ANALYSIS OF MHD FLOW OF BLOOD IN STENOSED ARTERIES WITH RADIALLY VARIABLE VISCOSITY AND PERIPHERAL PLASMA LAYER THICKNESS BY MEANS OF FROBENIUS METHOD

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ABSTRACT

A modest effort is made to study the unsteady two-fluid flow of blood in an arterial stenosis under the application of a transverse uniform magnetic field by adopting Frobenius series method. Plasma fluid in the region of peripheral layer and blood in the central region are considered as a Newtonian fluid in character. The Frobenius method has been adopted to solve the governing equation for the central region and obtain the expression of the velocity profile in that region. It is seen that when the values of hematocrit (h_m) and Hartmann number (M) are increasing, the decrease in forwarding flow velocity and the rise in reversal (backward) flow velocity are observed. The axial velocity is increased due to an increase in the thickness of plasma layer. It is evident that the analytical values of velocity obtained from the present study are compared with the corresponding numerical results obtained from work done by Ponalagusamy and Tamil Selvi (Meccanica, 48(2013), 2427-2438), and there is a good agreement between them.

KEY WORDS

Frobenius series method, Oscillatory Blood flow, Plasma layer thickness, Magnetic field, Stenosed artery.

1. INTRODUCTION

It is well known that human health is severely affected by a common disease which is termed as arteriosclerosis. The cardiovascular disease (arteriosclerosis or stenosis) that is often occurring is thought of uncommon and atypical growth; which grows at different sites of the cardiovascular flow system under diseased states. Even though exact reason accountable for the etiology of the initiation of this phenomenon are not yet clearly understood, it was systematically established by clinical and experimental observations that the normal blood flow is very much disordered by the existence of a mild stenosis in an artery and the chaotic flow further stimuluses the growth of the disease and arterial malformation, and change the local rheological behavior of blood [1, 2]. Since the hydrodynamic and hemodynamic factors play a pivotal role in the genesis and the proliferation of arteriosclerosis [3-6], the objective of the present analysis is driven to make available a model of blood and obtain precise information about the flow velocity.

The concept of magnetic field on the flow of blood through an artery is used by several researchers. Numerous investigations have endeavored on the flow of living fluids by taking into account magnetic force with the goal of its prominence to the medical meadow [7-9]. It is pertinent to point out here that separation of cells, reduction of blood loss during surgical procedure and provocation of occlusion of the feeding vessels of cancer tumors are the most remarkable implications of magnetic devices [10, 11, 12, 13]. Saedi Ardahaie et al. [14] have investigated the effect of magnetic field on the blood flow in a porous artery by assuming blood as a third-grade non-Newtonian fluid. It is observed that the increase in the magnetic force causes the nanofluid flow to impede [15]. Ponalagusamy and Priyadharshini [16] and Bhargava et al. [17] stated that we could use the applied magnetic field as a flow control mechanism in medical applications. Hence, investigating the pivotal role of the magnetic field on blood flow in an artery becomes significant.

It is experimentally observed by Bugliarello and Hayden [18] that there subsists a cell-free plasma layer near the arterial wall, while blood flows through tubes. Given this, it is desirable to characterize the flow of blood through arteries by a two-fluid model instead of a one-fluid model. Haynes [19] and Shukla et al. [20] developed a two-fluid model for the flow of blood comprising of a core region of suspension of blood cells as a Newtonian fluid and a cell-free layer of plasma as a Newtonian fluid of constant viscosity. Bugliarello and Sevilla [21] conducted experiments on blood flow and observed that there is a good agreement between experimentally and theoretically measured velocities by assuming core and peripheral fluids as Newtonian fluids with different viscosities. Numerous investigators [22-24] have analyzed the flow behavior of blood by representing blood as a two-fluid model and compared their theoretical findings with experimental observations. The pulsatile flow of blood is studied by Imaeda and Goodman [25], Ponalagusamy [26], El-Khatib and Damiano [27], Venkateshwarlu and Anand [28] and Ponalagusamy and Kawahara [29]. In these works, the influences of magnetic field, radially variable viscosity along with the thickness of plasma-layer are not examined.

