

Assessing Thermospheric Neutral Density Models using GEODYN’s Precision Orbit Determination

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June 25, 2023

Abstract

This study focuses on utilizing the increasing availability of satellite trajectory data from global navigation satellite system-enabled low-Earth orbiting satellites and their precision orbit determination (POD) solutions to expand and refine thermospheric model validation capabilities. The research introduces an updated interface for the GEODYN-II POD software, leveraging high-precision space geodetic POD to investigate satellite drag and assess density models. This work presents a case study to examine five models (NRLMSIS2.0, DTM2020, JB2008, TIEGCM, and CTIPe) using precise science orbit (PSO) solutions of the Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2). The PSO is used as tracking measurements to construct orbit fits, enabling an evaluation according to each model’s ability to redetermine the orbit. Relative in-track deviations, quantified by in-track residuals and root-mean-square errors (RMSe), are treated as proxies for model densities that differ from an unknown true density. The study investigates assumptions related to the treatment of the drag coefficient and leverages them to eliminate bias and effectively scale model density. Assessment results and interpretations are dictated by the timescale at which the scaling occurs. JB2008 requires the least scaling (~23%) to achieve orbit fits closely matching the PSO within an in-track RMSe of 9 m when scaled over two weeks and 4 m when scaled daily. The remaining models require substantial scaling of the mean density offset (~30-75%) to construct orbit fits that meet the aforementioned RMSe criteria. All models exhibit slight over or under sensitivity to geomagnetic activity according to trends in their 24-hour scaling factors.

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Key Points:

- Precision orbit determination solutions are expanded to study satellite drag and assess upper atmospheric density models using GEODYN.
- A proof-of-concept case study assessment of density models is presented using ICESat-2 precise science orbits and orbit fits
- Assessment results are provided for empirical (MSIS2, DTM2020, JB2008) and physics-based (TIEGCM, CTIPe) models for 14-days in November 2018

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 34 dictated by the timescale at which the scaling occurs. JB2008 requires the least scaling
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1 Plain Language Summary

40 This study utilizes the increasing availability of satellite trajectory data from low-
 41 Earth orbiting satellites and their precision orbit determination (POD) solutions to ex-
 42 pand thermospheric model validation capabilities. We introduce an updated interface
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 45 JB2008, TIEGCM, and CTIPe) using precise science orbit (PSO) solutions of the Ice,
 46 Cloud, and Land Elevation Satellite-2 (ICESat-2). GEODYN is used to construct or-
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 48 ability to redetermine the orbit. Relative deviations from the PSO, quantified by in-track
 49 residuals and root-mean-square errors (RMSe), serve as proxies for model densities that
 50 differ from an unknown true density. We investigate and leverage drag coefficient assump-
 51 tions to eliminate bias and scale model densities. JB2008 requires the least scaling (\sim
 52 -23%) to achieve orbit fits closely matching the PSO within an in-track RMSe of 9 m
 53 when scaled over two weeks and 4 m when scaled daily. The remaining models require
 54 substantial scaling ($\sim 30 - 75\%$) to meet the aforementioned RMSe criteria.

2 Introduction

55 With the drastic increase in commercial satellite launches, the need to address the
 56 challenges posed by satellite drag have come to the forefront of the scientific, operational,
 57 and commercial space communities (Muelhaupt et al., 2019; Berger et al., 2020; Thayer
 58 et al., 2021; Hejduk & Snow, 2018; Bussy-Virat et al., 2018). Shortly following the 36th
 59 launch of SpaceX’s Starlink constellation on 3 February 2022, 38 out of 49 satellites were
 60 lost due to the impacts of a modest geomagnetic storm that reached G1 intensity ear-
 61 lier that day (Berger et al., 2023; Fang et al., 2022; Hapgood et al., 2022). The satellites
 62 were placed into an initial orbit of 210 km after which they were intended to maneuver
 63 to an operational altitude of 500 km. While this low altitude plan lent itself to a quick
 64 de-orbit in the face of catastrophe, it exposed the satellites to the larger variations and
 65 uncertainties in neutral density associated with relatively meager space weather condi-
 66 tions. While this event happened at altitudes well below Starlink’s operational orbit, it
 67 has served as a potent example to the commercial space community of the need to bet-
 68 ter model and predict atmospheric drag, which represents the most significant hurdle pre-
 69 venting more accurate determination and prediction of trajectories in LEO.

70 Precision orbit determination (POD) programs are employed in both operational
 71 and research capacities to provide high-fidelity orbit trajectories of LEO satellites. The
 72 quality of such trajectories is directly dependent on the ability of a POD’s force model
 73 to realistically capture the conservative and non-conservative forces impacting a satel-
 74 lite’s orbit. Due to advancements in conservative force modeling, the largest source of
 75 error preventing more accurate orbit trajectories is now associated with non-conservative
 76 forces (Tapley et al., 2005; Reigber et al., 2006; Velicogna & Wahr, 2005). Of these, at-
 77 mospheric drag is the most variable and uncertain as a consequence of its reliance on mod-
 78 eling the thermospheric neutral mass density (ρ) variations and the satellite drag-coefficient
 79 (C_D) (Hejduk & Snow, 2018). The largest source of uncertainty is ρ , but for satellites
 80 with complex shapes, C_D can contribute to this uncertainty. Mehta et al. (2022) describes
 81 this issue as the interconnectedness of uncertain parameters, an extremely challenging
 82 problem to solve for the satellite drag community and one that has significant impact
 83 on the assumptions made in this work. The burden for achieving more precise and re-
 84 liable LEO nowcasting and forecasting largely relies on the ability of thermospheric den-
 85 sity models to accurately capture the behavior of neutral density and reliably predict it
 86 into the future. Adding to the problem, assessing the performance of density models presents
 87 a massive challenge due to the scarcity of data from satellite measurements, and the lack
 88 of absolute truth due to the complexity of interconnected uncertainties. This necessi-
 89 tates the community to seek alternative methods to add to the validation method reper-
 90 toire. The growing prevalence of global navigation satellite system (GNSS)-enabled low-
 91 Earth orbiting satellites and their POD solutions represents one such potential data source,
 92 and providing methods to take advantage of these datasets will help the community ex-
 93 pand and refine model validation capabilities.

94 POD programs such as the NASA Goddard Space Flight Center’s (GSFC) GEO-
 95 DYN II software (henceforth referred to as GEODYN) have been developed within the
 96 geodesy scientific community with the above challenges in mind—implementing techniques
 97 such as reduced-dynamics paired with extremely high quality tracking measurements from
 98 GNSS to mitigate the need for highly accurate non-conservative force models when per-
 99 forming non-predictive orbit determination. Through these means, centimeter level ra-
 100 dial accuracy has been demonstrated to produce precise science orbit (PSO) solutions
 101 for missions such as the Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2), which
 102 orbits at approximately 500 km (Thomas et al., 2021). These techniques—combined with
 103 GEODYN’s legacy of precise conservative force and measurement modeling, meticulous
 104 time systems, and accurate coordinate reference frames—have made the program a top-
 105 tier POD tool that is well-positioned to study thermospheric neutral density models and
 106 their distinct impacts on the estimation of satellite drag (Luthcke et al., 2003; Zelensky

107 et al., 2010; Lemoine et al., 2016; Loomis et al., 2019). This work aims to provide a method
 108 to improve the specification of satellite drag physics and the assessment of neutral den-
 109 sity model performance to help the Ionosphere-Thermosphere (IT) community advance
 110 model predictions, and consequently improve the accuracy of POD solutions.

111 This paper presents the development of a modernized Python interface for the GEO-
 112 DYN software, leveraging the high-precision nature of space geodetic POD, but refash-
 113 ioned to study satellite drag and to enable density model assessment. We make use of
 114 the well-specified, low-error ICESat-2 PSO to perform a case study assessment of five
 115 thermospheric density models, three of which are empirical while the other two are physics-
 116 based. The ICESat-2 PSO serves as tracking measurements to POD-based orbit fits in
 117 which the drag effects from density models are assessed according to each model’s abil-
 118 ity to redetermine the orbit. Implications regarding the treatment of the drag coefficient
 119 are investigated and discussed. This work reports an initial result using a fixed drag co-
 120 efficient of $C_D = 2.5$, followed by two methods for debiasing the assessment results us-
 121 ing a drag acceleration scaling factor over both a two-week and a daily time interval. Each
 122 model’s orbit fit contains relative in-track deviations, quantified by in-track residuals and
 123 root-mean-square errors from the ICESat-2 PSO, which are treated as proxies for model
 124 densities that differ from a true, unspecified density. By developing these methods, we
 125 aim to provide the community with the means to take advantage of emerging GNSS-tracked
 126 satellite datasets and POD solutions to objectively quantify density model performance.
 127 In addition, we hope to address deficiencies in non-conservative force modeling that may
 128 currently impede higher quality predictions of LEO trajectories. The presented model
 129 assessment results will be parsed into the Community Coordinated Modeling Center’s
 130 Comprehensive Assessment of Models and Events using Library Tools (CAMEL) frame-
 131 work, for community use.

132 Section 3 gives the necessary science background needed to understand our method-
 133 ology. Section 4 details the GEODYN software, provides information regarding the ICESat-
 134 2 POD solutions, and offers an overview description of the upper atmospheric density
 135 models that are assessed in this work. Section 5 details the methodology, the setup pro-
 136 cedure for conducting the model assessment, and the methods for debiasing the assess-
 137 ment results using drag acceleration scaling factors. Section 6 provides the results and
 138 discussion of the assessment using ICESat-2 PSO as a case study.

139 3 Background

140 The precision of a POD solution relies on the fidelity of the tracking measurement
 141 models, the quality of the tracking data, and the ability of the POD force model to cap-
 142 ture realistic accelerations acting on the satellite. In general, the force model defines the
 143 overall motion of a spacecraft by calculating the sum of all impacting forces, themselves
 144 being subdivided into conservative forces which are potential in nature, and non-conservative
 145 forces which act to dissipate the satellite’s orbital energy. Conservative forces captured
 146 by the GEODYN force model include the Earth’s static gravity field (geopotential), solid
 147 Earth and ocean tides, the effects of dynamic polar motion, the acceleration from time
 148 variable gravity, Third-body perturbations (primarily from the Sun and Moon), and con-
 149 tributions from general relativity. Recent improvements in conservative force modeling
 150 as well as advances in the internal measurement models have shifted the primary source
 151 of error in POD solutions to the non-conservative forces (Luthcke et al., 2006; Loomis
 152 et al., 2019; Reigber et al., 2006). The non-conservative forces modeled in GEODYN are
 153 atmospheric drag, solar radiation pressure (SRP), and Earth radiation pressure (ERP).
 154 As altitude decreases in the LEO regime, atmospheric drag increasingly becomes the largest
 155 non-gravitational force acting on satellites. In addition, the drag force’s dependence on
 156 the upper atmospheric neutral mass density makes it the most error-bound perturbing
 157 force (Hejduk & Snow, 2018). While force model errors can be circumvented via reduced-
 dynamics and high-quality tracking measurements, this technique is limited in its ap-

158 plication for the eventual goal of orbit prediction, which requires an improved, more-realistic
 159 force model (Tapley et al., 2004; Luthcke et al., 2019).

160 The drag force acting on a satellite of mass m_{sat} is proportional to the atmospheric
 161 neutral mass density ρ , the drag coefficient C_D , the projected area perpendicular to the
 162 flow direction A_{sat} , and the velocity of the satellite relative to the atmosphere \vec{V}_{rel} . The
 163 drag acceleration \vec{a}_D due to the drag force per unit mass acting on a satellite is given
 164 in Equation 1 as

$$165 \quad \vec{a}_D = -\frac{1}{2} \rho C_D \frac{A_{sat}}{m_{sat}} V_{rel}^2 \frac{\vec{V}_{rel}}{V_{rel}} \quad (1)$$

166 Physically, the total drag force acting on a satellite surface is given by the force due to
 167 incident atmospheric particles impacting the surface combined with the force from scatter-
 168 ed particles departing from the surface. These effects are represented by the drag co-
 169 efficient C_D , which depends on a satellite’s geometry and orientation, the material and
 170 surface temperature of the spacecraft, the local atmospheric composition, and gas-surface
 171 interactions and other effects (Bernstein & Pilinski, 2022). In the context of spacecraft
 172 dynamics, the C_D is generally characterized as either fixed, fitted, or physical. Fixed C_D
 173 uses a predetermined value that does not change. Fitted C_D is derived using some form
 174 of a fitting or filtering process and is typically updated over time (every few hours or or-
 175 bits). Physical C_D is computed by modeling the momentum and energy exchange be-
 176 tween the flow-field particles and the satellite (see Mehta et al. (2022) for more details).
 177 If not physically calculated, C_D ’s presence in Equation 1 may be thought of as a scal-
 178 ing factor that effectively serves to average out errors in the atmospheric density model
 179 and gas-surface interactions. In its base state, GEODYN can use either a fitted or fixed
 180 C_D . In the fitted case C_D is an adjustable parameter that accounts for mismodeled physics
 181 and for uncertainties in ρ associated with the upper atmospheric density model.

182 Earth’s upper atmosphere is driven by a broad range of external energy inputs, lead-
 183 ing to complex thermal, electromagnetic, and chemical processes that result in a ther-
 184 mospheric neutral mass density ρ that is highly dynamic and whose variability is diffi-
 185 cult to specify (Emmert, 2015). Upper-atmospheric density models are employed within
 186 POD force models to represent the complex behavior of ρ when calculating the force of
 187 satellite drag acting on a spacecraft, directly or indirectly through C_D . The three types
 188 of density models most commonly used by upper atmospheric communities are semi-empirical,
 189 physics-based, and data assimilative models. The simple yet effective semi-empirical mod-
 190 els are most commonly employed in POD force models since they offer excellent clima-
 191 tological pictures of upper atmospheric variability and are computationally inexpensive.
 192 Physics-based models are more complex, taking the form of general circulation models
 193 which solve the first-principle equations that govern the coupled thermosphere-ionosphere
 194 system. They are not typically used in POD geodetic settings due to the computational
 195 expense. A data assimilative technique can be used to calibrate modeled density and has
 196 given rise to data assimilative (also referred to as dynamically calibrated) models. These
 197 combine analyses from a multitude of space objects to produce corrections to empirical
 198 (and occasionally physics-based) thermospheric models. The most prominent example
 199 of assimilative thermospheric density models is the United States Space Force, High Ac-
 200 curacy Satellite Drag Model (HASDM) (Storz et al., 2005). It is a common practice in
 201 the IT modeling community to compare model performances against HASDM outputs
 202 since it performs real-time calibration using ~ 75 space objects.

203 Different models, and even model types, have varying degrees of performance un-
 204 der specified conditions. Individual model performances are known to depend greatly
 205 on the solar flux and geomagnetic conditions that drive them, and their respective strengths
 206 make some models better qualified for some scenarios than others. Semi-empirical mod-
 207 els are often computationally fast and accurate for climatological uses, but their abil-
 208 ity to accurately project into the future is closely tied to the fidelity of their drivers. Physics
 209 models offer great potential for forecasting, but lack the accuracy of semi-empirical mod-

210 els in near real-time scenarios (Shim et al., 2014; Sutton, 2018). The vast range in model
 211 performances makes the evaluation of models a critical goal for upper atmospheric sci-
 212 ence and satellite drag communities. The scarcity, and coupled uncertainty (via C_D un-
 213 certainty) of thermospheric density measurements makes this a significant challenge. The
 214 most common method for objectively quantifying a density model’s performance is to
 215 compare the sampled model outputs against satellite measurements, e.g. see Walterscheid
 216 et al. (2023)—usually in the form of accelerometer-derived densities from the Challeng-
 217 ing Minisatellite Payload (CHAMP), Gravity Recovery and Climate Experiment (GRACE)
 218 or Gravity Field and Steady-State Ocean Circulation (GOCE) missions (Bruinsma et
 219 al., 2004; Sutton et al., 2005; Doornbos et al., 2010; Mehta et al., 2017).

220 In a series of papers motivated to provide community organization for conducting
 221 model comparison and evaluation, Bruinsma et al. (2017, 2018, 2021) provide common-
 222 alities for inter-model scoring. They report on chosen observed density datasets, time
 223 periods of interest, and provide a scoring metric in the form of the mean, standard de-
 224 viation and root mean square error (RMSe) of the observation-to-model density ratios.
 225 He et al. (2018) similarly presents an assessment of several semi-empirical thermosphere
 226 models, focusing on their ability to reproduce spatial variations and capture complex fea-
 227 tures in thermosphere mass density. Shim et al. (2014) provides a systematic evaluation
 228 of thermospheric and ionospheric models, quantifying model performance using four skill
 229 scores calculated as functions of geomagnetic activity and geographic latitude: RMS er-
 230 ror, prediction efficiency, ratio of maximum-to-minimum, and ratio of maximum ampli-
 231 tude. Thayer et al. (2023) investigates the use of the day-to-night density ratio as a met-
 232 ric for representing the atmosphere’s response to large scale perturbations (i.e. the tran-
 233 sition from solar maximum to solar minimum), providing inter-model and model-to-observation
 234 comparisons, and unearthing discrepancies that are not observed between models and
 235 observations when viewed using more common metrics. Each of these reports makes use
 236 of the accelerometer-derived density data sets to objectively quantify model performance.

237 Through this work, we aim to contribute an additional method to the community
 238 in which accurately developed and well-honed POD tools can be leveraged for assessing
 239 density model performance. For the purposes of this paper, we make a distinction when
 240 referring to the different stages of model assessment. We use the term “assessment” to
 241 refer more generally to methods and results that offer insight into model performance.
 242 “Verification” refers to using other well-specified methods and datasets to confirm the
 243 fidelity of our methods and results. “Validation” refers to the act of objectively quan-
 244 tifying modeled densities against observed/derived values. This paper offers a verifica-
 245 tion of our method and results by comparing against the HASDM densities, and provides
 246 an example performance assessment using two-weeks of the ICESat-2 PSO as a case study.
 247 A more formal validation scheme is the eventual goal of this work, however, this requires
 248 additional considerations and is a source of continuing effort.

4 Program and Data Descriptions

4.1 GEODYN and the Pygeodyn Wrapper

249 The GEODYN-II program is a precision orbit determination and parameter esti-
 250 mation tool that has been used on every NASA geodetic Earth and planetary altime-
 251 ter mission since 1985. The program is used extensively for orbit determination, geode-
 252 tic parameter estimation, tracking instrument calibration, satellite orbit prediction, as
 253 well as for many other applied research studies in satellite geodesy (Pavlis et al., 2019;
 254 Luthcke et al., 2019). GEODYN is capable of ingesting essentially all types of tracking
 255 measurements, the most common of which include observations from global navigation
 256 satellite systems (GNSS) and satellite laser ranging (SLR), as well as post-processed or-
 257 bits in the form of orbit trajectories or precisely converted elements (PCE) (Pavlis et al.,
 258 2019; Lyon et al., 2004). GEODYN performs orbit propagation using Cowell’s method

259 of numerical integration, and performs data-reduction utilizing a Bayesian least-squares
 260 batch estimation process to optimally estimate parameters by minimizing the residuals
 261 between tracking data and orbit propagations (see Vallado (2013) for more information).
 262 GEODYN’s long history in geodetic applications has ensured the development of very
 263 precise conservative force and measurement models, as well as accurate time systems and
 264 coordinate reference frames, making the program a top-tier POD tool. With this under-
 265 standing, the errors found in the observed residuals between tracking data and deter-
 266 mined orbit are more related to uncertainties in the satellite specific non-conservative
 267 force models, rather than being related to the quality of measurement modeling or or-
 268 bit determination methods and tools. In the lower register of LEO, where atmospheric
 269 drag dominates, the observation residuals can provide valuable information on the drag
 270 model errors.

271 Pygeodyn is an internally-developed Python-based wrapper meant to offer improved
 272 user access to the FORTRAN-based GEODYN software. Pygeodyn offers users a stream-
 273 lined and simplified tool to navigate the complex steps for modifying, controlling, run-
 274 ning, and reading the various data sets and files that compose the GEODYN program.
 275 The main portion of GEODYN II is composed of two sequenced programs: GEODYN-
 276 IIS, a scheduling program and GEODYN-IIE, an execution program stage. The schedul-
 277 ing program reads and organizes input data, ancillary data files, and the user’s setup op-
 278 tions. The execution program then integrates the satellite trajectory and applies the se-
 279 lected models, performs orbit determination to provide computed observables, and uses
 280 the least squares scheme, along with any measured observables, to provide solutions for
 281 updated orbits as well as any requested geophysical parameters. The two stages com-
 282 municate via a series of binary files which are output from the scheduling program and
 283 fed into the execution program. Historically, adding atmospheric density models to GEO-
 284 DYN required modification to IIS as well as subsequent data tracking and modification
 285 to IIE, a series of complications that have been circumvented with our Pygeodyn tool.
 286 Pygeodyn gives the ability to switch between different atmospheric density models that
 287 have been connected to GEODYN-IIE without the need to modify GEODYN-IIS, sim-
 288 plifying the user experience for adding and selecting the models. Programming in Python
 289 has also afforded Pygeodyn the ability to interface with the NASA Goddard Commu-
 290 nity Coordinated Modeling Center’s (CCMC) Kamodo API (Ringuette et al., 2023), grant-
 291 ing access to their sophisticated model readers and allowing Pygeodyn to connect physics-
 292 based density model outputs to the POD scheme.

4.2 ICESat-2 PSO Solutions as Tracking Data

293 ICESat-2 flies in a near-circular, near-polar, low-Earth orbit at ~ 496 km altitude
 294 and an orbital period of 94.22 min. Details of the orbital parameters are reported in Luthcke
 295 et al. (2019). The ICESat-2 PSO (i.e., the science quality POD solutions), and their cor-
 296 responding setup files, are provided by the Geodesy and Geophysics Laboratory within
 297 NASA/GSFC, who maintain the GEODYN program and provide science quality POD
 298 for many NASA missions. ICESat-2 is an excellent platform for orbital drag-based model
 299 assessment because of its science requirements to have such high quality orbit solutions,
 300 as well as stable attitude specifications. The ICESat-2 PSO is reported by Thomas et
 301 al. (2021) as having a radial orbit accuracy of just below 1.5 cm over a 24-hour orbit solution—
 302 performing better than the mission requirement of 3 cm. These orbit solutions are gen-
 303 erated through the reduction of GNSS double-difference carrier phase observable resid-
 304 uals, and independently assessed using SLR measurement residual analysis. Technical
 305 details regarding the construction and analysis plan for the ICESat-2 PSO can be found
 306 in Luthcke et al. (2019).

307 The centimeter-level orbit of the PSO data is achieved using the previously men-
 308 tioned reduced-dynamics technique in which GEODYN solves for empirical acceleration
 309 parameters that describe the difference between the actual positions, i.e. those derived

310 from the GPS tracking measurements, and the positions that are calculated by the pro-
 311 gram’s physical force models and satellite propagator. The PSO data includes estima-
 312 tions of along-track and cross-track empirical accelerations every quarter of an orbit, ap-
 313 plying a nearest neighbor covariance constraint. With the use of reduced-dynamic em-
 314 pirical accelerations, it is possible to compensate for errors associated with using the MSIS86
 315 model to calculate the effects of atmospheric drag. Luthcke et al. (2019) notes that even
 316 though a reduced-dynamic approach is commonly employed by the geodesy community
 317 to overcome any inadequacies in a force model, the technique relies on an orbit solution
 318 that has already attained sufficient radial accuracy through the use of a high-quality phys-
 319 ical force model. Dense tracking measurements and the reduced-dynamic technique do
 320 not obviate the use of accurate orbit modeling, and improvements in the orbit fit will
 321 be realized when the force models are improved. We also note that while MSIS86 was
 322 used to estimate the drag accelerations in the ICESat-2 PSO, due to them being com-
 323 bined with additional empirical accelerations in the along-track and cross-track direc-
 324 tions no related bias is found that favors the MSIS series of models in the assessments
 325 reported in this work.

326 This work uses the ICESat-2 PSO as tracking measurement input to a data-reduction
 327 run of GEODYN—the goal being to assess the ability of each selected density model to
 328 re-determine the orbit of ICESat-2. A data-reduction run in GEODYN is one in which
 329 orbit parameters (i.e., initial conditions) and optionally geophysical parameters (such
 330 as gravitational coefficients or the drag coefficient) are adjusted to minimize residuals
 331 and provide an improved solution. This data reduction is computed over an orbital arc,
 332 a set time period for which continuous tracking data is available. The term “orbit fit”
 333 refers to the outputs of GEODYN runs in which the ICESat-2 PSO is the tracking data
 334 type and respective density models are used to iteratively re-determine the orbit.

335 The following capabilities for density model assessment are enabled by using GEO-
 336 DYN to construct orbit fits from ICESat-2 orbit solutions:

- 337 1. Leverage GEODYN’s high fidelity physical force models which have been honed
 338 by the program’s long legacy in space geodesy.
- 339 2. Perform data-reduction runs in which we compare the relative ability of each at-
 340 mospheric density model to re-determine the orbit of ICESat-2 given the isolated
 341 satellite drag effects.
- 342 3. Control the POD and force model parameters such that for each respective run,
 343 the only relative variable impacting the overall fit of the orbit solution for a given
 344 arc is the atmospheric density model used to estimate the drag term.
- 345 4. Control for relative errors between runs associated with an unknown drag coef-
 346 ficient by using a realistic fixed value of $C_D = 2.5$. This value is determined by
 347 physically calculating C_D using the Diffuse Reflection with Incomplete Accom-
 348 modation (DRIA) method along the orbit of ICESat-2, as is described in Section
 349 5.

4.3 Model Descriptions

350 This section provides a brief overview of the atmospheric density models that are
 351 used for verification and assessment. Table 1 lists the models, providing the Model ID
 352 used for referencing in this paper, the full name and version number, the run conditions
 353 based on the drivers, and the models’ spatial and temporal resolutions. The authors ac-
 354 knowledge that while there are a number of ways to improve a density model’s outputs
 355 at runtime (see Sutton (2018); Shim et al. (2014)), the outputs used in this work are in-
 356 tended to reflect typical community use, with each model being run according to the de-
 357 veloper’s operational instructions. Additional information regarding each model can be
 358 found in the references provided in the second column of Table 1.

