The mechanics of static non-planar faults in linear elasticity

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Abstract

Fault geometry is a key factor in controling the mechanics of faulting. However, there is currently limited theoretical knowledge regarding the effect of non-planar fault geometry on earthquake mechanics. Here, we address this gap by introducing an expansion of the relation between fault traction and slip, up to second order, relative to the deviation from a planar fault geometry. This expansion enables the separation of the effects of non-planarities from those of planar faults. This expansion is realised in the boundary integral equation, assuming a small fault slope. It provides an interpretation for the effect of complex fault geometry on fault traction, for any fault geometry and any slip distribution. Hence the results are also independent of the friction that applies on the fault. The findings confirm that fault geometry has a strong influence on in-plane faulting (mode II) by altering the normal traction on the fault and making it more resistant to slipping for any fault geometry. On the contrary, for out-of-plane faulting (mode III), fault geometry has a much smaller influence. Additionally, we analyse some singularities that arise for specific fault geometries often used in earthquake simulations and provide guidelines for their elimination. To conclude this study, we discuss the limits of the small strain approximation when non-planar faults are considered.













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#### Shear traction drag for different fault geometries



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-10 -8 -2 0 2 Position (km)

0-0.25 -0.2 -0.15 -0.1 -0.05 0 0.05 0.1 0.15 0.2 0.25 Difference from the theoretical shear traction drag (%) 10 8 6











For small strain, the elastic response is linear



For large displacement, the elastic response is non-linear







# The mechanics of static non-planar faults in linear elasticity

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August 2, 2023

### Abstract

Fault geometry is a key factor in controling the mechanics of faulting. However, 11 there is currently limited theoretical knowledge regarding the effect of non-planar 12 fault geometry on earthquake mechanics. Here, we address this gap by introducing 13 an expansion of the relation between fault traction and slip, up to second order, 14 relative to the deviation from a planar fault geometry. This expansion enables the 15 separation of the effects of non-planarities from those of planar faults. This expansion 16 is realised in the boundary integral equation, assuming a small fault slope. It provides 17 an interpretation for the effect of complex fault geometry on fault traction, for any 18 fault geometry and any slip distribution. Hence the results are also independent of 19 the friction that applies on the fault. The findings confirm that fault geometry has a 20 strong influence on in-plane faulting (mode II) by altering the normal traction on the 21 fault and making it more resistant to slipping for any fault geometry. On the contrary, 22 for out-of-plane faulting (mode III), fault geometry has a much smaller influence. 23

Additionally, we analyse some singularities that arise for specific fault geometries often used in earthquake simulations and provide guidelines for their elimination. To conclude this study, we discuss the limits of the small strain approximation when non-planar faults are considered.

# 28 1 Introduction



Figure 1: Description of the different modes of slip for a non-planar fault. Please note that we are using a slight abuse of language by calling them mode I, mode II, and mode III.

Understanding the mechanical effect of fault geometry is an important question for seis-29 mology, as it can influence various aspects of earthquake mechanics, such as the areas of 30 the fault that will nucleate, the size of the rupture, or the slip distribution (Aki, 1979; King 31 and Nabelek, 1985; Schwartz and Sibson, 1989; Klinger et al., 2006; Wesnousky, 2006, 2008; 32 Milliner et al., 2015). Previous theoretical efforts to understand the effect of fault non-33 planarities have been primarily focusing on particular fault geometries such as sinusoidale 34 geometry (Saucier et al., 1992; Chester and Chester, 2000), or rough faults (Dieterich and 35 Smith, 2009; Dunham et al., 2011; Fang and Dunham, 2013), limiting the generalizability 36

37 of the results.

In this paper, we extract the effect of non-planarities from the effect of planar fault by performing an expansion of the boundary integral equation. It makes the results presented in this paper very general as they do not depend on one specific geometry, or one specific slip distribution. It also makes the results presented here independent of the friction law that applies on the fault.

Our emphasis will be on the two modes of shearing fault: in-plane faulting and out-of-43 plane faulting. However, the results presented in this paper can also be applied to opening 44 faults, and the associated findings can be found in Appendix G. We will use a slight abuse 45 of language by sometimes calling mode I, mode II, and mode III what should respectively be 46 called in-plane opening, in-plane shearing, and out-of-plane shearing. Mode I corresponds 47 to pure opening, Mode II corresponds to a pure shear fault, where the direction of rupture 48 is parallel to the direction of slip. Mode III corresponds to a pure shear fault where the 49 direction of rupture is perpendicular to the direction of slip (see Fig. 1). The primary 50 objective of this paper is to provide a comprehensive explanation of the main effects of 51 non-planar fault geometry on the traction that applies on the fault. 52

# ⁵³ 2 Tractions on a non-planar fault



Figure 2: Definition of the parameters used in this study. The flat fault approximation is when the slope between any two points of the fault is considered small. This figure is modified from *Romanet et al.* (2020).

This paper derives exact semi-analytical solutions for the strains and stresses within a homogeneous, infinite, linear, static, two-dimensional (2D) medium caused by a finite, non-planar fault (see Appendix G, H and I). It can be viewed as the static, 2D extension of the previous work by *Romanet et al.* (2020), where the semi-analytical solution was derived for a fully-dynamic, three-dimensional (3D) medium using the Einstein notation.

The geometry of the fault is described by a function  $h(y_1)$  representing the height of the fault at given position  $\mathbf{y} = (y_1, y_2) = (y_1, h(y_1))$  (Fig. 2). The derivative of the height with respect to the coordinate  $y_1$  (the fault slope) is denoted as  $m(y_1) = \frac{d}{dy_1}h(y_1)$ . When the fault slope is small  $(m(y_1) << 1)$ , the second order derivative of height with respect to the coordinate  $y_1$  can be linked to the curvature along the fault  $\kappa^t$  by:

$$\kappa^{t}(y_{1}) = \frac{\frac{d}{dy_{1}}m(y_{1})}{(1+m^{2}(y_{1}))^{\frac{3}{2}}}$$

$$\simeq \frac{d}{dy_{1}}m(y_{1}) = \frac{d^{2}}{dy_{1}^{2}}h(y_{1}), \text{ if } m(y_{1}) << 1$$
(1)

The absolute value of the fault curvature  $|\kappa^t|$  can be interpreted geometrically as the inverse radius of the tangential circle to the fault (see Fig. 2). When the fault is locally flat, the curvature is zero ( $\kappa^t(y_1) = 0$ ).



Figure 3: A non-planar fault can be made equivalent to a planar fault with additional on-fault stresses correction. This is not working for off-fault stresses.

This paper demonstrates that the semi-analytical solution for the on-fault stresses can be significantly simplified when assuming a small fault slope. For the on-fault stresses, it is possible to identify the exact mechanical corrections that renders a planar fault mechanically equivalent to a non-planar fault by including additional terms that account for the non-planar geometry (see Fig. 3). In order not to distract the reader from the main points of this paper, most of the derivation is provided in appendix B. In the following, we just provide a summary of the principal steps involved in the derivation:

1. We start from the representation theorem (Aki and Richards (2002), chapter 2.5)

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which allows to calculate the displacement  $u_k$  anywhere in the medium by evaluating an integral over the fault involving the Green's function  $G_{kp}$  (*Tada and Yamashita*, 1997) and the displacement discontinuity  $\Delta u_i$  across the fault. The subscripts refer to specific component of the vector, for example  $u_k$  is the  $k^{\text{th}}$  component of the slip vector **u**.  $c_{ijpq}$  is the Hooke tensor, and **n** is the normal vector to the fault.

$$u_k(\mathbf{x}) = -\int_{\text{fault}} c_{ijpq} \Delta u_i(\mathbf{y}) n_j(\mathbf{y}) \frac{\partial}{\partial x_q} G_{kp}(\mathbf{x}, \mathbf{y}) dl(\mathbf{y})$$
(2)

2. By using the strain definition  $\epsilon_{cd} = \frac{1}{2} \left( \frac{\partial}{\partial x_c} u_d + \frac{\partial}{\partial x_d} u_c \right)$  and the Hooke's law ( $\sigma_{ab} = c_{abcd} \epsilon_{cd}$ ), we can obtain the stresses  $\sigma_{ab}$  at any point within the medium. It leads to an integral linking the stresses to the Green's function and the displacement discontinuity along the fault. Unfortunately the resulting integral is hypersingular for the on-fault stresses. This hypersingularity arises because of the second-order derivative of the Green's functions. As a result, conventional numerical integration technics cannot be used (*Koller et al.*, 1992; *Tada and Yamashita*, 1997).

$$\sigma_{ab}(\mathbf{x}) = -c_{abcd} \int_{\text{fault}} c_{ijpq} \Delta u_i(\mathbf{y}) n_j(\mathbf{y}) \frac{\partial}{\partial x_q} \frac{\partial}{\partial x_d} G_{cp}(\mathbf{x}, \mathbf{y}) dl(\mathbf{y})$$
(3)

3. We regularize this integral using the tangential differential operator (Bonnet, 1999;
Sato et al., 2020) and project the slip vector onto the fault (Romanet et al., 2020).
This makes appear the curvature term and the gradient term into the equation:

$$\sigma_{ij}(\mathbf{x}) = \int_{\text{fault}} \underbrace{\underbrace{c_{abcd}c_{ijpq} \frac{\partial}{\partial x_q} G_{cp}[n_d t_j - n_j t_d] t_i}_{\text{Kernel}} \underbrace{\frac{\partial}{\partial y^t} \Delta u(\mathbf{y})}_{\text{Gradient of slip}} \, \mathrm{d}l(\mathbf{y}) \\ + \underbrace{\int_{\text{fault}} \underbrace{c_{abcd}c_{ijpq} \frac{\partial}{\partial x_q} G_{cp}[n_d t_j - n_j t_d] n_i}_{\text{Kernel}} \underbrace{\kappa^t(\mathbf{y}) \Delta u(\mathbf{y})}_{\text{Curvature \times Slip}} \, \mathrm{d}l(\mathbf{y})}_{\text{Curvature term}}$$
(4)

This equation is now Cauchy integrable, and **no longer** hypersingular. In the gradient term,  $\frac{\partial}{\partial y^t} \Delta u(\mathbf{y})$  represents the derivative along the fault direction (see Appendix B.1). In the curvature term, the local curvature of the fault  $\kappa^t$  can be seen. The *t* upper-script emphasizes that the curvature is the one in the direction of the tangential vector to the fault  $\mathbf{t}$ .

4. For on-fault shear ( $\tau_{el} = t_i \sigma_{ij} n_j$ ) and normal ( $\sigma_{el} = n_i \sigma_{ij} n_j$ ) tractions only, we can develop the integrand that consists of Green's function and Hooke tensor while making the small slope approximation of the fault geometry (see Fig. 2, *Romanet et al.* (2020); *Romanet and Ozawa* (2021)). This approach allows to derive a simplified expansion for the elastic tractions on the fault due to a slip distribution:

$$\underbrace{\tau_{\rm el}(\mathbf{x})}_{\substack{\text{Full solution}\\\text{Elastic shear traction}}} = \underbrace{\tau_{\rm el}^{0}(x_{1})}_{0^{\rm th order}} + \underbrace{\tau_{\rm el}^{1}(x_{1})}_{1^{\rm st order}} + \underbrace{\cdots}_{\text{higher orders}}$$

$$\underbrace{\sigma_{\rm el}(\mathbf{x})}_{\substack{\text{Full solution}\\\text{Elastic normal traction}}} = \underbrace{\sigma_{\rm el}^{0}(x_{1})}_{0^{\rm th order}} + \underbrace{\sigma_{\rm el}^{1}(x_{1})}_{1^{\rm st order}} + \underbrace{\cdots}_{\text{higher orders}}$$
(5)

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The 0th order terms in the previous equation represent terms that are independent of the slope between any two points of the fault. The 1st order terms are terms whose integrands are proportional to the fault slope  $\propto \frac{x_2-y_2}{x_1-y_1}$ , and the 2nd order terms are terms whose integrands are proportional to the square of the fault slope  $\propto \left(\frac{x_2-y_2}{x_1-y_1}\right)^2$  and so on. This decomposition process may appear complex, but it offers two significant advantages when it comes to decomposing shear and normal traction:

- (a) Interpretability: Breaking down the tractions into terms based on fault slope
   provides a clearer understanding of the physical processes at play. Each order of
   terms corresponds to a specific level of importance regarding the effect of fault
   geometry, allowing for easier interpretation and analysis.
- (b) Approximation: The small slope approximation in the decomposition process enables us to simplify the mathematical expressions and calculations involved. This approximation is often valid in many practical scenarios, because faults are more or less linear structures. As a result, more computationally efficient numerical methods can be employed while still capturing essential aspects of fault behavior (*Romanet and Ozawa*, 2021).