Ponalagusamy and Tamil Selvi [30] have utilized finite difference scheme and computed results for several physical quantities of important physiological significance to have their quantitative measures concerning radially variable viscosity, hematocrit, plasma layer thickness, magnetic field and pulsatile Reynolds number. It is felt that having analytical expressions for flow variables such as velocity profile, flow rate, wall shear stress and flow resistance are essential in order to predict exact quantitative measures of above-mentioned flow variables. Hence, an attempt is made in the present study, which describes the blood flow in an axially symmetric and radially asymmetric stenosed artery consisting of blood (cells suspended in plasma) in the core region and a cell-free plasma fluid in the peripheral layer near the wall under a uniform magnetic field applied on the flow and obtains analytical expressions for flow variables.

2. FORMULATION OF THE PROBLEM

Consider an axially symmetric, oscillatory, laminar and fully developed flow of blood in arterial stenosis under the application of a uniformly applied magnetic field $\overline{H}_0^2$ (Figs.1 and 2). The flow of blood is characterized by a two-fluid model (a core of blood cells suspension surrounded by a peripheral layer of plasma). It is assumed that blood in the core region and the peripheral layer of plasma are both Newtonian fluids. We take the cylindrical coordinate system ($\bar{z}, \bar{r}, \bar{\phi}$) whose origin is situated on the blood vessel (stenosed artery) axis ($-$over a letter denotes the dimensional form of the corresponding quantity). The momentum equations governing the flow in dimensionless form are given by [2, 30]:

$$(\alpha^2/\rho_0)\frac{\partial u_c}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left\{1+\beta h_m (R_1^{m_2} - r^{m_2}\right\}\frac{\partial u_c}{\partial r}\right] - M^2 u_c, \tag{1}$$

in the core region,

$$\alpha^2 \frac{\partial u_p}{\partial t} = -\frac{\partial p}{\partial z} + \left\{\frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r}\frac{\partial u_p}{\partial r}\right\} - M^2 u_p \ , \tag{2}$$

in the peripheral plasma layer region,

where α^2 is the pulsatile Reynolds number, ρ_0 is the ratio between density of plasma fluid and density of core fluid, t is the time, p is the pressure, β is a constant, h_m is the hematocrit, m_2 is involved in the concentration profile, M is magnetic parameter or Hartmann number, u_c and u_p are axial velocities in the core and peripheral plasma regions respectively, and R_1 is the radius of the core region in the stenotic region (refer [30] for further details).

The non-dimensional boundary conditions are,

$$\left.\begin{array}{lll} u_p = 0 & at & r = R(z) \\ u_p = u_c & at & r = R_1(z) \\ \tau_p = \tau_c & at & r = R_1(z) \\ \dfrac{\partial u_c}{\partial r} = 0 & at & r = 0 \end{array}\right\}. \tag{3}$$

The mathematical expression for geometry of the stenosis (in non-dimensional form) is given by [30],

$$R(z) = 1 - B\,[L_0^{\,n_1-1}(z-d) - (z-d)^{n_1}], \qquad for \quad d \le z \le d + L_0, \tag{4}$$

= 1, otherwise

where $n_1(\geq 2)$ is the parameter determining the shape of the stenosis, $R(z)$ is the radius of the artery in the stenotic region, L_0 is the length of the stenosis, d denotes its location and B is expressed as,

$$B = \frac{\bar{\delta}_s}{\bar{R}_0 L_0^{\,n_1}} \frac{n_1^{\,n_1/(n_1-1)}}{(n_1-1)} \ .$$

Further $\bar{\delta}_s$ is the maximum height of the stenosis at $z = d + \frac{L_0}{n_1^{\,1/(n_1-1)}}$ such that $\frac{\bar{\delta}_s}{\bar{R}_0} << 1.0$ and $\bar{R}_0$ is the radius of the normal (uniform) artery. When n_1 = 2, the geometry of stenosis becomes symmetric at $z = d + \frac{L_0}{2}$.

