Emergence of efficient channel networks in fluvial landscapes

Dnyanesh Vijay Borse[1] and Basudev Biswal[1]

[1]Department of Civil Engineering, Indian Institute of Technology Bombay

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Abstract

Channel networks across fluvial landscapes are believed to have evolved to minimize energy expenditure[1–3], as evidenced by the similarities between computer-generated optimal channel networks (OCNs) and real networks[4,5]. However, the specific mechanisms driving energy minimization in fluvial landscapes remain largely elusive[6]. Here we propose that randomness has a profound role in landscape evolution[7] and that efficient channel networks emerge when the probability of a channel pixel changing its flow direction decreases with drainage area. The proposed probabilistic growth model then employs a power function to simulate channel-network evolution, with positive exponent (?) values leading to asymptotic decrease of energy expenditure. An interpretation of this result is energy minimization tendency of river networks is a result of landscape evolution following specific adaptive rules rather than being the cause of landscape evolution itself. A greater ? ensures a greater restriction on the role of randomness and thus results in a more stable channel network configuration, and vice versa. Interestingly, the most efficient networks are observed to emerge always at ? =0.5, suggesting that randomness plays an important but limited role in the emergence of efficient channel networks. The proposed framework holds promise for explaining the evolution of other tree-like networks in nature and for developing more efficient optimization methods for practical applications.

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Dnyanesh Borse[1], Basudev Biswal[1]

[1]Department of Civil Engineering, Indian Institute of Technology Bombay, Mumbai 400076, India
Dnyanesh Borse: dnyanesh.borse@iitb.ac.in (Corresponding Author)
Basudev Biswal: basudev@iitb.ac.in

Abstract

Channel networks across fluvial landscapes are believed to have evolved to minimize energy expenditure[1–3], as evidenced by the similarities between computer-generated optimal channel networks (OCNs) and real networks[4,5]. However, the specific mechanisms driving energy minimization in fluvial landscapes remain largely elusive[6]. Here we propose that randomness has a profound role in landscape evolution[7] and that efficient channel networks emerge when the probability of a channel pixel changing its flow direction decreases with drainage area. The proposed probabilistic growth model then employs a power function to simulate channel-network evolution, with positive exponent (η) values leading to asymptotic decrease of energy expenditure. An interpretation of this result is energy minimization tendency of river networks is a result of landscape evolution following specific adaptive rules rather than being the cause of landscape evolution itself. A greater η ensures a greater restriction on the role of randomness and thus results in a more stable channel network configuration, and vice versa. Interestingly, the most efficient networks are observed to emerge always at $\eta = 0.5$, suggesting that randomness plays an important but limited role in the emergence of efficient channel networks. The proposed framework holds promise for explaining the evolution of other tree-like networks in nature and for developing more efficient optimization methods for practical applications.

Main

Origins of fluvial channel networks continue to create curiosity among us because the processes leading to their formation are exceedingly complex and have not been fully understood yet. Nevertheless, networks across regions show remarkable statistical similarities[8,9], which is why most of the early models were statistical in nature[10]. Statistics-based approach can provide a diverse range of views on channel network structure. The random topology models explore possibilities of connecting nodes to form tree-like networks[11]. The statistical growth models allow network formation to begin at the outlet and gradually grow adding nodes to form tree-like networks resembling fluvial channel networks. The rationale behind these network growth models is that disturbance caused due to erosion propagates in the upstream direction[12,13]. It is also possible for a network growth model to produce networks with varying shapes and sizes and explain the scaling laws of river networks[14]. The main criticism of the statistics-based models is that they provide a very limited understanding of channel network evolution. Many studies have therefore attempted to use mass and momentum conservation equations for simulating channel networks[15,16]. However, these models too, do not explain the processes leading to the formation of fluvial channel networks properly as it is not possible to have detailed information on the initial conditions of a landscape. Moreover, the role of heterogeneity within a process-based model is typically handled statistically as it is not possible to do so in a fully mechanistic way[17].

