Tidally-induced acceleration as a potential cause of the Lunar Crustal Dichotomy

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Abstract

Analysis of GRAIL data revealed a lunar crustal dichotomy that can be described by a degree 1 spherical harmonic. A simple explanation of this observation is a superposition of the Moon's self-gravity and an external and constant acceleration. I explored the possibility that the Moon experienced a much greater prograde acceleration in the past (when it was considerably closer to the Earth). I use a simple density balance approximation to determine the approximate acceleration needed to produce the observed asymmetry: ~0.009 m/s2 (or ~1/2% of the Moon's self-gravity). In order to produce this acceleration, the Moon would need to be within ~13,000 km of the Earth. This is well within the Roche limit for completely fluid bodies (~18,350 km); for completely rigid bodies, the Roche limit is ~9,480 km. From this, it is extremely unlikely for tidally-induced acceleration to explain the observed crustal asymmetry.

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Tidally-induced acceleration as a potential cause of the Lunar Crustal Dichotomy

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Key Points:

- Tidally-induced acceleration could cause the observed lunar crustal dichotomy if the Moon is within ~10,000 km of the Earth.
- Reasonable parameters put this distance within the Roche limit for fluid bodies, and just outside the Roche limit for rigid bodies.
- This hypothesis requires an extreme level of rigidity that is unlikely to occur in order to place the Moon outside of the Roche limit.

Abstract

Analysis of GRAIL data revealed a lunar crustal dichotomy: the far side of the Moon has a thicker low density crust than the near side that (to first order) smoothly varies from one side to the other. Any hypothesis seeking to explain the lunar crustal asymmetry must also explain this smoothly varying distribution of crustal thickness. A simple explanation is the superposition of the Moon's self-gravity and an external and constant acceleration. I explored the possibility that the Moon experienced a much greater prograde acceleration in the past (when it was considerably closer to the Earth). This acceleration was caused by the Earth's tidal bulge, which is caused by the Moon's gravity. I use a simple density balance approximation to determine the approximate acceleration needed to produce the observed asymmetry: ~0.009 m/s^2 (or ~1/2% of the Moon's self-gravity). In order to produce this acceleration, the Moon would need to be within ~13,000 km of the Earth. This is well within the Roche limit for completely fluid bodies (~18,350 km); for completely rigid bodies (an extremely unlikely scenario), the Roche limit is ~9,480 km. From this, it is extremely unlikely for tidally-induced acceleration to explain the observed crustal asymmetry.

Plain Language Summary

1 Introduction

Analysis of GRAIL data revealed the presence of a lunar crustal dichotomy (often referred to as an asymmetry) in which the far side of the Moon has a thicker low density crust than the near side (Barr, 2016; National Academies of Sciences, 2022; Wieczorek et al., 2013). On the near side, crustal thicknesses are approximately 30-40 km; the far side, 50-60 km. To first order, this thickness smoothly varies from its maximum on the farside to its minimum on the nearside (see Figure 1); this is also known as a degree 1 spherical harmonic (Andrews-Hanna et al., 2023; Wieczorek et al., 2013). In a body dominated by self-gravity and without lateral plate motion, a largely uniform crustal thickness is expected. This deviation indicates one or more other processes are at work. The 2023-2032 Planetary Decadal Survey refers to this asymmetry as "one of the greatest outstanding mysteries regarding lunar early evolution" (National Academies of Sciences, 2022). As such, a number of hypotheses have been presented to explain this observation. Essentially all of these begin with one (or possibly more) giant impact into the proto-Earth (Barr, 2016; Canup et al., 2021; Tartèse et al., 2019). Garrick-Bethell et al. (2010) explain the thickened lunar highlands via asymmetric tidal heating in the early Moon. In another, a giant impactor moving at slow velocities can cause thinning of the lunar nearside crust (Zhu et al., 2019). A density inversion and degree-1 instability forming during the crystallization of the lunar magma ocean could create the asymmetry (Andrews-Hanna et al., 2023). In one hypothesis, simply the presence of a nearby hot Earth following the Moon-forming impact resulted in a primordial bulk compositional heterogeneity that then caused the observed crustal asymmetry (Roy et al., 2014).