The discretisation of the integrals needed for the calculation of the stresses (eq.(4)) and the on-fault shear and normal traction (eq. (5)) is done using a piecewise discretisation of the fault geometry, together with a piecewise discretisation of the slip (see Appendix 17).



### ¹¹⁹ 2.1 In-plane shear (mode II)

Figure 4: Assuming the fault geometry (as shown by the x-axis) and the slip distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$  (red curve), L being the length of the fault, the shear traction (upper panels) and the normal traction (lower panels) term can be decomposed into 0th order term and 1st order term. The 0th order and the 1st order (blue lines), are respectively calculated using the expressions (8) and (9) (more precisely the spectral version of these equations as given by (74) and (76) in appendix H). The full solution and associated gradient and curvature terms (black lines) are calculated using eq. (72).

By employing the small slope approximation, the shear and normal tractions can be expanded as:





$$\tau_{\rm el}^{0}(x_{1}) = -\frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_{1} - y_{1}} \frac{d}{dy_{1}} \Delta u^{t}(y_{1}) \right] dy_{1}$$
  
$$\sigma_{\rm el}^{0}(x_{1}) = -\frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_{1} - y_{1}} \kappa^{t}(y_{1}) \Delta u^{t}(y_{1}) \right] dy_{1}$$
(8)

$$\tau_{\rm el}^{1}(x_{1}) = \frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_{1})}{x_{1} - y_{1}} - \frac{x_{2} - y_{2}}{(x_{1} - y_{1})^{2}} \right] \kappa^{t}(y_{1}) \Delta u^{t}(y_{1}) \mathrm{d}y_{1}$$

$$\sigma_{\rm el}^{1}(x_{1}) = \frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) \int_{-\infty}^{+\infty} \left[ \frac{2m(x_{1})}{x_{1} - y_{1}} - \frac{m(y_{1})}{x_{1} - y_{1}} - \frac{x_{2} - y_{2}}{(x_{1} - y_{1})^{2}} \right] \frac{d}{dy_{1}} \Delta u^{t}(y_{1}) \mathrm{d}y_{1}$$
(9)

Contrary to the general expression given in eq. (5), the integrals in this decomposition are 121 over the line  $(dy_1)$  and not over the fault  $(dl(\mathbf{y}))$ . An example of this decomposition of shear 122 and normal traction into  $0^{\text{th}}$  order term and  $1^{\text{st}}$  order term is illustrated in figure 4. The 123 exemple considers a rough fault with an assumed slip distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$ , 124 where L is the length of the fault. It can be observed that for a rough fault, the  $0^{\text{th}}$ 125 and 1st order terms are already capturing a significant portion of the physics of the shear 126 and normal traction. In the subsequent section, a physical interpretation of the  $0^{\text{th}}$  and 127  $1^{\rm st}$  order terms for both shear and normal traction is provided. Additionally, analytical 128 results and possible other approximations will be presented. 129

130 The 0th order contribution to shear traction  $\tau_{el}^0$ : the planar fault response



Figure 5: Assuming a flat fault geometry and the slip distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$ (red curve), L being the length of the fault, the shear traction can be calculated analytically (black dash line, see appendix E). It is compared with the full solution as given by eq (72), and the 0th order solution as calculated by eq. (74) in the spectral domain. The 0th order solution for the shear traction is exactly the one for a planar fault. The slight difference between the analytical solution and the 0th order solution are coming from the fact that the calculation was performed in spectral domain, hence there is a periodic replication of the fault that slighly increase the shear traction of the 0th order solution.

The 0th order response of the shear traction, denoted as  $\tau_{\rm el}^0$ , is exactly the same as if the fault was flat (*Segall* (2010), section 4.7). In other words, It means that the main contribution for the shear traction of a non-planar fault is the flat fault response. In figure 5, the shear traction response for a rough fault that is subject to a slip distribution  $\Delta u(y_1) = (1-4\frac{y_1^2}{L^2})^{3/2}$  in meter, *L* being the length of the fault, can be seen. This represents the shear traction for a right lateral fault. Inside the fault, the shear traction exhibits a globally negative value, resulting in decreasing the shear traction on the fault. This overall shear traction reduction is usually called the shear stress drop in seismology. On the other
hand, outside the fault, the presence of the fault increases the shear traction.

¹⁴⁰ The 0th order contribution to normal traction  $\sigma_{el}^0$ : the main source of normal ¹⁴¹ traction variations



Figure 6: Zoom on the normal traction in Fig. 4. It can be seen that minimums and maximums of the normal traction correspond to area where the fault is locally flat ( $\kappa^t = 0$ ).

One of the main effect of non-planar geometry on an in-plane shear fault is the introduction of normal traction variations. This effect is well-documented in the literature (*Nielsen and Knopoff*, 1998; *Dunham et al.*, 2011; *Romanet et al.*, 2020; *Cattania and Segall*, 2021). Figure 6 provides a closer look at the normal traction depicted in figure 4, for a rough fault whose slip distribution is assumed. It can be seen that the maximums and minimums of normal traction correspond to the areas where the fault is locally flat ( $\kappa^t = 0$ ). This relationship can be analytically demonstrated with the 0th order elastic normal traction ¹⁴⁹ (which is the main contribution to the normal traction) given by:

$$\sigma_{\rm el}^0(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \kappa^t(y_1) \Delta u^t(y_1) \mathrm{d}y_1 \tag{10}$$

¹⁵⁰ To identify the areas of minimums and maximums of normal traction, we can differentiate ¹⁵¹ the previous expression and find where the derivative equals zero.

$$\frac{d}{dx_1}\sigma_{\rm el}^0(x_1) = \underbrace{\frac{\mu}{\pi} \left(1 - \frac{c_s^2}{c_p^2}\right)}_{>0} \int_{-\infty}^{+\infty} \underbrace{\frac{1}{(x_1 - y_1)^2}}_{>0} \kappa^t(y_1) \underbrace{\Delta u^t(y_1)}_{>0} \, \mathrm{d}y_1 \tag{11}$$

Since the slip is always positive, the only way that the previous integral to be 0 is if the curvature reverses sign. Moreover, due to the weight  $1/(x_1 - y_1)^2$ , the maximums and minimums of normal traction occur very close to the areas where the curvature changes sign. Although it may not precisely coincide with the location of the curvature reversing sign, it generally occurs very close. Similar reasoning can be applied for the extremas of the shear traction for pure opening faults (see appendix G).

Lastly, it is worth noting an interesting approximation for a rough fault that is not applicable to general fault geometries. If the slip distribution in Fourier domain has mainly low wavelength, so that it exists  $k_c$  such that  $\Delta u^t (k > k_c) = 0$ , and that the curvature along the fault in Fourier domain has mainly high wavelength such that  $\kappa^t (k < k_c) = 0$ , then the Hilbert transform of the curvature that multiplies the slip can be simplified using Bedrosian's theorem (*Bedrosian* (1963)):

$$\sigma_{\rm el}^0(x_1) \simeq -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \Delta u^t(x_1) \int_{-\infty}^{+\infty} \frac{\kappa^t(y_1)}{x_1 - y_1} \mathrm{d}y_1.$$
(12)

¹⁶⁴ Although this approximation is not entirely accurate as the slip and curvature do have ¹⁶⁵ overlapping bandwidth, it can be considered a reasonable assumption for rough faults, as 166 shown in Fig. 7.



Figure 7: Assuming the fault geometry and the slip distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$ , the full solution for normal traction (black curved) can be calculated using equation (72). The solution using the Bedrosian's approximation eq. (12) (blue dash curve) is very close to the full solution and allows to give some interpretations.

#### ¹⁶⁷ Scaling of slip vs geometry



Figure 8: Scaling of geometry and slip distribution for a static, in-plane shear (mode II) fault subject to a constant loading and that follows Coulomb friction. The black dash line is the planar fault solution. The solution for a non-planar fault geometry is shown as a blue line: a) a rough fault, b) a seamount fault geometry, c) an arctan fault geometry and d) a sum of two sinusoidal functions geometry.

First and foremost, please note that in this section, we are introducing a new approach by solving the mechanical problem using the momentum balance equation, without assuming a specific slip distribution along the fault. If the fault follows the Coulomb friction law (fbeing the friction coefficient) and that the loading can be considered constant at 0th order along the fault, we can write the momentum balance equation at 0th order as:

$$\underbrace{-f(\sigma_{\rm el}^0 + \sigma_{\rm load}^0)}_{\rm Friction} = \underbrace{\tau_{\rm el}^0}_{\substack{\rm Oher \ traction\\ due \ to \ elasticity}} + \underbrace{\tau_{\rm load}^0}_{\rm Loading} .$$
(13)

By utilizing the momentum balance equation mentioned above, it becomes possible to solve
for the distribution of slip along the fault.

This approach was applied for the different non-planar fault geometry in figure 8. The 175 figure illustrates a clear scaling relationship between the geometry of the fault and the slip 176 distribution: locally flat areas exhibit extrema of slip while highly curved areas show a 177 strong slip gradient. It can be seen that this scaling is not working near the edges of the 178 fault. Remarkably, the scaling appears to be linear and preserves the wavelength of the 179 fault geometry. Indeed, in figure 8-d, the fault geometry is a sum of two sinusoids with 180 different amplitude and wavelength. The amplitude and wavelength of the fault geometry 181 can be retrieved in the resulting slip distribution. 182

This scaling has been analytically derived in *Romanet et al.* (2020), and is presented again in this paper for consistency. This scaling arises from the relationship between the shear traction and the normal traction through the friction law. For an in-plane shear fault, as shown in the previous section, the shear traction  $\tau_{\rm el}^0$  is mainly controlled by the gradient term along the fault while the normal traction  $\sigma_{\rm el}^0$  is mainly controlled by the curvature term:

$$\tau_{\rm el}^{0}(x_{1}) = -\frac{\mu}{\pi} \left(1 - \frac{c_{s}^{2}}{c_{p}^{2}}\right) \int_{-\infty}^{+\infty} \left[\frac{1}{x_{1} - y_{1}} \frac{d}{dy_{1}} \Delta u^{t}(y_{1})\right] \mathrm{d}y_{1}$$
  
$$\sigma_{\rm el}^{0}(x_{1}) = -\frac{\mu}{\pi} \left(1 - \frac{c_{s}^{2}}{c_{p}^{2}}\right) \int_{-\infty}^{+\infty} \left[\frac{1}{x_{1} - y_{1}} \kappa^{t}(y_{1}) \Delta u^{t}(y_{1})\right] \mathrm{d}y_{1}$$
(14)

The Coulomb friction on the fault links the curvature term (the fault geometry) that controls normal traction and the gradient term (the slip distribution gradient) that controls shear traction. It is possible to invert the two Hilbert's transform, by using Chebitchev polynomials (*Segall* (2010), section 4.1), to obtain a non-linear ordinary differential equation:

$$\underbrace{\frac{d}{dx_1}\Delta u^t(x_1) + f\kappa^t(x_1)\Delta u^t(x_1)}_{\text{Gradient of slip}} = -\underbrace{\frac{f\sigma^{\text{load}} + \tau^{\text{load}}}{\mu(1 - c_s^2/c_p^2)}}_{\text{Effect of loading}} \underbrace{\frac{x_1 - L/2}{L\sqrt{(1 - \frac{4(x_1 - L/2)^2}{L^2})}}_{\text{Effect of loading}},$$
(15)

¹⁹⁴ In the center of fault the effect of loading cancels so that we obtained the scaling:

$$\frac{\mathrm{d}\Delta u^t}{\Delta u^t} = -f\mathrm{d}m.\tag{16}$$

where the property  $\kappa^t dx = dm$  has been used.