A completely different viewpoint was proposed by Leopold[18] that states landscapes evolve so as to form optimal channel network configuration. Although the optimality hypothesis is based on sound physics and has proven its worth in many scientific disciplines[19], it is quite unclear what exactly is optimized in the context of channel network evolution. Many objective functions have been proposed in the past to generate optimal channel networks (OCNs)[20], and it is not very uncommon to see contradictions[6]. The most widely accepted optimality hypothesis is that channel networks evolve to minimize total energy expenditure, quantitatively given as:

$$E \propto \sum(\Delta x_i \cdot Q_i{}^{\gamma}) \propto \sum(\Delta x_i \cdot A_i{}^{\gamma}) \quad (1)$$

where Δx_i is the length of the ith channel segment and Q_i is discharge through it, which is assumed to be proportional to the drainage area (A_i). The exponent γ characterizes the fluvial processes. Its value is typically observed to be close to 0.5 [21], implying that energy expenditure per unit channel-bed surface is spatially constant and that energy minimization also happens locally at every channel segment[21]. Numerous studies have been conducted using Equation (1) as the objective function, and the resulting OCNs have shown to capture the key statistical characteristics of real channel networks, suggesting the hypothesis is grounded well[5,22,23].

However, the OCN model sheds no light on the mechanisms behind the tendency of channel networks to become efficient. With simulation results from a process-based model accounting for erosion and deposition, Paik and Kumar[7] concluded that landscape heterogeneity leads to the formation of tree-like channel networks, thereby minimizing energy. However, they did not quantify the role of heterogeneity in channel network evolution. Moreover, they did not compare the energy expenditure of their simulated networks with that of OCNs. In fact, no comprehensive study so far, to our knowledge, has compared simulated networks with real networks in term of energy expenditure. In this study, we propose a probabilistic network growth model and compare the energy expenditure of the simulated networks with that of real channel networks and with the networks obtained using OCNet[4,24], a well-known model for generating OCNs.

The working of the model is given as follows. In each iteration, the proposed model assigns flow direction to all the pixels within a given planar boundary through a step-by-step procedure (refer to Methods). Each step involves selecting a pixel from those neighbouring the already evolved drainage network, based on a probabilistic function.

$$P_i \propto A_i{}^{\eta} \quad (2)$$

where P_i is the probability of the pixel i being selected in the step and A_i is the drainage area of the pixel. The flow direction for the pixel is assigned towards the neighbouring pixel of the already evolved network with the highest drainage area. The detailed methodology is described in the methods section.

A channel network evolves when the forces trying to change flow directions dominate the forces trying to preserve them. The parameter η is a numerical representation of the relative roles of these forces. When $\eta = 0$, forces of change or randomness dominates everywhere, resulting in the generation of an Eden-type network configuration[25,26] in each iteration that possesses no memory of the previous network configuration (Fig.1a). A positive η implies forces of change weakening with drainage area (Equation 2), ensuring a relatively greater stability for higher order channels (Fig.1a-f). As η increases, forces of change weaken and a greater portion of the initial network is retained (Fig.1g). The hierarchical reorganization of the drainage network is believed to occur through mechanisms such as valley migration and stream capture[27–29]. Energy expenditure (ΔE, expressed as % extra energy with respect to that given by OCNet) is observed to asymptotically decrease for any positive η (Fig.1h), suggesting the possibility of the proposed model explaining quite well the emergence of efficient channel networks in fluvial landscapes. The final, steady-state value of energy expenditure ΔE_s (ΔE after $N = 100$ here) decreases with η, with the emergence of the most efficient network configuration at $\eta = 0.5$, after which ΔE_s follows an increasing trend (Fig.1i). While the initial Eden-type network configuration shows ΔE_s approximately equal to 10%, the final network configuration obtained with $\eta = 0.5$ is as efficient as the OCNet (Fig.1i), supporting the notion that energy minimization is merely a consequence of landscapes following a few thumb rules to evolve.