359 We provide a verification using SET-HASDM, a data-assimilative model, and as-
360 sessment results for MSIS2, JB2008, DTM2020, TIEGCM, and CTIPe. The semi-empirical
361 models (MSIS2, JB2008, DTM2020) are interfaced directly into GEODYN's FORTRAN-
362 based source code. The physics-based models (TIEGCM and CTIPe) are interfaced to
363 GEODYN via the CCMC's Kamodo program which reads and interpolates the model
364 output files. These interpolated outputs are connected to GEODYN through the Pygeo-
365 dyn wrapper using an orbit cloud interpolation technique which is detailed in Appendix
366 C. In addition, physics-based models whose maximum altitude is below the orbit alti-
367 tude of ICESat-2 include a diffusive equilibrium extrapolation of the neutral densities
368 (see Chapter 10 of Schunk and Nagy (2009)). The use of Kamodo makes the analysis
369 techniques in this paper easily extensible to additional models. Any thermospheric model
370 that is supported by Kamodo, with the appropriate diffusive equilibrium extrapolation,
371 can be added to this and similar analyses in the future.

Models Assessed in this Report

Model ID	Full name/version	Drivers, (solar geomagnetic)	Resolution, (spatial time)
Semi-Empirical MSIS2	Naval Research Laboratory Mass Spectrometer and Incoherent Scatter (NRLMSIS 2.0), (Emmert et al., 2021)	$F_{10.7}$ A_p	
JB2008	Jacchia-Bowman 2008, (Bowman et al., 2008)	$F_{10.7}$, $S_{10.7}$, $M_{10.7}$, $Y_{10.7}$ A_p , Dst	
DTM2020	Drag Temperature Model 2020, operational mode (Bruinsma & Boniface, 2021)	$F_{10.7}$ and K_p	
Physics-Based TIEGCM	National Center for Atmospheric Research (NCAR) Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIEGCM; version 2.0), (Richmond et al., 1992; Qian et al., 2014; Sutton et al., 2015)	$F_{10.7}$, EUVAC proxy model (Solomon & Qian, 2005) K_p via the Heelis model (Heelis et al., 1982),	$5^\circ \times 5^\circ$, vertically specified by logarithmic pressure surfaces in half-scale height increments from ~ 97 km to ~ 800 km 1 minute time step, hourly output
CTIpe	Coupled Thermosphere Ionosphere Plasmasphere Electrodynamics (CTIpe; version 4.1) (G. Millward et al., 1996; G. H. Millward et al., 2001)	$F_{10.7}$, solar wind inputs via magnetic field, velocity, and density measurements from Advanced Composition Explorer (ACE) K_p , hemispheric power index from NOAA Polar Orbiting Environmental Satellite (POES)	2° lat. \times 18° lon., vertically specified by 15 logarithmic pressure surfaces from ~ 80 km to ~ 500 km 1 minute time step
Data-Assimilative SET-HASDM	Space Environment Technologies (SET) HASDM Database (Tobiska et al., 2021), which is derived from the US Space Force operational archive (Storz et al., 2005)		10° lat. \times 15° lon., 25 km altitude steps from 175 to 825 km. 3 hour time step

Table 1. A summary of the thermospheric density models used in this assessment.

5 Methodology

5.1 Setup for ICESat-2 Case Study

372 This method uses a satellite’s PSO as tracking measurements to construct dynamic
 373 POD-based orbit fits from different density models. The dynamic POD technique uses
 374 a batch least-squares approach to iteratively reduce errors between the propagating or-
 375 bit fit and the ingested PSO—GEODYN refers to this as data-reduction mode. The ini-
 376 tial conditions, and any other adjustable parameters, are iteratively estimated and up-
 377 dated to refine the orbit fit until it consistently reaches a convergence threshold. The
 378 remaining errors that persist between the PSO and a given model’s orbit fit are under-
 379 stood to be primarily due to atmospheric drag effects from the respective density model.
 380 This understanding is leveraged to investigate density model performance through the
 381 assessments that are presented in this paper. Figure 1 provides a visualization showing
 382 connections between the high-level datasets and processes. While the true density along
 383 the ICESat-2 orbit remains unknown, each model’s orbit fit contains in-track deviations
 384 from the ICESat-2 PSO, which are treated as proxies for model density deviations from
 385 the true density.

386 The GEODYN run setup that is used to construct the POD-based orbit fits is kept
 387 as similar as possible to the setup used by the team at NASA-GGL to produce the ICESat-
 388 2 PSO—meaning we modify only what is necessary to use PSO as the tracking measure-
 389 ment type, and to control the procedure such that drag is the only independent variable
 390 in each model’s run. An extended overview of GEODYN’s setup and force model param-
 391 eters for the model assessment runs is provided in the appendix in Table B1, with only
 392 the most impactful considerations being discussed here. In addition to each orbit fit us-
 393 ing the same background force models, the ICESat-2 external attitude information is also
 394 utilized to properly orient the spacecraft body and the solar array. The orbit fits are split
 395 into 24-hour, consecutive daily arcs. The arc length can theoretically be much shorter;
 396 however, orbit errors related to force model perturbations (i.e., drag) require propaga-
 397 tion time to accumulate. An arc length on the order of 1-2 orbital periods may not de-
 398 pict substantial trajectory deviations in the residuals, making 24-hour arcs a balanced
 399 choice to demonstrate this assessment method. In theory, reducing the arc lengths would
 400 provide more RMSE values over shorter times and would offer higher temporal resolu-
 401 tion towards understanding model performance, but at the cost of having accumulated
 402 less orbital error from the density model in the shorter propagation time. The choice of
 403 arc length and its ramifications on assessment results continues to be an area of study
 404 related to this work.

405 Other non-conservative forces that must be considered in addition to atmospheric
 406 drag are SRP and ERP which are both calculated by GEODYN according to the descrip-
 407 tions shown in Table B1, and the references therein. Using GEODYN and its high-fidelity
 408 force model ensures that the estimated SRP and ERP accelerations are more precise than
 409 what would be modeled by a standard satellite flythrough scheme. For each density model’s
 410 orbit fit the acceleration due to drag will vary according to the error in the respective
 411 model, while the contributions from SRP and ERP will remain consistent for each arc
 412 across each model run. For the orbit fits presented in this work, the variations of SRP
 413 and ERP were found to be small relative to the variable effects of drag. This being said,
 414 the magnitude of the SRP acceleration is often on par with that of the drag accelera-
 415 tion at the ICESat-2’s altitude. Errors related to mismodeling non-drag forces can po-
 416 tentially be transferred into the residuals, and as a result this method is presented as a
 417 relative assessment between controlled model runs rather than an absolute validation of
 418 performance.

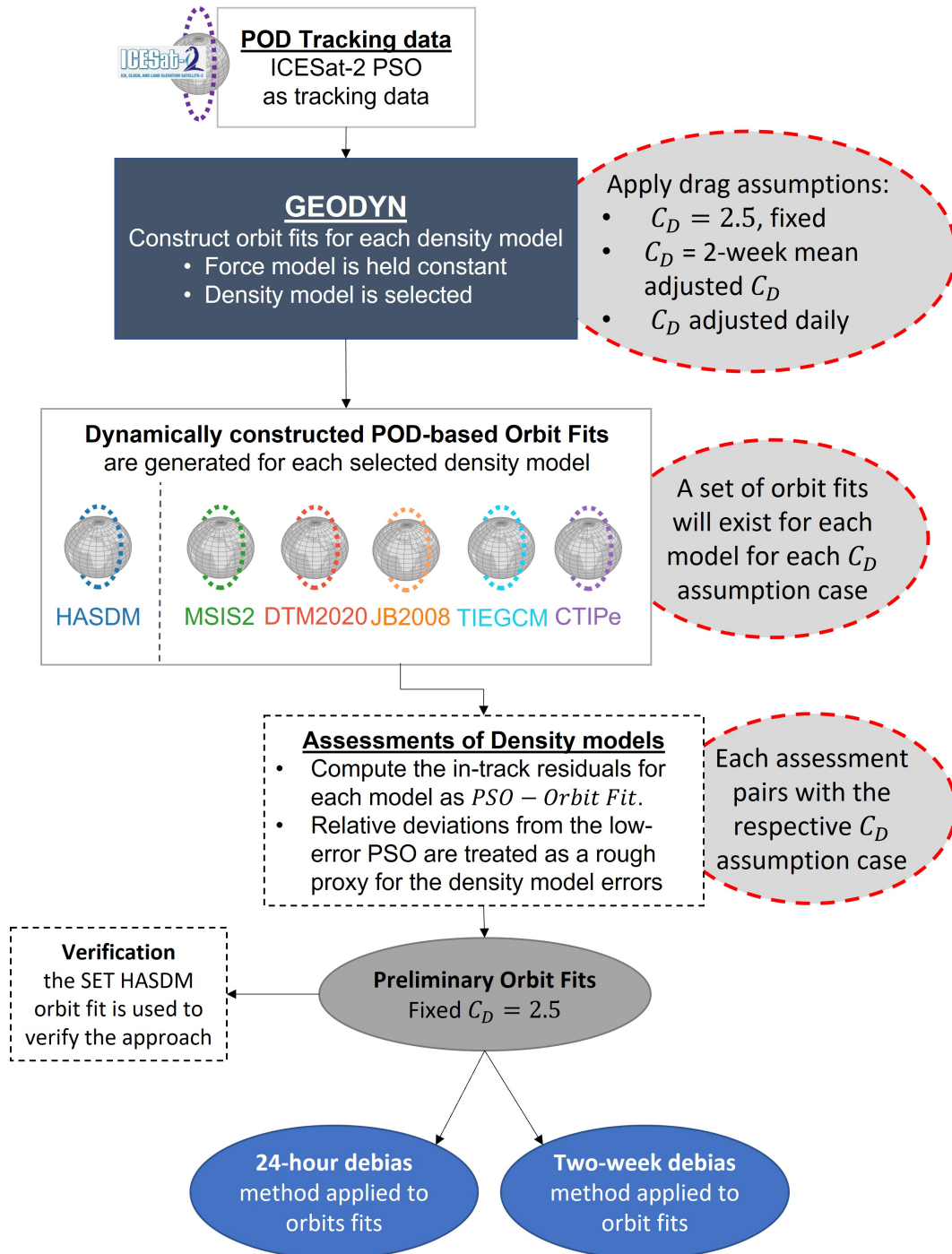


Figure 1. A flowchart visualizing the assessment process and how the datasets and POD methods fit together for model assessment.

5.2 Assessment Procedures

419 The interconnected, uncertain nature of C_D and ρ makes the absolute determina-
 420 tion and assessment of either value a very complex problem that is still an active field
 421 of research within this community. In this case study, the assessment procedure is split
 422 into three subsequent orbit fit methods, each based on assumptions made to character-

423 ize C_D when calculating the drag acceleration for each model during the orbit fit pro-
 424 cedure. In a preliminary orbit fit method, explicit biases are identified via orbit fits con-
 425 structed using a fixed C_D that is held constant across all models. The constant fixed C_D
 426 is chosen to be a physically realistic value of $C_D = 2.5$ based on the average result cal-
 427 culated from the Diffuse Reflection with Incomplete Accommodation (DRIA) model along
 428 the ICESat-2 orbit. This demonstrates that the method is sufficiently sensitive to rec-
 429 ognize differences in the drag effects between the models and provides an understand-
 430 ing of each model’s approximate mean density offset relative to the ICESat-2 PSO In
 431 an alternative orbit fitting method, GEODYN’s parameter estimation procedure is used
 432 to adjust the C_D for every 24-hour arc for the two-week period from the a priori esti-
 433 mate of $C_D = 2.5$. The mean-adjusted C_D over the two-weeks is then used as a fixed,
 434 model specific value that is constant for the time-period. This provides a fixed, unique
 435 C_D for each density model that effectively scales the density over the two weeks to ac-
 436 count for each model’s mean density offset and examine the model response to solar and
 437 geophysical dynamics. In a final orbit fitting method, the daily C_D adjustments are used
 438 without a two-week averaging. The residuals using these daily, model-specific C_D ad-
 439 justments provide an assessment of model performance on time periods less than a day.

440 The preliminary orbit fits use a fixed C_D of 2.5 that is constant with respect to each
 441 model. This enables direct model comparison, but subjects an assessment of the den-
 442 sity models to explicit biases depending on each model’s density offset relative to this
 443 C_D value. Each model’s sampled densities along the ICESat-2 orbit have an overall mean-
 444 density offset relative to each other. Fixing the C_D to a specific value will cause a par-
 445 ticular offset amount to be favored. For instance, $C_D = 3.5$ will produce favorable or-
 446 bit fits for models that trend a lower density, whereas $C_D = 1$ would favor models that
 447 trend towards higher densities. Due to these circumstances, the DRIA model is indepen-
 448 dently used to calculate a physically realistic value of the ICESat-2’s C_D along its or-
 449 bit. DRIA is a relatively simple, computationally fast model for capturing the gas-surface
 450 interactions between the upper atmosphere and a spacecraft. In the DRIA model, par-
 451 ticles are always reflected with a diffuse angular distribution, but their energy exchange
 452 with the surface varies depending on the value of the energy accommodation coefficient
 453 α . This work uses the Sentman’s closed-form solutions for the DRIA model as depicted
 454 in Equation 12 of Walker et al. (2014). The energy accommodation is assumed to be fixed
 455 at $\alpha = 0.89$ —a tenuous assumption based on the limited empirical data for α near 500
 456 km during solar minimum (Pilinski et al. (2010); Pilinski (2008)). This α value is likely
 457 higher than is realistic for this altitude and solar flux, therefore providing a lower limit
 458 for what a physically realistic drag coefficient might be; however, given the complex changes
 459 in atmospheric structure that occur in this altitudinal regime, this empirical value is still
 460 the most representative until further observations can be made. There are other phys-
 461 ical C_D models that could be used instead of DRIA, but choosing and assessing the C_D
 462 models quickly expands beyond the scope of this work. For the sake of being able to con-
 463 duct a model assessment as a proof-of-concept in this case study, this assumption is made
 464 with the intention to improve the treatment of C_D in future efforts. In future work, we
 465 aim to address this issue by implementing a physical satellite gas-surface interaction method
 466 to calculate the time-dependent drag coefficient, but even this will have associated as-
 467 sumptions and caveats. Constructing orbit fits with a fixed, common C_D of 2.5 for each
 468 model represents the type of method that is possible without being able to model the
 469 physical drag coefficient or without GEODYN’s capability to adjust the parameter. This
 470 adjustment procedure was performed for different a priori C_D values and found that the
 471 final adjusted C_D for each model was consistently the same.

472 The 24-hour debiasing method uses GEODYN’s parameter estimation capabilities
 473 to determine a daily fitted value of the C_D that accounts for accumulated errors from
 474 the force model over the 24-hour arc—the most prominent of which being due to den-
 475 sity uncertainty. In the field of space geodetic POD, C_D is often adjusted in conjunc-
 476 tion with reduced-dynamic empirical accelerations to account for disagreement between

477 the observed accelerations from tracking measurements, and calculated accelerations from
 478 uncertainties in the drag force model. This technique is used to get very low error, pre-
 479 cise orbit solutions, but limits the ability to distinguish errors that are specific to drag
 480 or the density models. By allowing only the C_D to adjust and match the orbit fit’s mod-
 481 eled accelerations with the PSO observation, density errors over the 24-hour period are
 482 incorporated into the adjustment. A density model that is found to be over-/under-estimating
 483 the density, will have a C_D that is adjusted to be smaller/larger in a non-physical way—
 484 effectively using C_D as a scaling term between the PSO observation and uncertainty in
 485 the density model orbit fits. In practice, the C_D also absorbs any errors from mismod-
 486 eled forces, but these are held constant in the model-to-model comparison. Each model
 487 is given an a priori estimate of $C_D = 2.5$ at the start of the 24-hour arc, which is al-
 488 lowed to adjust within a standard deviation of 10. The drag coefficient fitting occurs con-
 489 currently with the iterative orbit fit routine. Due to this non-physical use of C_D to ef-
 490 fectively debias the density, the term “drag acceleration scaling factor” is adopted. The
 491 24-hour scaling factor for each model (m) can be calculated for each arc (i) as,

$$f_{24,m,i} = C_{D,adj,m,i}/2.5 \quad (2)$$

492 The two-week debiasing method acts as a combination of the previous two meth-
 493 ods. The C_D adjustments for each model are averaged over the two-week period to pro-
 494 vide a mean adjusted C_D . Each model’s orbit fit is then re-determined using the mean
 495 adjusted C_D for each respective model as the fixed value for the two-week period. This
 496 assessment permits a scaling of the density models over an extended period of time, high-
 497 lighting errors in the orbit fits that are due to variations that take place on a longer time
 498 scale than 24-hours. This method is also motivated by the need to provide a scoring met-
 499 ric for each density model that can be parsed into the CCMC’s CAMEL model valida-
 500 tion infrastructure. While the $C_D = 2.5$ case is dominated by the model biases and the
 501 24-hour debiased case demonstrates a method to debias daily densities, the two-week de-
 502 biased case quantifies the ability of the models to capture dynamics caused by geomag-
 503 netic and solar activity over a more prolonged time period. This is a method that could
 504 be used in the future to assess model performance during individual stormtime periods.

5.3 Assessment Metrics

505 Using a PSO as tracking data makes use of GEODYN’s data-reduction mode com-
 506 bined with a dynamic technique for estimating the orbit of a satellite. This technique
 507 uses the trajectory input to estimate updates to the initial conditions which define the
 508 motion of the satellite, thus refining the orbit. The orbit residuals obtained in this setup
 509 are the absolute differences between the PSO and each density model’s orbit fit. Since
 510 other force model parameters are held constant between each density model’s run, the
 511 inter-comparison of the residuals contains information primarily corresponding to rel-
 512 ative errors in each density model’s ability to replicate the drag effects seen in the ICESat-
 513 2 PSO.

514 To best observe satellite drag effects, all output orbits are transformed from the
 515 J2000, geocentric inertial reference system to the NTW, orbit-aligned satellite coordi-
 516 nate system (Vallado, 2013). This system is composed of an in-track component \hat{T} that
 517 is parallel to the velocity vector, a normal component \hat{N} that is perpendicular to the ve-
 518 locity and nominally in the radial direction, and a cross-track component \hat{W} that is nor-
 519 mal to the orbit plane and completes the right-hand coordinate frame. The in-track com-
 520 ponent \hat{T} is parallel to the velocity vector direction and contains any indication that the
 521 spacecraft’s trajectory has changed since orbital energy dissipations from drag will im-
 522 pact in the velocity direction. Information regarding this transformation as well as sup-
 523 porting coordinate frame details can be found in Appendix A.

524 For any given arc, y_o is defined to be a component of the orbit from the PSO dataset
 525 in the NTW frame, and y_m to be the orbit fit for each density model m . The residuals
 526 for each component of the orbit and orbit fit are then calculated (in terms of the in-track
 527 component) as,

$$R_{m,T} = y_{o,T} - y_{m,T} \quad (3)$$

528 The root-mean-square error (RMSe) of the residuals represents the square root of the
 529 variance of the absolute difference in the two orbits, indicating how well the density model's
 530 orbit fit matches the PSO for that arc. For the in-track component, this is computed for
 531 every i^{th} time step of an arc with n time steps as,

$$\text{RMSe}_{m,T} = \sqrt{\frac{1}{n} \sum_i^n (y_{o,T,i} - y_{m,T,i})^2} \quad (4)$$

Figure 2 provides an example of the observation residuals for sample orbits over four,

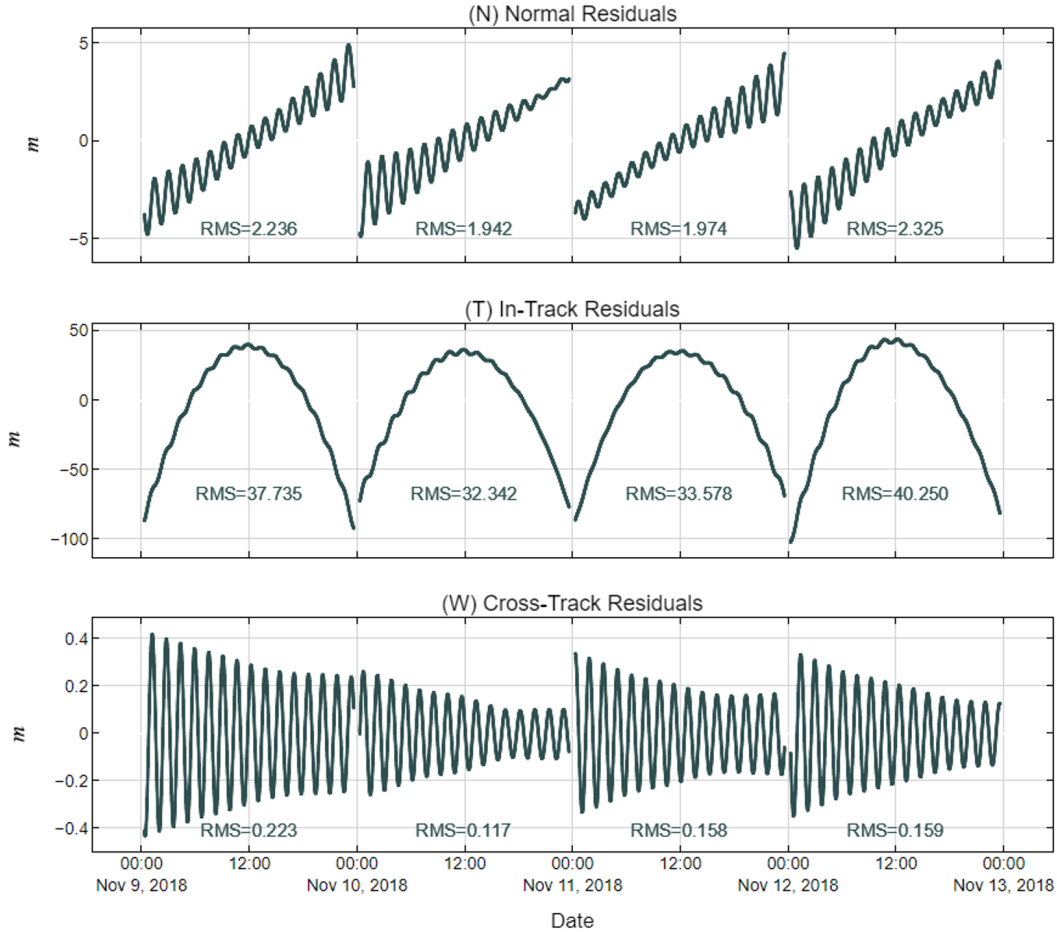


Figure 2. Depicted here are example observational residuals for each component of the NTW system across four 24-hour arcs using the MSIS2 model. The RMSe for each arc/component is given under each curve, showing that the majority of the residual variance (i.e. the orbit error due to drag) is contained within the in-track direction. Note that the vertical scale is different for each plot.

533 24-hour arcs. The normal \hat{N} (top panel) and cross-track \hat{W} (bottom panel) residuals are
 534 included in this figure only to demonstrate that the in-track component contains the ma-
 535 jority of the variance associated with residuals in this reference frame. The RMSe val-
 536 ues for each component and arc are included as an overlay.

537 During the POD process, GEODYN iteratively minimizes the discrepancies between
 538 the observed orbit (i.e., PSO) and computed orbit (i.e., orbit fits from a density model)
 539 across the entire 24-hour arc by adjusting the initial conditions to converge towards a
 540 computed trajectory. Since the minimization occurs across the entire arc, the resulting
 541 residuals take on the non-linear shapes shown in Figure 2. On a given arc and when com-
 542 paring the resulting orbit fits from each density model, the only variable that has been
 543 permitted to impact each orbit fit’s performance relative to the PSO is the drag effects
 544 from the selected density model. Therefore, we reason that the relative differences in the
 545 residuals for each orbit fit is indicative of density model performance. Other potential
 546 errors from mismodeled physics may persist in the residuals, but they are held constant
 547 between each model run and will impact the orbit fits consistently. If we run the same
 548 arc using the same force model and conditions, but only change the density model used
 549 to calculate satellite drag accelerations, the residuals will contain the errors related to
 550 the program attempting to reconcile errors in the density model. The RMSe for each or-
 551 bit fit represents a single value for how well the program can reconcile each model’s er-
 552 rors in density over the entire 24-hour arc.

553 As described in Section 4.2, the ICESat-2 PSO has been shown to have a radial or-
 554 bit accuracy of below 1.5 cm, generated through the reduction of GNSS double-difference
 555 carrier phase observable residuals and independently assessed using SLR measurement
 556 residual analysis (Thomas et al., 2021). The precision of the orbit solutions were also
 557 verified in all three components using orbit overlap analysis. Given this, relative devi-
 558 ations from the low-error PSO are treated as a rough proxy for the density model errors
 559 relative to some unknown true density. The true density value is obscured by the var-
 560 ious interconnected unknowns of C_D , SRP, and ERP and therefore remains unspecified.
 561 Over the course of a single arc, drag forces from each density model dissipate the satel-
 562 lite’s orbital energy at distinct rates, resulting in drag accelerations that are either greater
 563 or less than what is represented by the in-track position of the PSO. A strongly nega-
 564 tive in-track residual indicates a modeled density that is larger than truth, while a strongly
 565 positive in-track residual indicates a modeled density that is smaller than truth. Addi-
 566 tional details regarding the shape of the in-track residuals and the relationship between
 567 in-track position of the PSO and orbit fits and the density can be found in Figure A2
 568 of Appendix A.

569 The RMSe is the standard deviation of the residuals and serves as a measure of the
 570 difference between a respective orbit fit and the PSO over a single whole arc. Theoret-
 571 ically, an in-track RMSe of zero would mean no difference between an orbit fit and the
 572 PSO, indicating near-perfect agreement on average between the modeled density and the
 573 POD-based true density across the 24-hour arc. In this setup, perfect agreement for any
 574 model is unlikely since the residuals may additionally contain errors related to mismod-
 575 eled forces, as well as bias/offsets related to fixing the C_D to a common value for all mod-
 576 els. A further limitation of the metric is that the RMSe lacks information regarding timescales
 577 less than the arc length, and is unsigned, meaning it does not indicate if the modeled
 578 density is above or below the truth for a given arc. For these reasons, the in-track resid-
 579 uals and their respective RMSe values are assessed in conjunction with each other.