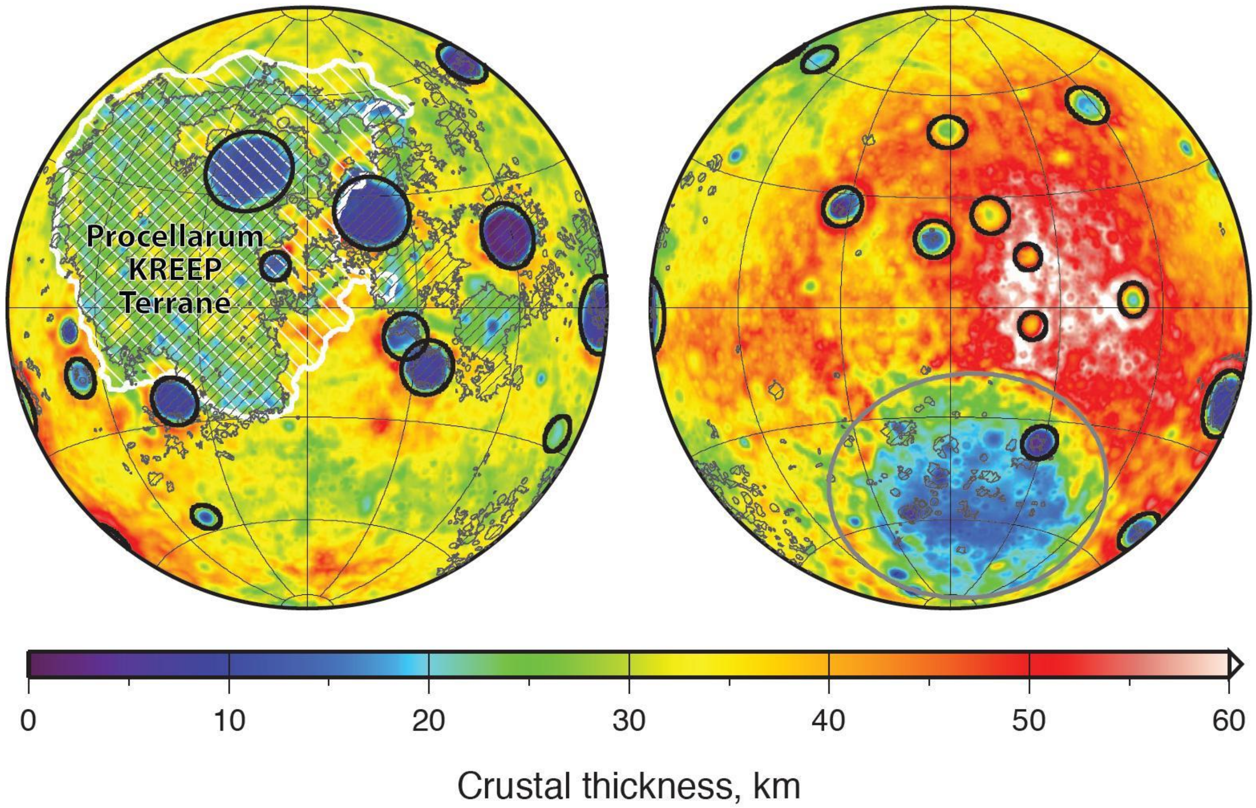

Figure 1. Lunar crustal thickness map derived from data acquired by the GRAIL mission. In this figure, we can see that the crustal thickness varies from its minimum on the nearside (left) to its maximum on the farside (right).
Image credit: NASA/JPL-Caltech/S. Miljkovic

In this brief letter, I present and cursorily explore the idea that the lunar crustal dichotomy is the result of acceleration induced by gravitational interactions between the Earth and Moon. We currently know that the Earth's tidal bulge causes a prograde acceleration of the Moon via the exchange of angular momentum (Canup et al., 2021). The gravity of the Moon distorts the Earth, creating a tidal bulge. The rotation of the Earth then moves the bulge prograde. Even though this bulge is rather small (only ~1 m today) and the lateral distance the bulge is displaced by the Earth's rotation is also small (at least relative to the current Earth/Moon distance), this tidal bulge produces a prograde acceleration of the Moon that results in the Moon's orbital distance from the Earth increasing at a rate of ~4 cm/yr. As the Moon moves further away from the Earth, the force exerted on the Moon by the Earth's tidal bulge shrinks, causing the prograde acceleration to shrink (and Earth's rotation to slow so as to conserve angular momentum and the bulge to shrink due to distance). Conversely, in the past, the Moon would have been closer to the Earth and this acceleration would have been stronger (and the Earth's rotation faster and bulge larger). The question becomes: was this past prograde acceleration sufficient to modify the thickness of the lunar crust? The observed smoothly varying crustal thickness is expected if this hypothesis is correct and is largely the impetus for this work.

