Long-wave isentropic ocean-atmosphere dynamics: providing faster-than-real-time, predictive modelling of the 2022 Hunga Tonga event

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Abstract

Starting from the fully-compressible Euler equations, a two-way-coupled system governing the long-wave behaviour of thin layers (with respect to the radius of Earth) representing the ocean and atmosphere, under an isentropic constraint, was derived. This approach incorporates bathymetry and topographic features as well as three-dimensional atmospheric non-uniformities through their depth-average over a spherical shell. Linear analysis of the obtained system yields two pairs of gravito-acoustic waves which are found to be representative of the fast-travelling atmospheric wave (with a propagation speed mainly governed by the atmospheric-layer-averaged speed of sound) and the slower-travelling gravity waves in the ocean (with a propagation speed mainly governed by local water depth). Remarkably, the 'Proudman resonance', observed in the forced shallow-water equation framework and invoked to justify, in part, observed large wave-heights, vanishes in favour of a continuous transition past the critical water depth, occurring when the two wave propagation speeds are closest. Two-dimensional non-linear global simulations were performed, using atmospheric conditions on the day, showcasing the predictive ability of the model. Local maxima of water-height disturbance in the farfield from the volcano, linked to the atmospheric wave deformation over time, are observed, emphasising the importance of the atmospheric-layer modelling and two-way coupling for any daylong predictions. An efficient implementation of the modelling strategy was carried out in the open source computational framework dNami to demonstrate the ability to perform faster-than-real-time simulations despite the additional equations in the governing system. Future work would see the strategy extended to incorporate additional layers and physics e.g. ocean and atmosphere stratification, interaction with the upper atmosphere.

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Motivation

The 2022 Hunga Tonga-Hunga Ha'apai eruption provoked worldwide confusion within tsunami warning systems and highlighted the need for predictive forecasting tools for coupled atmosphere-ocean events. Current one-way coupled forced shallow water equation (SWE) models provide unsatisfactory results motivating the present research which incorporates additional physics resulting in a two-way coupled (TWC) model.

Thin layers, long waves and two-way coupling

To derive the TWC model, long waves are considered to propagate in both the atmosphere and the ocean, which are comparatively thin relative to the wavelength of the waves, governed by the 3D compressible Euler equations.



Overview of the length scale comparisons considered in this twolayer model

An expansion of the equations based on the length-ratio parameters and a density-weighted vertical averaging yield equations governing the flow on 2D spherical shells. This systems solves the flow in both the atmosphere and the ocean allowing dynamic two-directional energy transfer. The density weighting yields layer-averaged atmospheric properties that converge to observed values once a sufficient atmospheric thickness is considered.



Local speed of sound (solid) and speed based on Favre-averaged temperature (dashed) versus atmospheric thickness using International Standard Atmosphere profiles

Two-way coupling is essential to providing **predictive** simulation tools for ocean-atmosphere interaction and understanding the physical mechanisms leading to large waves around the world.





Historical maximum sea height change at T+18hrs

Link to paper and open-source high-performance numerical code

No more Proudman resonance

The linear analysis of the TWC system reveals two pairs of gravito-acoustic eigenmodes, termed \mathscr{A} and \mathscr{G} , which are respectively representative of the fast-traveling atmospheric wave (Lamb wave) and the slower-travelling gravity waves in the ocean.

Finite A-mode induced sea height change

Unlike the forced SWE approach, which contains a singularity when atmospheric and gravity wave speeds match, referred to as the 'Proudman resonance', the sea height change induced by the \mathscr{A} mode remains finite for all ocean depths on Earth.

hPa	1
$[\mathrm{cm}/$	1
change	1
height	1
iduced sea	10

Energy transfer via refraction at steep changes in bathymetry is found to be significantly modified by the two-way coupling and is key to explaining unexpected early-arriving large waves around the world notably in the farfield region from the volcano (e.g. Atlantic ocean).



Simulation inputs

To perform predictive ~24h timescale simulations, the following inputs are required:

- State of the atmosphere (e.g. ERA5 fields on pressure levels) • Global bathymetry (e.g. GEBCO charts)
- Initial energy injection into water/atmosphere layers



Refraction: $\mathscr{A} \leftrightarrow \mathscr{G}$ **mode energy transfer**

Illustration of the refraction problem: as the atmospheric wave (orange) spreads away from the volcano, it encounters steep changes in water depth where energy can be transfer from $\mathscr{A} \rightarrow$ ${\mathscr G}$ which then propagates as would a regular tsunami.

• Polar ice coverage (e.g. NSIDC sea ice index)