6 Results and Discussion

580 This section is organized as follows: (1) the preliminary method for orbit fit con-
 581 struction using a fixed $C_D = 2.5$ is presented, and the orbit fit method is verified us-
 582 ing the SET-HASDM density database to determine baseline understanding; (2) an as-
 583 sessment of the semi-empirical and physics-based models is presented via orbit fit results
 584 that are debiased using a mean drag acceleration scaling factor over the full two week
 585 period; (3) an assessment of the semi-empirical and physics-based models is presented
 586 via orbit fit results that are debiased using a 24-hour drag acceleration scaling factor.

587 The specific conditions for producing density values for each model are detailed in
 588 Section 4.3. The authors acknowledge that while there are a number of ways to improve
 589 a density model's outputs at runtime (see Sutton (2018); Shim et al. (2014)), the out-
 590 puts used in this work are intended to reflect typical community use, with each model
 591 being run according to the developer's operational instructions. Results are presented
 592 by focusing on a two week time period from 9 November 2018 - 23 November 2018, pro-
 593 viding 14 adjacent daily arcs with no maneuver-based data gaps. The assessment con-
 594 ditions are for the altitude regime near ~ 490 km, in an atmosphere with very low so-
 595 lar flux, and low-to-minor geomagnetic activity. Note that this is a notoriously difficult
 596 altitude regime and activity condition for empirical models due to the minimal access
 597 to satellite density data at this altitude during times of prolonged solar minimum—increasing
 598 the potential value of this style of assessment for these models especially. Figure 3 shows
 599 low solar activity for the time period, both in terms of the magnitude and variation of
 600 solar EUV and FUV, as approximated by measurements of the 10.7 cm solar radio flux
 601 ($F_{10.7}$, top panel). The Kp geomagnetic index (bottom panel) depicts low-to-minor ge-
 602 omagnetic activity during the time of interest, with the two-week period being bookended
 603 by minor geomagnetic disturbances which reach no higher than $Kp = 4.3$. Only one
 604 minor-to-moderate disturbance occurs on 5 November 2018, four days before the period
 605 of interest, reaching a peak of $Kp = 5.7$. This event is mentioned only because of the
 606 possibility that its impact could be seen represented in the models as a density enhance-
 607 ment due to delayed heating and cooling effects.

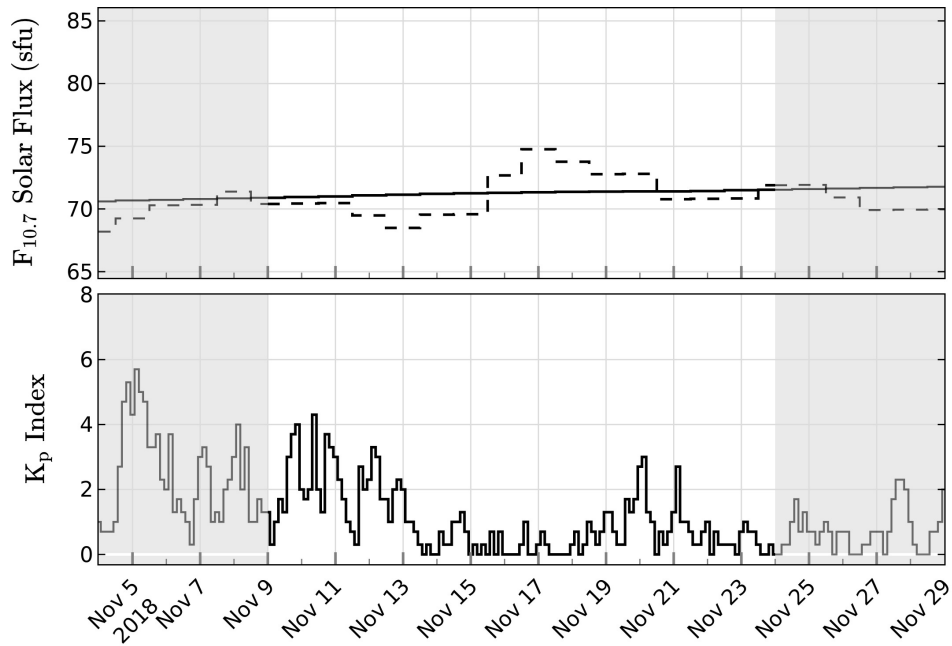


Figure 3. Top: observed solar $F_{10.7}$ radio flux. The dashed curve is the daily measured value from the Ottawa observatory normalized to 1 AU sun-earth distance; the solid curve is an 81-day (3 solar rotation) centered average. Bottom: the 3-hourly planetary magnetic index, K_p . Both panels depict the period of interest from 9 November 2018 - 23 November 2018. A few days before and after the period of interest are depicted in the shaded portions.

6.1 Preliminary Orbit Fits using a Fixed C_D of 2.5

608 The most straightforward way to construct the orbit fits is by calculating the ac-
 609 celeration due to drag from different density models using a fixed drag coefficient value
 610 for all arcs and models. This permits bias depending on the relationship between each
 611 model’s mean density and the chosen C_D , but importantly demonstrates that the method
 612 is sufficiently sensitive for recognizing differences in the drag effects between the mod-
 613 els. The in-track residual errors in this method should not be interpreted as indication
 614 of performance, but rather indicators of each model’s mean density offset relative to the
 615 true-unknown density. Figure 4, shows the fixed C_D assessment results for the semi-empirical
 616 and physics-based models. The top panel shows each model’s orbit averaged density along
 617 the orbit of ICESat-2, the middle panel shows the in-track residuals for each model and
 618 arc, and the bottom panel shows the in-track RMSe values.

619 The negative parabolic shape of the in-track residuals of MSIS2, TIEGCM, and JB2008
 620 indicate that these modeled densities are too high—i.e., these orbit fits experience more
 621 drag acceleration and their fits tend to lag behind the PSO. The positive parabolic shape
 622 of the in-track residuals of DTM2020 and CTIPe indicate modeled densities that are too
 623 low—i.e., the drag acceleration is lower and the orbit fits tend to be in front of the PSO.
 624 In reality the PSO-to-orbit fit relationship is slightly more complex over an arc, with the
 625 above being a generalization of the overall trend. A more detailed understanding of the
 626 orbit fit movement relative to the PSO can be found in Appendix A.

627 The orbit fits from SET-HASDM are separated for use as verification since it uses
 628 similar assumptions of a fixed drag coefficient, and a satellite drag data assimilation tech-
 629 nique in its internal workings. The SET-HASDM density database affords the oppor-
 630 tunity to access historical records of HASDM densities that have been corrected through
 631 the real-time data-assimilative calibration to ~ 80 low earth orbiters. The HASDM model
 632 is the operational standard used by the 18th Space Defense Squadron which is tasked
 633 with executing command and control over United States’ space assets and all resident
 634 space objects for sake of space situational awareness. Verification with SET-HASDM pro-
 635 vides a baseline understanding of the fidelity of the orbit fit results. Since the HASDM
 636 density values have already been effectively debiased in its data-assimilation scheme, we
 637 do not go through the steps of debiasing using the methods presented in Sections 6.2 and
 638 6.3. Referring to Figure 5, the HASDM model consistently depicts in-track RMSe val-
 639 ues that are on the order of 8.18 meters over the two-week period. The in-track resid-
 640 uals have a negative shape, indicating that the densities from SET-HASDM are slightly
 641 larger than what would be expected from the PSO. The results in Figure 5 are intended
 642 to serve as an approximate consistency check that our overall methodology, and more
 643 specifically our debiasing method, provide orbit fits with in-track errors in a reasonable
 644 range.

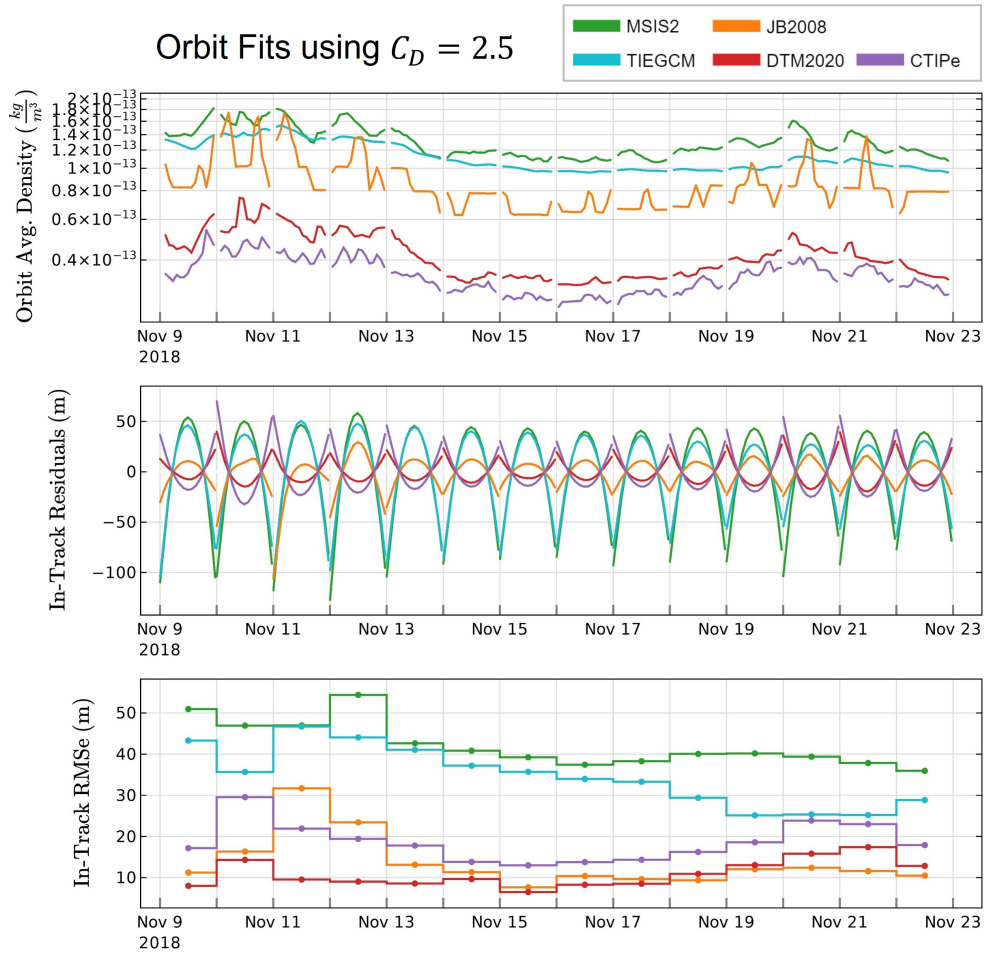


Figure 4. Assessment results given a fixed, common $C_D=2.5$ for MSIS2 (green), DTM2020 (red), JB2008 (orange), TIEGCM (cyan), and CTIPe (violet) during the two week time period containing 14, 24-hour arcs. Top: Orbit average neutral densities along the ICESat-2 orbit for each model. Middle: In-track orbit residuals for each arc. Bottom: In-track RMSe for each arc’s in-track residuals.

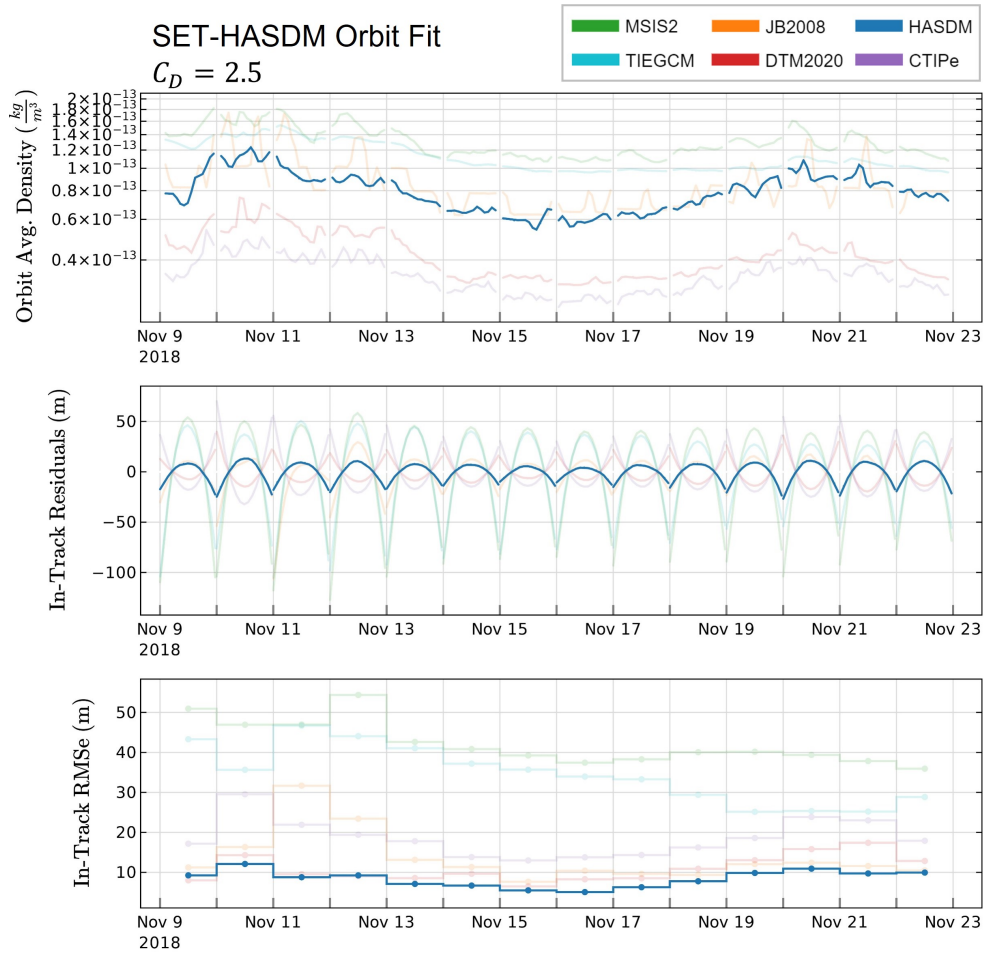


Figure 5. Verification results using SET-HASDM across 14 adjacent, 24-hour arcs from 9 November 2018 - 23 November 2018. Top: The solid blue curve depicts the neutral densities along the ICESat-2 orbit as an orbit average. Middle: In-track orbit residuals for each of the 14 adjacent, 24-hour arcs. Bottom: In-track RMSe values for each arc's in-track residuals. The range of the y-axes are chosen to facilitate comparison with Figure 4.

6.2 Debias using Two-Week Scaling Factor

The second orbit fit method debiases the density models using a mean-adjusted C_D over the two-week period (average values of the 24-hour adjusted C_D s shown later in Figure 8). This provides a fixed C_D that is unique for each density model, adjusted to account for biases due to each model's mean density offset over the time period (see Figure 4). The in-track residuals and RMSE values for this assessment, shown in Figure 6, quantify the error due to density variation over the two-week time period. The average adjusted C_D used to construct the two-week debiased orbit fits for each model is reported in Table 2.

As mentioned in Section 5.2, the DRIA calculations indicate that $C_D = 2.5$ is a realistic value if one assumes that a fixed energy accommodation of $\alpha = 0.89$ is reasonable—an assumption limited by lack of empirical observation. According to the DRIA model, 2.5 is a realistic lower limit for the drag coefficient. Looking to Table 2, we can see that the mean adjusted C_D for MSIS2, TIEGCM, and JB2008 are all well below this lower limit, offering further evidence that these models are, on average, over-estimating the density. The upper bound is slightly more difficult to estimate in this setup, but CTIPe's adjusted C_D of 4.3 is likely too high. This will need to be investigated further in the future.

After removing the bias from the models, the relative effects of the minor geomagnetic activity become more stark in the in-track residuals. Here the changing shape of the residual curves indicate whether the model is over or underestimating the effects of geomagnetic activity on the modeled density. For example, several of the models display downward-pointing curves during geomagnetic activity, indicating densities that are too low as compared to their quiet time densities. DTM2020's residuals show an anti-correlation to the geomagnetic activity, beginning with densities that are too high and ending the two weeks with densities that are too low, which may be reflective of an overly sensitive response to geomagnetic activity and the overall downward trend in Kp. The effects of a model poorly capturing density variations during geomagnetically active times are now better quantified by the in-track RMSE after two-week debiasing is applied.

Two-week Debiasing Method: $C_D = \text{Mean Adjusted } C_D$		
Model ID	Fixed C_D	Scaling Factor as % change
MSIS2	1.237	-49.861
TIEGCM	1.373	-44.899
JB2008	1.909	-22.967
DTM2020	3.351	34.903
CTIPe	4.368	75.663

Table 2. Summary of the assessment procedure assuming a fixed C_D that is equal to the average adjusted value for each model, assuming an a priori of $C_D = 2.5$. The second column reports mean-adjusted C_D used in each model's orbit fit construction. The third column reports the two-week scaling factor as a percent change for each model.

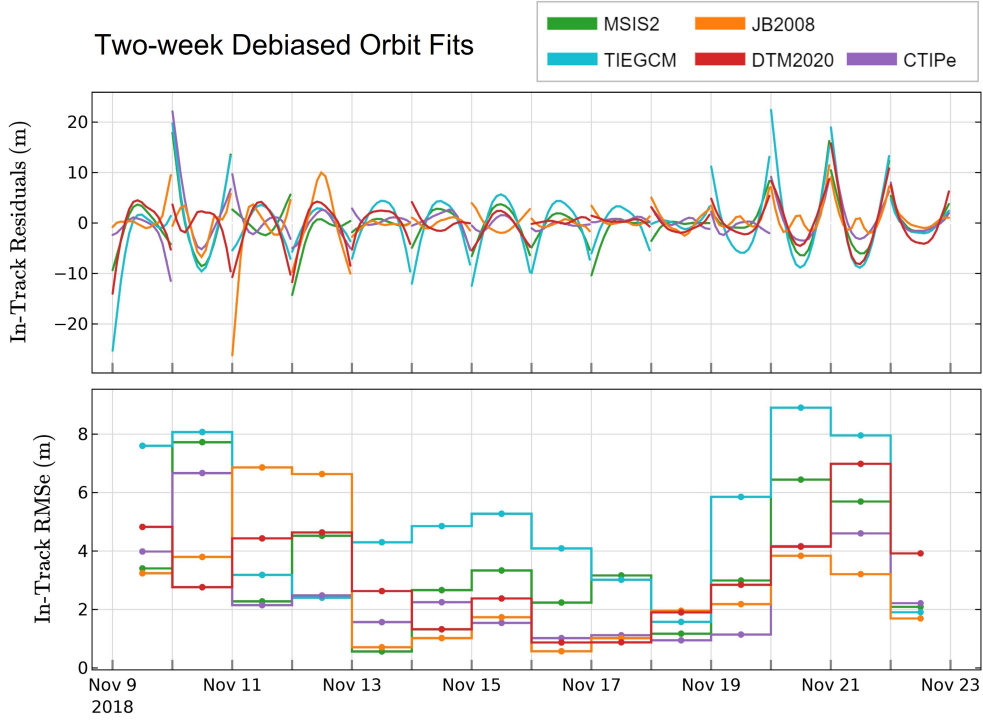


Figure 6. Assessment results for orbit fits that are debiased using two-week drag acceleration scaling factors. Top: Debiased in-track orbit residuals for each arc. Bottom: Debiased in-track RMSe for each arc’s in-track residuals.

6.3 Debias using 24-hour Scaling Factor

673 The 24-hour debiasing procedure described in Section 5.2, is used to scale the orbit
 674 fits and their residuals to a daily cadence. Figure 7 presents the resulting in-track
 675 residuals (top panel) and in-track RMSe values (bottom panel) for each 24-hour arc for
 676 each model. The 24-hour drag acceleration scaling factor is derived by adjusting the C_D
 677 from the a priori of 2.5 over each daily arc, absorbing the average density offset for that
 678 day. The debiasing effect is seen in the overall reduction in residual error from Figure
 679 4 to Figure 7. The calculated 24-hour scaling factors are presented in the top panel of
 680 Figure 8 as a percent change from the fixed value of 2.5. The bottom and right panels
 681 show the Kp index and Pearson’s correlation coefficient between each model’s scaling
 682 factors and the Kp , respectively.

683 The 24-hour scaling accounts for both the overall model bias and uncertainties in
 684 the density on timescales that are on the order of, or greater than, the chosen arc length
 685 of 24-hours (i.e. combination of mean density offset and daily geomagnetic variation).
 686 The remaining error depicted by the in-track residuals of Figure 7 are likely due to higher
 687 frequency variations in density that are not captured by the 24-hour debiasing (e.g. day-
 688 night variations in the neutral density). The correlation between Kp index and the scal-
 689 ing factors demonstrates how this metric can be used to determine how well a model ac-
 690 counts for geomagnetic activity. As shown by the scaling factors in Figure 8, MSIS2 ($R =$
 691 0.35), TIEGCM ($R = 0.29$), and CTIPe ($R = 0.19$) all exhibit a subtle positive correla-
 692 tion, indicating a slight underestimation of density enhancements from geomagnetic
 693 activity and resulting in the scaling factors being used to compensate for these errors.
 694 Contrarily, the inverse relationships shown by DTM2020 ($R = -0.36$) and JB2008 ($R =$

695 -0.3) indicate an overestimation of these models' densities during days of increased ac-
 696 tivity.

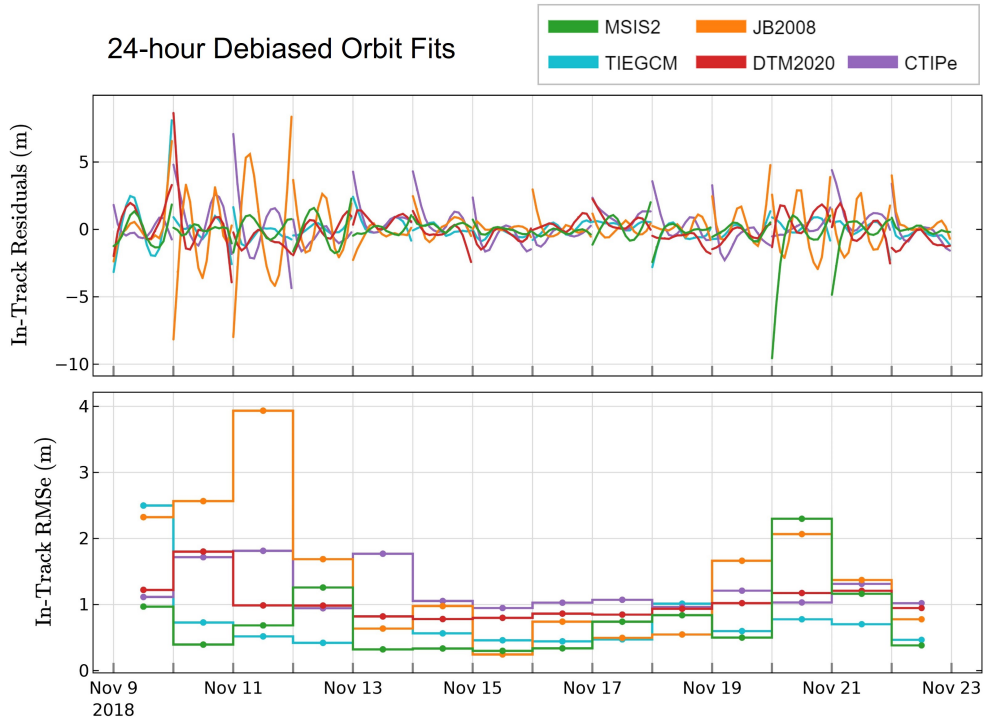


Figure 7. Assessment results for orbit fits using 24-hour drag acceleration scaling factors. Scaling factors are extracted in the least squares orbit fitting procedure by allowing the C_D to adjust once-per-arc to absorb observed errors between the PSO and the converging orbit fit. Top: In-track orbit residuals for each arc. Bottom: In-track RMSe for each arc's in-track residuals.

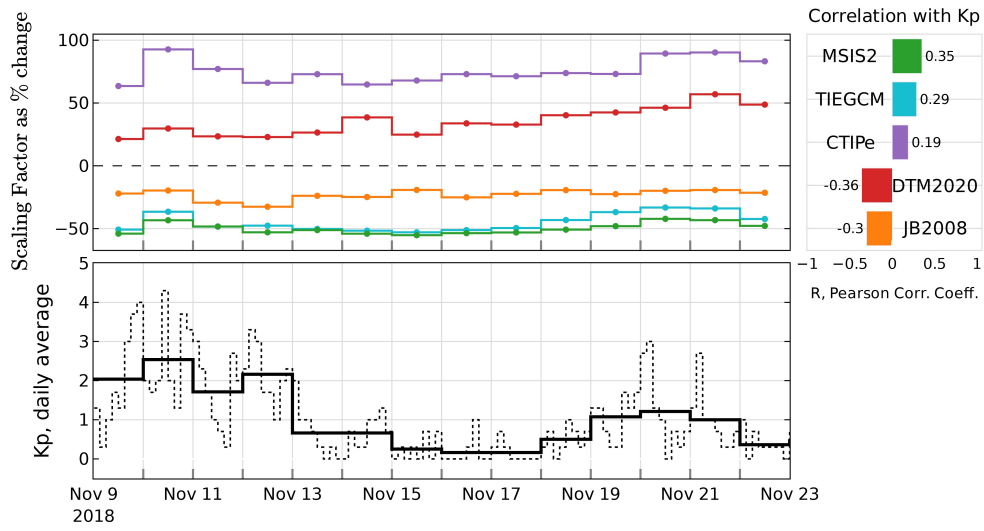


Figure 8. Top: Drag acceleration scaling factors extracted from the orbit fits shown in Figure 7, presented as a percent change from the fixed $C_D = 2.5$. Bottom left: The K_p index for the time period. Bottom right: Pearson’s correlation coefficient between the scaling factors and K_p .