This scaling exist only for in-plane shear fault (mode II) as shown in Fig. 8, but not for out-of-plane shear fault (mode III) as depicted in Fig. 13. Although his scaling has not been yet confirmed in observations (*Bruhat et al.*, 2020), it was verified for fully-dynamic simulations using rate and state friction (*Romanet and Ozawa*, 2021).

This scaling is an important theoretical finding as it provides a link between three crucial parameters of the fault, i.e. the fault geometry, the slip distribution and the friction coefficient.

### ²⁰³ The shear traction drag $au_{ ext{el}}^1$



Shear traction drag for different fault geometries

Figure 9: Calculation of the exact shear traction drag as expressed by equation (18) for different geometries. The fault geometry is shown as the x-axis. a) a finite rough fault. In the case of the rough fault geometry, the theoretical shear traction drag derived by *Fang and Dunham* (2013) is also shown as a black line. b) a finite fault with a seamount geometry, c) a finite fault with arctan geometry, and d) a finite fault whose geometry is the sum of two sinusoidal functions. The calculation is done for the prescribed slip distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$  in meter, where L is the length of the fault.

A key theoretical result on rough fault was obtained by *Dieterich and Smith* (2009) and 204 Fang and Dunham (2013). It states that a rough fault is harder to slip than a flat fault. In 205 other words, it says that a rough fault has an additional shear resistance when compared 206 to a flat fault. This term will be referred as the shear traction drag in the following. 207 Originally, it was derived as the 1st order effect* of a rough fault on the shear traction 208 They have shown that this term is mainly resisting movement. Figure 9 illustrates 209 the 1st order term for shear traction for different fault geometries when assuming the slip 210 distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$ . 211

$$\tau_{\rm el}^1(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \kappa^t(y_1) \Delta u^t(y_1) \mathrm{d}y_1 \tag{17}$$

This figure shows that the additional shear resistance coming from the shear traction drag 212 is not only valid for rough fault (Fig. 9, a), but for any non planar fault geometry (Fig. 213 9, b,c,d). It can be seen that as long as the fault is non-planar, this term is resisting to 214 movement. It leads to the very intuitive result that it is harder to slide a fault if it is 215 non-planar. Contrary to the expression given in Fang and Dunham (2013), the expression 216 presented here for the shear traction drag is exact for any fault geometry, and any slip 217 distribution. It can be viewed as the generalisation of their work. It is also independent 218 of the friction on the fault, making it a highly general result. Here are some properties of 219 the shear traction drag: 220

221 222

223

 The shear traction drag is valid for any geometry and any slip distribution. There is more resistance to slip for a non-planar fault compared to a planar fault (for in-plane shear -mode II-).

224 2. It is only associated with in-plane faulting (mode II), there is no shear traction drag
 *A 2nd order effect, but there is a mistake in their derivation.

225

for out-of-plane shearing (mode III).

3. Since the stresses in the Fourier domain in three dimensional can be understood as
 a combinaison of of mode II and mode III (*Geubelle and Rice*, 1995), the analytical
 result mentionned above can be used to calculate the shear traction drag in 3D.

4. Since the curvature term for shear traction as given by equation (4), and the 1st
order term (equation (9)) are the same at order 1, the two terms can both be used
to calculate the shear traction drag.

5. An interpretation is that for equivalent slip distribution, the shear stress drop will be
higher for a non-planar fault than for a planar fault. This interpretation may seem
to contradict the fact that the fault is more difficult to slip, but this is not the case.
To get an equivalent slip, the loading traction on a non-planar fault will be required
to be higher than on a planar fault, hence it is more difficult to slip on a non planar
fault.

6. This definition of the shear traction drag includes the result obtained by Fang and Dunham (2013) for the shear traction drag on a rough fault:  $\tau_{\rm th}^1 = -8\pi^3 \alpha^2 \mu \left(1 - \frac{c_s^2}{c_p^2}\right) \frac{\Delta u^t}{\lambda_{\rm min}}$ . The previous result was derived for a constant slip on an infinite friction less fault, where  $\alpha$  represents the amplitude to wavelength ratio, and  $\lambda_{\rm min}$  is the minimum roughness wavelength.

In the following, we will show that our result encompasses the result by Fang and Dunham (2013). Assuming that the slip is constant, and that the fault slope is small ( $m \ll 1$ ), it is possible to replace the curvature by the second order derivative of the height of the fault:  $\kappa^t(x) = \frac{m'(x)}{(1+m(x)^2)^{3/2}} \simeq m'(x)$ . The previous exact equation for the shear traction drag simplifies to

$$\tau_{\rm el}^1(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \Delta u^t \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] m'(y_1) \mathrm{d}y_1 \tag{18}$$

Another simplification can be made because the term  $\frac{x_2-y_2}{(x_1-y_1)^2}$  is usually small compare to  $\frac{m(y)}{x_1-y_1}$ . Then the equation becomes:

$$\tau_{\rm el}^1(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \Delta u^t \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} \right] m'(y_1) \mathrm{d}y_1 \tag{19}$$

²⁵⁰ Finally, using the Bedrosian theorem (*Bedrosian*, 1963), it can be obtained:

$$\tau_{\rm el}^1(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \Delta u^t(x_1) m(x_1) \int_{-\infty}^{+\infty} \left[ \frac{m'(y)}{x_1 - y_1} \right] \mathrm{d}y_1 \tag{20}$$

which is the same equation A61 as in *Fang and Dunham* (2013). Hence the mean roughness drag can be written as:

$$\tau_{\rm th}^1 = -8\pi^3 \alpha^2 \mu \left(1 - \frac{c_s^2}{c_p^2}\right) \frac{\Delta u^t}{\lambda_{\rm min}} \tag{21}$$

To further validate our derivations and assess the accuracy of the approximations made, 253 we conducted numerical tests comparing the two expressions. In figure 10-a, we computed 254 the shear traction drag (red line) using eq. (18) for an infinitely long fault with constant 255 slip. To achieve the infinite fault, we calculated the shear traction drag in the spectral 256 domain (eq. (76)), hence it is infinite with periodic replication. The theoretical expression 257 proposed by Fang and Dunham (2013) is shown in black. The mean value of our definition 258 for the shear traction drag closely aligns with the expression derived in Fang and Dunham 259 (2013). Additionally, in figure 10-b, we calculated the relative difference between the mean 260 of the exact expression (eq. (18)) and the theoretical expression (eq. (21)) for the shear 261 traction drag for 100,000 different rough fault profiles. The mean of the relative difference 262

is less that 1%, with a standard deviation of approximately 5%. The discrepancy can be explained by the neglect of the term  $\frac{x_2-y_2}{(x_1-y_1)^2}$ , and the utilization of the Bedrosian theorem whose assumptions are not entirely verified. These numerical comparisons further support the validity and accuracy of our derived expressions.

Finally, it can be demonstrated through an integration by parts that the expression for the shear traction drag , under the assumptions that  $\frac{x_2-y_2}{(x_1-y_1)^2}$  can be neglected and that the Bedrosian theorem can be applied, is always negative :

$$\begin{aligned} \tau_{\rm el}^{1}(x_{1}) &= \frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) \int_{-\infty}^{+\infty} \frac{1}{x_{1} - y_{1}} \Delta u^{t}(y_{1}) m(y_{1}) \frac{d}{dy_{1}} m(y_{1}) \mathrm{d}y_{1} \\ &= \frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) \Delta u^{t}(x_{1}) \int_{-\infty}^{+\infty} \frac{1}{x_{1} - y_{1}} \frac{1}{2} \frac{d}{dy_{1}} m^{2}(y_{1}) \mathrm{d}y_{1} \\ &= -\underbrace{\frac{\mu}{2\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right)}_{>0} \Delta u^{t}(x_{1}) \int_{-\infty}^{+\infty} \underbrace{\frac{1}{(x_{1} - y_{1})^{2}} m^{2}(y_{1})}_{>0} \mathrm{d}y_{1} \end{aligned}$$
(22)

The last expression, which is valid for any fault geometry, is always opposite sign of the slip  $\Delta u^t$ , hence resisting movement.



Figure 10: a) Exact -red line- (from eq (18)) and theoretical -black line- (from eq (21)) shear traction drag for an infinite rough fault. The exact shear traction drag was calculating in the Fourier domain (eq. (76), in appendix H) to make it infinite with periodic replication. b) In order to check the equivalence of the two expressions in the case of an infinite rough fault, we generated 100000 rough fault profiles and computed the distance of the mean value for the exact shear traction drag compared with the theoretical shear traction drag. The mean difference is less than 1% with a standard deviation of 5.5%.

### ²⁷² 1st order response of normal traction



Figure 11: The 1st order effect on normal traction. If can seen that the main effect is to slightly reduce the minimums and maximums of the normal traction. The full solution was calculated with the equation (72) in appendix H.

Although there is a 1st order response on normal traction, its effect is not as important as the shear traction drag. The main effect is to slightly oppose the effect of the 0th order, so that the full normal traction at maximum and minimums are slightly reduced compare to the 0th order effect. This effect can be seen on figure 11, where the maximums and minimums of normal traction at 0th order  $\sigma_{\rm el}^0$  are slightly reduced by the 1st order of normal traction  $\sigma_{\rm el}^1$ .
#### 279 2.2 Out-of-plane shear (mode III)



Figure 12: Total elastic traction for an out-of-plane fault (mode III). Assuming the fault geometry (as shown by the x-axis) and the slip distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$  in meter (red curve), the full solution, the gradient term and the curvature term (black lines) can be calculated using equation (77) (more precisely using the spectral expressions eq. (79) and eq. (81)). The 0th order and the 1st order (blue lines), are respectively calculated using the expressions (25) and (26).

Similarly as for the in-plane (mode II) case, the out-of-plane (mode III) shear and normal traction can be decomposed into 0th and 1st order responses.

$$\underbrace{\tau_{el}}_{\substack{\text{Elastic shear traction}\\\text{of non-planar fault}}} = \underbrace{\tau_{el}^{0}}_{\text{Planar fault response}} + \underbrace{\tau_{el}^{1}}_{=0} + \underbrace{\cdots}_{\text{Higher order terms}}$$

$$\underbrace{\sigma_{el}}_{\substack{el}\\=0} = \underbrace{\sigma_{el}^{0}}_{=0} + \underbrace{\sigma_{el}^{1}}_{=0} + \underbrace{\cdots}_{\substack{Higher order terms}}$$

$$(23)$$



However, in this case, only the shear traction at  $0^{\text{th}}$  order is non-zeros.

$$\tau_t^0(x_1) = \frac{\mu}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \frac{d}{dy_1} \Delta u^s(y_1) dy_1$$
  
$$\sigma_n^0(x_1) = 0$$
(25)

$$\tau_t^1(x_1) = 0$$
  
 $\sigma_n^1(x_1) = 0$ 
(26)

For an out-of-plane shear fault, the contribution of non-planar fault geometry to the stresses 281 is only a 2nd order effect. This means that fault geometry has a small effect for an out-282 of-plane shear fault (mode III). Figure 13 illustrates the slip distribution for non-planar 283 fault geometries with Coulomb friction and constant loading, using the same parameters 284 as in the in-plane shear case (mode II) shown in Figure 8. In the case of out-of-plane shear 285 (mode III), there is no scaling of the slip distribution with respect to fault geometry. The 286 slip distribution for the planar fault (dashed black line) perfectly overlaps with the slip 287 distributions for the non-planar faults (continuous blue lines). 288



Figure 13: Scaling of geometry and slip distribution for a static, out-of-plane shear (mode III) fault subject to a constant loading and that follows Coulomb friction. The black dash line is the planar fault solution. The slip distribution does not show any scaling with fault geometry. Fault geometry: a) a rough fault, b) a seamount fault geometry, c) an arctan fault geometry and d) a sum of two sinusoidal functions geometry.