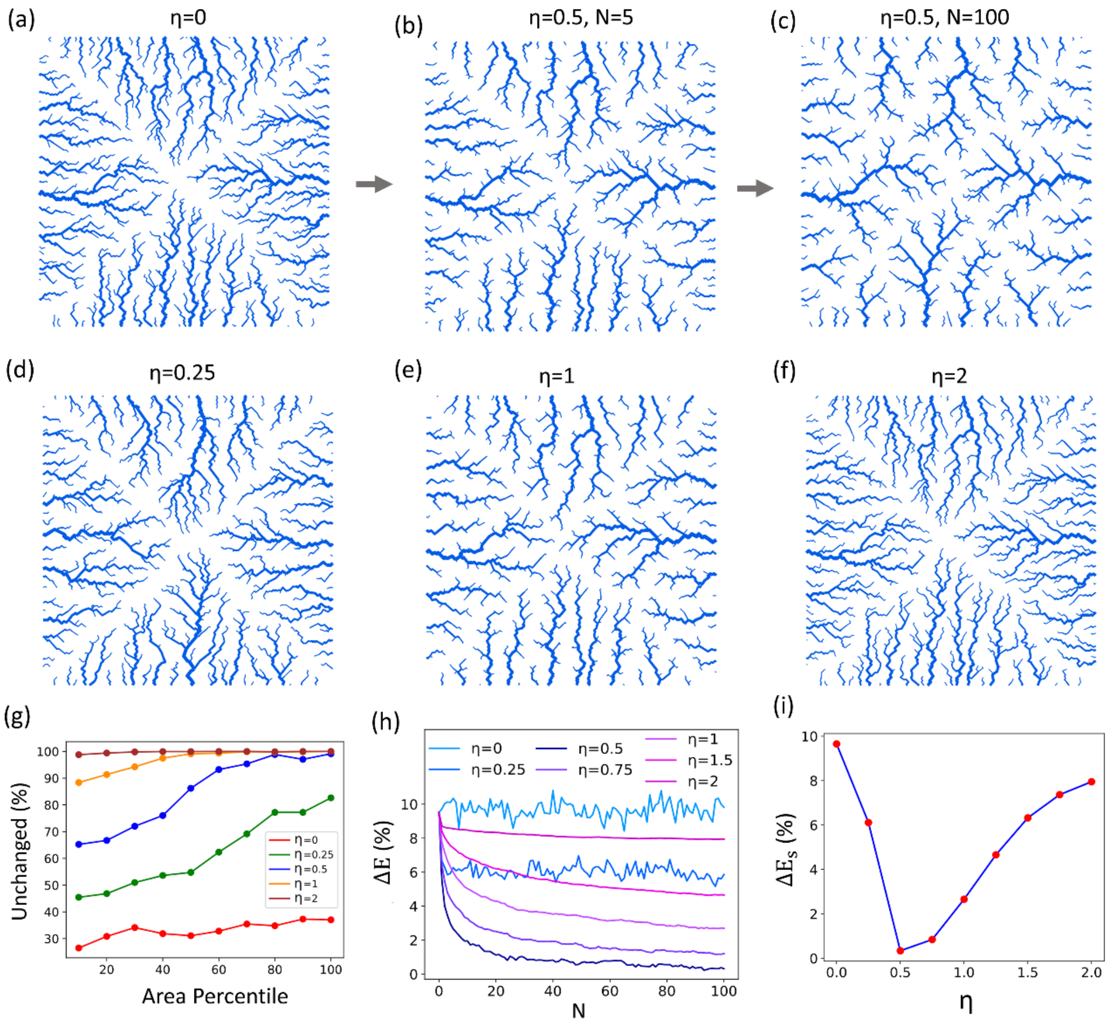

Figure 1: Evolution of drainage networks according to the proposed model
A sample Eden-type network obtained with $\eta = 0$ (a), which is allowed to evolve considering different η values: b) the networks with $\eta = 0.5$ for iteration $N = 5$ and c) for $N = 100$. The networks after $N = 100$ for η= 0.25,1 and 2 respectively are shown in (d), (e) and (f). (g) Percentage of total pixels that didn't change during the 100th iteration vs. drainage area percentile, indicating that pixels with higher drainage area are relatively more stable compared to pixels with lower drainage area and that the stability increases with η. (h) ΔE vs. N curves for different η values, which shows consistent decrease of ΔE, visible particularly for $\eta > 0$. The most efficient configuration is obtained for $\eta = 0.5$ (i) Variation in ΔE_s for resulting networks with different η. Each datapoint is median ΔE_s from an ensemble of 20 simulations