6.4 Discussion

697 While the in-track residuals of the ICESat-2 orbit fits offer an effective means of
 698 assessing the density models, most methods that use drag acceleration to study density
 699 are going to be limited by the complex interconnected uncertainties in the C_D and the
 700 density. For these reasons, we split our overall assessment into the three methods pre-
 701 sented, each of which help to further illuminate the performance of the models. For sake
 702 of brevity, only JB2008 and TIEGCM are given in-depth discussions that synthesize an
 703 understanding of model performance from their results. The reader will then be able to
 704 apply these discussions to the remaining models, which here are discussed in more gen-
 705 eral terms.

706 Considering the cumulative results for JB2008 (plotted in orange for all relevant
 707 figures), Figure 4 shows that with a fixed C_D of 2.5, the in-track residuals exhibit a neg-
 708 ative parabola shape, indicating that the JB2008 density provides larger drag accel-
 709 erations than what the PSO experiences. Figure 6 provides the assessment method in which
 710 the JB2008 densities are effectively scaled by $\sim -22\%$ for the full two-week period. In
 711 this case the in-track residuals better highlight the arcs in which JB2008 performs poorly
 712 relative to the other models, specifically on November 11th and 12th when the geomag-
 713 netic activity fluctuates around $Kp = 3$ after having been moderately elevated for sev-
 714 eral days. In the 24-hour debiased case, JB2008's daily scaling factors (shown in orange
 715 in Figure 8) effectively compensate for density variations on the order of or greater than
 716 24-hours. The mean density offset adjustment is $\sim -22\%$, and is seen to be inversely cor-
 717 related ($R = -0.3$) with the geomagnetic activity—indicating that JB2008 tends to over-
 718 estimate densities during active times. This effect can be clearly seen in JB2008's orbit
 719 averaged densities, represented by the orange line in the top panel of Figure 4, where
 720 the model provides much sharper density peaks than the other models during the times
 721 of slightly elevated geomagnetic activity. This effect is also clearly represented in the 24-
 722 hour debiased in-track RMSe values (bottom panel of Figure 7), which shows significant
 723 variance during active times between the PSO and the JB2008 orbit fit, even after the
 724 24-hour scaling. The higher RMSe values shown in Figure 7 on November 9th-13th in-
 725 dicate that JB2008's overestimation of density during active times is not sufficiently com-
 726 pensated by the 24-hour scaling factor, meaning that the variation likely occurs at a higher
 727 frequency. JB2008's estimation of quiet time density is among the best in this report,
 728 needing $\leq -22\%$ adjustment from the fixed C_D case to provide orbit fits that match the
 729 PSO to within 1 meter (during quiet times).

730 Considering the cumulative results for TIEGCM (plotted in cyan for all relevant
 731 figures), Figure 4 shows that with a fixed C_D of 2.5, the in-track residuals exhibit a neg-
 732 ative parabola shape, indicating that the TIEGCM density provides larger drag accel-
 733 erations than what the PSO experiences (i.e. the TIEGCM densities are too high). TIEGCM
 734 offers interesting results from Figure 6 of the two-week scaled case. The TIEGCM den-
 735 sities are effectively scaled down by $\sim -45\%$ over the two-week period to compensate for
 736 mean density offset. Since this value is the two week average from the scaling factors shown
 737 in Figure 8, it is skewed to only partially compensate for lack of geomagnetic sensitiv-
 738 ity (i.e., densities are scaled to be too low in active times, increase in RMSe on the book-
 739 ends of the period) and partially compensate for mean density offset in quiet times (den-
 740 sities are scaled to be too high in quiet times, increase in RMSe from Nov. 13th to 17th).
 741 The two-week scaled case is able to clearly show in the in-track residuals that TIEGCM
 742 struggles more than the other models to properly capture variation during the period
 743 of this study. TIEGCM's daily scaling factors (cyan line in Figure 8) effectively com-
 744 pensate for error in density variations on the order of or greater than 24-hours. The daily
 745 variation of the scaling factors is found to slightly correlate with the Kp ($R = 0.29$)—
 746 indicating that TIEGCM tends to subtly underestimate densities during active times.
 747 This is seen in TIEGCM's orbit averaged densities (cyan line in the top panel of Fig-
 748 ure 4) where the model provides significantly less sensitivity to the times of slightly el-

749 evated geomagnetic activity, and less variation overall. The 24-hour debiased in-track
 750 RMSE values (bottom panel of Figure 7), interestingly show very low variance between
 751 the PSO and the TIEGCM orbit fit after the 24-hour scaling. This is most likely explained
 752 by the overestimation of density during active times being sufficiently compensated for
 753 by the 24-hour scaling factors despite its orbit average densities seeming to lack much
 754 variation at all. This adds further suspicion to the higher frequency variations seen in
 755 models such as JB2008. It is possible that by moving to shorter arc lengths, such as 3-
 756 hours, the time series of scaling factors could better capture these variations, and this
 757 is a future goal of this work.

758 In general, MSIS2 and TIEGCM overestimate the density for all arcs, requiring $\sim -50\%$
 759 and $\sim -45\%$ scaling factors, respectively, to bring the in-track residuals to within two
 760 meters. DTM2020 and CTIPe both underestimate the density for all arcs, each requir-
 761 ing a $\sim 35\%$ and $\sim 76\%$ increase, respectively. JB2008 requires the least overall scaling,
 762 requiring only $\sim -23\%$ to bring the in-track residuals to within two meters during quiet
 763 times. All models capture the geomagnetic activity relatively well as demonstrated by
 764 their scaling factors not being very highly correlated to Kp. DTM2020 ($R = -0.36$) and
 765 JB2008's ($R = -0.3$) scaling factors are inversely correlated to Kp , indicating a slight
 766 over-sensitivity to geomagnetic activity during this time period, while TIEGCM ($R =$
 767 0.29), MSIS2 ($R = 0.35$), CTIPe ($R = 0.19$) all indicate an under-sensitivity. The scal-
 768 ing undergone for each model produces RMSE values that are comparable to that of the
 769 SET-HASDM orbit fit, which was separated out to serve as an approximate consistency
 770 check of our debiasing method due to its data-assimilative technique.

7 Conclusions and Future Work

771 This work presents the development of a modernized interface for the GEODYN-
 772 II POD software. The approach leverages the high-precision nature of space geodetic POD
 773 and an upgraded utility of the neutral density models to focus POD methods toward study-
 774 ing satellite drag and conducting density model assessment. The assessment method uses
 775 high-fidelity PSO as observed tracking measurements that are input into POD-based or-
 776 bit fits. The drag effects from each density model are assessed according to each model's
 777 ability to redetermine the satellite's orbit. Each density model's orbit fit contains rel-
 778 ative in-track deviations from the PSO which are treated as a proxy for model densities
 779 that differ from a true, unknown, density. These deviations are quantified with the in-
 780 track residuals and their RMS errors. We demonstrate the capabilities of this tool via
 781 a case study assessment of five thermospheric density models (MSIS2, DTM2020, JB2008,
 782 TIEGCM, and CTIPe, and a verification using SET-HASDM) using the ICESat-2 mis-
 783 sion PSO as the observed measurements. Preliminary orbit fits are constructed after de-
 784 termining a mean C_D from a physics-based solution. A fixed C_D of 2.5 is applied for all
 785 models before being debiased by adjusting the C_D to account for density errors in the
 786 drag acceleration. The debiasing is performed at two different cadences, 24-hours and
 787 two-weeks, with each method highlighting different temporal aspects of the model den-
 788 sity errors. The scaling factors extracted from the 24-hour and two-week debiasing meth-
 789 ods are well-equipped for use in improving forecasting and modeling methods. The 24-
 790 hour scaling factors provide a more accurate representation of the true density variations
 791 for each model, while the two-week scaling factors are computationally simpler and in-
 792 dicate more baseline density effects. In addition, the two-week extended time period scal-
 793 ing factors are compatible for parsing into the CCMC's CAMEL database to move in
 794 the direction of community-oriented model validation.

795 We continue our efforts on this project as we move in the direction of offering a more
 796 robust thermospheric model validation scheme. Possible improvements include improv-
 797 ing the non-conservative force modeling in GEODYN for ICESat-2 using a more real-
 798 istic 3-D model of the satellite shape that would account for self-shadowing and varia-
 799 tions in cross-sectional area with incidence angle e.g, as in March et al. (2019). The or-

800 bit determination for the primary science orbits, and the subsequent analyses described
 801 in this paper would have to use these improved geometry models. A further improve-
 802 ment would be to incorporate SLR measurements of ICESat-2 into the evaluations of the
 803 density models. One could include the SLR data along with the PSO trajectory data in
 804 the evaluation. See Thomas et al. (2021) for a description of these data for ICESat-2.
 805 Planned future work involves addressing the key constraints highlighted in the method-
 806 ology, the foremost of which is the need to evaluate the drag coefficient more frequently
 807 along the ICESat-2 orbit. Future work will also involve expanding the study to encom-
 808 pass the entire ICESat-2 mission time period. Additional expansion includes incorpo-
 809 rating additional satellites and constellations that may illuminate model performance
 810 within atmospheric regimes that lack observations of neutral density. We aim to make
 811 our expanded results available through the CCMC's CAMEL framework as well as through
 812 future publications.

813 The assumptions made in this paper are limited by the current status of unknowns
 814 between gas surface interaction research and thermospheric variability research. At this
 815 time, the true drag coefficient is not known for any satellite, and modeling the C_D will
 816 always introduce some inherent bias into the results. We aim to address this issue by im-
 817 plementing several of the satellite gas-surface interaction models currently used in the
 818 satellite drag community to calculate the time and compositionally-dependent drag co-
 819 efficient. Isolating the effects of the C_D will aid to better identify the various non-density
 820 related errors that may be present in the in-track residuals. Being able to distinguish
 821 these errors and accurately quantify the amount of deviation introduced by a given den-
 822 sity model will provide significant insight regarding model performance to the earth-space
 823 environment modeling community. As the ability to model C_D improves, the results pro-
 824 vided by this method will similarly become more valid. The Geospace Dynamics Con-
 825 constellation (GDC) is an upcoming NASA mission that is intended to help fill in the gaps
 826 of understanding gas-surface interactions by providing a stable platform with full mea-
 827 surements of neutral composition, density, and temperature along with a high fidelity
 828 POD in which cross validation of density model assessment is possible. As a result, in
 829 addition to providing its own neutral density observations that can be used for research,
 830 operations, and model validation, these advances expected from the GDC mission will
 831 improve the accuracy and usability of density proxies derived from POD solutions like
 832 those used here. These advances will effectively multiply our density observations to be
 833 able to use any satellite with sufficiently accurate GNSS positioning and knowledge of
 834 spacecraft parameters as a density observing platform.

835 This work provides a step in the direction of being able to use high-fidelity GNSS-
 836 enabled LEO satellite POD solutions to objectively quantify and validate thermospheric
 837 model performance. The strength of assessment using this method is its ability to iden-
 838 tify relative accuracy of the models in a way that is directly tied to operational use for
 839 orbit propagation. There are a multitude of uses for the tools and methods presented
 840 in this work, such as for density retrievals along the orbit of a satellite, which is a planned
 841 future effort; however, this report focuses specifically on model assessment. As work con-
 842 tinues to refine these methods and address the caveats presented in this paper, the re-
 843 sults of model assessments using this technique will continue to become better suited to
 844 aid satellite operators when choosing a model that will perform best under specified con-
 845 ditions. Having a multitude of methods for assessing upper atmospheric models under
 846 various conditions helps model developers refine the models themselves, making them
 847 better suited for orbit prediction.

8 Open Research

848 The ICESat-2 POD solutions, their corresponding setup files, and the GEODYN
 849 II software are provided by the Geodesy and Geophysics Laboratory within NASA-GSFC.
 850 Simulation results for the CTIPE model have been provided by the Community Coor-

851 dinated Modeling Center at Goddard Space Flight Center through their publicly avail-
852 able simulation services (<https://ccmc.gsfc.nasa.gov>). Orbit fly-throughs of the TIEGCM
853 simulation results and relevant codes used to produce the results in this paper are avail-
854 able at Zenodo via <https://doi.org/10.5281/zenodo.8015368> (Waldron et al., 2023). The
855 SET HASDM density data are provided for scientific use by Space Environment Tech-
856 nologies.

857 **Acknowledgments**

858 The authors acknowledge the generous support of the Community Coordinated Mod-
859 eling Center under grant number 80NSSC21K1747 and support from the University of
860 Colorado at Boulder Grand Challenge Grant: Space Weather Technology, Research, and
861 Education Center (SWx TREC). This material is partially based upon work supported
862 by the National Aeronautics and Space Administration under Grant 80NSSC21K1554
863 issued through the Heliophysics Division Space Weather Science Application initiative
864 and the Geospace Dynamics Constellation Contract No. 80GSFC22CA012. The authors
865 acknowledge the CCMC-Kamodo Team who designed the Kamodo API, permitting the
866 ability to interface physics-based model outputs to the POD scheme.

Appendix A Coordinate System to Study Drag

867 GEODYN's input and output trajectories make use of the J2000 inertial reference
 868 system. The \hat{X} , \hat{Y} , and \hat{Z} components of the inertial coordinate system offer limited in-
 869 formation on how a satellite's orbit is impacted by atmospheric drag, leading us to con-
 870 vert to the more suitable Satellite Coordinate System. Two coordinate frames suited for
 871 this assessment are the NTW and RSW frames, with differences between the two being
 872 highlighted in Figure A1. We make use of the NTW system, which aligns with the or-
 873 bit plane and is composed of an in-track component \hat{T} that is parallel to the velocity vec-
 874 tor \vec{v} , a normal component \hat{N} that is perpendicular to the velocity and nominally in the
 875 radial direction, and a cross-track component \hat{W} that is normal to the orbit plane and
 876 completes the right-hand coordinate frame. Being parallel to the velocity vector means
 877 that the in-track component \hat{T} will contain any indication that the spacecraft's trajec-
 878 tory has changed since orbital energy dissipations from drag will impact in the velocity
 879 direction.

880 Figure A2 contains additional visualization related to the shape of the in-track resid-
 881 uals and how it relates to the movement of the PSO relative to the orbit-fit satellite. The
 882 overall shape of the in-track residuals is a result of the batch-least squares fitting rou-
 883 tine as it attempts to minimize the distance between the PSO and the orbit fit across
 884 the whole arc.

885 Variations in the in-track component \hat{T} are not the same as variations in the along-
 886 track component \hat{S} of the RSW system. In-track variations act in the direction of the
 887 velocity vector, whereas along-track variations are merely along, but not necessarily par-
 888 allel, to the direction of the velocity vector. We make the distinction to use the NTW
 889 system rather than the RSW system whose radial component is often used to assess or-
 890 bit accuracy in geodetic POD studies. The NTW coordinate system is described in (Vallado,
 891 2013) to have the following unit vectors and transformation:

$$892 \quad \hat{T} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad (\text{A1})$$

$$893 \quad \hat{W} = \frac{\mathbf{r} \times \mathbf{v}}{|\mathbf{r} \times \mathbf{v}|} \quad (\text{A2})$$

$$894 \quad \hat{N} = \hat{T} \times \hat{W} \quad (\text{A3})$$

$$895 \quad \mathbf{r}_{XYZ} = \begin{bmatrix} \hat{N} \\ \hat{T} \\ \hat{W} \end{bmatrix} \mathbf{r}_{NTW} \quad (\text{A4})$$

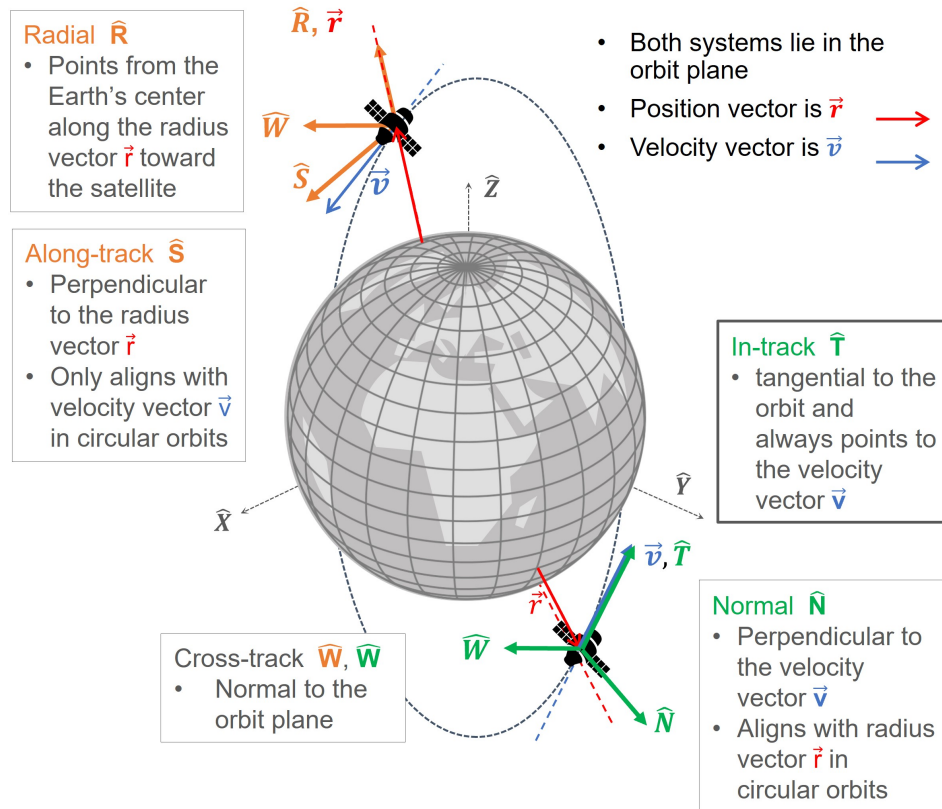
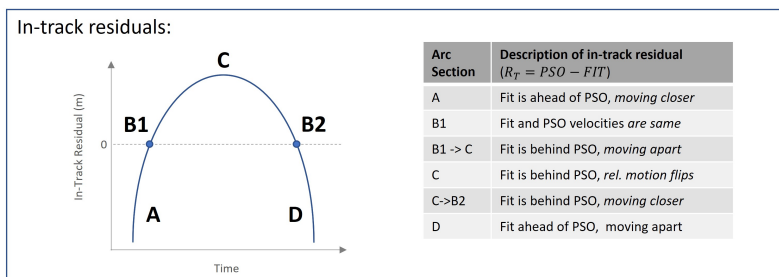


Figure A1. The above is a schematic showing the NTW and RSW satellite coordinate systems and details regarding their components. The NTW system's in-track component is parallel to the velocity vector, making it an effective tool for assessing relative effects due to atmospheric drag.

Scenario: Density from model is higher than truth

Fit orbit experiences higher drag acceleration than PSO



In-Track residuals, visualized: $Residual = PSO - FIT$ (note in-track component is parallel to velocity vector)

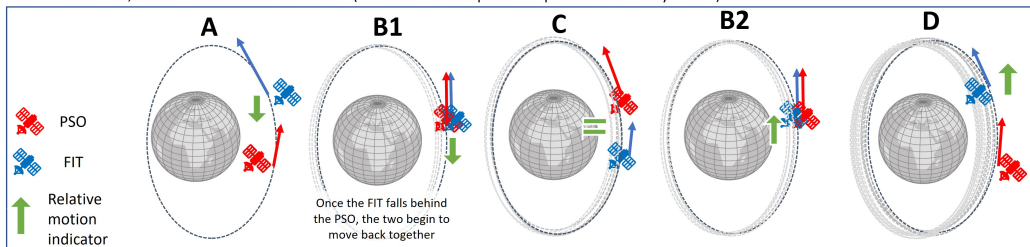


Figure A2. Above is a diagram which explores the circumstances that result in the parabolic shape of the in-track residuals. Representative in-track residuals for a single arc is given as a negative quadratic curve in the top panel. The bottom panel provides corresponding frames of schematics for each marked point to depict how the orbit fit and PSO are positioned relative to each other.

Appendix B GEODYN Run Setup for Orbit Fits

896 This section provides a summarized overview, in the form of Table B1, of the run
897 setup used in GEODYN to produce the orbit fits. A similar overview is given in Table
898 1 of Thomas et al. (2021), which details the most relevant and important constants, mod-
899 els, and standards used to produce the ICESat-2 PSO.

 GEODYN's Setup to Produce POD-based Orbit Fits

Satellite geometry	Panel model based on pre-launch geometry and satellite surface optical properties. 14 panels make up a Box Wing model to calculate time varying area.
ICESat-2 Attitude	Telemetered spacecraft body-fixed reference frame to inertial reference frame quaternions. Telemetered solar array drive angles (for force modeling)
Non-Conservative Forces	
Atmospheric Density Models	Modified for comparison; see Table 1 for the list of assessed density models
Earth Radiation	Knocke 2nd degree zonal spherical harmonic of Earth's albedo and emissivity (Knocke et al., 1988)
Solar Radiation Pressure	Solar radiation incident on plate model (Luthcke et al., 2019; Marshall & Luthcke, 1994)
Conservative Forces	
Geopotential gravity	EIGEN6C, tide-free (Foerste et al., 2014)
Time variable gravity	Contribution from atmosphere, non-tidal oceans, hydrology, and ice; Developed from GRACE models
Earth, Pole, and Ocean tides	IERS2010 Conventions (Petit & Luzum, 2010)
Planetary ephemerides (N-Body)	JPL DE430 (Folkner et al., 2014)
Relativistic corrections	IERS2010
General Reference Frame and Constants	
Conventional inertial system	J2000 geocentric; mean equator and equinox of 2000 JAN 01 12:00:00; IERS2010
Precession - Nutation	IAU 2000A precession-nutation model
Earth Orientation Parameters	IERS 08 C04 (Bizouard & Gambis, 2011), IERS2010 conventions for diurnal, semidiurnal, and long period tidal effects on polar motion and UT1
Numerical integration	Cowell predictor-corrector; fixed and variable step; equations of motion and variational equations.
Estimation method	Partitioned Bayesian least squares.
GEODYN Controlled Setup Information	
Tracking data type	PCE (orbit trajectory) using ICESat-2 PSO
POD technique	Dynamic data reduction (no empirical accelerations)
Arc Length	24 hours
Adjusted Parameters	Initial conditions only: $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$
Force model parameters	$C_D = \text{fixed}$; $C_R = 1$ (not adjusted)
Integration/orbit step	10 seconds

Table B1. A summarized overview of the GEODYN run setup for using the program to conduct density model assessment. Many of the above parameters are summarized from (Thomas et al., 2021) and (Luthcke et al., 2019).

Appendix C Orbit Uncertainty Interpolation Technique and the Kamodo Interface

900 Kamodo is a CCMC tool for access, interpolation, and visualization of space weather
 901 models and data in Python (Ringuette et al., 2023). Kamodo allows model developers
 902 to represent simulation results as mathematical functions which may be manipulated di-
 903 rectly by end users. Kamodo handles unit conversion transparently and supports inter-
 904 active science discovery through Jupyter notebooks with minimal coding in Python. Kamodo
 905 is chosen for this project due to its ability to offer model agnostic methods for reading
 906 data output from different model sources. Kamodo is called using its Satellite Flythrough
 907 capabilities, in which a user is able to sample the models with satellite ephemeris and
 908 return requested values from the chosen model. The orbit is pre-initialized in GEODYN
 909 using MSIS2 to get an a priori estimate for the orbit coordinates. Then using the a pri-
 910 ori orbit, extend out the uncertainty of the coordinates to create a cube of possible val-
 911 ues centered on the orbit. This approach accounts for possible model output differences
 912 as the orbit iteratively converges towards a solution. Finally, we plug the orbit and its
 913 uncertainty cubes into Kamodo to interpolate the model densities at all requested points.
 914 By doing this, the orbit density values from the physics model can be quickly ingested
 915 into the POD program. Figure C1 visualizes this procedure.

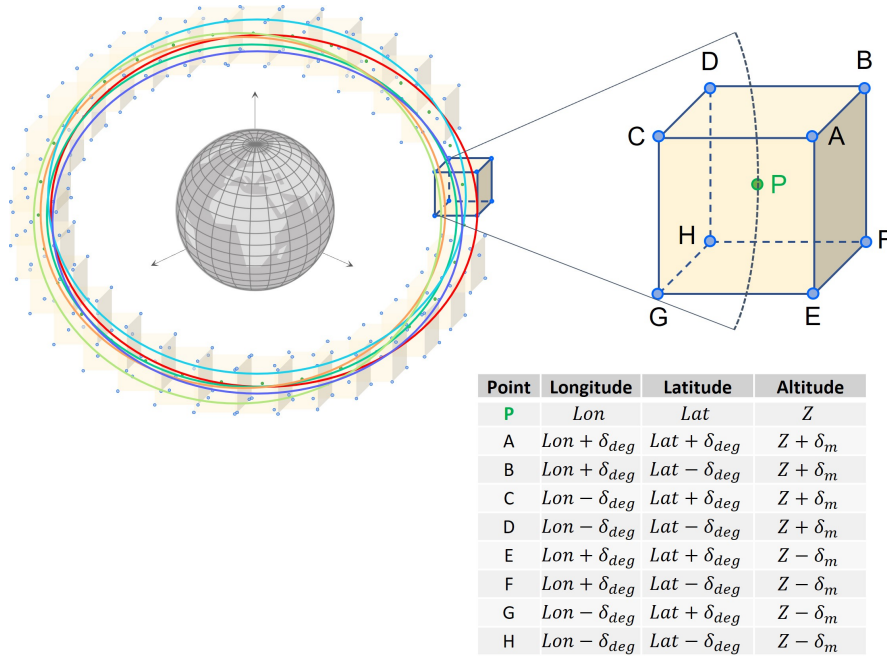


Figure C1. A representative schematic showing the constructed “cube of uncertainty” that surrounds a given coordinate along the orbit of a satellite. Each point that makes up this cube will contain modeled neutral density values between which we can interpolate in GEODYN as the orbit drifts from the a priori orbit. This figure also demonstrates how perturbations due to different density models, represented here by different colored orbits, may necessitate a range of uncertainty for the satellite’s indexed location.