#### 289 3 Discussion

3.1 Physical singularities that appear on non-planar fault
3.1.1 Non-zeros slip at kinks of the fault.



Figure 14: Figure showing the grid dependency of a kinked faults. a) The geometry of the fault and the momentum equation. b) The slip distribution versus position along the fault. The slip distribution is changing with gridsize. Please note the only physically acceptable slip at the kink is zero. c) The shear traction distribution versus the position along the fault. The shear traction is closely following the normal traction because they are linked by the friction law. It is also diverging with the refinement of the mesh. d) The normal traction distribution versus position along the fault.

²⁹² Many literature sources make the mistake of modeling kinks while assuming small strain ²⁹³ elasticity and no opening (*Tada and Yamashita*, 1996; *Aochi et al.*, 2000a; *Duan and* ²⁹⁴ *Oglesby*, 2005; *Ely et al.*, 2009; *Lozos et al.*, 2011; *Fukuyama and Hok*, 2015; *Sathiaku-*²⁹⁵ *mar and Barbot*, 2021). A kink on a fault with non-zeros slip create a 1/r singularity, ²⁹⁶ where r is the distance from the kink. This is a non-physical singularity that results to ²⁹⁷ infinite strain energy near the kink. We can demonstrate this result in the small slope ²⁹⁸ approximation, for the 0th order on normal traction:

$$\sigma_{\rm el}^0(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_1 - y_1} \frac{d}{dy_1} m(y_1) \Delta u^t(y_1) \right] \mathrm{d}y_1 \tag{27}$$

If we assume that the curvature is  $\kappa^t(y_1) = \frac{d}{dy_1}m(y_1)$ . For a kink at the position  $y_1 = 0$ , the slope is discontinuous so that we can assume that  $m(y_1) = AH(y_1)$ , where H is the Heaviside function and A the fault slope after the kink. In this case, the previous equation becomes:

$$\sigma_{\rm el}^{0}(x_{1}) = -\frac{\mu}{\pi} \left(1 - \frac{c_{s}^{2}}{c_{p}^{2}}\right) \int_{-\infty}^{+\infty} \left[\frac{A\Delta u(y_{1})}{x_{1} - y_{1}}\delta(y_{1})\right] \mathrm{d}y_{1}$$
$$= -\frac{\mu}{\pi} \left(1 - \frac{c_{s}^{2}}{c_{p}^{2}}\right) \frac{A\Delta u^{t}(0)}{x_{1}}$$
(28)

This reveals a  $1/x_1$  singularity of normal traction on the fault. One straightforward way to eliminate this singularity is by imposing the slip to be null at the kink  $\Delta u^t(0) = 0$ , which is evident from the previous equation. Another approach would be to allow for fault opening or adding a third fault and imposing the closure of slip at the kink (*Andrews*, 1989). In the boundary element method, the maximum normal traction concentration due to the discretization is typically proportional to the gridsize  $\Delta s$ . Thus, the maximum normal ³⁰⁹ traction on the fault is:

$$\sigma_{\rm el}^0(x_1 = 0) \propto -\frac{\mu}{\pi} \left(1 - \frac{c_s^2}{c_p^2}\right) \frac{A\Delta u^t(0)}{\Delta s}$$
(29)

The problem is that the usual gridsize for the modeling of fault is of the order  $\Delta s \sim 1000m$ , 310 as a result, the stress concentration at the kink, which is normally infinite, is of the order 311 of  $\sigma_{\rm el}^0(x_1) \simeq 1$  MPa for typical shear modulus  $\mu$  (30 GPa), P-wave velocity (5000 m/s) and 312 S wave velocity (3000 m/s) and a slip at the kink of  $\Delta u^t(0) = 1$ m. This is why the issue of 313 diverging traction at kinks has remained unknown in numerical work so far. An example 314 of this effect is shown in figure 14, where the slip, normal traction and shear traction was 315 calculated for a fault following Coulomb friction with constant loading. The maximum 316 values of normal traction and shear traction are largely underestimate ( $\sim 1$ MPa) for the 317 gridsize  $\Delta s = 500$  m. It is worth noting that adding plasticity or viscosity is not an entirely 318 satisfying solution. In most of seismological research, the small strain approximation is used 319 together with plasticity/viscosity and the plastic/viscous effects are driven by the linear 320 elastic interaction. If the elastic interactions are underestimated, the plastic/viscous effect 321 will also be significantly underestimated. It means all the quantitative work done using 322 kinks in linear elasticity (with the small strain assumption) is grid-dependent and does not 323 converge numerically. 324

Finally, it is still possible to observe that modeling a kink is still possible in out-of-plane faulting (mode III), because there is no curvature term.

#### 327 3.1.2 Discontinuous curvature along the fault

A discontinuity in the curvature of the fault also creates a singularity in stresses. However, this singularity can be compensated by a singularity in the slip derivative (hence the slip will still be continuous). Using the scaling of slip vs curvature -that is valid only when the fault follows Coulomb friction- (eq. (16)):

$$\frac{d}{dx_1}\Delta u^t(x_1) = -f\kappa^t(x_1)\Delta u^t(x_1).$$
(30)

So that if the curvature has a discontinuity of Amplitude *B* at  $x_1 = 0$ ,  $\kappa^t(x_1) = BH(x_1)$ , the discontinuity in slip gradient will be:  $\frac{d}{dx_1}\Delta u^t(x_1) = -fBH(x_1)\Delta u^t(x_1)$ . This effect is well observed on figure 15-b, where there is a discontinuity in slip gradient due to the discontinuity of the curvature along the fault.



Figure 15: Figure showing the grid dependency of a bended fault with discontinuous curvature. a) The geometry of the fault and the momentum equation. b) The slip distribution versus position along the fault. The slip distribution is changing with gridsize. c) The shear traction distribution versus the position along the fault. The shear traction is closely following the normal traction because they are linked by the friction law. It is also diverging with the refinement of the mesh. d) The normal traction distribution vs position along the fault. The normal traction is diverging with the refinement of the mesh.

The discontinuity in curvature creates a singularity of the stresses that is  $\propto \log(x_1)$  when  $x_1 \rightarrow 0$ . Because this singularity is less strong than  $1/\sqrt{(x_1)}$ , this discontinuity in curvature does not lead to infinite strain energy, hence it is a physically acceptable singularity. Let's show the previous result using the small slope approximation and assuming constant slip distribution. For a bended fault like in Fig. 15-a, the curvature can be written  $\kappa^t(y_1) =$  $B(H(y_1 - a) - H(y_1 + b))$ , where B is the curvature in the bended portion of the fault. It leads to the normal traction distribution (Hilbert transform of a characteristic function):

$$\sigma_{\rm el}^{0}(x_{1}) = -\frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) \Delta u^{t} \int_{-\infty}^{+\infty} \left[ \frac{1}{x_{1} - y_{1}} B(H(y_{1} - a) - H(y_{1} + b)) \right] dy_{1}$$

$$= -\frac{\mu}{\pi} \left( 1 - \frac{c_{s}^{2}}{c_{p}^{2}} \right) B \Delta u^{t} \log \left| \frac{x_{1} - a}{x_{1} - b} \right|$$
(31)

Where we found a log singularity when  $x_1 \to a$  or  $x_1 \to b$ . The last comment is that, again, there is no such a problem in out-of-plane shear, because the stresses and strains are independent of the curvature term (see eq. 77 in appendix I).

#### ³⁴⁶ 3.2 Limits of linear elasticity



Figure 16: Figure showing the limits of linear elasticity in the modeling of non-planar fault geometries.

The small strain approximation is commonly used in seismology and is necessary to linearises the strain tensor with the displacement field. However, when considering non planar fault geometry, the strains and stresses will keep increasing with on-going slip breaking the small strain approximation. At a certain point, the small strain approximation becomes invalid as depicted in figure 16. In such cases, finite elasticity should be considered to prevent the strains and stresses from growing without bounds (*Romanet et al.*, 2020; *Tal*, 2023). Various approaches have been proposed in the litterature to address the issue of ³⁵⁴ unbounded increase or decrease in normal traction with ongoing slip (*Duan and Oglesby*,
³⁵⁵ 2005; *Dunham et al.*, 2011; *Heimisson*, 2020; *Cattania and Segall*, 2021), for example the
³⁵⁶ inclusion of viscosity or plasticity while retaining the small strain approximation of linear
³⁵⁷ elasticity. However, this approach may lead to a significant overestimation of the effect of
³⁵⁸ plasticity and viscosity in fault mechanics.

While we acknowledge the likelihood of non-linear anelastic phenomena occurring (such as damage, plasticity, or viscosity), we disagree with the interpretation that they are a necessary condition for preventing the stresses and strains from growing indefinitely with ongoing slip on non-planar faults (*Dieterich and Smith*, 2009; *Shi and Day*, 2013), as this does not consider the effect of finite elasticity.

It is important to recognize that small strain elasticity is violated when dealing with very small asperities, and even regularization techniques may not provide accurate results. One possible direction is to move towards finite elasticity and thoroughly test the limits of linear elasticity in current earthquake simulations.

#### 368 4 Conclusion

	Opening fault (mode I)	In-plane shear fault (mode II)	Out-of-plane shear fault (mode III)
Shear traction $\tau^0$ (0 th order)	Mainly controlled by the fault geometry $\propto \kappa^t \Delta u^n$	Same as the planar fault response	Same as the planar fault response
Shear traction $\tau^1$ (1 st order)	Nearly no effect (depends on the gradient of slip)	Shear traction drag, the fault is resisting slip	= 0 No effect of fault geometry
Normal traction $\sigma^0$ (0 th order)	Same as the planar fault response	Mainly controlled by the fault geometry $\propto \kappa^t \Delta u^t$	= 0 No effect of fault geometry
Normal traction $\sigma^1$ (1 st order)	Normal traction drag, the fault is resisting opening	Slightly reduce the minimums and maximums of normal traction changes	= 0

Table 1: Summary of the results. Please note that there is an effect of geometry for out-of-plane shear, but this effect only appears at  $2^{nd}$  order.

This work advances the limited theoretical knowledge regarding the effect of non-planar fault geometries on earthquake mechanics. By expanding the relation of fault traction and slip, up to second order relative to the deviation from a planar fault, this study allows for the interpretation of complex fault geometry and its impact on fault traction (see Table 1).

The results of this study confirm that fault geometry plays a significant role in inplane faulting (mode II) by modifying the normal traction on the fault and increasing its resistance to slipping. We provide a general expression independent of fault geometry and fault slip for the shear traction drag (*Fang and Dunham*, 2013), making it a general result. We also provide some useful simplification for the effect of rough fault on normal traction, which can be simplified as an Hilbert transform of the curvature by using Bedrosian's theorem.