The model's outcomes are influenced by its initial conditions. In an island resembling a square pyramid with $\Delta E = 70\%$, where the flow from every pixel is directed toward the nearest border pixel, the first iteration results in the formation of a network configuration with ΔE very close to that of an Eden-type network, irrespective of the value of η. This observation indicates a negligible role heterogeneity (represented by η, see Equation (2)) when branching has not formed yet. Fig.2 shows network configurations obtained with the model using different initial network configurations. Network configurations with high initial ΔE show a steep decrease of ΔE with N (Fig.2a-h). On the other hand, the network configuration obtained with OCNet as initial condition showed an increase of ΔE (Fig.2i-l). For the network configuration obtained from OCNet (initial $\Delta E = 0$), ΔE first increased and then continued to decrease to attain a steady state for $\eta = 0.5$ (Fig.2l). A possible explanation could be that the organizations of the most efficient network configurations of the proposed model and OCNet have certain different key properties. It also underlines the fact that the proposed model works differently.

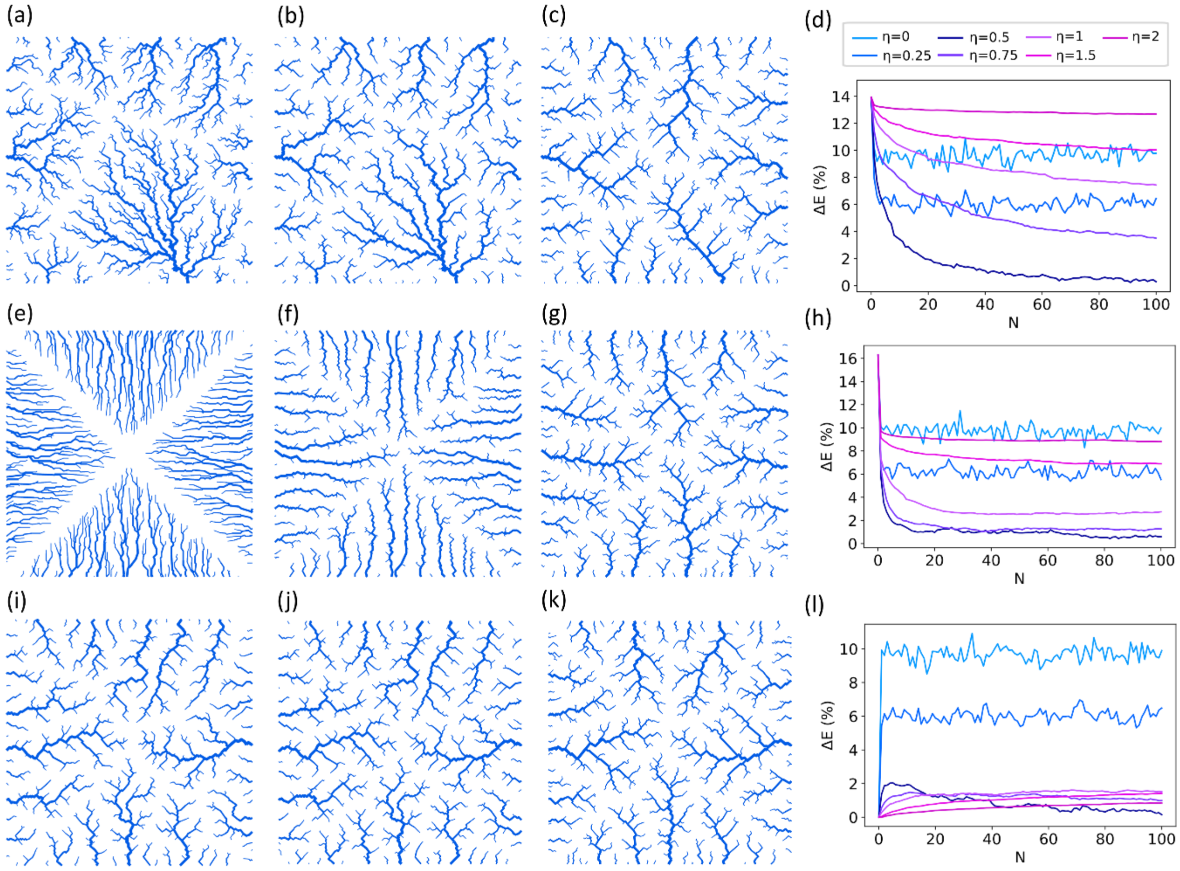

Figure 2: Model with different initial conditions. (a), (e) and (i) show networks obtained from a probabilistic model[14], Paik and Kumar's model[7] and OCNet[24] model, respectively, which are allowed to evolve using the proposed model. (b), (f) and (j) show resulting networks with $\eta = 1$ from initial conditions corresponding to (a), (e), and (i), respectively, after $N = 100$. Similarly (c), (g) and (k) show resulting networks with $\eta = 0.5$ and (d), (h) and (l) show the corresponding energy profiles.

The ΔE_s is independent of initial conditions for $\eta \leq 0.5$ and the resulting network configurations are quite indistinguishable (Fig.3). On the other hand, for values of η greater than 0.5, the evolved network configurations share a lot of similarities with the initial configurations and the ΔE_s is influenced greatly by the initial conditions (Fig.2). That is to say $\eta = 0.5$ acts as a