Introduction

This file contains three text sections and one table. Text S1 provides details on the CDF-t method used for bias correcting the CESM2 data. Text S2 and Table S1 describe the overlapping ratio and parameter setting used in storm tracking method STARCH. Text S3 shows the mathematical derivation to transform a Generalized Gamma distribution to a Gamma distribution.

Text S1. CESM2 Bias Correction

The CDF-t method adjusts the CDF of an atmospheric variable from CESM2 to match the CDF from ERA5. Unlike quantile-quantile (QQ) mapping, it accounts for CDF changes between historical and future GCM simulations (Famien et al., 2018). A transformation *T* is obtained by linear interpolation to map the CDF of the atmospheric variable from CESM2 (F_{Gh}) to the CDF from ERA5 (F_{Sh}) over the calibration period (1979-2021). *T* is then applied to the CESM2 CDF for an early period (1951-1978) or future period (2022-2050), F_{Gf} , to generate an adjusted CDF F_{Sf} , which represents the "unobserved" ERA5 CDF for those periods. For the early and future periods, QQ mapping is performed between the CESM2 CDF F_{Gf} and adjusted CDF F_{Sf} to correct the CESM2 data. For the calibration period, QQ mapping is applied directly between F_{Gh} and F_{Sh} . This CDF-t method was applied to CESM2 precipitation, precipitable water, and IVT data for winter (Dec-Jan) and spring (Mar-May). More details on the CDF-t method can be found in Vrac et al. (2012).

Text S2. STARCH Overlapping Ratio and Parameter Setting

The overlapping ratio between two consecutive IVT objects was defined as follows (Liu & Wright, 2022):

$$R(t_1, i, j) = \frac{A}{A_i(t_1)} + \frac{A}{A_j(t_0)}$$
(S1)

where $R(t_1, i, j) \subset [0, 2]$ is the overlapping ratio, A is the overlapping area between IVT objects i and j, $A_i(t_1)$ is the area of IVT object i at time step t_1 , $A_j(t_0)$ is the area of IVT object j at time step t_0 ; all the areas are measured by pixels.

The parameter setting in STARCH is shown in Table S1. More details about the parameters and their functions can be found in (Liu & Wright, 2022) and STARCH repository (https://github.com/lorenliu13/starch/tree/v1.0.1).

Dataset	M orph radius <i>R_m</i> (pixel)	H igh threshold (kg/m/s)	L ow threshold (kg/m/s)	Overlapping ratio threshold (-)
ERA5/CESM2	1	500	250	0.2

Table S1. Parameter setting in STARCH.

Text S3. Transformation of Generalized Gamma Distribution to Gamma Distribution

The probability density function (PDF) of a generalized gamma distribution can be written as:

$$f_X(x,a,c,b) = \frac{c}{b\Gamma(a)} \left(\frac{x}{b}\right)^{ca-1} \exp\left(-\left(\frac{x}{b}\right)^c\right)$$
(S2)

where shape parameters a > 0, c > 0 and scale parameter b > 0.

Applying a transformation $y = g(x) = x^c$, the PDF of y can be derived following the transformation of single random variable:

$$f_{Y}(y) = f_{X}(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|$$
(S3)

For the first term $f_x(g^{-1}(y))z$, we have:

$$f_x(g^{-1}(y)) = f_x(y^{1/c}) = \frac{c}{y^{1/c}\Gamma(a)} (\frac{y}{b^c})^a \exp(-\frac{y}{b^c})$$
(S4)

For the second term, since $x = g^{-1}(y) = y^{1/c}$, we have:

$$|\frac{d}{dy}g^{-1}(y)| = \frac{1}{c}y^{1/c-1}$$
(S5)

Combining two terms and let a' = a and $b' = b^c$, we will get:

$$f_{Y}(y,a',b') = \frac{1}{b'\Gamma(a')} (\frac{y}{b'})^{a'-1} \exp(-\frac{y}{b'}) \sim Gamma(a',b')$$
(S6)