³⁸¹ Conversely, for out-of-plane faulting (mode III), the influence of fault geometry is neg-³⁸² ligible. There is no effect of fault geometry up to the second order. We also showed that, ³⁸³ in this case, there is no scaling between the slip distribution and the geometry.

The paper also examines singularities that arise in specific fault geometries commonly used in earthquake simulations and provides guidelines for their elimination.

³⁸⁶ Ultimately, this study highlights the limitations of the small strain approximation when ³⁸⁷ considering non-planar faults, emphasizing the need to consider finite elasticity for more ³⁸⁸ accurate modeling of non-planar faults.

#### ³⁸⁹ Availability statement

The mathematica code used to develop the equation is available as supplementary material.
No data was used in this research.

#### 392 Acknowledgments

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## ³⁹⁹ A Table of parameters

Symbol	Description	
h	Height of the fault	
m	Slope of the fault	
x	Point of evaluation	
$x_d$	$d^{\rm th}$ component of the point ${f x}$	
У	Point of evaluation, usually over which an integral is performed	
$y_d$	$d^{\rm th}$ component of the point <b>y</b>	
$\kappa^t$	Curvature of the fault	
$u_k$	$k^{\text{th}}$ component of the displacement vector	
$c_{ijpq}$	ijpq component of the Hooke tensor	
$\Delta u_i^n$	$i^{\text{th}}$ component of the slip vector for opening (mode I)	
$\Delta u_i^t$	$i^{\text{th}}$ component of the slip vector for in-plane shear (mode II)	
$\Delta u_i^s$	$i^{\text{th}}$ component of the slip vector for out-of-plane shear (mode III)	
$n_j$	$j^{\text{th}}$ of the vector normal to the fault	
$G_{kp}$	kp component of the Green's functions	
l	Length along the fault	
$\epsilon_{cd}$	cd component of the strain tensor	
$\sigma_{ab}$	ab component of the stress tensor	
$t_{j}$	$j^{\rm th}$ component of the tangential vector to the fault	
$rac{\partial}{\partial y^t}$	Derivative along the fault $\left(\frac{\partial}{\partial y^t} = t_1 \frac{\partial}{\partial x_1} + t_2 \frac{\partial}{\partial x_2}\right)$	
$ au_{\mathrm{el}}$	Total elastic shear traction along the fault	
$\tau_{\rm el}^0$	Elastic shear traction at $0^{\text{th}}$ order along the fault	
$ au_{ m el}^1$	Elastic shear traction at $1^{\text{th}}$ order along the fault	
$\sigma_{ m el}$	Total elastic normal traction along the fault	
$\sigma_{ m el}^0$	Elastic normal traction at $0^{\text{th}}$ order along the fault	
$\sigma_{\rm el}^1$	Elastic normal traction at 1 th order along the fault	
L	Length of the fault, used for the prescribed slip definition	
$\mu$	Shear modulus	
$C_S$	Shear wave speed	
$c_p$	Compressional wave speed	

Table 2: Table of parameters

Symbol	Description
$\sigma_{\rm load}^0$	Constant normal traction loading
$\tau_{\rm load}^0$	Constant shear traction loading
f	Coulomb friction parameter
$ au_{\mathrm{th}}^1$	Theoretical shear traction drag as given in Fang and Dunham $(2013)$
$\alpha$	Amplitude to wavelength ratio for a self-similar fault
$\lambda_{\min}$	Minimum roughness wavelength
H	Heaviside function
δ	Dirac function
λ	Lamé's first parameter
θ	Angle between the axis $1$ and the fault (see Fig. 18)

Table 3: Table of parameters (continue)

#### $_{400}$ B Derivation

⁴⁰¹ In the following, the convention for stress is chosen as tension positive and compression
 ⁴⁰² negative.

#### 403 B.1 Formulas

Here, we just recall some formulas that will be used in the following section for the deriva-tion.

406 The momentum balance equation:

$$c_{ijpq}\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_q}G_{pn}(\mathbf{x},\mathbf{y}) = 0$$
(32)

407 Symmetry of the Green's function:

$$\frac{\partial}{\partial x_q} G_{ij}(\mathbf{x}, \mathbf{y}) = -\frac{\partial}{\partial y_q} G_{ij}(\mathbf{x}, \mathbf{y})$$
(33)

⁴⁰⁸ Definition of the tangential differential operator on a function f (Bonnet, 1999):

$$D_{ij}[f(\mathbf{y})] = n_i(\mathbf{y})\frac{\partial}{\partial y_j}f(\mathbf{y}) - n_j(\mathbf{y})\frac{\partial}{\partial y_i}f(\mathbf{y})$$
(34)

 $_{409}$  Definition of the derivative along the fault, if t is the tangential vector to the fault:

$$\frac{\partial}{\partial y^t}f = t_1 \frac{\partial}{\partial y_1}f + t_2 \frac{\partial}{\partial y_2}f \tag{35}$$

The expression for the Green's function can be found in *Tada and Yamashita* (1997):

$$G_{ij}(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi\mu} [\gamma_i \gamma_j (1 - c_s^2/c_p^2) - \delta_{ij} (1 + c_s^2/c_p^2) \log(r)], \text{ for } i, j \in \{1, 2\}$$

$$G_{33}(\mathbf{x}, \mathbf{y}) = \frac{-1}{2\pi\mu} \log(r)$$
(36)

where  $\gamma_1 = \frac{x_1 - y_1}{r}$ ,  $\gamma_2 = \frac{x_2 - y_2}{r}$ , and  $r = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$  is the distance between the points **x** and **y**.

#### 413 B.2 Derivation

⁴¹⁴ We start from the representation theorem (*Tada and Yamashita*, 1997):

$$u_k(\mathbf{x}) = -\int_{\text{fault}} c_{ijpq} n_j(\mathbf{y}) \Delta u_i(\mathbf{y}) \frac{\partial}{\partial x_q} G_{kp}(\mathbf{x}, \mathbf{y}) dl(\mathbf{y})$$
(37)

415 From the previous equation, by deriving under the integral it can be obtained:

$$\frac{\partial}{\partial x_l} u_k(\mathbf{x}) = -\int_{\text{fault}} c_{ijpq} n_j(\mathbf{y}) \Delta u_i(\mathbf{y}) \frac{\partial}{\partial x_q} \left[ \frac{\partial}{\partial x_l} G_{kp}(\mathbf{x}, \mathbf{y}) \right] dl(\mathbf{y})$$
(38)

⁴¹⁶ The reason to do so is that both the strains  $(\epsilon_{kl} = \frac{1}{2} \left( \frac{\partial}{\partial x_k} u_l + \frac{\partial}{\partial x_l} u_k \right))$  and the stresses  $(\sigma_{ab} =$ ⁴¹⁷  $c_{abkl}\epsilon_{kl})$  can be obtained by linear combinaison of the previous equation. To regularise this integral we need to work on the integrand of the previous integral. Using the symmetry of second derivatives, we can replace the derivatives with respect to x with derivatives with respect to y.

$$\frac{\partial}{\partial x_l} u_k(\mathbf{x}) = -\int_{\text{fault}} c_{ijpq} n_j(\mathbf{y}) \frac{\partial}{\partial y_q} \left[ \frac{\partial}{\partial y_l} G_{kp}(\mathbf{x}, \mathbf{y}) \right] dl(\mathbf{y})$$
(39)

⁴²¹ Then, we used the definition of the tangential differential operator together with the mo-⁴²² memtum balance equation (eq. (32)):

$$c_{ijpq}n_{j}(y)\frac{\partial}{\partial y_{l}}\frac{\partial}{\partial y_{q}}G_{kp} = c_{ijpq}\left[D_{jl}\frac{\partial}{\partial y_{q}}G_{kp} + n_{l}(y)\frac{\partial}{\partial y_{j}}\frac{\partial}{\partial y_{q}}G_{kp}\right]$$

$$= c_{ijpq}D_{jl}\frac{\partial}{\partial y_{q}}G_{kp}$$
(40)

Please note that this derivation is due to Daisuke Sato (*Sato et al.*, 2020) for the 2D case. The equivalent 3D case derivation was done by Marc Bonnet (*Bonnet*, 1999), and can also be found in *Romanet et al.* (2020).

We then developed the differential tangential operator in the local coordinate system of the fault, and used the fact that the derivative perpendicular to the fault direction are null:

$$D_{jk}[f(y)] = \left[ n_j(y) \frac{\partial}{\partial y_k} - n_k(y) \frac{\partial}{\partial y_j} \right] f(y)$$
  
=  $\left[ n_j(y) \left( t_k(y) \frac{\partial}{\partial y^t} + n_k(y) \frac{\partial}{\partial y^n} \right) - n_k(y) \left( t_j(y) \frac{\partial}{\partial y^t} + n_j(y) \frac{\partial}{\partial y^n} \right) \right] f(y)$  (41)  
=  $\left[ n_j(y) t_k(y) - n_k(y) t_j(y) \right] \frac{\partial}{\partial y^t} f(y)$ 

It can be checked easily that the previous equation is 0 for any pair in (1, 2), because  $t_{12} = n_2$  and  $t_2 = -n_1$ .

Then, we replace the integrand in eq. (39), using eq. (40) and eq. (41):

$$\frac{\partial}{\partial x_l} u_k(\mathbf{x}) = -\int_{\text{fault}} c_{ijpq} \Delta u_i [n_j(\mathbf{y}) t_l(\mathbf{y}) - n_l(\mathbf{y}) t_j(\mathbf{y})] \frac{\partial}{\partial y^t} \frac{\partial}{\partial y_q} G_{kp}(\mathbf{x}, \mathbf{y}) dl(\mathbf{y})$$
(42)

Finally, it is possible to perform an integration by parts to regulatize the hypersingular integral given previously (eq. (43)):

$$\frac{\partial}{\partial x_l} u_k(\mathbf{x}) = -\int_{\text{fault}} c_{ijpq} [n_j(\mathbf{y}) t_l(\mathbf{y}) - n_l(\mathbf{y}) t_j(\mathbf{y})] \frac{\partial}{\partial y^t} \Delta u_i \frac{\partial}{\partial y_q} G_{kp}(\mathbf{x}, \mathbf{y}) dl(\mathbf{y})$$
(43)

⁴³⁴ For numerical calculation and the discretisation of the fault slip, it is best to use eq. (43).
⁴³⁵ However for facilitate interpretation, the slip vector can be projected on the local basis as:

$$\Delta u_i = \Delta u^n n_i, \text{ for mode I}$$

$$\Delta u_i = \Delta u^t t_i, \text{ for mode II}$$

$$\Delta u_i = \Delta u^s s_i, \text{ for mode III}$$
(44)

where  $\Delta u^n$ ,  $\Delta u^t$ , and  $\Delta u^s$  are respectively the opening, the in-plane slip and the out-ofplane slip. The derivative of slip with respect to the direction of the fault  $\frac{\partial}{\partial y^t} \Delta u_i$  can then be replaced by:

$$\frac{\partial}{\partial y^{t}} [\Delta u^{n} n_{i}] = n_{i} \frac{\partial}{\partial y^{t}} \Delta u^{n} - t_{i} \Delta u^{n} \kappa^{t}, \text{ for mode I}$$

$$\frac{\partial}{\partial y^{t}} [\Delta u^{t} t_{i}] = t_{i} \frac{\partial}{\partial y^{t}} \Delta u^{t} + n_{i} \Delta u^{n} \kappa^{t}, \text{ for mode II}$$

$$\frac{\partial}{\partial y^{t}} [\Delta u^{s} s_{i}] = s_{i} \frac{\partial}{\partial y^{t}} \Delta u^{s} = \delta_{3i} \frac{\partial}{\partial y^{t}} \Delta u^{s}, \text{ for mode III}$$
(45)

439 where  $\delta_{3i}$  is the Kronecker delta.