threshold, below which heterogeneity determines ΔE_s, and above which initial conditions influence ΔE_s; the degree of this influence increases with η. The most efficient network configuration always appears when $\eta = 0.5$ with ΔE_s quite close to that given by OCNet (Fig.3), implying that the proposed algorithm provides a possible explanation for the emergence of efficient channel networks. Interestingly, the assertion $\eta = 0.5$ leading to OCN formation is true for any other γ (Equation.1) as long as its value falls between 0 and 1, which indicates the robustness of the optimality hypothesis.

Although the model does not explicitly take time into account, ΔE of a landscape is expected to decrease with time before it reaches a steady state (Fig.1h). Thus, if a landscape is relatively young, it is expected to show high ΔE. The island Hawaii, which is only about 0.5 million years old[30], has $\Delta E = 31.7\%$. In comparison, the 5-million year old island Kauai[30] shows just 8.5% ΔE (Fig.4a-b). Since both islands are located in the same geographical region and are expected to exhibit similar evolutionary trajectories, it is quite certain that Hawaii is at an early evolutionary stage. However, the nearly one billion year old Tasmania[31], expected to exhibit an optimal channel configuration, shows

$\Delta E = 9.8\%$ (Fig.4c). In fact, for none of the ten islands studied here $\Delta E \approx 0$ (Table S1), which suggests real landscapes may not be evolving with the intention of attaining a state of optimality, and thus the value of η cannot be simply assumed to be 0.5. This also means energy minimization is an outcome of landscape evolution rather than the cause of landscape evolution[6]. The above observations pose the challenge of predicting the future evolutionary trajectory of a landscape. What is the value of η we need to select for a landscape? For a given non-zero ΔE_s, there are two possible values of η (Fig.3). Geological and tectonic constraints are believed to be responsible for suboptimal network configurations in real landscapes[25], meaning $\eta > 0.5$ condition is more likely. Future studies need to focus on exploring all possibilities.

