One-Minute Resolution GOES-R Observations of Lamb and Gravity Waves Triggered by the Hunga Tonga-Hunga Ha'apai Eruptions on 15 January 2022

Akos Horvath¹, Sharon L. Vadas², Claudia Christine Stephan³, and Stefan Alexander Buehler⁴

¹University of Hamburg ²NorthWest Research Associates, Inc. ³Max Planck Institute for Meteorology ⁴Universität Hamburg

January 18, 2024

1 2	One-Minute Resolution GOES-R Observations of Lamb and Gravity Waves Triggered by the Hunga Tonga-Hunga Ha'apai Eruptions on 15 January 2022
3	
4	Ákos Horváth ¹ , Sharon L. Vadas ² , Claudia C. Stephan ³ , and Stefan A. Buehler ¹
5 6 7	¹ Meteorological Institute, Universität Hamburg, Hamburg, Germany, ² Northwest Research Associates, Boulder, Colorado, USA, ³ Leibniz Institute of Atmospheric Physics, University of Rostock, Kühlungsborn, Germany
8	
9 10	Corresponding author: Ákos Horváth (hfakos@gmail.com)
11	
12	Key Points:
13 14 15	• Propagation of surface pressure anomalies explains the Lamb waveform derivative patterns seen in brightness temperature image differences
16 17 18	• 1-min mesoscale imagery depicts the short-period variations within the long-period wave packet envelope captured in full disk imagery
19 20	• The Lamb wave train appears dispersive, main pulse is followed by waves with decreasing wavelength of ~40–80 km and period of ~2.1–4.2 min
21	
22	

23 Abstract

- 24 We use high temporal-resolution mesoscale imagery from the Geostationary Operational
- 25 Environmental Satellite-R (GOES-R) series to track the Lamb and gravity waves generated by
- the 15 January 2022 Hunga Tonga-Hunga Ha'apai eruption. The 1-min cadence of these limited
- 27 area (~1,000×1,000 km²) brightness temperatures ensures an order of magnitude better temporal
- sampling than full-disk imagery available at 10-min or 15-min cadence. The wave patterns are
- visualized in brightness temperature image differences, which represent the time derivative of
- the full waveform with the level of temporal aliasing being determined by the imaging cadence.
- 31 Consequently, the mesoscale data highlight short-period variations, while the full-disk data
- 32 capture the long-period wave packet envelope. The full temperature anomaly waveform,
- however, can be reconstructed reasonably well from the mesoscale waveform derivatives. The reconstructed temperature anomaly waveform essentially traces the surface pressure anomaly
- \sim waveform. The 1-min imagery reveals waves with \sim 40–80 km wavelengths, which trail the
- primary Lamb pulse emitted at ~04:29 UTC. Their estimated propagation speed is \sim 315±15 m s⁻
- primary Lamb pulse emitted at ~04:29 UTC. Their estimated propagation speed is ~ 315 ± 15 m s⁻¹, resulting in typical periods of 2.1–4.2 min. Weaker Lamb waves were also generated by the
- \sim last major eruption at ~08:40–08:45 UTC, which were, however, only identified in the near field
- but not in the far field. We also noted wind effects such as mean flow advection in the
- 40 propagation of concentric gravity wave rings and observed gravity waves traveling near their
- 41 theoretical maximum speed.

42 Plain Language Summary

- 43 The record-setting eruption of the Hunga Tonga-Hunga Ha'apai volcano on 15 January 2022 was
- observed by geostationary satellites, which take an image of the full Earth disk every 10–15 min.
- 45 Several smaller areas (~1,000 km on a side) are, however, imaged every 1 min. The eruption
- 46 generated various waves in the atmosphere, including acoustic waves traveling at the speed of
- 47 sound and slower gravity waves. These atmospheric waves can be tracked by the subtle
- brightness temperature changes they cause in the images. We show that the 1-min images used in
- 49 our study capture finer details of the wave patterns and allow a better estimation of wave
- 50 properties than the relatively infrequent full disk images. The high temporal frequency imagery
- also allows to determine the eruption sequence more precisely and reveal how the background
- 52 winds affect the propagation of the waves.

53 **1 Introduction**

- On 15 January 2022, the Hunga Tonga-Hunga Ha'apai submarine volcano (20.54°S, 175.38°W, hereafter HTHH) experienced a climactic eruption which produced a plume with overshooting tops reaching record-setting altitudes of 55–57 km (Carr et al., 2022; Proud et al., 2022). The eruption generated worldwide tsunamis (Kubota et al., 2022; Purkis et al., 2023), lofted unprecedented amounts of water vapor directly into the stratosphere to heights of 30–40 km (Khaykin et al., 2022; Millán et al. 2022; Randel et al., 2023), and perturbed the global electric circuit via extreme lightning activity (Bór et al., 2023).
- 61 Global seismic and microbarometer observations revealed that the eruption emitted a 62 wide spectrum of acoustic and gravity waves, including audible sound, infrasound, and internal 63 gravity waves (Matoza et al., 2022; Vergoz et al., 2022), which also caused significant changes 64 in the ionosphere and thermosphere (Harding et al., 2022; Vadas et al., 2023a). Most prominent

among them was a large-amplitude Lamb wave, which circled the Earth several times (Amores
et al., 2022; Heki, 2022; Otsuka, 2022; Wright et al., 2022).