For a purely oscillatory flow, we shall take the pressure gradient of the form,

$$-\frac{\partial p}{\partial z} = P_s \, e^{it} \ , \tag{5}$$

where P_s is the constant pressure gradient. In view of equation (5), we undertake the solutions for $u_c(r,t)$ and $u_p(r,t)$ as,

$$u_c(r,t) = u_{cs}(r) e^{it} \ ,$$
$$u_p(r,t) = u_{ps}(r) e^{it} \ . \tag{6}$$

3. SOLUTION

The axial velocity in the peripheral plasma region is given by [30],

$$u_p(r,t) = \frac{P_s}{M_1^2} \left\{ \frac{J_0(M_1 r)}{J_0(M_1 R)} - 1 \right\} e^{it} \ , \tag{7}$$

where

$$M_1^2 = \left(M^2 + \frac{i\alpha^2}{\rho_0} \right).$$

Using equation(6), equation(1) becomes,

$$\left[1 + \beta\, h_m\, (R_1^{m_2} - r^{m_2})\right] \frac{\partial^2 u_{cs}}{\partial r^2} + \frac{1}{r}\left[1 + \beta\, h_m\, (R_1^{m_2} - \ (m_2 + 1)\ r^{m_2}\right] \frac{\partial u_{cs}}{\partial r}$$
$$- \left[M^2 + \frac{\alpha^2\, i\, \omega}{\rho_0} \right] u_{cs} + P_s = 0. \qquad (8)$$

The Frobenius series method is adopted to obtain the analytical solution of equation (8). To apply Frobenius technique, the velocity in the core region u_{cs} is bounded at $r = 0$. The only suitable series solution of equation (8) may be expressed as,

$$u_{cs} = C \sum_{m=0}^{\infty} E_m\, r^m - \frac{P_s}{4\beta_1} \sum_{m=0}^{\infty} F_m\, r^{m+2}\,, \qquad (9)$$

where C, E_m, F_m are arbitrary unknown constants, $\beta_1 = 1 + \beta_2\, R_1^{m_2}$ and $\beta_2 = \ \beta\, h_m$. It is observed from equation (9) that the first and second terms of the right-hand side of equation (9) are solutions corresponding to homogeneous and non-homogeneous parts of equation (8) respectively. Using equation (9), the analytic solution of equation (8) is obtained by tedious mathematical steps involved in the Frobenius method as,

$$u_{cs} = \frac{\left[u_{ps} + \frac{P_s}{4\beta_1} \sum_{m=0}^{\infty} F_m\, R_1^{m+2} \right] \sum_{m=0}^{\infty} E_m\, r^m - \frac{P_s}{4\beta_1} \sum_{m=0}^{\infty} E_m\, R_1^m \sum_{m=0}^{\infty} F_m\, r^{m+2}}{\sum_{m=0}^{\infty} E_m\, R_1^m}\,, \qquad (10)$$

where

$$E_{m+1} = \frac{\beta_2\, (m+1)(m+1-m_2)\, E_{m+1-m_2} + M_1^2\, E_{m-1}}{\beta_1 (m+1)^2},$$

$$F_{m+1} = \frac{\beta_2 (m+3)(m+3-m_2) F_{m+1-m_2} + M_1^{\ 2} F_{m-1}}{\beta_1 (m+3)^2},$$

with $E_0 = F_0 = 1$ and $E_{-m} = F_{-m} = 0.$

The axial velocity in the core region is expressed as,

$$u_c(r,t) = u_{cs}\, e^{it}. \tag{11}$$

Using analytic expressions for velocities in the plasma layer and core regions respectively given by equations (7 & 11), one can readily obtain analytic expressions for flow rate, wall shear stress and resistance to flow by employing the corresponding definitions mentioned in [30].

4. RESULTS AND DISCUSSION

The motivation of the present work is to obtain the analytic solution for axial velocity in the core region by adopting Frobenius method. For a computational purpose, the values of the parameters are considered as follows [2, 30, 31,32]: $h_m = 0.4$, 0.6, 0.8; $\rho_0 = 1.0$; $\alpha^2 = 2.5, 5.0$; $\delta_s = 0.2$; $M = 0.6, 1.0$; $\beta = 1.0$; $P_s = 10$; $\gamma = 0.8, 0.9, 1.0$; $t = 45^\circ, 135^\circ, 225^\circ$ and 315°. In the present study, exact values of axial velocity with respect to radial distance are computed for different values of hematocrit (h_m), pulsatile Reynolds number (α^2), Hartmann number (M), peripheral plasma layer thickness $\delta\,(=1-\gamma)$ and time t as illustrated through Tables (1-7).