By treating the accelerations that the Moon experiences as a superposition of its own self-gravity and the tidal acceleration caused by the tidal bulge of the Earth (see Figure 2), it is possible to extract the tidally induced acceleration necessary to produce the observed dichotomous crustal thicknesses. I break this apart into two extremely simple pieces: (1) a density balance approximation meant to relate differences in crustal thickness to differences in gravitational acceleration, and (2) a calculation of the torque exerted by the Earth on the Moon, which is then converted to acceleration. In reality, the Moon also experiences a third major gravity field in the form of the Earth's "static" gravity (that caused by the spheroidal mass of the Earth). This is excluded as it will not change the distribution of material within the Moon; rather, it only distorts the shape of the Moon.

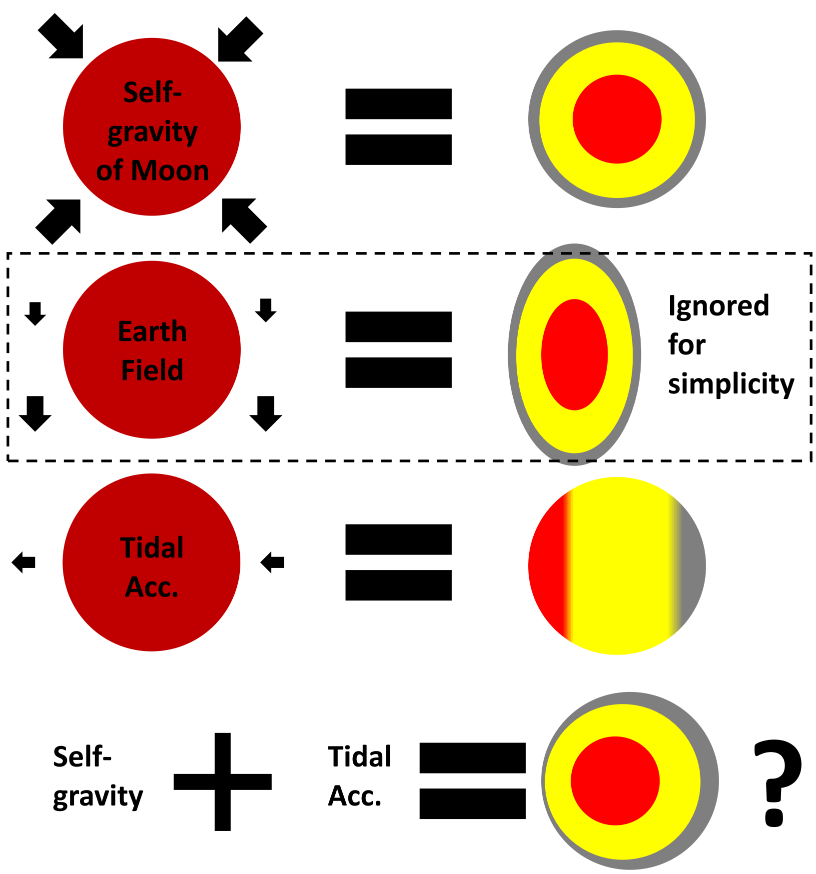

Figure 2. Visual depiction of hypothesis. On the right side of the equal sign, grey depicts crustal material, yellow mantle material, and red core material. Self-gravity is expected to produce concentric layers with uniform thickness; the "static" Earth field only distorts the shape of the Moon; if only tidal-bulge induced acceleration acts on the Moon (i.e., self-gravity is ignored), the material layers as shown. If the Earth field (and a number of other effects) can be ignored, self-gravity and tidal-bulge induced acceleration should result in an asymmetric distribution of crustal thickness.

2 Methods

I make use of relatively simple analytical equations to estimate the necessary prograde acceleration the Moon must experience to cause the observed dichotomy and the proximity necessary for the Moon to experience this acceleration.