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Assessing Thermospheric Neutral Density Models using GEODYN's Precision Orbit Determination

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Key Points:

- Precision orbit determination solutions are expanded to study satellite drag and assess upper atmospheric density models using GEODYN.
- A proof-of-concept case study assessment of density models is presented using ICESat-2 precise science orbits and orbit fits
- Assessment results are provided for empirical (MSIS2, DTM2020, JB2008) and physics-based (TIEGCM, CTIPe) models for 14-days in November 2018

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19 **Abstract**

20 This study focuses on utilizing the increasing availability of satellite trajectory data
 21 from global navigation satellite system-enabled low-Earth orbiting satellites and their
 22 precision orbit determination (POD) solutions to expand and refine thermospheric model
 23 validation capabilities. The research introduces an updated interface for the GEODYN-
 24 II POD software, leveraging high-precision space geodetic POD to investigate satellite
 25 drag and assess density models. This work presents a case study to examine five mod-
 26 els (NRLMSIS2.0, DTM2020, JB2008, TIEGCM, and CTIPe) using precise science or-
 27 bit (PSO) solutions of the Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2). The
 28 PSO is used as tracking measurements to construct orbit fits, enabling an evaluation ac-
 29 cording to each model’s ability to redetermine the orbit. Relative in-track deviations,
 30 quantified by in-track residuals and root-mean-square errors (RMSe), are treated as prox-
 31 ies for model densities that differ from an unknown true density. The study investigates
 32 assumptions related to the treatment of the drag coefficient and leverages them to elim-
 33 inate bias and effectively scale model density. Assessment results and interpretations are
 34 dictated by the timescale at which the scaling occurs. JB2008 requires the least scaling
 35 ($\sim -23\%$) to achieve orbit fits closely matching the PSO within an in-track RMSe of
 36 9 m when scaled over two weeks and 4 m when scaled daily. The remaining models re-
 37 quire substantial scaling of the mean density offset ($\sim 30-75\%$) to construct orbit fits
 38 that meet the aforementioned RMSe criteria. All models exhibit slight over or under sen-
 39 sitivity to geomagnetic activity according to trends in their 24-hour scaling factors.

1 Plain Language Summary

40 This study utilizes the increasing availability of satellite trajectory data from low-
 41 Earth orbiting satellites and their precision orbit determination (POD) solutions to ex-
 42 pand thermospheric model validation capabilities. We introduce an updated interface
 43 for the GEODYN-II POD software to investigate satellite drag and assess density mod-
 44 els. This work presents a case study assessment of five models (NRLMSIS2.0, DTM2020,
 45 JB2008, TIEGCM, and CTIPe) using precise science orbit (PSO) solutions of the Ice,
 46 Cloud, and Land Elevation Satellite-2 (ICESat-2). GEODYN is used to construct or-
 47 bit fits to the PSO via the five models, enabling an evaluation according to each model’s
 48 ability to redetermine the orbit. Relative deviations from the PSO, quantified by in-track
 49 residuals and root-mean-square errors (RMSe), serve as proxies for model densities that
 50 differ from an unknown true density. We investigate and leverage drag coefficient assump-
 51 tions to eliminate bias and scale model densities. JB2008 requires the least scaling (\sim
 52 -23%) to achieve orbit fits closely matching the PSO within an in-track RMSe of 9 m
 53 when scaled over two weeks and 4 m when scaled daily. The remaining models require
 54 substantial scaling ($\sim 30 - 75\%$) to meet the aforementioned RMSe criteria.

2 Introduction

55 With the drastic increase in commercial satellite launches, the need to address the
 56 challenges posed by satellite drag have come to the forefront of the scientific, operational,
 57 and commercial space communities (Muelhaupt et al., 2019; Berger et al., 2020; Thayer
 58 et al., 2021; Hejduk & Snow, 2018; Bussy-Virat et al., 2018). Shortly following the 36th
 59 launch of SpaceX’s Starlink constellation on 3 February 2022, 38 out of 49 satellites were
 60 lost due to the impacts of a modest geomagnetic storm that reached G1 intensity ear-
 61 lier that day (Berger et al., 2023; Fang et al., 2022; Hapgood et al., 2022). The satellites
 62 were placed into an initial orbit of 210 km after which they were intended to maneuver
 63 to an operational altitude of 500 km. While this low altitude plan lent itself to a quick
 64 de-orbit in the face of catastrophe, it exposed the satellites to the larger variations and
 65 uncertainties in neutral density associated with relatively meager space weather condi-
 66 tions. While this event happened at altitudes well below Starlink’s operational orbit, it
 67 has served as a potent example to the commercial space community of the need to bet-
 68 ter model and predict atmospheric drag, which represents the most significant hurdle pre-
 69 venting more accurate determination and prediction of trajectories in LEO.

70 Precision orbit determination (POD) programs are employed in both operational
 71 and research capacities to provide high-fidelity orbit trajectories of LEO satellites. The
 72 quality of such trajectories is directly dependent on the ability of a POD’s force model
 73 to realistically capture the conservative and non-conservative forces impacting a satel-
 74 lite’s orbit. Due to advancements in conservative force modeling, the largest source of
 75 error preventing more accurate orbit trajectories is now associated with non-conservative
 76 forces (Tapley et al., 2005; Reigber et al., 2006; Velicogna & Wahr, 2005). Of these, at-
 77 mospheric drag is the most variable and uncertain as a consequence of its reliance on mod-
 78 eling the thermospheric neutral mass density (ρ) variations and the satellite drag-coefficient
 79 (C_D) (Hejduk & Snow, 2018). The largest source of uncertainty is ρ , but for satellites
 80 with complex shapes, C_D can contribute to this uncertainty. Mehta et al. (2022) describes
 81 this issue as the interconnectedness of uncertain parameters, an extremely challenging
 82 problem to solve for the satellite drag community and one that has significant impact
 83 on the assumptions made in this work. The burden for achieving more precise and re-
 84 liable LEO nowcasting and forecasting largely relies on the ability of thermospheric den-
 85 sity models to accurately capture the behavior of neutral density and reliably predict it
 86 into the future. Adding to the problem, assessing the performance of density models presents
 87 a massive challenge due to the scarcity of data from satellite measurements, and the lack
 88 of absolute truth due to the complexity of interconnected uncertainties. This necessi-
 89 tates the community to seek alternative methods to add to the validation method reper-
 90 toire. The growing prevalence of global navigation satellite system (GNSS)-enabled low-
 91 Earth orbiting satellites and their POD solutions represents one such potential data source,
 92 and providing methods to take advantage of these datasets will help the community ex-
 93 pand and refine model validation capabilities.

94 POD programs such as the NASA Goddard Space Flight Center’s (GSFC) GEO-
 95 DYN II software (henceforth referred to as GEODYN) have been developed within the
 96 geodesy scientific community with the above challenges in mind—implementing techniques
 97 such as reduced-dynamics paired with extremely high quality tracking measurements from
 98 GNSS to mitigate the need for highly accurate non-conservative force models when per-
 99 forming non-predictive orbit determination. Through these means, centimeter level ra-
 100 dial accuracy has been demonstrated to produce precise science orbit (PSO) solutions
 101 for missions such as the Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2), which
 102 orbits at approximately 500 km (Thomas et al., 2021). These techniques—combined with
 103 GEODYN’s legacy of precise conservative force and measurement modeling, meticulous
 104 time systems, and accurate coordinate reference frames—have made the program a top-
 105 tier POD tool that is well-positioned to study thermospheric neutral density models and
 106 their distinct impacts on the estimation of satellite drag (Luthcke et al., 2003; Zelensky

et al., 2010; Lemoine et al., 2016; Loomis et al., 2019). This work aims to provide a method to improve the specification of satellite drag physics and the assessment of neutral density model performance to help the Ionosphere-Thermosphere (IT) community advance model predictions, and consequently improve the accuracy of POD solutions.

This paper presents the development of a modernized Python interface for the GEODYN software, leveraging the high-precision nature of space geodetic POD, but refashioned to study satellite drag and to enable density model assessment. We make use of the well-specified, low-error ICESat-2 PSO to perform a case study assessment of five thermospheric density models, three of which are empirical while the other two are physics-based. The ICESat-2 PSO serves as tracking measurements to POD-based orbit fits in which the drag effects from density models are assessed according to each model’s ability to redetermine the orbit. Implications regarding the treatment of the drag coefficient are investigated and discussed. This work reports an initial result using a fixed drag coefficient of $C_D = 2.5$, followed by two methods for debiasing the assessment results using a drag acceleration scaling factor over both a two-week and a daily time interval. Each model’s orbit fit contains relative in-track deviations, quantified by in-track residuals and root-mean-square errors from the ICESat-2 PSO, which are treated as proxies for model densities that differ from a true, unspecified density. By developing these methods, we aim to provide the community with the means to take advantage of emerging GNSS-tracked satellite datasets and POD solutions to objectively quantify density model performance. In addition, we hope to address deficiencies in non-conservative force modeling that may currently impede higher quality predictions of LEO trajectories. The presented model assessment results will be parsed into the Community Coordinated Modeling Center’s Comprehensive Assessment of Models and Events using Library Tools (CAMEL) framework, for community use.

Section 3 gives the necessary science background needed to understand our methodology. Section 4 details the GEODYN software, provides information regarding the ICESat-2 POD solutions, and offers an overview description of the upper atmospheric density models that are assessed in this work. Section 5 details the methodology, the setup procedure for conducting the model assessment, and the methods for debiasing the assessment results using drag acceleration scaling factors. Section 6 provides the results and discussion of the assessment using ICESat-2 PSO as a case study.

3 Background

The precision of a POD solution relies on the fidelity of the tracking measurement models, the quality of the tracking data, and the ability of the POD force model to capture realistic accelerations acting on the satellite. In general, the force model defines the overall motion of a spacecraft by calculating the sum of all impacting forces, themselves being subdivided into conservative forces which are potential in nature, and non-conservative forces which act to dissipate the satellite’s orbital energy. Conservative forces captured by the GEODYN force model include the Earth’s static gravity field (geopotential), solid Earth and ocean tides, the effects of dynamic polar motion, the acceleration from time variable gravity, Third-body perturbations (primarily from the Sun and Moon), and contributions from general relativity. Recent improvements in conservative force modeling as well as advances in the internal measurement models have shifted the primary source of error in POD solutions to the non-conservative forces (Luthcke et al., 2006; Loomis et al., 2019; Reigber et al., 2006). The non-conservative forces modeled in GEODYN are atmospheric drag, solar radiation pressure (SRP), and Earth radiation pressure (ERP). As altitude decreases in the LEO regime, atmospheric drag increasingly becomes the largest non-gravitational force acting on satellites. In addition, the drag force’s dependence on the upper atmospheric neutral mass density makes it the most error-bound perturbing force (Hejduk & Snow, 2018). While force model errors can be circumvented via reduced-dynamics and high-quality tracking measurements, this technique is limited in its ap-

158 plication for the eventual goal of orbit prediction, which requires an improved, more-realistic
 159 force model (Tapley et al., 2004; Luthcke et al., 2019).

160 The drag force acting on a satellite of mass m_{sat} is proportional to the atmospheric
 161 neutral mass density ρ , the drag coefficient C_D , the projected area perpendicular to the
 162 flow direction A_{sat} , and the velocity of the satellite relative to the atmosphere \vec{V}_{rel} . The
 163 drag acceleration \vec{a}_D due to the drag force per unit mass acting on a satellite is given
 164 in Equation 1 as

$$165 \quad \vec{a}_D = -\frac{1}{2} \rho C_D \frac{A_{sat}}{m_{sat}} V_{rel}^2 \frac{\vec{V}_{rel}}{V_{rel}} \quad (1)$$

166 Physically, the total drag force acting on a satellite surface is given by the force due to
 167 incident atmospheric particles impacting the surface combined with the force from scat-
 168 tered particles departing from the surface. These effects are represented by the drag co-
 169 efficient C_D , which depends on a satellite’s geometry and orientation, the material and
 170 surface temperature of the spacecraft, the local atmospheric composition, and gas-surface
 171 interactions and other effects (Bernstein & Pilinski, 2022). In the context of spacecraft
 172 dynamics, the C_D is generally characterized as either fixed, fitted, or physical. Fixed C_D
 173 uses a predetermined value that does not change. Fitted C_D is derived using some form
 174 of a fitting or filtering process and is typically updated over time (every few hours or or-
 175 bits). Physical C_D is computed by modeling the momentum and energy exchange be-
 176 tween the flow-field particles and the satellite (see Mehta et al. (2022) for more details).
 177 If not physically calculated, C_D ’s presence in Equation 1 may be thought of as a scal-
 178 ing factor that effectively serves to average out errors in the atmospheric density model
 179 and gas-surface interactions. In its base state, GEODYN can use either a fitted or fixed
 180 C_D . In the fitted case C_D is an adjustable parameter that accounts for mismodeled physics
 181 and for uncertainties in ρ associated with the upper atmospheric density model.

182 Earth’s upper atmosphere is driven by a broad range of external energy inputs, lead-
 183 ing to complex thermal, electromagnetic, and chemical processes that result in a ther-
 184 mospheric neutral mass density ρ that is highly dynamic and whose variability is diffi-
 185 cult to specify (Emmert, 2015). Upper-atmospheric density models are employed within
 186 POD force models to represent the complex behavior of ρ when calculating the force of
 187 satellite drag acting on a spacecraft, directly or indirectly through C_D . The three types
 188 of density models most commonly used by upper atmospheric communities are semi-empirical,
 189 physics-based, and data assimilative models. The simple yet effective semi-empirical mod-
 190 els are most commonly employed in POD force models since they offer excellent clima-
 191 tological pictures of upper atmospheric variability and are computationally inexpensive.
 192 Physics-based models are more complex, taking the form of general circulation models
 193 which solve the first-principle equations that govern the coupled thermosphere-ionosphere
 194 system. They are not typically used in POD geodetic settings due to the computational
 195 expense. A data assimilative technique can be used to calibrate modeled density and has
 196 given rise to data assimilative (also referred to as dynamically calibrated) models. These
 197 combine analyses from a multitude of space objects to produce corrections to empirical
 198 (and occasionally physics-based) thermospheric models. The most prominent example
 199 of assimilative thermospheric density models is the United States Space Force, High Ac-
 200 curacy Satellite Drag Model (HASDM) (Storz et al., 2005). It is a common practice in
 201 the IT modeling community to compare model performances against HASDM outputs
 202 since it performs real-time calibration using ~ 75 space objects.

203 Different models, and even model types, have varying degrees of performance un-
 204 der specified conditions. Individual model performances are known to depend greatly
 205 on the solar flux and geomagnetic conditions that drive them, and their respective strengths
 206 make some models better qualified for some scenarios than others. Semi-empirical mod-
 207 els are often computationally fast and accurate for climatological uses, but their abil-
 208 ity to accurately project into the future is closely tied to the fidelity of their drivers. Physics
 209 models offer great potential for forecasting, but lack the accuracy of semi-empirical mod-

210 els in near real-time scenarios (Shim et al., 2014; Sutton, 2018). The vast range in model
 211 performances makes the evaluation of models a critical goal for upper atmospheric sci-
 212 ence and satellite drag communities. The scarcity, and coupled uncertainty (via C_D un-
 213 certainty) of thermospheric density measurements makes this a significant challenge. The
 214 most common method for objectively quantifying a density model’s performance is to
 215 compare the sampled model outputs against satellite measurements, e.g. see Walterscheid
 216 et al. (2023)—usually in the form of accelerometer-derived densities from the Challeng-
 217 ing Minisatellite Payload (CHAMP), Gravity Recovery and Climate Experiment (GRACE)
 218 or Gravity Field and Steady-State Ocean Circulation (GOCE) missions (Bruinsma et
 219 al., 2004; Sutton et al., 2005; Doornbos et al., 2010; Mehta et al., 2017).

220 In a series of papers motivated to provide community organization for conducting
 221 model comparison and evaluation, Bruinsma et al. (2017, 2018, 2021) provide common-
 222 alities for inter-model scoring. They report on chosen observed density datasets, time
 223 periods of interest, and provide a scoring metric in the form of the mean, standard de-
 224 viation and root mean square error (RMSe) of the observation-to-model density ratios.
 225 He et al. (2018) similarly presents an assessment of several semi-empirical thermosphere
 226 models, focusing on their ability to reproduce spatial variations and capture complex fea-
 227 tures in thermosphere mass density. Shim et al. (2014) provides a systematic evaluation
 228 of thermospheric and ionospheric models, quantifying model performance using four skill
 229 scores calculated as functions of geomagnetic activity and geographic latitude: RMS er-
 230 ror, prediction efficiency, ratio of maximum-to-minimum, and ratio of maximum ampli-
 231 tude. Thayer et al. (2023) investigates the use of the day-to-night density ratio as a met-
 232 ric for representing the atmosphere’s response to large scale perturbations (i.e. the tran-
 233 sition from solar maximum to solar minimum), providing inter-model and model-to-observation
 234 comparisons, and unearthing discrepancies that are not observed between models and
 235 observations when viewed using more common metrics. Each of these reports makes use
 236 of the accelerometer-derived density data sets to objectively quantify model performance.

237 Through this work, we aim to contribute an additional method to the community
 238 in which accurately developed and well-honed POD tools can be leveraged for assessing
 239 density model performance. For the purposes of this paper, we make a distinction when
 240 referring to the different stages of model assessment. We use the term “assessment” to
 241 refer more generally to methods and results that offer insight into model performance.
 242 “Verification” refers to using other well-specified methods and datasets to confirm the
 243 fidelity of our methods and results. “Validation” refers to the act of objectively quan-
 244 tifying modeled densities against observed/derived values. This paper offers a verifica-
 245 tion of our method and results by comparing against the HASDM densities, and provides
 246 an example performance assessment using two-weeks of the ICESat-2 PSO as a case study.
 247 A more formal validation scheme is the eventual goal of this work, however, this requires
 248 additional considerations and is a source of continuing effort.

4 Program and Data Descriptions

4.1 GEODYN and the Pygeodyn Wrapper

249 The GEODYN-II program is a precision orbit determination and parameter esti-
 250 mation tool that has been used on every NASA geodetic Earth and planetary altime-
 251 ter mission since 1985. The program is used extensively for orbit determination, geode-
 252 tic parameter estimation, tracking instrument calibration, satellite orbit prediction, as
 253 well as for many other applied research studies in satellite geodesy (Pavlis et al., 2019;
 254 Luthcke et al., 2019). GEODYN is capable of ingesting essentially all types of tracking
 255 measurements, the most common of which include observations from global navigation
 256 satellite systems (GNSS) and satellite laser ranging (SLR), as well as post-processed or-
 257 bits in the form of orbit trajectories or precisely converted elements (PCE) (Pavlis et al.,
 258 2019; Lyon et al., 2004). GEODYN performs orbit propagation using Cowell’s method

259 of numerical integration, and performs data-reduction utilizing a Bayesian least-squares
 260 batch estimation process to optimally estimate parameters by minimizing the residuals
 261 between tracking data and orbit propagations (see Vallado (2013) for more information).
 262 GEODYN’s long history in geodetic applications has ensured the development of very
 263 precise conservative force and measurement models, as well as accurate time systems and
 264 coordinate reference frames, making the program a top-tier POD tool. With this under-
 265 standing, the errors found in the observed residuals between tracking data and deter-
 266 mined orbit are more related to uncertainties in the satellite specific non-conservative
 267 force models, rather than being related to the quality of measurement modeling or or-
 268 bit determination methods and tools. In the lower register of LEO, where atmospheric
 269 drag dominates, the observation residuals can provide valuable information on the drag
 270 model errors.

271 Pygeodyn is an internally-developed Python-based wrapper meant to offer improved
 272 user access to the FORTRAN-based GEODYN software. Pygeodyn offers users a stream-
 273 lined and simplified tool to navigate the complex steps for modifying, controlling, run-
 274 ning, and reading the various data sets and files that compose the GEODYN program.
 275 The main portion of GEODYN II is composed of two sequenced programs: GEODYN-
 276 IIS, a scheduling program and GEODYN-IIE, an execution program stage. The schedul-
 277 ing program reads and organizes input data, ancillary data files, and the user’s setup op-
 278 tions. The execution program then integrates the satellite trajectory and applies the se-
 279 lected models, performs orbit determination to provide computed observables, and uses
 280 the least squares scheme, along with any measured observables, to provide solutions for
 281 updated orbits as well as any requested geophysical parameters. The two stages com-
 282 municate via a series of binary files which are output from the scheduling program and
 283 fed into the execution program. Historically, adding atmospheric density models to GEO-
 284 DYN required modification to IIS as well as subsequent data tracking and modification
 285 to IIE, a series of complications that have been circumvented with our Pygeodyn tool.
 286 Pygeodyn gives the ability to switch between different atmospheric density models that
 287 have been connected to GEODYN-IIE without the need to modify GEODYN-IIS, sim-
 288 plifying the user experience for adding and selecting the models. Programming in Python
 289 has also afforded Pygeodyn the ability to interface with the NASA Goddard Commu-
 290 nity Coordinated Modeling Center’s (CCMC) Kamodo API (Ringuette et al., 2023), grant-
 291 ing access to their sophisticated model readers and allowing Pygeodyn to connect physics-
 292 based density model outputs to the POD scheme.

4.2 ICESat-2 PSO Solutions as Tracking Data

293 ICESat-2 flies in a near-circular, near-polar, low-Earth orbit at ~ 496 km altitude
 294 and an orbital period of 94.22 min. Details of the orbital parameters are reported in Luthcke
 295 et al. (2019). The ICESat-2 PSO (i.e., the science quality POD solutions), and their cor-
 296 responding setup files, are provided by the Geodesy and Geophysics Laboratory within
 297 NASA/GSFC, who maintain the GEODYN program and provide science quality POD
 298 for many NASA missions. ICESat-2 is an excellent platform for orbital drag-based model
 299 assessment because of its science requirements to have such high quality orbit solutions,
 300 as well as stable attitude specifications. The ICESat-2 PSO is reported by Thomas et
 301 al. (2021) as having a radial orbit accuracy of just below 1.5 cm over a 24-hour orbit solution—
 302 performing better than the mission requirement of 3 cm. These orbit solutions are gen-
 303 erated through the reduction of GNSS double-difference carrier phase observable resid-
 304 uals, and independently assessed using SLR measurement residual analysis. Technical
 305 details regarding the construction and analysis plan for the ICESat-2 PSO can be found
 306 in Luthcke et al. (2019).

307 The centimeter-level orbit of the PSO data is achieved using the previously men-
 308 tioned reduced-dynamics technique in which GEODYN solves for empirical acceleration
 309 parameters that describe the difference between the actual positions, i.e. those derived

310 from the GPS tracking measurements, and the positions that are calculated by the pro-
 311 gram’s physical force models and satellite propagator. The PSO data includes estima-
 312 tions of along-track and cross-track empirical accelerations every quarter of an orbit, ap-
 313 plying a nearest neighbor covariance constraint. With the use of reduced-dynamic em-
 314 pirical accelerations, it is possible to compensate for errors associated with using the MSIS86
 315 model to calculate the effects of atmospheric drag. Luthcke et al. (2019) notes that even
 316 though a reduced-dynamic approach is commonly employed by the geodesy community
 317 to overcome any inadequacies in a force model, the technique relies on an orbit solution
 318 that has already attained sufficient radial accuracy through the use of a high-quality phys-
 319 ical force model. Dense tracking measurements and the reduced-dynamic technique do
 320 not obviate the use of accurate orbit modeling, and improvements in the orbit fit will
 321 be realized when the force models are improved. We also note that while MSIS86 was
 322 used to estimate the drag accelerations in the ICESat-2 PSO, due to them being com-
 323 bined with additional empirical accelerations in the along-track and cross-track direc-
 324 tions no related bias is found that favors the MSIS series of models in the assessments
 325 reported in this work.

326 This work uses the ICESat-2 PSO as tracking measurement input to a data-reduction
 327 run of GEODYN—the goal being to assess the ability of each selected density model to
 328 re-determine the orbit of ICESat-2. A data-reduction run in GEODYN is one in which
 329 orbit parameters (i.e., initial conditions) and optionally geophysical parameters (such
 330 as gravitational coefficients or the drag coefficient) are adjusted to minimize residuals
 331 and provide an improved solution. This data reduction is computed over an orbital arc,
 332 a set time period for which continuous tracking data is available. The term “orbit fit”
 333 refers to the outputs of GEODYN runs in which the ICESat-2 PSO is the tracking data
 334 type and respective density models are used to iteratively re-determine the orbit.

335 The following capabilities for density model assessment are enabled by using GEO-
 336 DYN to construct orbit fits from ICESat-2 orbit solutions:

- 337 1. Leverage GEODYN’s high fidelity physical force models which have been honed
 338 by the program’s long legacy in space geodesy.
- 339 2. Perform data-reduction runs in which we compare the relative ability of each at-
 340 mospheric density model to re-determine the orbit of ICESat-2 given the isolated
 341 satellite drag effects.
- 342 3. Control the POD and force model parameters such that for each respective run,
 343 the only relative variable impacting the overall fit of the orbit solution for a given
 344 arc is the atmospheric density model used to estimate the drag term.
- 345 4. Control for relative errors between runs associated with an unknown drag coef-
 346 ficient by using a realistic fixed value of $C_D = 2.5$. This value is determined by
 347 physically calculating C_D using the Diffuse Reflection with Incomplete Accom-
 348 modation (DRIA) method along the orbit of ICESat-2, as is described in Section
 349 5.

4.3 Model Descriptions

350 This section provides a brief overview of the atmospheric density models that are
 351 used for verification and assessment. Table 1 lists the models, providing the Model ID
 352 used for referencing in this paper, the full name and version number, the run conditions
 353 based on the drivers, and the models’ spatial and temporal resolutions. The authors ac-
 354 knowledge that while there are a number of ways to improve a density model’s outputs
 355 at runtime (see Sutton (2018); Shim et al. (2014)), the outputs used in this work are in-
 356 tended to reflect typical community use, with each model being run according to the de-
 357 veloper’s operational instructions. Additional information regarding each model can be
 358 found in the references provided in the second column of Table 1.