The radial distribution of velocity with time (t) at the mid-point of the stenotic region by considering other parameters involved in the present study are held fixed, is tabulated in Tables 1 and 2. It is seen that the increase in the pulsatile Reynolds number (α^2 or Womersley frequency) leads to the increase in the velocity. Another remarkable result is regarding the variation of velocity with the plasma layer thickness and is shown in Tables 3 and 4. The magnitude of velocity is higher for a two-fluid model ($\gamma < 1.0$) as compared to that of the one-fluid model ($\gamma = 1.0$). Furthermore, the percentage of decrease of the velocity along the radial direction (r) is found to be higher in the presence of plasma layer thickness ($\gamma < 1.0$) than that of the case of absence of plasma layer thickness ($\gamma = 1.0$). The same trend has been noticed for the time (t) considered in the present work.

Tables 4 and 5 reveal the effects of Hartmann number (M) on the velocity distribution. The forwarding flow velocity (a positive value of velocity) is found to be decreased with the increasing value of M, but the reversal flow velocity (a negative value of velocity) is increased. These behaviors are attributed due to the fact that the applied magnetic field induces a body force known as the Lorentz force which opposes the forwarding flow velocity of blood and enhances the reversal flow velocity of blood in the core region. The forward and backward flow velocities are decreased with the increase in the hematocrit (or the concentration) of blood cells in the core region (Tables 6 and 7). The reason is that the increase in the hematocrit (h_m) of blood cells tends to increase the viscous force causing lower the flow velocity. It is pertinent to point out that a comparative analysis between the analytic values of velocity obtained from the present study and the corresponding numerical results obtained by Ponalagusamy and Tamil Selvi [30] is made and it is witnessed that there is a good agreement between them.

5. CONCLUSION

The analytic values (or exact values) of axial velocity are computed from the present investigation by taking a sum of required terms in power series (the Frobenius series) solution so that the magnitude of the difference between the current value and its previous value becomes less than 10^{-6}. Using the numerical technique with Thomas algorithm adopted by Ponalagusamy and Tamilselvi [30], the numerical values of velocity are computed by taking the step size $\Delta r = 0.01$in the radial direction. The analytic and numerical values of velocity are tabulated in Tables (1-7). A comparison between the analytic and numerical values of velocity is found to be fairly good. But the order of time complexity of the present method (the Frobenius series method) is $o(1)$ and the order of time complexity of Thomas algorithm becomes $o(n)$, where the number of the unknown is n. At this juncture, it is pertinent to point out that for a practical problem (blood flow phenomenon); an analytic solution (or a closed form solution) can bring out the exact mechanism and physical impacts of the problem model under consideration. The precise information obtained from the analytical solutions of the mathematical model (whenever it is possible to obtain) over that of numerical solution could lead to the development of diagnostic tools for effective treatment of patients suffering from severe diseases [31].

r	t = 45^0		t = 135^0		t = 225^0		t = 315^0	
	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)
0.00	1.1312	1.1306	-0.9145	-0.9143	-1.1312	-1.1306	0.9145	0.9143
0.08	1.1189	1.1187	-0.9091	-0.9090	-1.1189	-1.1187	0.9091	0.9090
0.16	1.0818	1.0817	-0.8927	-0.8927	-1.0818	-1.0817	0.8927	0.8927
0.24	1.0196	1.0195	-0.8647	-0.8646	-1.0196	-1.0195	0.8647	0.8646
0.32	0.9316	0.9316	-0.8239	-0.8239	-0.9317	-0.9316	0.8239	0.8239
0.40	0.8170	0.8169	-0.7688	-0.7688	-0.8170	-0.8169	0.7688	0.7688
0.48	0.6743	0.6743	-0.6971	-0.6971	-0.6743	-0.6743	0.6971	0.6971
0.56	0.5021	0.5021	-0.6058	-0.6058	-0.5021	-0.5021	0.6058	0.6058
0.64	0.2984	0.2984	-0.4910	-0.4910	-0.2984	-0.2984	0.4910	0.4910
0.72	0.1617	0.1617	-0.2561	-0.2561	-0.1617	-0.1617	0.2561	0.2561
0.75	0.1041	0.1041	-0.1624	-0.1624	-0.1041	-0.1041	0.1624	0.1624
0.78	0.0429	0.0429	-0.0659	-0.0659	-0.0429	-0.0429	0.0659	0.0659
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 1. Comparison between Numerical solution and Power-series solution for velocity distribution with time t at the mid-point of stenosed tube ($\alpha^2 = 2.5$, M = 0.6, $\gamma = 0.8$ and $h_m = 0.4$)