2.1 Density balance approximation

To acquire the minimum prograde acceleration, I use two simplistic relations. The first is an extremely simplistic, ad hoc approximation given by equation 1.

$$\frac{\Delta g}{g_M} = \frac{\Delta CT}{r_M}\ ; \ \Delta g = [(g_M + g_{TIA}) - (g_M - g_{TIA})];\ \Delta CT = CT_{Far} - CT_{Near} \qquad \text{(eq. 1)}$$

In this equation, Δg is the difference in gravity experienced by the near and far sides; ΔCT is the difference in crustal thickness given as 20 km; g_M is the self-gravity of the Moon with a value of 1.62 m/s^2; r_M is the radius of the Moon which is 1737.4 km; g_{TIA} is the tidally induced acceleration that modifies the lunar gravity field. Solving for g_{TIA} gives equation 2:

$$g_{TIA} = \frac{g_M}{2}\frac{CT_{Far}-CT_{Near}}{r_M} = \frac{g_M}{2}\frac{\Delta CT}{r_M} \qquad \text{(eq. 2)}$$

To corroborate this ad hoc methodology, I also used a simple density balance:

$$\rho g_a\, h_a = \rho g_b\, h_b;\;\; g_a = g_M - g_{TIA}, g_b = g_M - g_{TIA};\; h_a = r_M + CT_{Near}, h_b = r_M + CT_{Far}$$

In this simple density balance, ρ is density, h is the height above some reference frame, and the other symbols are as defined before. Therefore:

$$g_{TIA} = g_M\,\frac{h_a - h_b}{h_a + h_b} = g_M\,\frac{\Delta CT}{2r_M + CT_{Near} + CT_{Far}} \qquad \text{(eq. 3)}$$

With these estimates, I move on to determining the approximate orbital distance necessary to produce this acceleration.

2.2 Torque exerted by the Earth on the Moon

The tidal bulge of the Earth produces a torque on the Moon. A simple analytical equation (given by equation 4 below) gives this torque:

$$\tau = \frac{3}{2}\,G\,m_M^2\,k_2\,\frac{r_E^5}{A^6}\,\sin(2\alpha) \qquad \text{(eq. 4)}$$

In equation 4, τ is torque; G is the gravitational constant, 6.6743×10^{-11} m^3/(kg s^2); m_M is the mass of the Moon, 7.347×10^{22} kg; k_2 is the tidal love number; r_E is the radius of the Earth, 6,371 km; A is the Earth-Moon distance for which to solve; and α is the tidal bulge lag (~3° today).

I explored parameter space for the tidal love number (k_2), Earth-Moon distance (A), and tidal lag (α). The love number, k (note that $k \approx \frac{2}{3}\,k_2$), ranges from a minimum of 0 for a completely rigid body or one in which the mass is concentrated towards its center, and reaches a maximum of 1.5 for a body of isotropic density. Increasing the love number has the effect of linearly increasing the required torque; conversely, as the love number becomes vanishingly small, so too does the torque produced. The love number of Earth is currently 0.3531 (Ćuk et al., 2016), but was likely higher in the past before differentiation took place. Under the lunar synestia hypothesis, the Moon was completely molten and likely well-mixed after its initial formation as it cooled via radiative processes. Although a value of 1.5 is likely unrealistic (pressure considerations alone result in the Moon's center having greater density even if it were completely compositionally isotropic), it represents a useful end-member of the love numbers to explore. I use the following love numbers: 0.001, 0.3, 0.6, 0.9, 1.2, and 1.5. I vary Earth-Moon distance from 5,000 to 100,000 km in 1 km increments (though my results indicate an appreciable torque only within ~20,000 km). For tidal bulge lag, I used values of 3, 10, 20, 30, 40, and 45°. This set begins with 3° as that is the approximate value observed today and ends with 45° as this provides the theoretical maximum torque possible (sin(2×45°)=1).

This method for calculating the torque the Earth exerts on the Moon overestimates the value of today's torque by a small degree: 4.4×10^{16} Nm calculated versus 3.9×10^{16} Nm observed (MacDonald, 1964). Because of this, I scale the calculated torques of the past by the ratio of today's actual torque divided by the calculated torque of today.

Once the torque that the Earth exerts on the Moon is acquired, it can be easily converted to an acceleration using the standard equation for torque: $\tau = m\ a\ r\ sin(\theta) \rightarrow a = \frac{\tau}{m\ rsi\,n(\theta)} \rightarrow g_{TIA} = \frac{\tau}{m_M Asi\,n(\theta)}$, in which m_M is the mass of the Moon, and θ is the angle between the Moon and Earth's tidal bulge.