#### $_{440}$ C Discretization



Figure 17: Figure showing the discretisation of the fault.

The discretisation is using the same strategy as in *Romanet et al.* (2020). It consists into discretizing the slip and the tangential vector as constant over a straight line. It is convenient for this part to use the curvilinear abcisse  $\xi$  instead of the position along the fault  $\mathbf{y}(\xi)$ . Also, to simplify the demonstration, we will just keep the scalar t instead of one component of the tangential vector  $t_i$ .

$$t(\xi) = \sum_{j} t_{j} [H(\xi - \xi_{j}) - H(\xi - \xi_{j+1})]$$
  

$$\Delta u(\xi) = \sum_{j} \Delta u_{j} [H(\xi - \xi_{j}) - H(\xi - \xi_{j+1})]$$
(46)

The discretisation is using a piecewise contant slip and a straight element over.

$$\int_{\text{fault}} K(\mathbf{x}, \mathbf{y}) \frac{\partial}{\partial y^{t}} [t\Delta u] dl(\mathbf{y}) =$$

$$\int_{\text{fault}} K(\mathbf{x}, \mathbf{y}) t \frac{\partial}{\partial y^{t}} [\Delta u] dl(\mathbf{y}) +$$

$$\int_{\text{fault}} K(\mathbf{x}, \mathbf{y}) \Delta u \frac{\partial}{\partial y^{t}} [t] dl(\mathbf{y}) =$$

$$\int_{\text{fault}} K(\mathbf{x}, \mathbf{y}) t \frac{\partial}{\partial y^{t}} \left[ \sum_{j} \Delta u_{j} [H(\xi - \xi_{j}) - H(\xi - \xi_{j+1})] \right] dl(\mathbf{y}) +$$

$$\int_{\text{fault}} K(\mathbf{x}, \mathbf{y}) \Delta u \frac{\partial}{\partial y^{t}} \left[ \sum_{j} t_{j} [H(\xi - \xi_{j}) - H(\xi - \xi_{j+1})] \right] dl(\mathbf{y}) =$$

$$\sum_{j} [K[\mathbf{x}, \mathbf{y}(\xi_{j})] t(\xi_{j}) - K[\mathbf{x}, \mathbf{y}(\xi_{j+1})] t(\xi_{j+1})] \Delta u_{j} +$$

$$\sum_{j} [K[\mathbf{x}, \mathbf{y}(\xi_{j})] \Delta u(\xi_{j}) - K[\mathbf{x}, \mathbf{y}(\xi_{j+1})] \Delta u(\xi_{j+1})] t_{j}$$
(47)

447 We can then replace  $t(\xi_j)$  and  $\Delta u(\xi_j)$  by (H(0) = 1/2):

$$t(\xi_j) = (1 - 0.5)t_{j-1} + 0.5t_j = \frac{t_{j-1} + t_j}{2}$$

$$\Delta u(\xi_j) = \frac{\Delta u_{j-1} + \Delta u_j}{2}$$
(48)

448 which gives:

$$\int_{\text{fault}} K(\mathbf{x}, \mathbf{y}) \frac{\partial}{\partial y^{t}} [t\Delta u] \, dl(\mathbf{y}) = \sum_{j} [K[\mathbf{x}, \mathbf{y}(\xi_{j})] \frac{t_{j} + t_{j-1}}{2} - K[\mathbf{x}, \mathbf{y}(\xi_{j+1})] \frac{t_{j+1} + t_{j}}{2}] \Delta u_{j} + \sum_{j} [K[\mathbf{x}, \mathbf{y}(\xi_{j})] \frac{\Delta u_{j} + \Delta u_{j-1}}{2} - K[\mathbf{x}, \mathbf{y}(\xi_{j+1})] \frac{\Delta u_{j+1} + \Delta u_{j}}{2}] t_{j}$$
(49)
$$\underbrace{\sum_{j} [K[\mathbf{x}, \mathbf{y}(\xi_{j})] \frac{\Delta u_{j} + \Delta u_{j-1}}{2} - K[\mathbf{x}, \mathbf{y}(\xi_{j+1})] \frac{\Delta u_{j+1} + \Delta u_{j}}{2}] t_{j}}_{\text{Curvature term}}$$

⁴⁴⁹ This way of discretising the integral allows for the separation of the curvature and gradient

450 terms.

#### 451 D The small slope approximation

# 452 D.1 It transforms the integrals along the fault to integral along 453 a straight line

Starting from a regularized boundary element method, and applying the small slope approximation, it is possible to obtain the 0th and 1st order solution of the stress on the fault due to a slip distribution for the different mode of slip. Please notice that all the integrals that were along the fault become along the  $y_1$  axis only. This is due to the fact that if  $y_2 = h(y_1)$  and the slope derivative is small  $\frac{d}{dy_1}h(y_1) \ll 0$ , and changing the variable of the integral:

$$\int_{\text{fault}} f(y_1) dl(y_1) = \int_{-\infty}^{+\infty} f(y_1) \sqrt{1 + h'^2(y_1)} dy_1$$
  

$$\simeq \int_{-\infty}^{+\infty} f(y_1) (1 + h'^2(y_1)/2) dy_1$$
  

$$\simeq \int_{-\infty}^{+\infty} f(y_1) dy_1$$
(50)

# 460 D.2 It provides an expression for the normal and tangential vec 461 tor to the fault



Figure 18: Figure showing the simplification that happens in the small slope approximation for the normal and tangential vectors.

The normal and tangential vectors can be simplified in the small slope approximation (seeFig. 18), indeed:

$$m(y_1) = \tan[\theta(y_1)] \simeq \theta(y_1) \tag{51}$$

⁴⁶⁴ so that in the small slope approximation:

$$\mathbf{n}(y_1) = \begin{pmatrix} -\sin(\theta(y_1)) \\ \cos(\theta(y_1)) \end{pmatrix} \simeq \begin{pmatrix} -m(y_1) \\ 1 \end{pmatrix}$$

$$\mathbf{t}(y_1) = \begin{pmatrix} \cos(\theta(y_1)) \\ \sin(\theta(y_1)) \end{pmatrix} \simeq \begin{pmatrix} 1 \\ m(y_1) \end{pmatrix}$$
(52)

# ⁴⁶⁵ E A particular solution for the semi-analytical formu ⁴⁶⁶ lation

⁴⁶⁷ The slip/opening distribution for the result section is chosen as following:

$$\Delta u(y_1) = (1 - y_1^2)^{3/2} \tag{53}$$

The reason for this particular form of slip (or opening) distribution compared to the classic distribution in linear fracture mechanics  $(\Delta u(y_1) = \sqrt{1-y_1^2})$  is because it brings finite stress concentrations outside the fault (hence the numerical result are converging) and that there is an analytical solution for the stresses in the case of planar fault:

$$\frac{1}{2\pi} \int_{-1}^{1} \frac{1}{x_1 - y_1} \frac{\mathrm{d}}{\mathrm{d}y_1} \Delta u(y_1) \mathrm{d}y_1 = \frac{3}{4} \begin{cases} 1 - 2x_1^2 - 2x_1\sqrt{x_1^2 - 1}, & \text{if } x_1 < -1\\ 1 - 2x_1^2, & \text{if } -1 \le x_1 \le 1, \text{ inside the fault}\\ 1 - 2x_1^2 + 2x_1\sqrt{x_1^2 - 1}, & \text{if } x_1 > 1 \end{cases}$$

$$(54)$$

The previous solution was used to check our numerical scheme against the analytical formulation. Obtaining this solution is not straightforward. For the case  $-1 < x_1 < 1$ , the solution can be obtained by using Chebychev polynomial (*Mason and Handscomb* (2002), section 9.5.1). The solution outside  $|x_1| > 1$  can be obtained by doing a first change of variable  $y_1 \rightarrow sin(\theta)$  and checking that  $\frac{\sin(\theta)\cos^2(\theta)}{x_1-\sin(\theta)}$  is symmetric at  $\pi/2$ . That allows us to make the integration over the whole circle and take half the value. If we start from the following integral:

$$\frac{1}{2\pi} \int_{-1}^{1} \frac{1}{x_1 - y_1} \frac{\mathrm{d}}{\mathrm{d}y_1} \Delta u(y_1) \mathrm{d}y_1 = -\frac{3}{2\pi} \int_{-1}^{1} \frac{y_1 \sqrt{1 - y_1^2}}{x_1 - y_1} \mathrm{d}y_1 \tag{55}$$

479 And then do the first change of variable:

$$\frac{1}{2\pi} \int_{-1}^{1} \frac{1}{x_1 - y_1} \frac{\mathrm{d}}{\mathrm{d}y_1} \Delta u(y_1) \mathrm{d}y_1 = -\frac{3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{\sin(\theta) \cos(\theta) |\cos(\theta)| \cos(\theta)|}{x_1 - \sin(\theta)} \mathrm{d}\theta$$
$$= -\frac{3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{\sin(\theta) \cos^2(\theta)}{x_1 - \sin(\theta)} \mathrm{d}\theta$$
$$= -\frac{3}{4\pi} \int_{-\pi/2}^{3\pi/2} \frac{\sin(\theta) \cos^2(\theta)}{x_1 - \sin(\theta)} \mathrm{d}\theta$$
(56)

A second change of variable is using  $z = e^{i\theta}$ , where  $\sin(\theta) = \frac{z-1/z}{2i}$  and  $\cos(\theta) = \frac{z+1/z}{2}$ .

$$-\frac{3}{4\pi} \int_{-\pi/2}^{3\pi/2} \frac{\sin(\theta)\cos^2(\theta)}{x_1 - \sin(\theta)} d\theta = -\frac{3i}{16\pi} \int_C \frac{(z^2 - 1)(1 + z^2)^2}{z^3 [z - (ix + \sqrt{1 - x^2})][z - (ix - \sqrt{1 - x^2})]} dz$$
(57)

For x < -1, there are two poles at z = 0 and  $z = i(x + \sqrt{x^2 - 1})$ , so that applying the residue theorem gives:

$$\frac{1}{2\pi} \int_{-1}^{1} \frac{1}{x_1 - y_1} \frac{\mathrm{d}}{\mathrm{d}y_1} \Delta u(y_1) \mathrm{d}y_1 = 2\pi i \mathrm{Res}(f, z = 0) + 2\pi i \mathrm{Res}(f, z = i(x_1 + \sqrt{x_1^2 - 1}))$$

$$= \frac{3}{4} (1 - 2x_1^2 - 2x_1\sqrt{x_1^2 - 1})$$
(58)

483 where  $\operatorname{Res}(f,z)$  is the residue of f in z

For x > 1, there are also two poles at z = 0 and  $z = i(x_1 - \sqrt{x_1^2 - 1})$ , so that the residue theorem yields:

$$\frac{1}{2\pi} \int_{-1}^{1} \frac{1}{x_1 - y_1} \frac{\mathrm{d}}{\mathrm{d}y_1} \Delta u(y_1) \mathrm{d}y_1 = 2\pi i \mathrm{Res}(f, z = 0) + 2\pi i \mathrm{Res}(f, z = i(x_1 - \sqrt{x_1^2 - 1}))$$

$$= \frac{3}{4} (1 - 2x_1^2 + 2x_1\sqrt{x_1^2 - 1})$$
(59)

#### 486 F Definition of the Fourier Transform

In the following part, the 0th and 1st order solution for both normal and shear traction are
provided. Their expression depends on the choose of the definition of the Fourier transform.
We define the Fourier transform as following:

$$\mathcal{F}[f](k) = \int_{-\infty}^{+\infty} f(x_1) e^{-ikx_1} dx_1$$
(60)

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$$\mathcal{F}^{-1}[f](x_1) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(x_1) e^{ikx_1} dk$$
(61)

⁴⁹¹ where k is the wavenuber and is linked to the wavelength  $\lambda$  by  $k = 2\pi/\lambda$ . In particular, ⁴⁹² we will use the two Fourier transform:

$$\mathcal{F}[1/x_1](k) = -i\pi \operatorname{sign}(k)$$

$$\mathcal{F}[1/x_1^2](k) = -\pi |k|$$
(62)

#### ⁴⁹³ G In-plane opening (mode I)

⁴⁹⁴ The mode I correspond to opening. One common assumption for an opening fault is the ⁴⁹⁵ traction free condition at the surface.