r	t = 45^0		t = 135^0		t = 225^0		t = 315^0	
	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)
0.00	1.1357	1.1350	-0.7133	-0.7132	-1.1357	-1.1350	0.7133	0.7132
0.08	1.1217	1.1214	-0.7117	-0.7117	-1.1217	-1.1214	0.7117	0.7117
0.16	1.0793	1.0791	-0.7066	-0.7066	-1.0793	-1.0791	0.7066	0.7066
0.24	1.0080	1.0079	-0.6971	-0.6970	-1.0080	-1.0079	0.6971	0.6971
0.32	0.9070	0.9069	-0.6814	-0.6814	-0.9070	-0.9069	0.6814	0.6814
0.40	0.7751	0.7750	-0.6571	-0.6571	-0.7751	-0.7750	0.6571	0.6571
0.48	0.6106	0.6106	-0.6207	-0.6207	-0.6106	-0.6106	0.6207	0.6207
0.56	0.4117	0.4117	-0.5676	-0.5676	-0.4117	-0.4117	0.5676	0.5676
0.64	0.1761	0.1761	-0.4916	-0.4916	-0.1761	-0.1761	0.4916	0.4916
0.72	0.1019	0.1019	-0.2566	-0.2566	-0.1019	-0.1019	0.2566	0.2566
0.75	0.0672	0.0672	-0.1628	-0.1628	-0.0672	-0.0672	0.1628	0.1628
0.78	0.0284	0.0284	-0.0660	-0.0660	-0.0284	-0.0284	0.0660	0.0660
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2. Comparison between Numerical solution and Power-series solution for velocity distribution with time t at the mid-point of stenosed tube (α^2 = 5, M = 0.6, γ = 0.8 and h_m = 0.4)

r	t = 45^0		t = 135^0		t = 225^0		t = 315^0	
	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)
0.00	1.1177	1.1171	-0.5344	-0.5344	-1.1177	-1.1171	0.5344	0.5344
0.08	1.1063	1.1060	-0.5332	-0.5332	-1.1063	-1.1060	0.5332	0.5332
0.16	1.0718	1.0716	-0.5291	-0.5291	-1.0718	-1.0716	0.5291	0.5291
0.24	1.0137	1.0136	-0.5214	-0.5214	-1.0137	-1.0136	0.5214	0.5214
0.32	0.9313	0.9311	-0.5089	-0.5089	-0.9313	-0.9311	0.5089	0.5089
0.40	0.8232	0.8231	-0.4894	-0.4894	-0.8232	-0.8231	0.4894	0.4894
0.48	0.6880	0.6879	-0.4603	-0.4603	-0.6880	-0.6879	0.4603	0.4603
0.56	0.5238	0.5237	-0.4178	-0.4177	-0.5238	-0.5237	0.4178	0.4177
0.64	0.3280	0.3280	-0.3568	-0.3568	-0.3280	-0.3280	0.3568	0.3568
0.72	0.0982	0.0982	-0.2707	-0.2707	-0.0982	-0.0982	0.2707	0.2707
0.75	0.0649	0.0649	-0.1715	-0.1715	-0.0649	-0.0649	0.1715	0.1715
0.78	0.0275	0.0275	-0.0695	-0.0695	-0.0275	-0.0275	0.0695	0.0695
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3. Comparison between Numerical solution and Power-series solution for velocity distribution with time t at the mid-point of stenosed tube (α^2 = 5, M = 1.0, γ = 0.9 and h_m = 0.4)