The Lamb wave is a special acoustic wave that is hydrostatically balanced in the vertical 67 direction and propagates in the horizontal direction only. The ideal Lamb wave is a mode of an 68 isothermal and windless atmosphere, where it propagates non-dispersively and isotropically with 69 a nominal sound speed of \sim 312 m s⁻¹ (Bretherton, 1969). A similar mode exists in the real 70 atmosphere and travels as an edge wave guided by the surface with most of its energy 71 concentrated in the troposphere (Garrett, 1969). Real Lamb waves do show dispersion and 72 waveform distortion, whose magnitude depends mostly on the global winds and to a lesser 73 degree on the vertical temperature structure and topography encountered along the travel path 74 (Garrett, 1969; Sepúlveda et al., 2023). Under standard atmospheric conditions, the phase speed 75 of Lamb waves can easily vary between 294–319 m s⁻¹ (Garrett, 1969). Lamb waves propagate 76 very efficiently because their attenuation distance largely exceeds the circumference of the Earth. 77 They appear as a pseudo-mode bridging the gap between the acoustic and gravity modes and can 78 exist at all periods ranging from a few minutes to about a day (Francis, 1973). 79

80 Several studies tracked the global propagation of the primary Lamb wave emitted by HTHH using infrared brightness temperatures from geostationary satellites (Amores et al., 2022; 81 Matoza et al., 2022; Otsuka, 2022; Winn et al., 2023). All of these studies used full disk (FD) 82 imagery available either at 10-min cadence from the Advanced Baseline Imager (ABI) aboard 83 84 the Geostationary Operational Environmental Satellite-R series (GOES-R) and the Advanced Himawari Imager (AHI) aboard the Himawari-8 satellite or at 15-min cadence from the Spinning 85 Enhanced Visible and Infrared Imager (SEVIRI) aboard the Meteosat Second Generation 86 satellites. From such FD imagery, the horizontal phase speed, horizontal wavelength, and 87 ground-based period of the Lamb wave were estimated as $c_H \approx 303-323$ m s⁻¹, $\lambda_H \approx 400-500$ 88 km and $\tau_R \approx 20-30$ min, respectively. 89

Studies analyzing surface pressure records or numerically simulating the atmospheric waves and the triggered meteotsunamis arrived at similar or even longer Lamb wavelengths and periods. The far-field envelope of the complex pressure wave packet can be roughly approximated by an *N*-wave or a positive triangular pulse of 400–900 km width and 20–50 min duration (Heinrich et al., 2023; Kubota et al., 2022; Matoza et al., 2022; Vergoz et al., 2022; Watada et al., 2023; Winn et al., 2023).

96 In this study, we take advantage of the GOES-R mesoscale scans, which offer a limitedarea (~1,000×1,000 km²) view of the HTHH Lamb and gravity waves, but at an order of 97 magnitude better temporal resolution of 1 min. We show that the long-period wave cannot be 98 directly extracted from brightness temperature imagery against a highly varying background. The 99 commonly used visualization technique of differencing image sequences characterizes instead 100 the waveform time derivatives and thus constitutes a high-pass filter. Temporal aliasing due to a 101 102 reduced sampling frequency, on the other hand, represents a compensating factor. As a result of these opposing effects, the FD difference imagery captures the long-period envelope of the full 103 104 wave packet. However, it misses important high-frequency variations within the waveform, which can only be obtained from the mesoscale imagery. 105

The paper is organized as follows. Section 2 introduces the ABI mesoscale data. Section
 3 explains the emergence of the specific wave patterns that can be visualized in brightness
 temperature image differences. Section 4 analyzes the first two passages of the Lamb waves and

109 provides further examples of gravity waves captured in the high-resolution data. A discussion

and conclusions are given in Section 5.

111 2 ABI 1-minute mesoscale imagery

112 2.1 Mesoscale domains

To identify and track atmospheric waves generated by the HTHH eruption, we used 113 infrared (IR) imagery from ABI aboard GOES-17 (GOES-West) and GOES-16 (GOES-East). 114 The vertical near-side perspective views of the Earth disk from the geostationary vantage points 115 of GOES-17 and GOES-16 are depicted in Figures 1a and 1b, respectively. GOES-17, stationed 116 at 137.2°W, observes the Pacific including HTHH, as well as Central and North America. The 117 coverage area of GOES-16, stationed at 75.2°W, is centered on the Americas, but also includes 118 the Eastern Pacific and parts of the Atlantic. The current ABI imaging Mode 6 scans the full disk 119 120 every 10 minutes, the Pacific US (PACUS, GOES-17) and Continental US (CONUS, GOES-16) sectors every 5 minutes, and two mesoscale domains (M1, M2) per satellite every minute. 121

All previous studies used the 10-minute FD imagery for the global tracking of the main Lamb wave. The PACUS and CONUS scans offer an improved 5-minute temporal resolution over a ~5,000×3,000 km² sector covering the latitudes of the US (15°N–50°N). In this study, however, we exploit the mesoscale scans, which provide the highest cadence of 1 minute, albeit in a limited area.