3 Results & Discussion

My cursory results and examination indicate that this hypothesis is most likely wrong. Although the Earth's tidal bulge can induce the necessary acceleration, it requires an impossibly close proximity. Figure 3 encapsulates the results of this work.

The estimates of the necessary tidally-induced acceleration from equations 2 and 3 agree well. The ad hoc equation 2 gives a required a g_{TIA} of 0.009324 m/s^2 or ~0.576% of the Moon's self-gravity; equation 3 gives 0.00911 m/s^2 or ~0.563% of the Moon's self-gravity. I use the larger value as the minimum acceleration required.

For all of the parameter space explored, an extremely close proximity is required for the tidal bulge of the Earth to cause an acceleration strong enough to produce the observed lunar crustal asymmetry. Tidal lag angle has very little effect on the distance at which a particular tidally-induced acceleration occurs. This should be expected as the part of equation 4 that tidal lag angle affects only ranges from 0 to 1. The love number has a stronger effect due to its larger range of values and linear effect on the equation. A higher tidal love number increases the required distance while also requiring a more fluid and more homogenously mixed Earth and Moon. The tidal love number – both for the Earth and Moon – was almost certainly higher in the past than today's value of ~0.3531 (Ćuk et al., 2016). This makes the proximity required for today's tidal bulge lag angle and tidal love numbers (~9,500 km) an underestimate when compared to the likely conditions when the Moon initialy formed. That being said, doubling (or even tripling) the modern tidal love number has the relatively small effect of increasing the required distance from ~9,400 km to ~10,550 km (or ~11,300 km). Above 13,000 km, there is no combination of love numbers or tidal lag angles that produce the required acceleration.

This is an extremely close proximity that violates Roche limit requirements. The Roche limit is the distance at which a smaller body orbiting a larger one will be torn apart by tidal forces. In the particular case of the impact-generated disk for the Earth/Moon system, material within this distance will accrete to form the Earth, and outside of this distance the Moon (Barr, 2016). For the Earth/Moon system, the Roche limit is given as approximately 3 Earth radii (Barr, 2016), though the Roche limit for an entirely fluid body can be approximated as: $d_f \approx 2.44\, r_E \left(\frac{\rho_E}{\rho_M}\right)^{\frac{1}{3}}$. In this equation, d_f is the Roche limit for a fluid body, r_E is the radius of the Earth, ρ_E is the average density of the Earth (5.5 g/cm3), and ρ_M is the average density of the Moon (3.34 g/cm^3). This gives a Roche limit of ~18,357 km, which is significantly larger than the distance required by this hypothesis. If the Earth and Moon are considered rigid bodies, this hypothesis no longer violates the Roche limit. For entirely rigid bodies, the Roche limit can be