r	t = 45^0		t = 135^0		t = 225^0		t = 315^0	
	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)
0.00	1.1067	1.1060	-0.3686	-0.3686	-1.1067	-1.1060	0.3686	0.3686
0.08	1.0967	1.0964	-0.3671	-0.3671	-1.0967	-1.0964	0.3671	0.3671
0.16	1.0667	1.0665	-0.3623	-0.3624	-1.0667	-1.0665	0.3623	0.3624
0.24	1.0163	1.0160	-0.3537	-0.3537	-1.0163	-1.0160	0.3537	0.3537
0.32	0.9447	0.9445	-0.3400	-0.3401	-0.9447	-0.9445	0.3400	0.3401
0.40	0.8512	0.8510	-0.3196	-0.3196	-0.8512	-0.8510	0.3196	0.3197
0.48	0.7343	0.7341	-0.2901	-0.2901	-0.7343	-0.7341	0.2901	0.2902
0.56	0.5927	0.5925	-0.2484	-0.2484	-0.5927	-0.5925	0.2484	0.2484
0.64	0.4246	0.4244	-0.1901	-0.1901	-0.4246	-0.4244	0.1901	0.1901
0.72	0.2278	0.2276	-0.1098	-0.1098	-0.2278	-0.2276	0.1098	0.1098
0.75	0.1461	0.1460	-0.0725	-0.0725	-0.1461	-0.1460	0.0725	0.0725
0.78	0.0600	0.0599	-0.0307	-0.0307	-0.0600	-0.0599	0.0307	0.0307
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4. Comparison between Numerical solution and Power-series solution for velocity distribution with time t at the mid-point of stenosed tube (α^2 = 5, M = 1.0, γ = 1.0 and h_m = 0.4)

r	t = 45^0		t = 135^0		t = 225^0		t = 315^0	
	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)	Numerical Solution	Series Solution (m =10)
0.00	1.1762	1.1769	-0.3610	-0.3611	-1.1762	-1.1769	0.3610	0.3611
0.08	1.1658	1.1662	-0.3597	-0.3597	-1.1658	-1.1662	0.3597	0.3597
0.16	1.1334	1.1337	-0.3554	-0.3554	-1.1334	-1.1337	0.3554	0.3554
0.24	1.0789	1.0792	-0.3474	-0.3474	-1.0789	-1.0792	0.3474	0.3474
0.32	1.0018	1.0021	-0.3346	-0.3346	-1.0018	-1.0021	0.3346	0.3346
0.40	0.9012	0.9014	-0.3152	-0.3152	-0.9012	-0.9014	0.3152	0.3152
0.48	0.7760	0.7762	-0.2867	-0.2868	-0.7760	-0.7762	0.2867	0.2868
0.56	0.6249	0.6251	-0.2460	-0.2460	-0.6249	-0.6251	0.2460	0.2460
0.64	0.4464	0.4465	-0.1887	-0.1887	-0.4464	-0.4465	0.1887	0.1887
0.72	0.2387	0.2388	-0.1091	-0.1091	-0.2387	-0.2388	0.1091	0.1092
0.75	0.1529	0.1530	-0.0721	-0.0722	-0.1529	-0.1530	0.0721	0.0722
0.78	0.0627	0.0627	-0.0305	-0.0305	-0.0627	-0.0627	0.0305	0.0306
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 5. Comparison between Numerical solution and Power-series solution for velocity distribution with time t at the mid-point of stenosed tube ($\alpha^2 = 5$, M = 0.6, $\gamma = 1.0$ and $h_m = 0.4$)