359 We provide a verification using SET-HASDM, a data-assimilative model, and as-
360 sessment results for MSIS2, JB2008, DTM2020, TIEGCM, and CTIPe. The semi-empirical
361 models (MSIS2, JB2008, DTM2020) are interfaced directly into GEODYN's FORTRAN-
362 based source code. The physics-based models (TIEGCM and CTIPe) are interfaced to
363 GEODYN via the CCMC's Kamodo program which reads and interpolates the model
364 output files. These interpolated outputs are connected to GEODYN through the Pygeo-
365 dyn wrapper using an orbit cloud interpolation technique which is detailed in Appendix
366 C. In addition, physics-based models whose maximum altitude is below the orbit alti-
367 tude of ICESat-2 include a diffusive equilibrium extrapolation of the neutral densities
368 (see Chapter 10 of Schunk and Nagy (2009)). The use of Kamodo makes the analysis
369 techniques in this paper easily extensible to additional models. Any thermospheric model
370 that is supported by Kamodo, with the appropriate diffusive equilibrium extrapolation,
371 can be added to this and similar analyses in the future.

Models Assessed in this Report

Model ID	Full name/version	Drivers, (solar geomagnetic)	Resolution, (spatial time)
Semi-Empirical MSIS2	Naval Research Laboratory Mass Spectrometer and Incoherent Scatter (NRLMSIS 2.0), (Emmert et al., 2021)	$F_{10.7}$ A_p	
JB2008	Jacchia-Bowman 2008, (Bowman et al., 2008)	$F_{10.7}$, $S_{10.7}$, $M_{10.7}$, $Y_{10.7}$ A_p , Dst	
DTM2020	Drag Temperature Model 2020, operational mode (Bruinsma & Boniface, 2021)	$F_{10.7}$ and K_p	
Physics-Based TIEGCM	National Center for Atmospheric Research (NCAR) Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIEGCM; version 2.0), (Richmond et al., 1992; Qian et al., 2014; Sutton et al., 2015)	$F_{10.7}$, EUVAC proxy model (Solomon & Qian, 2005) K_p via the Heelis model (Heelis et al., 1982),	$5^\circ \times 5^\circ$, vertically specified by logarithmic pressure surfaces in half-scale height increments from ~ 97 km to ~ 800 km 1 minute time step, hourly output
CTIpe	Coupled Thermosphere Ionosphere Plasmasphere Electrodynamics (CTIpe; version 4.1) (G. Millward et al., 1996; G. H. Millward et al., 2001)	$F_{10.7}$, solar wind inputs via magnetic field, velocity, and density measurements from Advanced Composition Explorer (ACE) K_p , hemispheric power index from NOAA Polar Orbiting Environmental Satellite (POES)	2° lat. \times 18° lon., vertically specified by 15 logarithmic pressure surfaces from ~ 80 km to ~ 500 km 1 minute time step
Data-Assimilative SET-HASDM	Space Environment Technologies (SET) HASDM Database (Tobiska et al., 2021), which is derived from the US Space Force operational archive (Storz et al., 2005)		10° lat. \times 15° lon., 25 km altitude steps from 175 to 825 km. 3 hour time step

Table 1. A summary of the thermospheric density models used in this assessment.

5 Methodology

5.1 Setup for ICESat-2 Case Study

372 This method uses a satellite’s PSO as tracking measurements to construct dynamic
 373 POD-based orbit fits from different density models. The dynamic POD technique uses
 374 a batch least-squares approach to iteratively reduce errors between the propagating or-
 375 bit fit and the ingested PSO—GEODYN refers to this as data-reduction mode. The ini-
 376 tial conditions, and any other adjustable parameters, are iteratively estimated and up-
 377 dated to refine the orbit fit until it consistently reaches a convergence threshold. The
 378 remaining errors that persist between the PSO and a given model’s orbit fit are under-
 379 stood to be primarily due to atmospheric drag effects from the respective density model.
 380 This understanding is leveraged to investigate density model performance through the
 381 assessments that are presented in this paper. Figure 1 provides a visualization showing
 382 connections between the high-level datasets and processes. While the true density along
 383 the ICESat-2 orbit remains unknown, each model’s orbit fit contains in-track deviations
 384 from the ICESat-2 PSO, which are treated as proxies for model density deviations from
 385 the true density.

386 The GEODYN run setup that is used to construct the POD-based orbit fits is kept
 387 as similar as possible to the setup used by the team at NASA-GGL to produce the ICESat-
 388 2 PSO—meaning we modify only what is necessary to use PSO as the tracking measure-
 389 ment type, and to control the procedure such that drag is the only independent variable
 390 in each model’s run. An extended overview of GEODYN’s setup and force model param-
 391 eters for the model assessment runs is provided in the appendix in Table B1, with only
 392 the most impactful considerations being discussed here. In addition to each orbit fit us-
 393 ing the same background force models, the ICESat-2 external attitude information is also
 394 utilized to properly orient the spacecraft body and the solar array. The orbit fits are split
 395 into 24-hour, consecutive daily arcs. The arc length can theoretically be much shorter;
 396 however, orbit errors related to force model perturbations (i.e., drag) require propaga-
 397 tion time to accumulate. An arc length on the order of 1-2 orbital periods may not de-
 398 pict substantial trajectory deviations in the residuals, making 24-hour arcs a balanced
 399 choice to demonstrate this assessment method. In theory, reducing the arc lengths would
 400 provide more RMSE values over shorter times and would offer higher temporal resolu-
 401 tion towards understanding model performance, but at the cost of having accumulated
 402 less orbital error from the density model in the shorter propagation time. The choice of
 403 arc length and its ramifications on assessment results continues to be an area of study
 404 related to this work.

405 Other non-conservative forces that must be considered in addition to atmospheric
 406 drag are SRP and ERP which are both calculated by GEODYN according to the descrip-
 407 tions shown in Table B1, and the references therein. Using GEODYN and its high-fidelity
 408 force model ensures that the estimated SRP and ERP accelerations are more precise than
 409 what would be modeled by a standard satellite flythrough scheme. For each density model’s
 410 orbit fit the acceleration due to drag will vary according to the error in the respective
 411 model, while the contributions from SRP and ERP will remain consistent for each arc
 412 across each model run. For the orbit fits presented in this work, the variations of SRP
 413 and ERP were found to be small relative to the variable effects of drag. This being said,
 414 the magnitude of the SRP acceleration is often on par with that of the drag accelera-
 415 tion at the ICESat-2’s altitude. Errors related to mismodeling non-drag forces can po-
 416 tentially be transferred into the residuals, and as a result this method is presented as a
 417 relative assessment between controlled model runs rather than an absolute validation of
 418 performance.

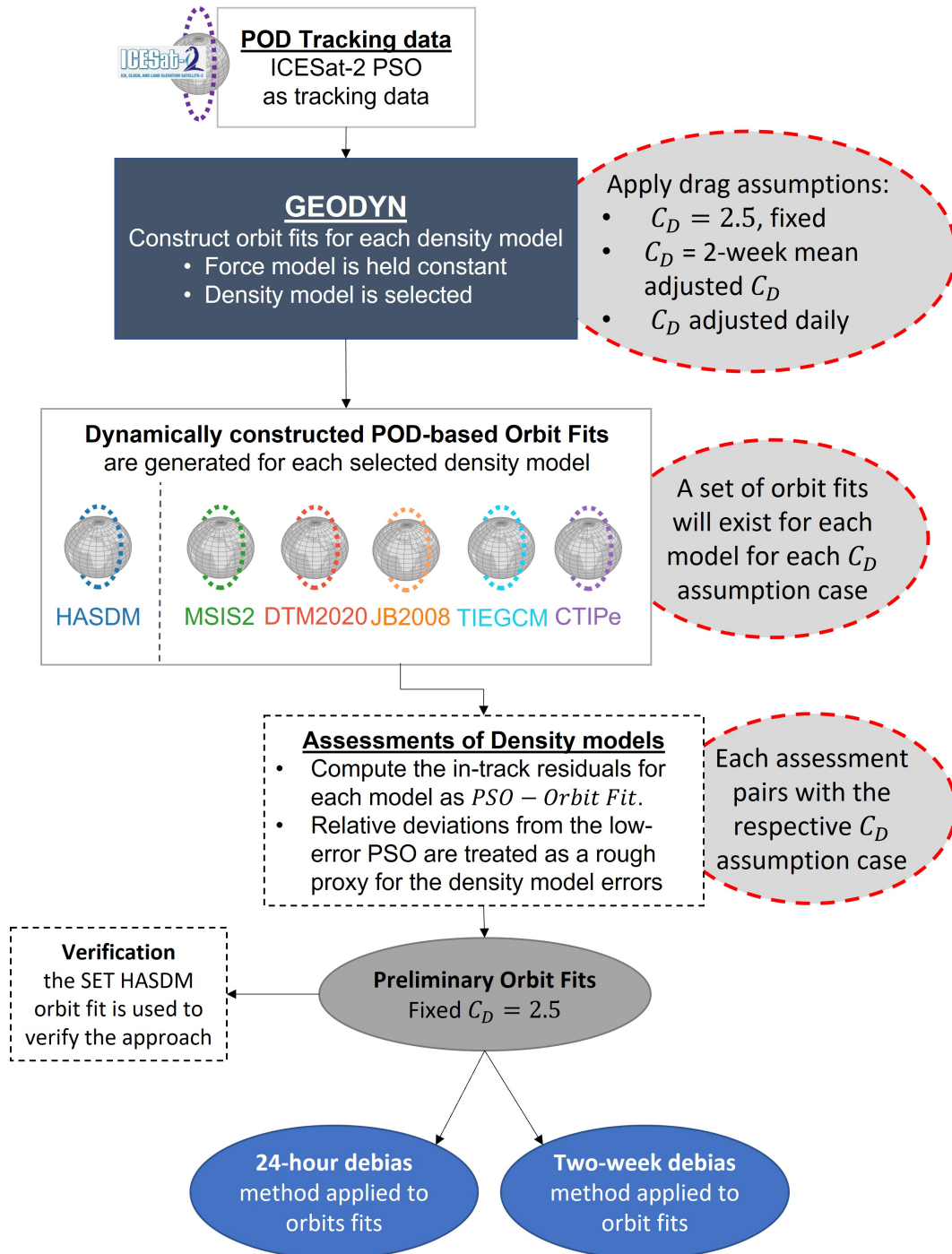


Figure 1. A flowchart visualizing the assessment process and how the datasets and POD methods fit together for model assessment.

5.2 Assessment Procedures

419 The interconnected, uncertain nature of C_D and ρ makes the absolute determina-
 420 tion and assessment of either value a very complex problem that is still an active field
 421 of research within this community. In this case study, the assessment procedure is split
 422 into three subsequent orbit fit methods, each based on assumptions made to character-

423 ize C_D when calculating the drag acceleration for each model during the orbit fit pro-
 424 cedure. In a preliminary orbit fit method, explicit biases are identified via orbit fits con-
 425 structed using a fixed C_D that is held constant across all models. The constant fixed C_D
 426 is chosen to be a physically realistic value of $C_D = 2.5$ based on the average result cal-
 427 culated from the Diffuse Reflection with Incomplete Accommodation (DRIA) model along
 428 the ICESat-2 orbit. This demonstrates that the method is sufficiently sensitive to rec-
 429 ognize differences in the drag effects between the models and provides an understand-
 430 ing of each model’s approximate mean density offset relative to the ICESat-2 PSO In
 431 an alternative orbit fitting method, GEODYN’s parameter estimation procedure is used
 432 to adjust the C_D for every 24-hour arc for the two-week period from the a priori esti-
 433 mate of $C_D = 2.5$. The mean-adjusted C_D over the two-weeks is then used as a fixed,
 434 model specific value that is constant for the time-period. This provides a fixed, unique
 435 C_D for each density model that effectively scales the density over the two weeks to ac-
 436 count for each model’s mean density offset and examine the model response to solar and
 437 geophysical dynamics. In a final orbit fitting method, the daily C_D adjustments are used
 438 without a two-week averaging. The residuals using these daily, model-specific C_D ad-
 439 justments provide an assessment of model performance on time periods less than a day.

440 The preliminary orbit fits use a fixed C_D of 2.5 that is constant with respect to each
 441 model. This enables direct model comparison, but subjects an assessment of the den-
 442 sity models to explicit biases depending on each model’s density offset relative to this
 443 C_D value. Each model’s sampled densities along the ICESat-2 orbit have an overall mean-
 444 density offset relative to each other. Fixing the C_D to a specific value will cause a par-
 445 ticular offset amount to be favored. For instance, $C_D = 3.5$ will produce favorable or-
 446 bit fits for models that trend a lower density, whereas $C_D = 1$ would favor models that
 447 trend towards higher densities. Due to these circumstances, the DRIA model is indepen-
 448 dently used to calculate a physically realistic value of the ICESat-2’s C_D along its or-
 449 bit. DRIA is a relatively simple, computationally fast model for capturing the gas-surface
 450 interactions between the upper atmosphere and a spacecraft. In the DRIA model, par-
 451 ticles are always reflected with a diffuse angular distribution, but their energy exchange
 452 with the surface varies depending on the value of the energy accommodation coefficient
 453 α . This work uses the Sentman’s closed-form solutions for the DRIA model as depicted
 454 in Equation 12 of Walker et al. (2014). The energy accommodation is assumed to be fixed
 455 at $\alpha = 0.89$ —a tenuous assumption based on the limited empirical data for α near 500
 456 km during solar minimum (Pilinski et al. (2010); Pilinski (2008)). This α value is likely
 457 higher than is realistic for this altitude and solar flux, therefore providing a lower limit
 458 for what a physically realistic drag coefficient might be; however, given the complex changes
 459 in atmospheric structure that occur in this altitudinal regime, this empirical value is still
 460 the most representative until further observations can be made. There are other phys-
 461 ical C_D models that could be used instead of DRIA, but choosing and assessing the C_D
 462 models quickly expands beyond the scope of this work. For the sake of being able to con-
 463 duct a model assessment as a proof-of-concept in this case study, this assumption is made
 464 with the intention to improve the treatment of C_D in future efforts. In future work, we
 465 aim to address this issue by implementing a physical satellite gas-surface interaction method
 466 to calculate the time-dependent drag coefficient, but even this will have associated as-
 467 sumptions and caveats. Constructing orbit fits with a fixed, common C_D of 2.5 for each
 468 model represents the type of method that is possible without being able to model the
 469 physical drag coefficient or without GEODYN’s capability to adjust the parameter. This
 470 adjustment procedure was performed for different a priori C_D values and found that the
 471 final adjusted C_D for each model was consistently the same.

472 The 24-hour debiasing method uses GEODYN’s parameter estimation capabilities
 473 to determine a daily fitted value of the C_D that accounts for accumulated errors from
 474 the force model over the 24-hour arc—the most prominent of which being due to den-
 475 sity uncertainty. In the field of space geodetic POD, C_D is often adjusted in conjunc-
 476 tion with reduced-dynamic empirical accelerations to account for disagreement between

477 the observed accelerations from tracking measurements, and calculated accelerations from
 478 uncertainties in the drag force model. This technique is used to get very low error, pre-
 479 cise orbit solutions, but limits the ability to distinguish errors that are specific to drag
 480 or the density models. By allowing only the C_D to adjust and match the orbit fit’s mod-
 481 eled accelerations with the PSO observation, density errors over the 24-hour period are
 482 incorporated into the adjustment. A density model that is found to be over-/under-estimating
 483 the density, will have a C_D that is adjusted to be smaller/larger in a non-physical way—
 484 effectively using C_D as a scaling term between the PSO observation and uncertainty in
 485 the density model orbit fits. In practice, the C_D also absorbs any errors from mismod-
 486 eled forces, but these are held constant in the model-to-model comparison. Each model
 487 is given an a priori estimate of $C_D = 2.5$ at the start of the 24-hour arc, which is al-
 488 lowed to adjust within a standard deviation of 10. The drag coefficient fitting occurs con-
 489 currently with the iterative orbit fit routine. Due to this non-physical use of C_D to ef-
 490 fectively debias the density, the term “drag acceleration scaling factor” is adopted. The
 491 24-hour scaling factor for each model (m) can be calculated for each arc (i) as,

$$f_{24,m,i} = C_{D,adj,m,i}/2.5 \quad (2)$$

492 The two-week debiasing method acts as a combination of the previous two meth-
 493 ods. The C_D adjustments for each model are averaged over the two-week period to pro-
 494 vide a mean adjusted C_D . Each model’s orbit fit is then re-determined using the mean
 495 adjusted C_D for each respective model as the fixed value for the two-week period. This
 496 assessment permits a scaling of the density models over an extended period of time, high-
 497 lighting errors in the orbit fits that are due to variations that take place on a longer time
 498 scale than 24-hours. This method is also motivated by the need to provide a scoring met-
 499 ric for each density model that can be parsed into the CCMC’s CAMEL model valida-
 500 tion infrastructure. While the $C_D = 2.5$ case is dominated by the model biases and the
 501 24-hour debiased case demonstrates a method to debias daily densities, the two-week de-
 502 biased case quantifies the ability of the models to capture dynamics caused by geomag-
 503 netic and solar activity over a more prolonged time period. This is a method that could
 504 be used in the future to assess model performance during individual stormtime periods.

5.3 Assessment Metrics

505 Using a PSO as tracking data makes use of GEODYN’s data-reduction mode com-
 506 bined with a dynamic technique for estimating the orbit of a satellite. This technique
 507 uses the trajectory input to estimate updates to the initial conditions which define the
 508 motion of the satellite, thus refining the orbit. The orbit residuals obtained in this setup
 509 are the absolute differences between the PSO and each density model’s orbit fit. Since
 510 other force model parameters are held constant between each density model’s run, the
 511 inter-comparison of the residuals contains information primarily corresponding to rela-
 512 tive errors in each density model’s ability to replicate the drag effects seen in the ICESat-
 513 2 PSO.

514 To best observe satellite drag effects, all output orbits are transformed from the
 515 J2000, geocentric inertial reference system to the NTW, orbit-aligned satellite coordi-
 516 nate system (Vallado, 2013). This system is composed of an in-track component \hat{T} that
 517 is parallel to the velocity vector, a normal component \hat{N} that is perpendicular to the ve-
 518 locity and nominally in the radial direction, and a cross-track component \hat{W} that is nor-
 519 mal to the orbit plane and completes the right-hand coordinate frame. The in-track com-
 520 ponent \hat{T} is parallel to the velocity vector direction and contains any indication that the
 521 spacecraft’s trajectory has changed since orbital energy dissipations from drag will im-
 522 pact in the velocity direction. Information regarding this transformation as well as sup-
 523 porting coordinate frame details can be found in Appendix A.

524 For any given arc, y_o is defined to be a component of the orbit from the PSO dataset
 525 in the NTW frame, and y_m to be the orbit fit for each density model m . The residuals
 526 for each component of the orbit and orbit fit are then calculated (in terms of the in-track
 527 component) as,

$$R_{m,T} = y_{o,T} - y_{m,T} \quad (3)$$

528 The root-mean-square error (RMSe) of the residuals represents the square root of the
 529 variance of the absolute difference in the two orbits, indicating how well the density model's
 530 orbit fit matches the PSO for that arc. For the in-track component, this is computed for
 531 every i^{th} time step of an arc with n time steps as,

$$\text{RMSe}_{m,T} = \sqrt{\frac{1}{n} \sum_i^n (y_{o,T,i} - y_{m,T,i})^2} \quad (4)$$

Figure 2 provides an example of the observation residuals for sample orbits over four,

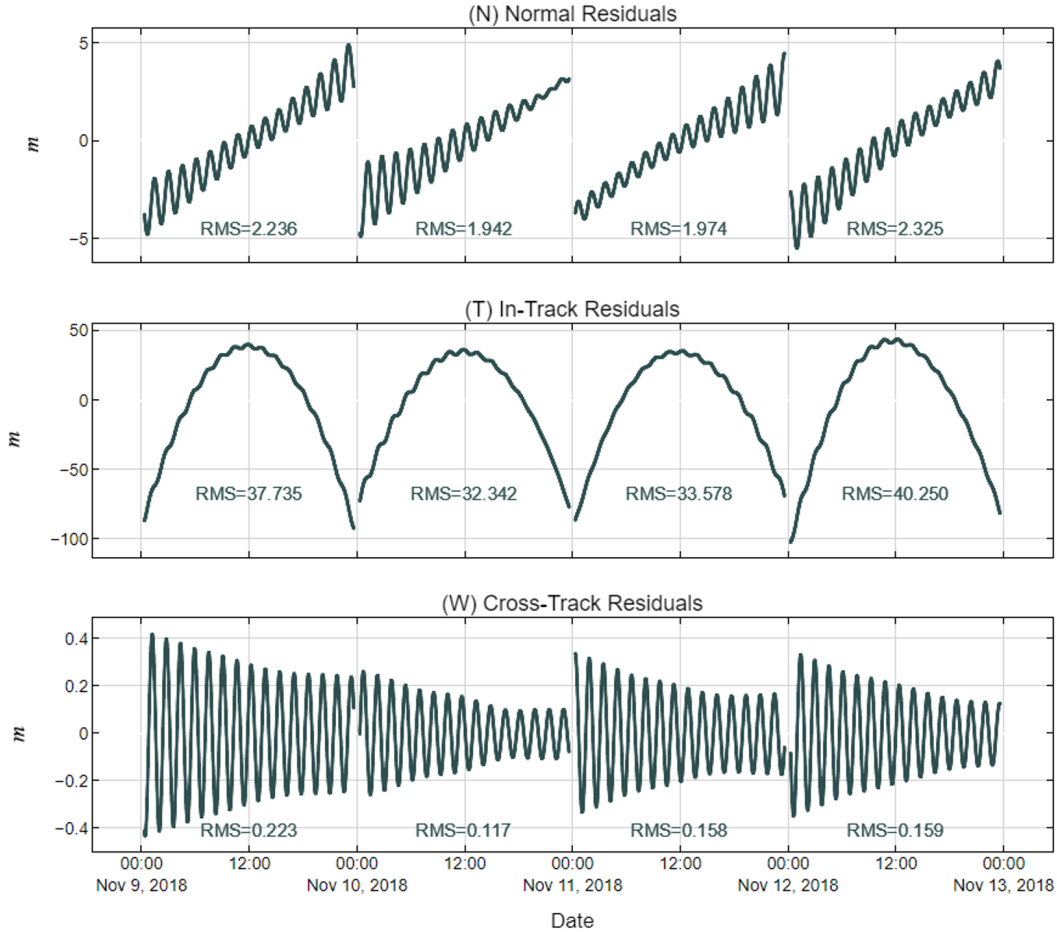


Figure 2. Depicted here are example observational residuals for each component of the NTW system across four 24-hour arcs using the MSIS2 model. The RMSe for each arc/component is given under each curve, showing that the majority of the residual variance (i.e. the orbit error due to drag) is contained within the in-track direction. Note that the vertical scale is different for each plot.

533 24-hour arcs. The normal \hat{N} (top panel) and cross-track \hat{W} (bottom panel) residuals are
 534 included in this figure only to demonstrate that the in-track component contains the ma-
 535 jority of the variance associated with residuals in this reference frame. The RMSe val-
 536 ues for each component and arc are included as an overlay.

537 During the POD process, GEODYN iteratively minimizes the discrepancies between
 538 the observed orbit (i.e., PSO) and computed orbit (i.e., orbit fits from a density model)
 539 across the entire 24-hour arc by adjusting the initial conditions to converge towards a
 540 computed trajectory. Since the minimization occurs across the entire arc, the resulting
 541 residuals take on the non-linear shapes shown in Figure 2. On a given arc and when com-
 542 paring the resulting orbit fits from each density model, the only variable that has been
 543 permitted to impact each orbit fit’s performance relative to the PSO is the drag effects
 544 from the selected density model. Therefore, we reason that the relative differences in the
 545 residuals for each orbit fit is indicative of density model performance. Other potential
 546 errors from mismodeled physics may persist in the residuals, but they are held constant
 547 between each model run and will impact the orbit fits consistently. If we run the same
 548 arc using the same force model and conditions, but only change the density model used
 549 to calculate satellite drag accelerations, the residuals will contain the errors related to
 550 the program attempting to reconcile errors in the density model. The RMSe for each or-
 551 bit fit represents a single value for how well the program can reconcile each model’s er-
 552 rors in density over the entire 24-hour arc.

553 As described in Section 4.2, the ICESat-2 PSO has been shown to have a radial or-
 554 bit accuracy of below 1.5 cm, generated through the reduction of GNSS double-difference
 555 carrier phase observable residuals and independently assessed using SLR measurement
 556 residual analysis (Thomas et al., 2021). The precision of the orbit solutions were also
 557 verified in all three components using orbit overlap analysis. Given this, relative devi-
 558 ations from the low-error PSO are treated as a rough proxy for the density model errors
 559 relative to some unknown true density. The true density value is obscured by the var-
 560 ious interconnected unknowns of C_D , SRP, and ERP and therefore remains unspecified.
 561 Over the course of a single arc, drag forces from each density model dissipate the satel-
 562 lite’s orbital energy at distinct rates, resulting in drag accelerations that are either greater
 563 or less than what is represented by the in-track position of the PSO. A strongly nega-
 564 tive in-track residual indicates a modeled density that is larger than truth, while a strongly
 565 positive in-track residual indicates a modeled density that is smaller than truth. Addi-
 566 tional details regarding the shape of the in-track residuals and the relationship between
 567 in-track position of the PSO and orbit fits and the density can be found in Figure A2
 568 of Appendix A.

569 The RMSe is the standard deviation of the residuals and serves as a measure of the
 570 difference between a respective orbit fit and the PSO over a single whole arc. Theoret-
 571 ically, an in-track RMSe of zero would mean no difference between an orbit fit and the
 572 PSO, indicating near-perfect agreement on average between the modeled density and the
 573 POD-based true density across the 24-hour arc. In this setup, perfect agreement for any
 574 model is unlikely since the residuals may additionally contain errors related to mismod-
 575 eled forces, as well as bias/offsets related to fixing the C_D to a common value for all mod-
 576 els. A further limitation of the metric is that the RMSe lacks information regarding timescales
 577 less than the arc length, and is unsigned, meaning it does not indicate if the modeled
 578 density is above or below the truth for a given arc. For these reasons, the in-track resid-
 579 uals and their respective RMSe values are assessed in conjunction with each other.