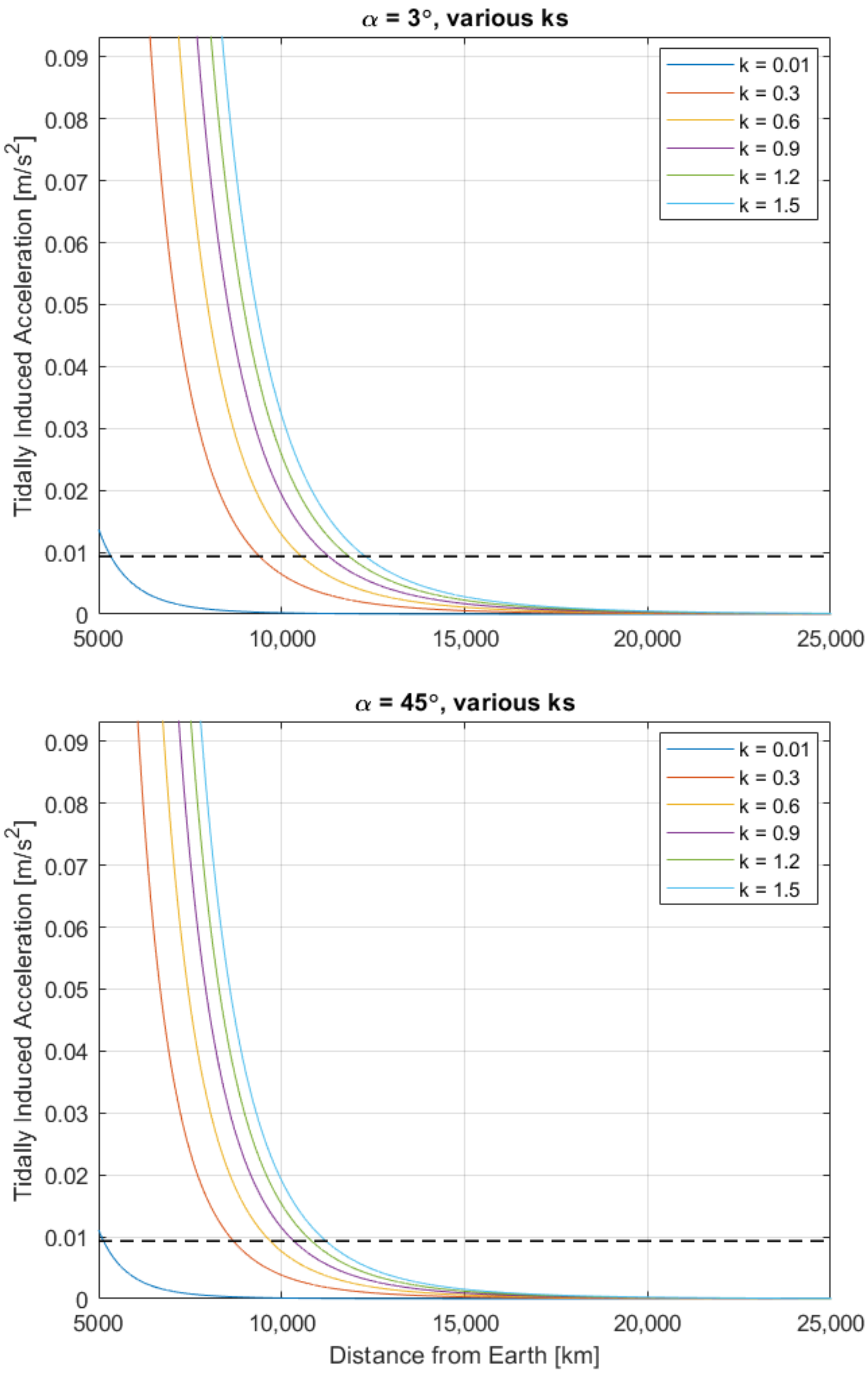

Figure 3. Tidally induced acceleration versus distance for multiple love numbers. Top: for a tidal bulge angle of 3°; Bottom: 45°. The black, horizontal dashed line is the approximate tidal-bulge induced acceleration necessary to produce the observed crustal dichotomy. Distance has the greatest importance, followed by the love number. The tidal lag angle has little effect.

approximated using the following equations: $d_{R_1} \approx r_E \left(2\frac{\rho_E}{\rho_M}\right)^{\frac{1}{3}}$ *or* $d_{R_2} \approx r_M \left(2\frac{m_E}{m_M}\right)^{\frac{1}{3}}$. In these equations, m_E is the mass of the Earth (5.97×10^{24} kg), and other symbols are as described previously. These equations give a Roche limit for a rigid body of 9,479 and 9,481 km respectively. This is marginally less than the Earth/Moon distance required to produce the requisite acceleration for reasonable parameter values. This might give a person hope for this hypothesis. However, it is impossible for the Earth/Moon system to be entirely rigid.

Even today, after 4.5 GYA of cooling, neither the Earth nor the Moon is completely rigid (Zhang, 1992). A number of hypotheses require an initially completely molten Moon (e.g., (Lock et al., 2018); also see works described in (Barr, 2016; Canup et al., 2021)). In essentially all hypotheses for lunar formation, at least some significant portion of the Moon is fluidized. This renders the smaller Roche limit, $d_{R_{1-2}}$, that the tidally induced acceleration hypothesis requires very unlikely. Furthermore, even if the Earth and Moon could be considered nearly completely rigid, that rigidity reduces the size of the tidal bulge the Moon can produce on the Earth and therefore the tidal acceleration the Earth can produce on the Moon, requiring an even smaller Earth/Moon distance. This can be seen in Figure 3 for $k = 0.001$.

3.1 Caveats

In this sub-section, I describe some of the limitations of the methodology I used. Although the analysis of the Roche limit eliminates this hypothesis, I consider it useful to explore some of the other complications that would arise to put this hypothesis into proper context.