r	t = 45^0		t = 135^0		t = 225^0		t = 315^0	
	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)
0.00	1.0792	1.0785	-0.5385	-0.5385	-1.0792	-1.0785	0.5385	0.5385
0.08	1.0685	1.0683	-0.5371	-0.5371	-1.0685	-1.0683	0.5371	0.5371
0.16	1.0365	1.0363	-0.5327	-0.5327	-1.0365	-1.0363	0.5327	0.5327
0.24	0.9824	0.9822	-0.5245	-0.5245	-0.9824	-0.9822	0.5245	0.5245
0.32	0.9052	0.9051	-0.5113	-0.5113	-0.9052	-0.9051	0.5113	0.5114
0.40	0.8034	0.8033	-0.4913	-0.4914	-0.8034	-0.8033	0.4913	0.4914
0.48	0.6750	0.6748	-0.4619	-0.4619	-0.6750	-0.6748	0.4619	0.4619
0.56	0.5171	0.5170	-0.4191	-0.4191	-0.5171	-0.5170	0.4191	0.4191
0.64	0.3263	0.3262	-0.3578	-0.3578	-0.3263	-0.3262	0.3578	0.3579
0.72	0.0982	0.0982	-0.2707	-0.2707	-0.0982	-0.0982	0.2707	0.2707
0.75	0.0649	0.0649	-0.1715	-0.1715	-0.0649	-0.0649	0.1715	0.1715
0.78	0.0275	0.0275	-0.0695	-0.0695	-0.0275	-0.0275	0.0695	0.0695
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 6. Comparison between Numerical solution and Power-series solution for velocity distribution with time t at the mid-point of stenosed tube (α^2 = 5, M = 1.0, γ = 0.9 and h_m = 0.6)

r	t = 45^0		t = 135^0		t = 225^0		t = 315^0	
	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)	Numerical Solution	Series Solution (m =12)
0.00	1.0439	1.0434	-0.5414	-0.5413	-1.0439	-1.0434	0.5314	0.5313
0.08	1.0340	1.0338	-0.5399	-0.5399	-1.0340	-1.0338	0.5299	0.5299
0.16	1.0041	1.0040	-0.5352	-0.5352	-1.0041	-1.0040	0.5252	0.5252
0.24	0.9535	0.9534	-0.5266	-0.5266	-0.9535	-0.9534	0.5166	0.5166
0.32	0.8810	0.8809	-0.5131	-0.5131	-0.8810	-0.8809	0.5031	0.5031
0.40	0.7848	0.7847	-0.4927	-0.4928	-0.7848	-0.7847	0.4827	0.4828
0.48	0.6624	0.6623	-0.4630	-0.4631	-0.6624	-0.6623	0.4530	0.4531
0.56	0.5105	0.5105	-0.4202	-0.4202	-0.5105	-0.5105	0.4102	0.4102
0.64	0.3245	0.3245	-0.3588	-0.3588	-0.3245	-0.3245	0.3488	0.3488
0.72	0.0982	0.0982	-0.2707	-0.2707	-0.0982	-0.0982	0.2707	0.2707
0.75	0.0649	0.0649	-0.1715	-0.1715	-0.0649	-0.0649	0.1715	0.1715
0.78	0.0275	0.0275	-0.0695	-0.0695	-0.0275	-0.0275	0.0695	0.0695
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 7. Comparison between Numerical solution and Power-series solution for velocity distribution with time t at the mid-point of stenosed tube (α^2 = 5, M = 1.0, γ = 0.9 and h_m = 0.8)

Conflict of Interest: The author declares that no conflict of interest is raised.

Nomenclature

(bar indicates the dimensional form of a corresponding quantity)

d	location of stenosis
$\overline{H}_0^{\,2}$	strength of the magnetic field
h_m	hematocrit
J_0	Bessel function of the first kind of order zero
L_0	length of the stenosis
m_2	power parameter in the concentration profile
M	Hartmann number (or Magnetic parameter)
n_1	the parameter that determines the shape of the stenosis
p	pressure
p_s	constant pressure gradient
r	radial direction
$\overline{R}_0$	the radius of the normal artery
$R(z)$	the radius of an artery in the stenotic region
$R_1(z)$	the radius of the core region in the stenotic region
t	time
u_c, u_p	fluid velocities in the core and plasma regions respectively
u_{cs}	core fluid velocity under steady flow condition

u_{ps} plasma fluid velocity under steady flow condition

z axial direction

Greek symbols

α^2 Womersley number or pulsatile Reynolds number

β parameter involved in the concentration profile

ρ_0 the ratio between the density of plasma fluid and density of core fluid

δ plasma layer thickness

$\bar{\delta}_s$ the maximum height of the stenosis

γ the ratio of radii of the core region and artery in the stenotic region

τ_c, τ_p shear stresses in the core and plasma regions respectively

ω the frequency of oscillations

Subscripts

c core region

p plasma region

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