6 Results and Discussion

580 This section is organized as follows: (1) the preliminary method for orbit fit con-
 581 struction using a fixed $C_D = 2.5$ is presented, and the orbit fit method is verified us-
 582 ing the SET-HASDM density database to determine baseline understanding; (2) an as-
 583 sessment of the semi-empirical and physics-based models is presented via orbit fit results
 584 that are debiased using a mean drag acceleration scaling factor over the full two week
 585 period; (3) an assessment of the semi-empirical and physics-based models is presented
 586 via orbit fit results that are debiased using a 24-hour drag acceleration scaling factor.

587 The specific conditions for producing density values for each model are detailed in
 588 Section 4.3. The authors acknowledge that while there are a number of ways to improve
 589 a density model’s outputs at runtime (see Sutton (2018); Shim et al. (2014)), the out-
 590 puts used in this work are intended to reflect typical community use, with each model
 591 being run according to the developer’s operational instructions. Results are presented
 592 by focusing on a two week time period from 9 November 2018 - 23 November 2018, pro-
 593 viding 14 adjacent daily arcs with no maneuver-based data gaps. The assessment con-
 594 ditions are for the altitude regime near ~ 490 km, in an atmosphere with very low so-
 595 lar flux, and low-to-minor geomagnetic activity. Note that this is a notoriously difficult
 596 altitude regime and activity condition for empirical models due to the minimal access
 597 to satellite density data at this altitude during times of prolonged solar minimum—increasing
 598 the potential value of this style of assessment for these models especially. Figure 3 shows
 599 low solar activity for the time period, both in terms of the magnitude and variation of
 600 solar EUV and FUV, as approximated by measurements of the 10.7 cm solar radio flux
 601 ($F_{10.7}$, top panel). The Kp geomagnetic index (bottom panel) depicts low-to-minor ge-
 602 omagnetic activity during the time of interest, with the two-week period being bookended
 603 by minor geomagnetic disturbances which reach no higher than $Kp = 4.3$. Only one
 604 minor-to-moderate disturbance occurs on 5 November 2018, four days before the period
 605 of interest, reaching a peak of $Kp = 5.7$. This event is mentioned only because of the
 606 possibility that its impact could be seen represented in the models as a density enhance-
 607 ment due to delayed heating and cooling effects.

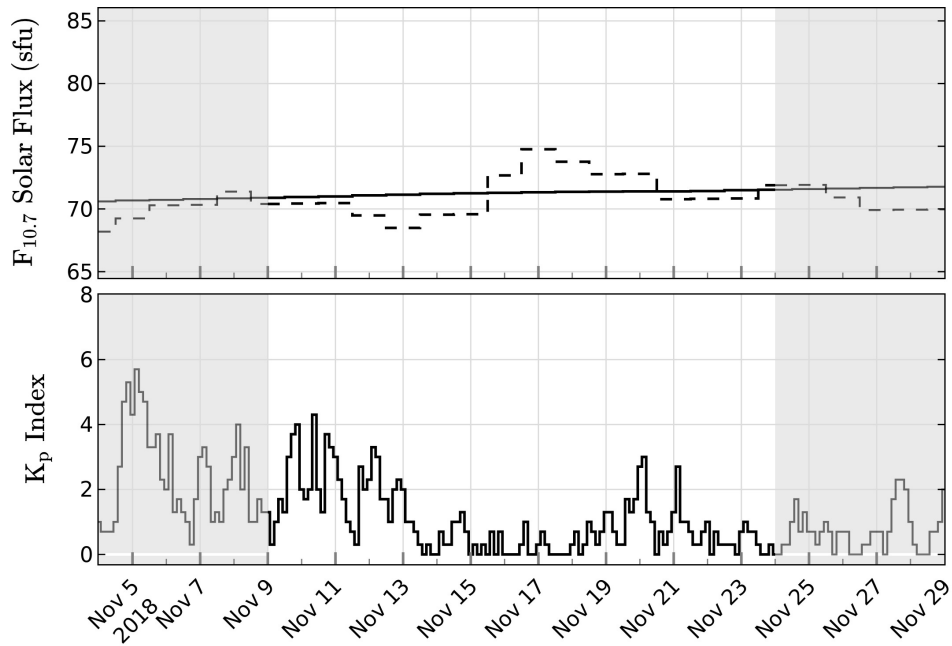


Figure 3. Top: observed solar $F_{10.7}$ radio flux. The dashed curve is the daily measured value from the Ottawa observatory normalized to 1 AU sun-earth distance; the solid curve is an 81-day (3 solar rotation) centered average. Bottom: the 3-hourly planetary magnetic index, K_p . Both panels depict the period of interest from 9 November 2018 - 23 November 2018. A few days before and after the period of interest are depicted in the shaded portions.

6.1 Preliminary Orbit Fits using a Fixed C_D of 2.5

608 The most straightforward way to construct the orbit fits is by calculating the ac-
 609 celeration due to drag from different density models using a fixed drag coefficient value
 610 for all arcs and models. This permits bias depending on the relationship between each
 611 model’s mean density and the chosen C_D , but importantly demonstrates that the method
 612 is sufficiently sensitive for recognizing differences in the drag effects between the mod-
 613 els. The in-track residual errors in this method should not be interpreted as indication
 614 of performance, but rather indicators of each model’s mean density offset relative to the
 615 true-unknown density. Figure 4, shows the fixed C_D assessment results for the semi-empirical
 616 and physics-based models. The top panel shows each model’s orbit averaged density along
 617 the orbit of ICESat-2, the middle panel shows the in-track residuals for each model and
 618 arc, and the bottom panel shows the in-track RMSe values.

619 The negative parabolic shape of the in-track residuals of MSIS2, TIEGCM, and JB2008
 620 indicate that these modeled densities are too high—i.e., these orbit fits experience more
 621 drag acceleration and their fits tend to lag behind the PSO. The positive parabolic shape
 622 of the in-track residuals of DTM2020 and CTIPe indicate modeled densities that are too
 623 low—i.e., the drag acceleration is lower and the orbit fits tend to be in front of the PSO.
 624 In reality the PSO-to-orbit fit relationship is slightly more complex over an arc, with the
 625 above being a generalization of the overall trend. A more detailed understanding of the
 626 orbit fit movement relative to the PSO can be found in Appendix A.

627 The orbit fits from SET-HASDM are separated for use as verification since it uses
 628 similar assumptions of a fixed drag coefficient, and a satellite drag data assimilation tech-
 629 nique in its internal workings. The SET-HASDM density database affords the oppor-
 630 tunity to access historical records of HASDM densities that have been corrected through
 631 the real-time data-assimilative calibration to ~ 80 low earth orbiters. The HASDM model
 632 is the operational standard used by the 18th Space Defense Squadron which is tasked
 633 with executing command and control over United States’ space assets and all resident
 634 space objects for sake of space situational awareness. Verification with SET-HASDM pro-
 635 vides a baseline understanding of the fidelity of the orbit fit results. Since the HASDM
 636 density values have already been effectively debiased in its data-assimilation scheme, we
 637 do not go through the steps of debiasing using the methods presented in Sections 6.2 and
 638 6.3. Referring to Figure 5, the HASDM model consistently depicts in-track RMSe val-
 639 ues that are on the order of 8.18 meters over the two-week period. The in-track resid-
 640 uals have a negative shape, indicating that the densities from SET-HASDM are slightly
 641 larger than what would be expected from the PSO. The results in Figure 5 are intended
 642 to serve as an approximate consistency check that our overall methodology, and more
 643 specifically our debiasing method, provide orbit fits with in-track errors in a reasonable
 644 range.

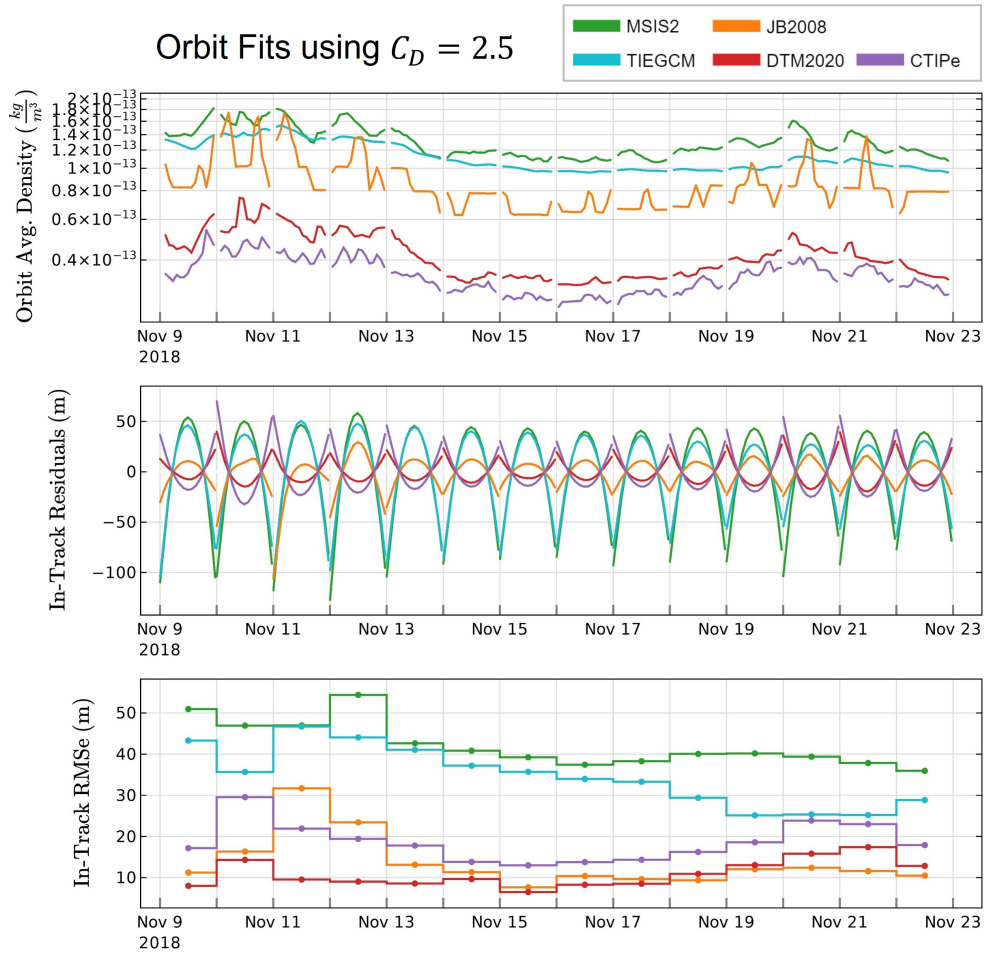


Figure 4. Assessment results given a fixed, common $C_D=2.5$ for MSIS2 (green), DTM2020 (red), JB2008 (orange), TIEGCM (cyan), and CTIPe (violet) during the two week time period containing 14, 24-hour arcs. Top: Orbit average neutral densities along the ICESat-2 orbit for each model. Middle: In-track orbit residuals for each arc. Bottom: In-track RMSe for each arc’s in-track residuals.

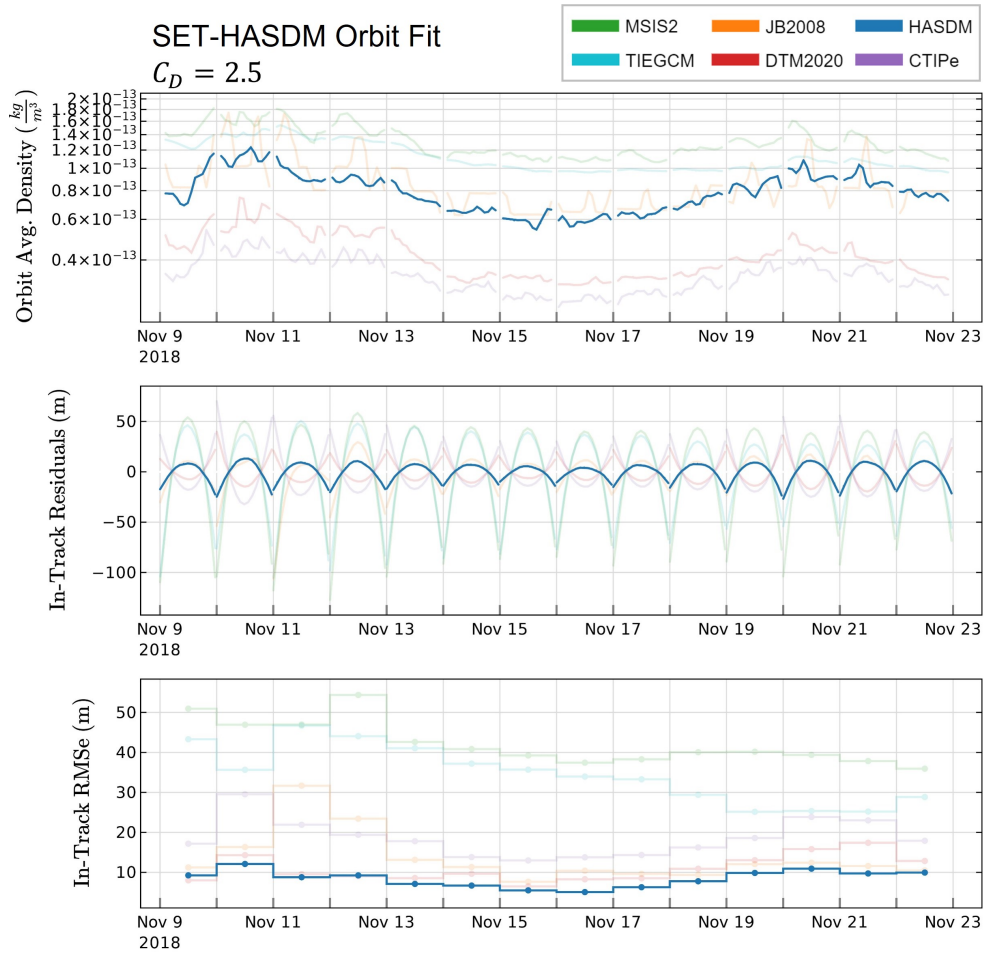


Figure 5. Verification results using SET-HASDM across 14 adjacent, 24-hour arcs from 9 November 2018 - 23 November 2018. Top: The solid blue curve depicts the neutral densities along the ICESat-2 orbit as an orbit average. Middle: In-track orbit residuals for each of the 14 adjacent, 24-hour arcs. Bottom: In-track RMSe values for each arc's in-track residuals. The range of the y-axes are chosen to facilitate comparison with Figure 4.

6.2 Debias using Two-Week Scaling Factor

645 The second orbit fit method debiases the density models using a mean-adjusted C_D
 646 over the two-week period (average values of the 24-hour adjusted C_D s shown later in Fig-
 647 ure 8). This provides a fixed C_D that is unique for each density model, adjusted to ac-
 648 count for biases due to each model’s mean density offset over the time period (see Fig-
 649 ure 4). The in-track residuals and RMSe values for this assessment, shown in Figure 6,
 650 quantify the error due to density variation over the two-week time period. The average
 651 adjusted C_D used to construct the two-week debiased orbit fits for each model is reported
 652 in Table 2.

653 As mentioned in Section 5.2, the DRIA calculations indicate that $C_D = 2.5$ is a
 654 realistic value if one assumes that a fixed energy accommodation of $\alpha = 0.89$ is reasonable—
 655 an assumption limited by lack of empirical observation. According to the DRIA model,
 656 2.5 is a realistic lower limit for the drag coefficient. Looking to Table 2, we can see that
 657 the mean adjusted C_D for MSIS2, TIEGCM, and JB2008 are all well below this lower
 658 limit, offering further evidence that these models are, on average, over-estimating the
 659 density. The upper bound is slightly more difficult to estimate in this setup, but CTIPe’s
 660 adjusted C_D of 4.3 is likely too high. This will need to be investigated further in the fu-
 661 ture.

662 After removing the bias from the models, the relative effects of the minor geomag-
 663 netic activity become more stark in the in-track residuals. Here the changing shape of
 664 the residual curves indicate whether the model is over or underestimating the effects of
 665 geomagnetic activity on the modeled density. For example, several of the models dis-
 666 play downward-pointing curves during geomagnetic activity, indicating densities that are
 667 too low as compared to their quiet time densities. DTM2020’s residuals show an anti-
 668 correlation to the geomagnetic activity, beginning with densities that are too high and
 669 ending the two weeks with densities that are too low, which may be reflective of an overly
 670 sensitive response to geomagnetic activity and the overall downward trend in Kp. The
 671 effects of a model poorly capturing density variations during geomagnetically active times
 672 are now better quantified by the in-track RMSe after two-week debiasing is applied.

Two-week Debiasing Method: $C_D = \text{Mean Adjusted } C_D$		
Model ID	Fixed C_D	Scaling Factor as % change
MSIS2	1.237	-49.861
TIEGCM	1.373	-44.899
JB2008	1.909	-22.967
DTM2020	3.351	34.903
CTIPe	4.368	75.663

Table 2. Summary of the assessment procedure assuming a fixed C_D that is equal to the average adjusted value for each model, assuming an a priori of $C_D = 2.5$. The second column reports mean-adjusted C_D used in each model’s orbit fit construction. The third column reports the two-week scaling factor as a percent change for each model.

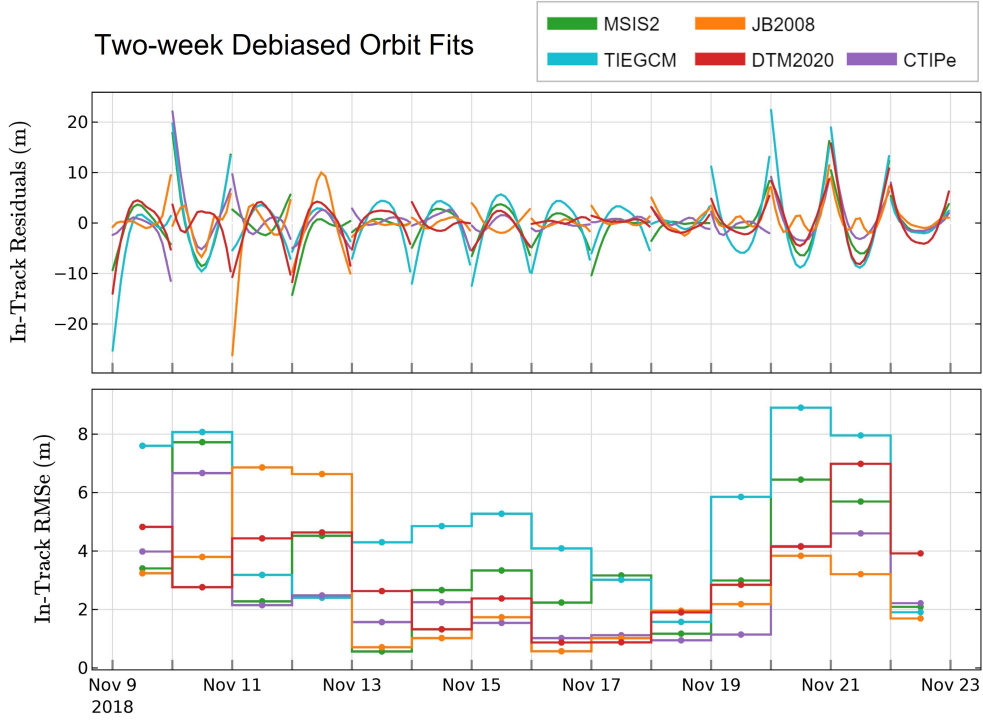


Figure 6. Assessment results for orbit fits that are debiased using two-week drag acceleration scaling factors. Top: Debiased in-track orbit residuals for each arc. Bottom: Debiased in-track RMSe for each arc’s in-track residuals.

6.3 Debias using 24-hour Scaling Factor

673 The 24-hour debiasing procedure described in Section 5.2, is used to scale the orbit
 674 fits and their residuals to a daily cadence. Figure 7 presents the resulting in-track
 675 residuals (top panel) and in-track RMSe values (bottom panel) for each 24-hour arc for
 676 each model. The 24-hour drag acceleration scaling factor is derived by adjusting the C_D
 677 from the a priori of 2.5 over each daily arc, absorbing the average density offset for that
 678 day. The debiasing effect is seen in the overall reduction in residual error from Figure
 679 4 to Figure 7. The calculated 24-hour scaling factors are presented in the top panel of
 680 Figure 8 as a percent change from the fixed value of 2.5. The bottom and right panels
 681 show the Kp index and Pearson’s correlation coefficient between each model’s scaling
 682 factors and the Kp , respectively.

683 The 24-hour scaling accounts for both the overall model bias and uncertainties in
 684 the density on timescales that are on the order of, or greater than, the chosen arc length
 685 of 24-hours (i.e. combination of mean density offset and daily geomagnetic variation).
 686 The remaining error depicted by the in-track residuals of Figure 7 are likely due to higher
 687 frequency variations in density that are not captured by the 24-hour debiasing (e.g. day-
 688 night variations in the neutral density). The correlation between Kp index and the scal-
 689 ing factors demonstrates how this metric can be used to determine how well a model ac-
 690 counts for geomagnetic activity. As shown by the scaling factors in Figure 8, MSIS2 ($R =$
 691 0.35), TIEGCM ($R = 0.29$), and CTIPe ($R = 0.19$) all exhibit a subtle positive correla-
 692 tion, indicating a slight underestimation of density enhancements from geomagnetic
 693 activity and resulting in the scaling factors being used to compensate for these errors.
 694 Contrarily, the inverse relationships shown by DTM2020 ($R = -0.36$) and JB2008 ($R =$

695 -0.3) indicate an overestimation of these models' densities during days of increased ac-
 696 tivity.

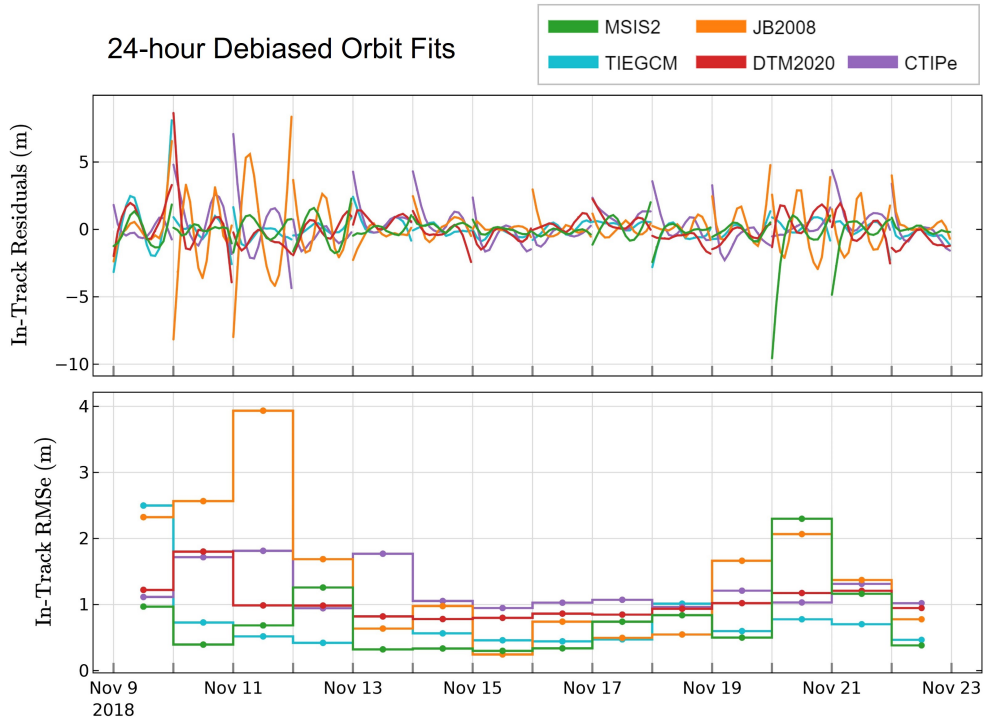


Figure 7. Assessment results for orbit fits using 24-hour drag acceleration scaling factors. Scaling factors are extracted in the least squares orbit fitting procedure by allowing the C_D to adjust once-per-arc to absorb observed errors between the PSO and the converging orbit fit. Top: In-track orbit residuals for each arc. Bottom: In-track RMSe for each arc's in-track residuals.

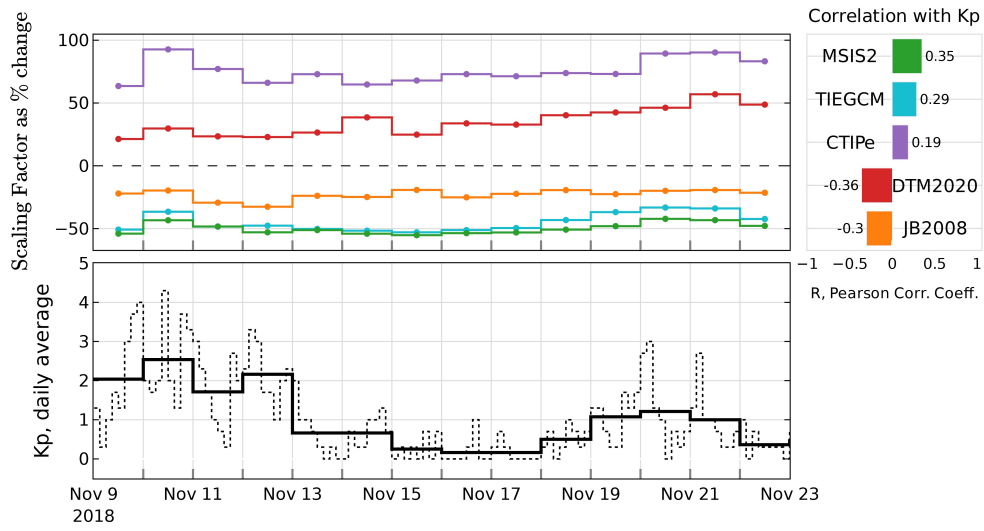


Figure 8. Top: Drag acceleration scaling factors extracted from the orbit fits shown in Figure 7, presented as a percent change from the fixed $C_D = 2.5$. Bottom left: The K_p index for the time period. Bottom right: Pearson’s correlation coefficient between the scaling factors and K_p .