I assume the Moon is formed via a giant impact that requires it to fully differentiate (i.e., the Moon is at least partially fluid (possibly entirely fluid)). In part, I assume a differentiated Moon because contemporary hypotheses for lunar formation require it; this hypothesis would not work if the Moon is a captured object or somehow formed beside the Earth. Also because it simplifies the physics determining how material is redistributed. More complex mechanisms than 'simple' gravity-driven flotation can be at play (e.g., magma ocean convection).

I treated the Earth/Moon system as a simple 2D point system. This neglects the effects of the Earth's obliquity and the Moon's eccentricity and inclination, as well as the effects of other bodies (such as the Sun's gravity) on the Earth and Moon.

The density balance used to acquire the necessary acceleration is a gross approximation that will require improvement if this hypothesis is to be pursued further. Equation 1 was simply an ad hoc approximation created by balancing the units involved. Although equation 3 is more steeped in actual physics, it suffers from approximations as well in that it falsely inflates the thickness of the Moon. I simply add the crustal thickness on top of the Moon's radius. That does not reflect reality, though I consider it an acceptable first order approximation for two reasons. The first is that the thickness of the crust in comparison to the full radius of the Moon is very small, suggesting that the error produced by this approximation will also be very small. The second is the surprisingly good agreement between the ad hoc approach and the density balance approximation. Though sufficient for a first order test, more refined testing of this hypothesis would require a better estimate of the tidally induced acceleration necessary to produce the observed dichotomy.

The equations used to calculate the Roche limits are both very simplistic approximations. They either assume completely rigid bodies or completely fluid bodies, neither of which accurately describes the Earth/Moon system now or in the distant past. More exact and/or numerical methods may reveal that the Roche limit is more relaxed than determined here. However, to make this hypothesis viable, such an extreme degree of change is necessary that I consider it effectively impossible.

I assume the Moon reaches synchronous rotation instantly and keeps the same side facing the Earth for all times. This might reflect reality accurately, but has significant possible range.

Roy et al. (2014) state the Moon's rotation becomes synchronous after only ~100 days; another source gives 10,000 years (Makarov, 2013); a third source gives several million years (Peale, 1977). If the Moon is non-synchronously rotating, the tidal acceleration on the Moon would no longer concentrate low density material at the prograde pole and reduce low density material at the retrograde pole. Instead, the rotation would cause the acceleration to average out to a uniform value around the equator, resulting in no crustal asymmetry and therefore no evidence of the tidal acceleration.

Further, I neglect the effects of viscosity and the time required for material in the liquid Moon to redistribute itself; in other words, I assume an instantaneous jump from initial conditions to their final state. The crystallization and flotation of material in a lunar magma ocean is a non-trivial problem. Working within this hypothesis, the retrograde pole would experience greater total downward acceleration (by the Moon's reference frame) and therefore a faster rate of flotation of anorthite crystals to the surface. This is because gravity drives the rate of separation in a fluid medium. This means that the retrograde pole would initially have a thicker low-density crust. The crust of the prograde pole would require time to "catch up" to and then surpass the thickness of the crust of the retrograde pole. Fully understanding this requires thorough exploration of cooling and crystallization of the lunar magma ocean and the time frames over which it occurred. This also ignores any internal convection within the Moon (which likely further complicate the flotation process).

4 Conclusions

Ultimately, the idea that tidally induced acceleration could cause the observed lunar crustal asymmetry seems highly unlikely. This hypothesis requires an extremely close proximity of ~10,000 km that – once Roche limits are considered – then requires the Earth and Moon be nearly completely rigid bodies in order to prevent tidal stresses from ripping the Moon apart. Neither body exhibits this state today even after 4.5 GYA of cooling. Even if the two bodies were completely rigid in the distant past, that rigidity would then require an even smaller Earth/Moon distance that would put the outer edge of both bodies within the other, a physical impossibility. These results render alternative hypotheses as far more plausible for the cause of the lunar crustal dichotomy. It would therefore be more fruitful to explore the body of alternative hypotheses until they are disproven or found to be similarly unlikely.

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Open Research

This project did not produce any new data. All data used in this project (e.g., tidal love numbers, mass of the Moon, etc.) are publicly available information. Figure 3 was created using MATLAB.

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