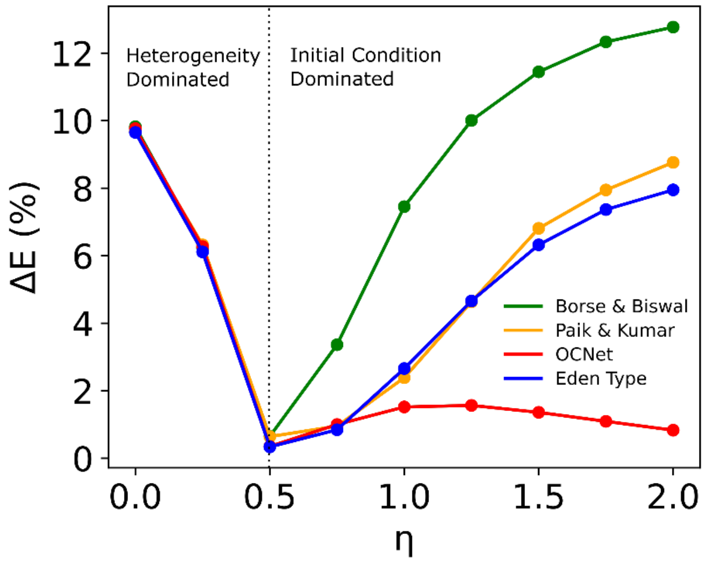

Figure 3: Steady-state (N=100) ΔE_s vs. η from different initial conditions. When $\eta \leq 0.5$ (heterogeneity dominated systems), ΔE_s is independent of initial conditions, whereas initial condition has a profound influence on ΔE_s when $\eta > 0.5$. Note that each data point in the plot is median ΔE_s from an ensemble of 20 simulations.

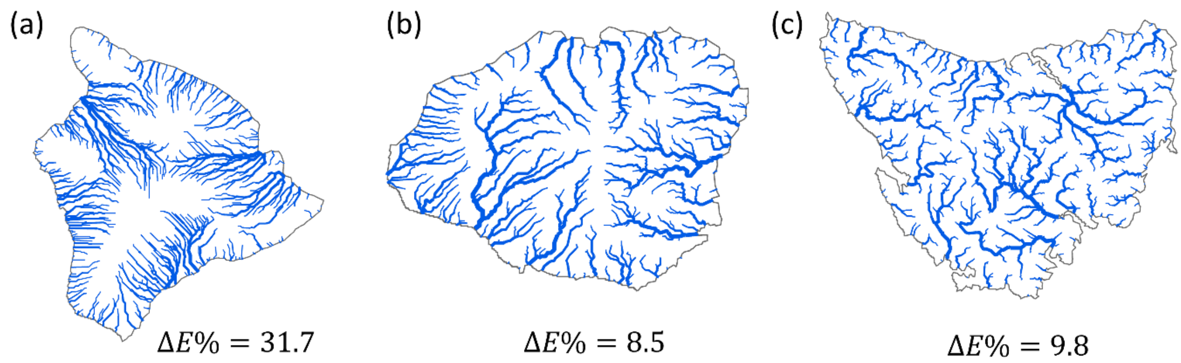

Figure 4: Energy Expenditure of real-world channel networks. (a), (c)&(e) shows real networks of Hawaii, Kauai and Tasmania Islands, respectively.

The main discussion point of this study is the role of randomness, reflected in terms of landscape heterogeneity, in the emergence of efficient channel networks. Although Paik and Kumar[7] also recognized the role of randomness and observed increasing efficiency with evolution, the evolved channel networks obtained by their model are not efficient (Fig.2e and Fig.2h). The difference is our study sees a rather limited role of randomness as highlighted by the observation that channel networks obtained with $\eta < 0.5$ are not that efficient. This resonates quite well with the observation made by

Watts and Strogatz[32] that limited randomness is a 'necessary condition' for the emergence of small-world networks. Nevertheless, our study provides a much broader picture by revealing the hierarchical influence of randomness on the flow direction of a channel segment. The condition $\eta = 0.5$ always leads to the most efficient network configuration, even though the model does not employ an optimization scheme. The model thus holds the potential to be used as an optimization algorithm for practical problems concerning network optimization. While our analyses are restricted to river networks, the idea that specific adaptive rules representing basic physical processes give rise to networks exhibiting tendency to minimize transportation efficiency may be applicable to other physical and biological networks, such as vascular, root and respiratory networks[2,33,34]