6.4 Discussion

697 While the in-track residuals of the ICESat-2 orbit fits offer an effective means of
 698 assessing the density models, most methods that use drag acceleration to study density
 699 are going to be limited by the complex interconnected uncertainties in the C_D and the
 700 density. For these reasons, we split our overall assessment into the three methods pre-
 701 sented, each of which help to further illuminate the performance of the models. For sake
 702 of brevity, only JB2008 and TIEGCM are given in-depth discussions that synthesize an
 703 understanding of model performance from their results. The reader will then be able to
 704 apply these discussions to the remaining models, which here are discussed in more gen-
 705 eral terms.

706 Considering the cumulative results for JB2008 (plotted in orange for all relevant
 707 figures), Figure 4 shows that with a fixed C_D of 2.5, the in-track residuals exhibit a neg-
 708 ative parabola shape, indicating that the JB2008 density provides larger drag accel-
 709 erations than what the PSO experiences. Figure 6 provides the assessment method in which
 710 the JB2008 densities are effectively scaled by $\sim -22\%$ for the full two-week period. In
 711 this case the in-track residuals better highlight the arcs in which JB2008 performs poorly
 712 relative to the other models, specifically on November 11th and 12th when the geomag-
 713 netic activity fluctuates around $Kp = 3$ after having been moderately elevated for sev-
 714 eral days. In the 24-hour debiased case, JB2008's daily scaling factors (shown in orange
 715 in Figure 8) effectively compensate for density variations on the order of or greater than
 716 24-hours. The mean density offset adjustment is $\sim -22\%$, and is seen to be inversely cor-
 717 related ($R = -0.3$) with the geomagnetic activity—indicating that JB2008 tends to over-
 718 estimate densities during active times. This effect can be clearly seen in JB2008's orbit
 719 averaged densities, represented by the orange line in the top panel of Figure 4, where
 720 the model provides much sharper density peaks than the other models during the times
 721 of slightly elevated geomagnetic activity. This effect is also clearly represented in the 24-
 722 hour debiased in-track RMSe values (bottom panel of Figure 7), which shows significant
 723 variance during active times between the PSO and the JB2008 orbit fit, even after the
 724 24-hour scaling. The higher RMSe values shown in Figure 7 on November 9th-13th in-
 725 dicate that JB2008's overestimation of density during active times is not sufficiently com-
 726 pensated by the 24-hour scaling factor, meaning that the variation likely occurs at a higher
 727 frequency. JB2008's estimation of quiet time density is among the best in this report,
 728 needing $\leq -22\%$ adjustment from the fixed C_D case to provide orbit fits that match the
 729 PSO to within 1 meter (during quiet times).

730 Considering the cumulative results for TIEGCM (plotted in cyan for all relevant
 731 figures), Figure 4 shows that with a fixed C_D of 2.5, the in-track residuals exhibit a neg-
 732 ative parabola shape, indicating that the TIEGCM density provides larger drag accel-
 733 erations than what the PSO experiences (i.e. the TIEGCM densities are too high). TIEGCM
 734 offers interesting results from Figure 6 of the two-week scaled case. The TIEGCM den-
 735 sities are effectively scaled down by $\sim -45\%$ over the two-week period to compensate for
 736 mean density offset. Since this value is the two week average from the scaling factors shown
 737 in Figure 8, it is skewed to only partially compensate for lack of geomagnetic sensitiv-
 738 ity (i.e., densities are scaled to be too low in active times, increase in RMSe on the book-
 739 ends of the period) and partially compensate for mean density offset in quiet times (den-
 740 sities are scaled to be too high in quiet times, increase in RMSe from Nov. 13th to 17th).
 741 The two-week scaled case is able to clearly show in the in-track residuals that TIEGCM
 742 struggles more than the other models to properly capture variation during the period
 743 of this study. TIEGCM's daily scaling factors (cyan line in Figure 8) effectively com-
 744 pensate for error in density variations on the order of or greater than 24-hours. The daily
 745 variation of the scaling factors is found to slightly correlate with the Kp ($R = 0.29$)—
 746 indicating that TIEGCM tends to subtly underestimate densities during active times.
 747 This is seen in TIEGCM's orbit averaged densities (cyan line in the top panel of Fig-
 748 ure 4) where the model provides significantly less sensitivity to the times of slightly el-

749 evated geomagnetic activity, and less variation overall. The 24-hour debiased in-track
 750 RMSE values (bottom panel of Figure 7), interestingly show very low variance between
 751 the PSO and the TIEGCM orbit fit after the 24-hour scaling. This is most likely explained
 752 by the overestimation of density during active times being sufficiently compensated for
 753 by the 24-hour scaling factors despite its orbit average densities seeming to lack much
 754 variation at all. This adds further suspicion to the higher frequency variations seen in
 755 models such as JB2008. It is possible that by moving to shorter arc lengths, such as 3-
 756 hours, the time series of scaling factors could better capture these variations, and this
 757 is a future goal of this work.

758 In general, MSIS2 and TIEGCM overestimate the density for all arcs, requiring $\sim -50\%$
 759 and $\sim -45\%$ scaling factors, respectively, to bring the in-track residuals to within two
 760 meters. DTM2020 and CTIPe both underestimate the density for all arcs, each requir-
 761 ing a $\sim 35\%$ and $\sim 76\%$ increase, respectively. JB2008 requires the least overall scaling,
 762 requiring only $\sim -23\%$ to bring the in-track residuals to within two meters during quiet
 763 times. All models capture the geomagnetic activity relatively well as demonstrated by
 764 their scaling factors not being very highly correlated to Kp. DTM2020 ($R = -0.36$) and
 765 JB2008's ($R = -0.3$) scaling factors are inversely correlated to Kp , indicating a slight
 766 over-sensitivity to geomagnetic activity during this time period, while TIEGCM ($R =$
 767 0.29), MSIS2 ($R = 0.35$), CTIPe ($R = 0.19$) all indicate an under-sensitivity. The scal-
 768 ing undergone for each model produces RMSE values that are comparable to that of the
 769 SET-HASDM orbit fit, which was separated out to serve as an approximate consistency
 770 check of our debiasing method due to its data-assimilative technique.

7 Conclusions and Future Work

771 This work presents the development of a modernized interface for the GEODYN-
 772 II POD software. The approach leverages the high-precision nature of space geodetic POD
 773 and an upgraded utility of the neutral density models to focus POD methods toward study-
 774 ing satellite drag and conducting density model assessment. The assessment method uses
 775 high-fidelity PSO as observed tracking measurements that are input into POD-based or-
 776 bit fits. The drag effects from each density model are assessed according to each model's
 777 ability to redetermine the satellite's orbit. Each density model's orbit fit contains rel-
 778 ative in-track deviations from the PSO which are treated as a proxy for model densities
 779 that differ from a true, unknown, density. These deviations are quantified with the in-
 780 track residuals and their RMS errors. We demonstrate the capabilities of this tool via
 781 a case study assessment of five thermospheric density models (MSIS2, DTM2020, JB2008,
 782 TIEGCM, and CTIPe, and a verification using SET-HASDM) using the ICESat-2 mis-
 783 sion PSO as the observed measurements. Preliminary orbit fits are constructed after de-
 784 termining a mean C_D from a physics-based solution. A fixed C_D of 2.5 is applied for all
 785 models before being debiased by adjusting the C_D to account for density errors in the
 786 drag acceleration. The debiasing is performed at two different cadences, 24-hours and
 787 two-weeks, with each method highlighting different temporal aspects of the model den-
 788 sity errors. The scaling factors extracted from the 24-hour and two-week debiasing meth-
 789 ods are well-equipped for use in improving forecasting and modeling methods. The 24-
 790 hour scaling factors provide a more accurate representation of the true density variations
 791 for each model, while the two-week scaling factors are computationally simpler and in-
 792 dicate more baseline density effects. In addition, the two-week extended time period scal-
 793 ing factors are compatible for parsing into the CCMC's CAMEL database to move in
 794 the direction of community-oriented model validation.

795 We continue our efforts on this project as we move in the direction of offering a more
 796 robust thermospheric model validation scheme. Possible improvements include improv-
 797 ing the non-conservative force modeling in GEODYN for ICESat-2 using a more real-
 798 istic 3-D model of the satellite shape that would account for self-shadowing and varia-
 799 tions in cross-sectional area with incidence angle e.g, as in March et al. (2019). The or-

800 bit determination for the primary science orbits, and the subsequent analyses described
 801 in this paper would have to use these improved geometry models. A further improve-
 802 ment would be to incorporate SLR measurements of ICESat-2 into the evaluations of the
 803 density models. One could include the SLR data along with the PSO trajectory data in
 804 the evaluation. See Thomas et al. (2021) for a description of these data for ICESat-2.
 805 Planned future work involves addressing the key constraints highlighted in the method-
 806 ology, the foremost of which is the need to evaluate the drag coefficient more frequently
 807 along the ICESat-2 orbit. Future work will also involve expanding the study to encom-
 808 pass the entire ICESat-2 mission time period. Additional expansion includes incorpo-
 809 rating additional satellites and constellations that may illuminate model performance
 810 within atmospheric regimes that lack observations of neutral density. We aim to make
 811 our expanded results available through the CCMC’s CAMEL framework as well as through
 812 future publications.

813 The assumptions made in this paper are limited by the current status of unknowns
 814 between gas surface interaction research and thermospheric variability research. At this
 815 time, the true drag coefficient is not known for any satellite, and modeling the C_D will
 816 always introduce some inherent bias into the results. We aim to address this issue by im-
 817 plementing several of the satellite gas-surface interaction models currently used in the
 818 satellite drag community to calculate the time and compositionally-dependent drag co-
 819 efficient. Isolating the effects of the C_D will aid to better identify the various non-density
 820 related errors that may be present in the in-track residuals. Being able to distinguish
 821 these errors and accurately quantify the amount of deviation introduced by a given den-
 822 sity model will provide significant insight regarding model performance to the earth-space
 823 environment modeling community. As the ability to model C_D improves, the results pro-
 824 vided by this method will similarly become more valid. The Geospace Dynamics Con-
 825 constellation (GDC) is an upcoming NASA mission that is intended to help fill in the gaps
 826 of understanding gas-surface interactions by providing a stable platform with full mea-
 827 surements of neutral composition, density, and temperature along with a high fidelity
 828 POD in which cross validation of density model assessment is possible. As a result, in
 829 addition to providing its own neutral density observations that can be used for research,
 830 operations, and model validation, these advances expected from the GDC mission will
 831 improve the accuracy and usability of density proxies derived from POD solutions like
 832 those used here. These advances will effectively multiply our density observations to be
 833 able to use any satellite with sufficiently accurate GNSS positioning and knowledge of
 834 spacecraft parameters as a density observing platform.

835 This work provides a step in the direction of being able to use high-fidelity GNSS-
 836 enabled LEO satellite POD solutions to objectively quantify and validate thermospheric
 837 model performance. The strength of assessment using this method is its ability to iden-
 838 tify relative accuracy of the models in a way that is directly tied to operational use for
 839 orbit propagation. There are a multitude of uses for the tools and methods presented
 840 in this work, such as for density retrievals along the orbit of a satellite, which is a planned
 841 future effort; however, this report focuses specifically on model assessment. As work con-
 842 tinues to refine these methods and address the caveats presented in this paper, the re-
 843 sults of model assessments using this technique will continue to become better suited to
 844 aid satellite operators when choosing a model that will perform best under specified con-
 845 ditions. Having a multitude of methods for assessing upper atmospheric models under
 846 various conditions helps model developers refine the models themselves, making them
 847 better suited for orbit prediction.

8 Open Research

848 The ICESat-2 POD solutions, their corresponding setup files, and the GEODYN
 849 II software are provided by the Geodesy and Geophysics Laboratory within NASA-GSFC.
 850 Simulation results for the CTIPE model have been provided by the Community Coor-

851 dinated Modeling Center at Goddard Space Flight Center through their publicly avail-
852 able simulation services (<https://ccmc.gsfc.nasa.gov>). Orbit fly-throughs of the TIEGCM
853 simulation results and relevant codes used to produce the results in this paper are avail-
854 able at Zenodo via <https://doi.org/10.5281/zenodo.8015368> (Waldron et al., 2023). The
855 SET HASDM density data are provided for scientific use by Space Environment Tech-
856 nologies.

857 **Acknowledgments**

858 The authors acknowledge the generous support of the Community Coordinated Mod-
859 eling Center under grant number 80NSSC21K1747 and support from the University of
860 Colorado at Boulder Grand Challenge Grant: Space Weather Technology, Research, and
861 Education Center (SWx TREC). This material is partially based upon work supported
862 by the National Aeronautics and Space Administration under Grant 80NSSC21K1554
863 issued through the Heliophysics Division Space Weather Science Application initiative
864 and the Geospace Dynamics Constellation Contract No. 80GSFC22CA012. The authors
865 acknowledge the CCMC-Kamodo Team who designed the Kamodo API, permitting the
866 ability to interface physics-based model outputs to the POD scheme.

Appendix A Coordinate System to Study Drag

867 GEODYN's input and output trajectories make use of the J2000 inertial reference
 868 system. The \hat{X} , \hat{Y} , and \hat{Z} components of the inertial coordinate system offer limited in-
 869 formation on how a satellite's orbit is impacted by atmospheric drag, leading us to con-
 870 vert to the more suitable Satellite Coordinate System. Two coordinate frames suited for
 871 this assessment are the NTW and RSW frames, with differences between the two being
 872 highlighted in Figure A1. We make use of the NTW system, which aligns with the or-
 873 bit plane and is composed of an in-track component \hat{T} that is parallel to the velocity vec-
 874 tor \vec{v} , a normal component \hat{N} that is perpendicular to the velocity and nominally in the
 875 radial direction, and a cross-track component \hat{W} that is normal to the orbit plane and
 876 completes the right-hand coordinate frame. Being parallel to the velocity vector means
 877 that the in-track component \hat{T} will contain any indication that the spacecraft's trajec-
 878 tory has changed since orbital energy dissipations from drag will impact in the velocity
 879 direction.

880 Figure A2 contains additional visualization related to the shape of the in-track resid-
 881 uals and how it relates to the movement of the PSO relative to the orbit-fit satellite. The
 882 overall shape of the in-track residuals is a result of the batch-least squares fitting rou-
 883 tine as it attempts to minimize the distance between the PSO and the orbit fit across
 884 the whole arc.

885 Variations in the in-track component \hat{T} are not the same as variations in the along-
 886 track component \hat{S} of the RSW system. In-track variations act in the direction of the
 887 velocity vector, whereas along-track variations are merely along, but not necessarily par-
 888 allel, to the direction of the velocity vector. We make the distinction to use the NTW
 889 system rather than the RSW system whose radial component is often used to assess or-
 890 bit accuracy in geodetic POD studies. The NTW coordinate system is described in (Vallado,
 891 2013) to have the following unit vectors and transformation:

$$892 \quad \hat{T} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad (\text{A1})$$

$$893 \quad \hat{W} = \frac{\mathbf{r} \times \mathbf{v}}{|\mathbf{r} \times \mathbf{v}|} \quad (\text{A2})$$

$$894 \quad \hat{N} = \hat{T} \times \hat{W} \quad (\text{A3})$$

$$895 \quad \mathbf{r}_{XYZ} = \begin{bmatrix} \hat{N} \\ \hat{T} \\ \hat{W} \end{bmatrix} \mathbf{r}_{NTW} \quad (\text{A4})$$

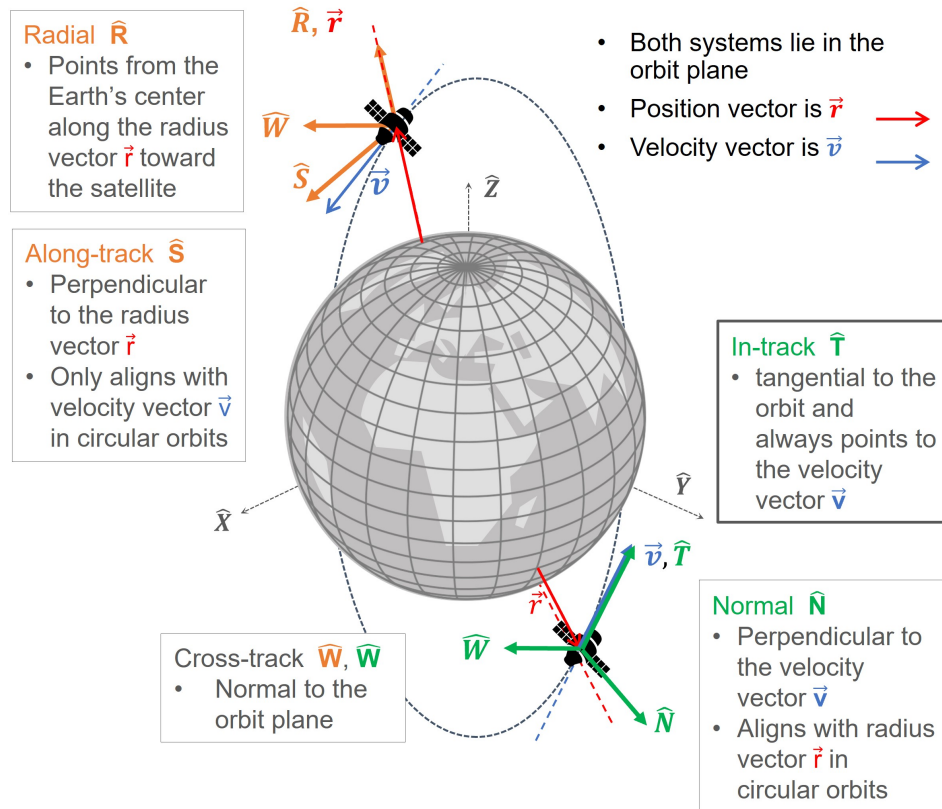
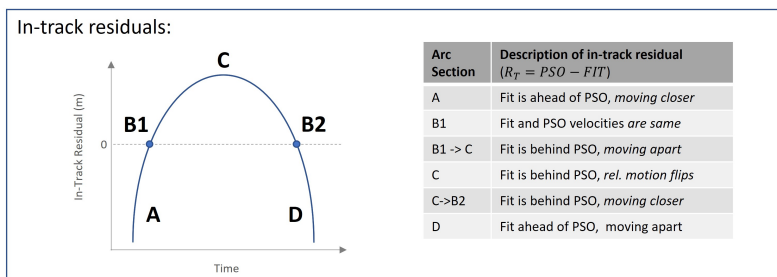


Figure A1. The above is a schematic showing the NTW and RSW satellite coordinate systems and details regarding their components. The NTW system's in-track component is parallel to the velocity vector, making it an effective tool for assessing relative effects due to atmospheric drag.

Scenario: Density from model is higher than truth

Fit orbit experiences higher drag acceleration than PSO



In-Track residuals, visualized: $Residual = PSO - FIT$ (note in-track component is parallel to velocity vector)

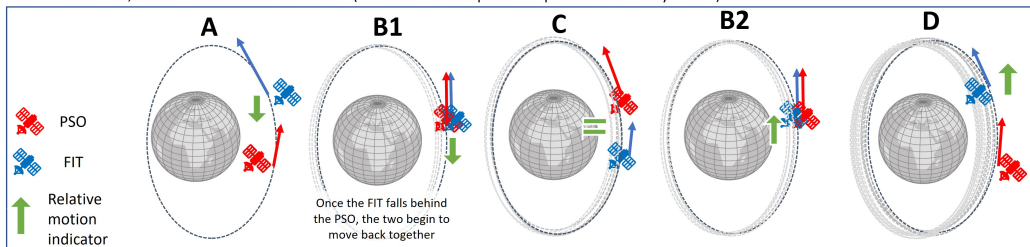


Figure A2. Above is a diagram which explores the circumstances that result in the parabolic shape of the in-track residuals. Representative in-track residuals for a single arc is given as a negative quadratic curve in the top panel. The bottom panel provides corresponding frames of schematics for each marked point to depict how the orbit fit and PSO are positioned relative to each other.

Appendix B GEODYN Run Setup for Orbit Fits

896 This section provides a summarized overview, in the form of Table B1, of the run
897 setup used in GEODYN to produce the orbit fits. A similar overview is given in Table
898 1 of Thomas et al. (2021), which details the most relevant and important constants, mod-
899 els, and standards used to produce the ICESat-2 PSO.

 GEODYN's Setup to Produce POD-based Orbit Fits

Satellite geometry	Panel model based on pre-launch geometry and satellite surface optical properties. 14 panels make up a Box Wing model to calculate time varying area.
ICESat-2 Attitude	Telemetered spacecraft body-fixed reference frame to inertial reference frame quaternions. Telemetered solar array drive angles (for force modeling)
Non-Conservative Forces	
Atmospheric Density Models	Modified for comparison; see Table 1 for the list of assessed density models
Earth Radiation	Knocke 2nd degree zonal spherical harmonic of Earth's albedo and emissivity (Knocke et al., 1988)
Solar Radiation Pressure	Solar radiation incident on plate model (Luthcke et al., 2019; Marshall & Luthcke, 1994)
Conservative Forces	
Geopotential gravity	EIGEN6C, tide-free (Foerste et al., 2014)
Time variable gravity	Contribution from atmosphere, non-tidal oceans, hydrology, and ice; Developed from GRACE models
Earth, Pole, and Ocean tides	IERS2010 Conventions (Petit & Luzum, 2010)
Planetary ephemerides (N-Body)	JPL DE430 (Folkner et al., 2014)
Relativistic corrections	IERS2010
General Reference Frame and Constants	
Conventional inertial system	J2000 geocentric; mean equator and equinox of 2000 JAN 01 12:00:00; IERS2010
Precession - Nutation	IAU 2000A precession-nutation model
Earth Orientation Parameters	IERS 08 C04 (Bizouard & Gambis, 2011), IERS2010 conventions for diurnal, semidiurnal, and long period tidal effects on polar motion and UT1
Numerical integration	Cowell predictor-corrector; fixed and variable step; equations of motion and variational equations.
Estimation method	Partitioned Bayesian least squares.
GEODYN Controlled Setup Information	
Tracking data type	PCE (orbit trajectory) using ICESat-2 PSO
POD technique	Dynamic data reduction (no empirical accelerations)
Arc Length	24 hours
Adjusted Parameters	Initial conditions only: $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$
Force model parameters	$C_D = \text{fixed}; C_R = 1$ (not adjusted)
Integration/orbit step	10 seconds

Table B1. A summarized overview of the GEODYN run setup for using the program to conduct density model assessment. Many of the above parameters are summarized from (Thomas et al., 2021) and (Luthcke et al., 2019).

Appendix C Orbit Uncertainty Interpolation Technique and the Kamodo Interface

900 Kamodo is a CCMC tool for access, interpolation, and visualization of space weather
 901 models and data in Python (Ringuette et al., 2023). Kamodo allows model developers
 902 to represent simulation results as mathematical functions which may be manipulated di-
 903 rectly by end users. Kamodo handles unit conversion transparently and supports inter-
 904 active science discovery through Jupyter notebooks with minimal coding in Python. Kamodo
 905 is chosen for this project due to its ability to offer model agnostic methods for reading
 906 data output from different model sources. Kamodo is called using its Satellite Flythrough
 907 capabilities, in which a user is able to sample the models with satellite ephemeris and
 908 return requested values from the chosen model. The orbit is pre-initialized in GEODYN
 909 using MSIS2 to get an a priori estimate for the orbit coordinates. Then using the a pri-
 910 ori orbit, extend out the uncertainty of the coordinates to create a cube of possible val-
 911 ues centered on the orbit. This approach accounts for possible model output differences
 912 as the orbit iteratively converges towards a solution. Finally, we plug the orbit and its
 913 uncertainty cubes into Kamodo to interpolate the model densities at all requested points.
 914 By doing this, the orbit density values from the physics model can be quickly ingested
 915 into the POD program. Figure C1 visualizes this procedure.

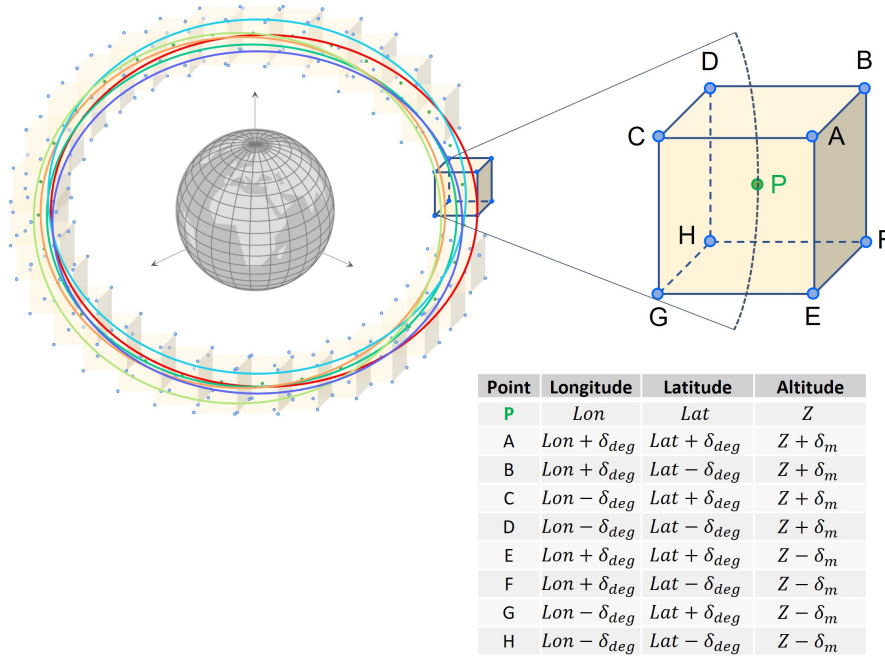


Figure C1. A representative schematic showing the constructed “cube of uncertainty” that surrounds a given coordinate along the orbit of a satellite. Each point that makes up this cube will contain modeled neutral density values between which we can interpolate in GEODYN as the orbit drifts from the a priori orbit. This figure also demonstrates how perturbations due to different density models, represented here by different colored orbits, may necessitate a range of uncertainty for the satellite’s indexed location.

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