Figure 19: Assuming the fault geometry and the opening distribution  $\Delta u(y_1) = (1 - 4\frac{y_1^2}{L^2})^{3/2}$ in meter (red curve), the full solution, the gradient term and the curvature term can be calculated using equation (67). The 0th order and the 1st order, are respectively calculated using the expressions (65) and (66).

496

497

By making the small slope approximation is can be shown that the main contribution for the shear and normal tractions are:





$$\tau_t^0(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \kappa^t(y_1) \Delta u^n(y_1) dy_1$$
  
$$\sigma_n^0(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \frac{d}{dy_1} \Delta u^n(y_1) dy_1$$
(65)

$$\tau_t^1(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \frac{d}{dy_1} \Delta u^n(y_1) \mathrm{d}y_1$$

$$\sigma_n^1(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{-2m(x_1)}{x_1 - y_1} + \frac{m(y_1)}{x_1 - y_1} + \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \kappa^t(y_1) \Delta u^n(y_1) \mathrm{d}y_1$$
(66)

On figure 19 an example of a in-plane opening fault is given for a rough geometry. 499 An interesting feature is that the on-fault shear traction mainly (0th order) depends upon 500 the geometry, because it the 0th order shear traction depends on the local curvature (a 501 geometrical parameter) that multiplies the opening. On the contrary, the normal traction 502 depends mainly on the derivative of opening along the fault. That means that it is more 503 dependent on the opening distribution than the fault geometry. By looking at the 0th order 504 solution, it is interesting to see that the result of opening on a non -planar fault leads to 505 the apparition of shear traction on this fault. If we assume traction free condition, the 506 only solution to remove the shear traction that appears it that mode II is also involve. 507 One result from that is that mode I on a non-planar fault, with the traction free condition 508 cannot exist without the existence of mode II. 509

#### 510 G.1 Full solution

 $_{511}$  In the following  $\ast$  represents the convolution operation.

$$\epsilon_{11}(\mathbf{x}) = \frac{1}{2\pi(\lambda + 2\mu)} \int_{\text{fault}} \left[ n_1 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(3\gamma_1^2 + \gamma_2^2)) + n_2 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_2^2 - \gamma_1^2)) \right] \frac{d}{dy^t} \Delta u^n(\mathbf{y}) dl(\mathbf{y}) \\ + \frac{1}{2\pi(\lambda + 2\mu)} \int_{\text{fault}} \left[ -n_2 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(3\gamma_1^2 + \gamma_2^2)) + n_1 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_2^2 - \gamma_1^2)) \right] \kappa^t(\mathbf{y}) \Delta u^n(\mathbf{y}) dl(\mathbf{y})$$

$$\epsilon_{22}(\mathbf{x}) = -\frac{1}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_2 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_1^2 + 3\gamma_2^2)) + n_1 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(\gamma_1^2 - \gamma_2^2)) \right] \frac{d}{dy^t} \Delta u^n(\mathbf{y}) dl(\mathbf{y}) - \frac{1}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_1 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_1^2 + 3\gamma_2^2)) - n_2 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(\gamma_1^2 - \gamma_2^2)) \right] \kappa^t(\mathbf{y}) \Delta u^n(\mathbf{y}) dl(\mathbf{y})$$

$$\epsilon_{12}(\mathbf{x}) = -\frac{\lambda + \mu}{2\pi(\lambda + 2\mu)} \int_{\text{fault}} \left[ (n_1 \frac{\gamma_1}{r} + n_2 \frac{\gamma_2}{r})(\gamma_1^2 - \gamma_2^2) \right] \frac{d}{dy^t} \Delta u^n(\mathbf{y}) dl(\mathbf{y}) - \frac{\lambda + \mu}{2\pi(\lambda + 2\mu)} \int_{\text{fault}} \left[ (-n_2 \frac{\gamma_1}{r} + n_1 \frac{\gamma_2}{r})(\gamma_1^2 - \gamma_2^2) \right] \kappa^t(\mathbf{y}) \Delta u^n(\mathbf{y}) dl(\mathbf{y})$$

$$\sigma_{11}(\mathbf{x}) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ n_1(\mathbf{y}) (3\gamma_1^2 + \gamma_2^2) \frac{\gamma_2}{r} - n_2(\mathbf{y}) (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^n(\mathbf{y}) \mathrm{d}l(\mathbf{y}) - \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ n_2(\mathbf{y}) (3\gamma_1^2 + \gamma_2^2) \frac{\gamma_2}{r} + n_1(\mathbf{y}) (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \kappa^t(\mathbf{y}) \Delta u^n(\mathbf{y}) \mathrm{d}l(\mathbf{y})$$

$$\sigma_{22}(\mathbf{x}) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ n_2(\mathbf{y})(\gamma_1^2 + 3\gamma_2^2) \frac{\gamma_1}{r} + n_1(\mathbf{y})(\gamma_1^2 - \gamma_2^2) \frac{\gamma_2}{r} \right] \frac{d}{dy^t} \Delta u^n(\mathbf{y}) \mathrm{d}l(\mathbf{y}) - \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ n_1(\mathbf{y})(\gamma_1^2 + 3\gamma_2^2) \frac{\gamma_1}{r} - n_2(\mathbf{y})(\gamma_1^2 - \gamma_2^2) \frac{\gamma_2}{r} \right] \kappa^t(\mathbf{y}) \Delta u^n(\mathbf{y}) \mathrm{d}l(\mathbf{y})$$

$$\sigma_{12}(\mathbf{x}) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ \left( n_1 \gamma_1 + n_2 \gamma_2 \right) \left( \frac{\gamma_1^2}{r} - \frac{\gamma_2^2}{r} \right) \right] \frac{d}{dy^t} \Delta u^n(\mathbf{y}) dl(\mathbf{y}) - \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ \left( n_1 \gamma_2 - n_2 \gamma_1 \right) \left( \frac{\gamma_1^2}{r} - \frac{\gamma_2^2}{r} \right) \right] \kappa^t(\mathbf{y}) \Delta u^n(\mathbf{y}) dl(\mathbf{y})$$

$$\tau_t(\mathbf{x}) = n_1(\mathbf{x})n_2(\mathbf{x})(\sigma_{11}(\mathbf{x}) - \sigma_{22}(\mathbf{x})) + (n_2^2 \mathbf{x}^3) - n_1^2(\mathbf{x}))\sigma_{12}(\mathbf{x})$$
  
$$\sigma_n(\mathbf{x}) = n_1^2(\mathbf{x})\sigma_{11}(\mathbf{x}) + 2n_1(\mathbf{x})n_2(\mathbf{x})\sigma_{12}(\mathbf{x}) + n_2^2(\mathbf{x})\sigma_{22}(\mathbf{x})$$

(67)

#### 512 G.2 Zeroth Order

#### 513 G.2.1 Space-time

$$\sigma_{11}(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \frac{d}{dy^t} \Delta u^n(y_1) dy_1$$
  

$$\sigma_{22}(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \frac{d}{dy^t} \Delta u^n(y_1) dy_1$$
  

$$\sigma_{12}(x_1) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \kappa^t(y_1) \Delta u^n(y_1) dy_1$$
  

$$\tau_t(x_1) = \sigma_{12}^0$$
  

$$= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \kappa^t(y_1) \Delta u^n(y_1) dy_1$$
  

$$\sigma_n(x_1) = \sigma_{22}^0$$
  

$$= -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \frac{d}{dy^t} \Delta u^n(y_1) dy_1$$
  
(68)

#### 514 G.2.2 Spectral-time

$$\sigma_{11}(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \pi |k| \mathcal{F}[\Delta u^n]$$

$$\sigma_{22}(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \pi |k| \mathcal{F}[\Delta u^n]$$

$$\sigma_{12}(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) i\pi \operatorname{sign}(k) \mathcal{F}[\kappa^t(y_1) \Delta u^n(y_1)]$$

$$\tau_t(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) i\pi \operatorname{sign}(k) \mathcal{F}[\kappa^t(y_1) \Delta u^n(y_1)]$$

$$\sigma_n(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \pi |k| \mathcal{F}[\Delta u^n]$$
(69)

#### 515 G.3 First Order

#### 516 G.3.1 Space-time

$$\begin{aligned} \sigma_{11}(x_1) &= -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ 3 \frac{x_2 - y_2}{(x_1 - y_1)^2} - \frac{m(y_1)}{x_1 - y_1} \right] \kappa^t(y_1) \Delta u^n(y_1) dy_1 \\ \sigma_{22}(x_1) &= -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ -\frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \kappa^t(y_1) \Delta u^n(y_1) dy_1 \\ \sigma_{12}(x_1) &= -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ -\frac{m(y_1)}{x_1 - y_1} + \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \frac{d}{dy^t} \Delta u^n(y_1) dy_1 \\ \tau_t(x_1) &= -m(\sigma_{11}^0 - \sigma_{22}^0) + \sigma_{12}^1 \\ &= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \frac{d}{dy^t} \Delta u^n(y_1) dy_1 \end{aligned}$$
(70)  
$$&= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \frac{d}{dy^t} \Delta u^n(y_1) dy_1 \\ \sigma_n(x_1) &= -2m\sigma_{12}^0 + \sigma_{22}^1 \\ &= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{-2m(x_1)}{x_1 - y_1} + \frac{m(y_1)}{x_1 - y_1} + \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \kappa^t(y_1) \Delta u^n(y_1) dy_1 \end{aligned}$$

#### 517 G.3.2 Spectral-time

$$\sigma_{11}(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( -3\pi |k| (h * \mathcal{F}[\kappa^t \Delta u^n] - \mathcal{F}[h\kappa^t \Delta u^n]) + i\pi \operatorname{sign}(k) \mathcal{F}[m\kappa^t \Delta u^n] \right)$$

$$\sigma_{22}(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( i\pi \operatorname{sign}(k) \mathcal{F}[m\kappa^t \Delta u^n] + \pi |k| (h * \mathcal{F}[\kappa^t \Delta u^n] - \mathcal{F}[h\kappa^t \Delta u^n])) \right)$$

$$\sigma_{12}(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( i\pi \operatorname{sign}(k) \mathcal{F}\left[m\frac{d}{dy_1} \Delta u^n\right] - \pi |k| \left( h * \mathcal{F}\left[\frac{d}{dy_1} \Delta u^n\right] - \mathcal{F}\left[h\frac{d}{dy_1} \Delta u^n\right] \right) \right)$$

$$\tau_t(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( -i\pi \operatorname{sign}(k) \mathcal{F}\left[m\frac{d}{dy_1} \Delta u^n\right] + \pi |k| \left( h * \mathcal{F}\left[\frac{d}{dy_1} \Delta u^n\right] - \mathcal{F}\left[h\frac{d}{dy_1} \Delta u^n\right] \right) \right)$$

$$\sigma_n(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( 2i\pi \operatorname{sign}(k)m * \mathcal{F}\left[\kappa^t \Delta u^n\right] - i\pi \operatorname{sign}(k) \mathcal{F}\left[m\kappa^t \Delta u^n\right]$$

$$-\pi |k| (h * \mathcal{F}\left[\kappa^t \Delta u^n\right] - \mathcal{F}\left[h\kappa^t \Delta u^n\right]))$$

$$(71)$$

## 518 H In-plane shear (mode II)

In the following,  $\gamma_1 = \frac{x_1 - y_1}{r}$ ,  $\gamma_2 = \frac{x_2 - y_2}{r}$ , and * represents the convolution operation.