Methods

The model is demonstrated using a 250×250 planar matrix that represents a hypothetical landscape. The model can be applied to any loopless flow conditions. During each iteration, the model assigns flow directions to all pixels by selecting one pixel at a time. The flow direction of a pixel can be oriented toward any one of its eight adjacent pixels, and its drainage area is quantified by flow accumulation, representing the total number of pixels flowing into it. All boundary pixels are considered as outlets. Initially, these outlet pixels are designated as "evolved pixels," while their neighbouring pixels are termed "potential pixels." In each computational step, a pixel is selected from the list of potential pixels using the power function (Equation.2). The chosen pixel is then assigned a flow direction towards the adjacent evolved pixel with the highest flow accumulation. This is because a pixel with higher flow accumulation would experience greater erosion, and thus would be at a lower elevation compared to adjacent pixels. This selected potential pixel is now reclassified as "evolved pixel" to evolved pixel and its neighbouring unevolved pixels are added to the list of potential pixels (Fig.S1). This process continues to assign flow directions to all the pixels. This constitutes as a single iteration and the model performs this all over again for the next iteration. The resulting flow directions of one iteration serve as input for the next iteration.

The model demonstrates network evolution from different initial conditions. The network shown in Fig.2a was generated using a probabilistic model proposed by Borse and Biswal[14]. This model, implemented on a 250×250 grid, simulates the probabilistic headward growth of channel networks which is assumed to be proportional to the pixel's length to the outlet. The network shown in Fig 2e was generated using Paik and Kumar's model[7], applied on a pyramidal-shaped 250×250 grid with parameter values similar to those mentioned in the study. This model simulates landscape evolution through mass balance processes and incorporates a randomly distributed surface resistance parameter.

To visualize these river networks, we have set the flow accumulation threshold at 50 for all cases. To compare the energy expenditure of real islands with any model, we need same sized grids. We obtained the island's digital elevation data from the SRTM 1 arc second global dataset and resampled it to a smaller size comparable to the already used square matrix. This resampled DEM was used to delineate networks and calculate the energy expenditure (Fig.4). We executed the OCNet provided by Carraro et.al.,[24] with default parameter settings to obtain OCN for the corresponding resampled grids. The excess energy ΔE for a particular network is calculated in reference to energy expenditure (Equation.1 for $\gamma = 0.5$) of OCN within the same boundary as $\Delta E = \frac{E - E_{OCNet}}{E_{OCNet}} \times 100$.

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Author Contributions

Author 1: Conceptualization, Methodology, Formal analysis, Data curation, Writing
Author 2: Ideation, Supervision, Formal analysis, Writing

Declaration of Competing interests

We have no conflicts of interest to disclose.

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Supplementary Information

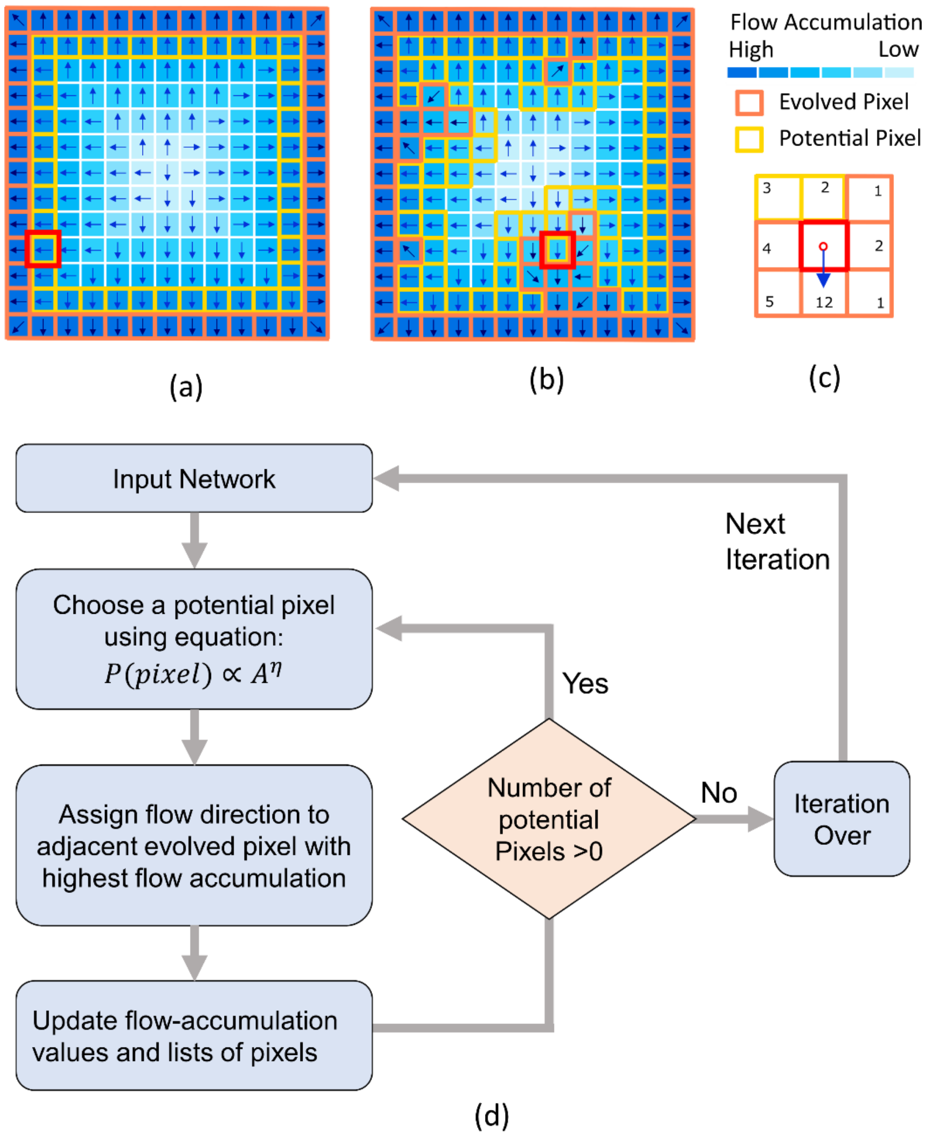

Figure S1: Model algorithm explained with a sample example with pyramid-like initial flow conditions (a). At the start of the iteration, outlets are “Evolved pixels” and neighboring to them are “Potential pixels.” In each step, a potential pixel would be chosen with probability obtained using equation 2. Let’s assume the pixel highlighted in red is selected in the first step (a), then it would be assigned flow direction towards the neighboring evolved pixel with the highest flow accumulation value. Thus, its updated flow direction would be toward northwest. After this, the flow accumulation values for corresponding pixels would be updated, and the three neighboring unevolved pixels would be reclassified as potential pixels. An intermediate step in the computation would look like (b), with a sample chosen potential pixel highlighted. The assignment of drainage direction for this highlighted potential pixel is shown in (c). Once all pixels are assigned, the iteration is over. The same process can be followed all over again for the next iteration as shown in (d). Note that the model can be simulated with any other initial loopless flow conditions.

Sl. no.	Island	Location	ΔE (%)
1	Barbados	Atlantic Ocean	11.4
2	Cyprus	Mediterranean Sea	13.1
3	Grenada	Caribbean Sea	5.6
4	Hawaii (the big island)	Pacific Ocean	29.14
5	Jeju	Yellow Sea	13.6
6	Kauai	Pacific Ocean	6.41
7	Mauritius	Indian Ocean	18.03
8	Cape Verde	Atlantic Ocean	12.92
9	Reunion	Indian Ocean	26.31
10	Tasmania	South Pacific Ocean	9.92

Table S1: ΔE (%) calculated for the different islands.