#### 520 H.1 Full solution

$$\begin{split} \epsilon_{11}(\mathbf{x}) &= \frac{1}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_2 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(3\gamma_1^2 + \gamma_2^2)) - n_1 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_2^2 - \gamma_1^2)) \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &+ \frac{1}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_1 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(3\gamma_1^2 + \gamma_2^2)) + n_2 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_2^2 - \gamma_1^2)) \right] \kappa^t(\mathbf{y}) \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ \epsilon_{22}(\mathbf{x}) &= \frac{1}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_1 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_1^2 + 3\gamma_2^2)) - n_2 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(\gamma_1^2 - \gamma_2^2)) \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{1}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_2 \frac{\gamma_1}{r} (2\lambda\gamma_2^2 + \mu(\gamma_1^2 + 3\gamma_2^2)) + n_1 \frac{\gamma_2}{r} (2\lambda\gamma_1^2 + \mu(\gamma_1^2 - \gamma_2^2)) \right] \kappa^t(\mathbf{y}) \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ \epsilon_{12}(\mathbf{x}) &= -\frac{\lambda+\mu}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ \left( n_2 \frac{\gamma_1}{r} - n_1 \frac{\gamma_2}{r} \right) (\gamma_1^2 - \gamma_2^2) \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{\lambda+\mu}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ \left( n_1 \frac{\gamma_1}{r} + n_2 \frac{\gamma_2}{r} \right) (\gamma_1^2 - \gamma_2^2) \right] \kappa^t(\mathbf{y}) \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{\lambda+\mu}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_2 (3\gamma_1^2 + \gamma_2^2) \frac{\gamma_2}{r} + n_1 (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{\lambda+\mu}{2\pi(\lambda+2\mu)} \int_{\text{fault}} \left[ n_1 (3\gamma_1^2 + \gamma_2^2) \frac{\gamma_2}{r} - n_2 (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &+ \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ n_1 (3\gamma_1^2 + \gamma_2^2) \frac{\gamma_2}{r} - n_2 (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &+ \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ n_1 (\gamma_1^2 + 3\gamma_2^2) \frac{\gamma_1}{r} - n_2 (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ n_1 (\gamma_1^2 + 3\gamma_2^2) \frac{\gamma_1}{r} + n_2 (\gamma_2^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) du^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ (n_1 \frac{\gamma_1}{r} - n_2 \frac{\gamma_1}{r}) (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) du^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ (n_1 \frac{\gamma_1}{r} - n_2 \frac{\gamma_1}{r}) (\gamma_1^2 - \gamma_2^2) \frac{\gamma_1}{r} \right] \frac{d}{dy^t} \Delta u^t(\mathbf{y}) du^t(\mathbf{y}) dl(\mathbf{y}) \\ &- \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{\text{fault}} \left[ (n$$

#### 521 H.2 Zeroth Order

#### 522 H.2.1 Space-time

$$\sigma_{11}(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_1 - y_1} \kappa^t(y_1) \Delta u^t(y_1) \right] dy_1$$
  

$$\sigma_{22}(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_1 - y_1} \kappa^t(y_1) \Delta u^t(y_1) \right] dy_1$$
  

$$\sigma_{12}(x_1) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_1 - y_1} \frac{d}{dy^t} \Delta u^t(y_1) \right] dy_1$$
  

$$\tau_t(x_1) = \sigma_{12}^0$$
  

$$= -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_1 - y_1} \frac{d}{dy^t} \Delta u^t(y_1) \right] dy_1$$
  

$$\sigma_n(x_1) = \sigma_{22}^0$$
  

$$= -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{1}{x_1 - y_1} \kappa^t(y_1) \Delta u^t(y_1) \right] dy_1$$
  
(73)

#### 523 H.2.2 Spectral-time

$$\sigma_{11}(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) i\pi \operatorname{sign}(k) \mathcal{F}[\kappa^t \Delta u^t]$$

$$\sigma_{22}(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) i\pi \operatorname{sign}(k) \mathcal{F}[\kappa^t \Delta u^t]$$

$$\sigma_{12}(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \pi |k| \mathcal{F}[\Delta u^t]$$

$$\tau_t(k) = -\frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \pi |k| \mathcal{F}[\Delta u^t]$$

$$\sigma_n(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) i\pi \operatorname{sign}(k) \mathcal{F}[\kappa^t \Delta u^t]$$
(74)

#### 524 H.3 First Order

#### 525 H.3.1 Space-time

$$\begin{aligned} \sigma_{11}(x_1) &= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ 3 \frac{x_2 - y_2}{(x_1 - y_1)^2} - \frac{m(y_1)}{x_1 - y_1} \right] \frac{d}{dy^t} \Delta u^t(y_1) dy_1 \\ \sigma_{22}(x_1) &= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ -\frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \frac{d}{dy^t} \Delta u^t(y_1) dy_1 \\ \sigma_{12}(x_1) &= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \kappa^t(y_1) \Delta u^t(y_1) dy_1 \\ \tau_t(x_1) &= -m(\sigma_{11}^0 - \sigma_{22}^0) + \sigma_{12}^1 \\ &= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \kappa^t(y_1) \Delta u^t(y_1) dy_1 \end{aligned}$$
(75)  
$$&= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \kappa^t(y_1) \Delta u^t(y_1) dy_1 \\ \sigma_n(x_1) &= -2m\sigma_{12}^0 + \sigma_{22}^1 \\ &= \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \int_{-\infty}^{+\infty} \left[ \frac{2m(x_1)}{x_1 - y_1} - \frac{m(y_1)}{x_1 - y_1} - \frac{x_2 - y_2}{(x_1 - y_1)^2} \right] \frac{d}{dy^t} \Delta u^t(y_1) dy_1 \end{aligned}$$

#### 526 H.3.2 Spectral-time

$$\sigma_{11}(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( -3\pi |k| \left( h * \mathcal{F} \left[ \frac{d}{y_1} \Delta u^t \right] - \mathcal{F} \left[ h \frac{d}{y_1} \Delta u^t \right] \right) + i\pi \operatorname{sign}(k) \mathcal{F} \left[ m \frac{d}{y_1} \Delta u^t \right] \right) \right)$$

$$\sigma_{22}(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( i\pi \operatorname{sign}(k) \mathcal{F} \left[ m \frac{d}{y_1} \Delta u^t \right] + \pi |k| \left( h * \mathcal{F} \left[ \frac{d}{y_1} \Delta u^t \right] - \mathcal{F} \left[ h \frac{d}{y_1} \Delta u^t \right] \right) \right) \right)$$

$$\sigma_{12}(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( -i\pi \operatorname{sign}(k) \mathcal{F} [m\kappa^t \Delta u^t] + \pi |k| (h * \mathcal{F} [\kappa^t \Delta u^t] - \mathcal{F} [h\kappa^t \Delta u^t]) \right)$$

$$\tau_t(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( -i\pi \operatorname{sign}(k) \mathcal{F} [m\kappa^t \Delta u^t] + \pi |k| (h * \mathcal{F} [\kappa^t \Delta u^t] - \mathcal{F} [h\kappa^t \Delta u^t]) \right)$$

$$\sigma_n(k) = \frac{\mu}{\pi} \left( 1 - \frac{c_s^2}{c_p^2} \right) \left( -2i\pi \operatorname{sign}(k) m * \mathcal{F} \left[ \frac{d}{dy_1} \Delta u^t \right] + i\pi \operatorname{sign}(k) \mathcal{F} \left[ m \frac{d}{dy_1} \Delta u^t \right] \right)$$

$$(76)$$

## 527 I Out-of-plane (mode III)

In the following,  $\gamma_1 = \frac{x_1 - y_1}{r}$ ,  $\gamma_2 = \frac{x_2 - y_2}{r}$ , and * represents the convolution operation.

#### 529 I.1 Full solution

$$\epsilon_{13}(\mathbf{x}) = \frac{1}{4\pi} \int_{\text{fault}} \frac{\gamma_2}{r} \frac{d}{dy^t} \Delta u^s(\mathbf{y}) dl(\mathbf{y})$$

$$\epsilon_{23}(\mathbf{x}) = -\frac{1}{4\pi} \int_{\text{fault}} \frac{\gamma_1}{r} \frac{d}{dy^t} \Delta u^s(\mathbf{y}) dl(\mathbf{y})$$

$$\sigma_{13}(\mathbf{x}) = \frac{\mu}{2\pi} \int_{\text{fault}} \frac{\gamma_2}{r} \frac{d}{dy^t} \Delta u^s(\mathbf{y}) dl(\mathbf{y})$$

$$\sigma_{23}(\mathbf{x}) = -\frac{\mu}{2\pi} \int_{\text{fault}} \frac{\gamma_1}{r} \frac{d}{dy^t} \Delta u^s(\mathbf{y}) dl(\mathbf{y})$$

$$\tau_t(\mathbf{x}) = \frac{\mu}{2\pi} \int_{\text{fault}} \frac{\gamma_1}{r} \frac{d}{dy^t} \Delta u^s(\mathbf{y}) dl(\mathbf{y})$$
(77)

$$\sigma_n(\mathbf{x}) = 0$$

#### 530 I.2 Zeroth Order

531 I.2.1 Space-time

$$\sigma_{13}(x_1) = 0$$
  

$$\sigma_{23}(x_1) = -\frac{\mu}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \frac{d}{dy_1} \Delta u^s(y_1) dy_1$$
  

$$\tau_t(x_1) = -\frac{\mu}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{x_1 - y_1} \frac{d}{dy_1} \Delta u^s(y_1) dy_1$$
  

$$\sigma_n(x_1) = 0$$
(78)

#### 532 I.2.2 Spectral-time

$$\sigma_{13}(k) = 0$$
  

$$\sigma_{23}(k) = -\frac{\mu}{2\pi}\pi |k|\mathcal{F}[\Delta u^{s}]$$
  

$$\tau_{t}(k) = -\frac{\mu}{2\pi}\pi |k|\mathcal{F}[\Delta u^{s}]$$
  

$$\sigma_{n}(k) = 0$$
(79)

#### 533 I.3 First Order

#### 534 I.3.1 Space-time

$$\sigma_{13}(x_1) = \frac{\mu}{2\pi} \int_{-\infty}^{+\infty} \frac{x_2 - y_2}{(x_1 - y_1)^2} \frac{d}{dy^t} \Delta u^s(y_1) dy_1$$
  

$$\sigma_{23}(x_1) = 0$$
  

$$\tau_t(x_1) = 0$$
  

$$\sigma_n(x_1) = 0$$
  
(80)

#### 535 I.3.2 Spectral-time

$$\sigma_{13}(k) = -\frac{\mu}{2\pi} \left( \pi kh * \mathcal{F}[\Delta u^s] + \pi i k^2 \mathcal{F}[h\Delta u^s] \right)$$
  

$$\sigma_{23}(k) = 0$$
  

$$\tau_t(k) = 0$$
  

$$\sigma_n(k) = 0$$
  
(81)

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Figure 1.



changing along the fault

The slip direction is changing along the fault Figure 2.



Figure 3.



Mechanically equivalent on-fault stresses Planar fault

# Correcting terms for non-planar fault

Figure 4.



Figure 5.



Figure 6.



Figure 7.



Figure 8.



Figure 9.

# Shear traction drag for different fault geometries



Figure 10.



Figure 11.



Figure 12.

## Total elastic traction





## 0th order

## 1st order

Figure 13.



Figure 14.



Figure 15.



Figure 16.



#### For small strain, the elastic response is linear



### For large displacement, the elastic response is non-linear



Figure 17.




Figure 18.



Figure19.