127 A full disk IR image is given on a 5424×5424 fixed grid rectified to the Geodetic 128 Reference System 1980 (GRS80) ellipsoid. The fixed grid has an angular sampling distance of 129 56 μ rad, which corresponds to a spatial resolution of ~2 km at the subsatellite point. A mesoscale 130 domain is a 500×500-pixel square subset of an FD image, covering a ~1,000×1,000 km² area at 131 the subsatellite point. Each mesoscale domain can be independently retargeted from its default 132 position to follow evolving features of interest, such as severe weather or volcanic eruptions.

The ABI mesoscale domains were moved several times during 15–16 January 2022. 133 Figure 1 shows, in chronological order, only those locations that were used for tracking the 134 waves triggered by the eruptions. The GOES-17 M2 domain was centered on American Samoa 135 to the northeast of HTHH between 02:41-05:59 UTC on 15 January. The midpoint and southern 136 edge of this domain were respectively 930 km and 155 km from the volcano. Although this 137 138 domain unfortunately missed the rising eruption column, it did however capture the earliest phases of wave propagation. In this domain, we used atmospheric surface pressure data collected 139 in the National Park of American Samoa, Pago Pago (blue star in Figure 1) to explain the 140 observed temperature anomaly waveform by the pressure anomaly waveform. 141

After a 1-hour gap, the GOES-17 M1 domain was centered on HTHH at 07:05 UTC on 143 15 January and kept there for a little over 2 days. This domain observed the spreading of the 144 stratospheric and near-tropopause plumes produced by the main eruption at ~04:29 UTC and 145 also captured the later smaller eruptions. These two domains provide a rich data source for the 146 analysis of near-field wave phenomena.

The GOES-17 M2 domain was subsequently moved back to its default location over Alaska (~9,000 km from HTHH) and allowed the far-field observation of the A1 minor arc westto-east wave passage between 11–14 UTC on 15 January. Additional far-field observations of the A1 wave passage over the southwest and southeast US were provided by the GOES-16 M2 domain (~9,500 km from HTHH) between 12–15 UTC and the GOES-16 M1 domain (~11,000

152 km from HTHH) between 15–17 UTC on 15 January. Surface pressure data at Midland

International Air and Space Port (Midland, Texas; red star in Figure 1) located in the GOES-16
 M1 domain was also analyzed.

The antipodal A2 major arc east-to-west wave passage on 16 January was first observed in the GOES-16 M2 domain, relocated to the southeastern US, and the GOES-16 M1 domain between 06–08 UTC. These two GOES-16 mesoscale domains cover nearly the same longitude range and half overlap with M2 being north of M1. The A2 wave passage was then imaged in the default GOES-17 M2 domain over Alaska between 08–11 UTC. Finally, the GOES-17 M1 domain captured the confluence and interference of the returning A2 waves over HTHH between 16:00–18:00 UTC.

Note that while the mesoscale scans are square images in the native fixed grid, they generally do not correspond to square areas in longitude-latitude space. As shown by the equirectangular projection in Figure 1c, the map distortion increases with increasing distance from the equatorial subsatellite point. As a result, the map distortion is largest for the default M2 domain over Alaska, which is near the limb of the GOES-17 full disk (in fact, its top left northwestern corner contains space pixels). There is significant east-west stretching at such oblique view angles, especially poleward of 60°N.

A related caveat is that although the fixed grid is a regular grid in the satellite coordinate 169 system with an angular spacing of 56 µrad in both the east-west and north-south scan directions, 170 it is an irregular grid in ellipsoid-projected distance. The nominal grid spacing (or ground 171 resolution) is ~2 km only near the subsatellite point. Grid spacing generally increases with 172 distance from the subsatellite point and shows significant variations within a given domain too. 173 The horizontal distance between two neighboring image pixels is largest in the most obliquely-174 viewed GOES-17 M2 domain over Alaska, where it typically is in the range of 3–15 km. Grid 175 spacing is finer in the rest of the domains, usually between 2.5–6.0 km. Therefore, when 176 177 estimating the propagation speed of the observed waves, it is important to calculate ellipsoidprojected distances from the actual geodetic latitude and longitude of image pixels-we use the 178 GRS80 ellipsoid. 179

180 2.2 Brightness temperatures

Spectral radiances were converted to the equivalent black body brightness temperature,
henceforth simply brightness temperature (*BT*), as described in the *GOES R Series Product Definition and Users' Guide* (GOES-R PUG L1B Vol 3 Rev 2.2, 2019). We track waves directly
in the fixed grid square images (500×500 pixels) to avoid unnecessary remapping and
interpolation. The waves are detectable in all IR channels to a varying degree. For illustrations in
this paper, we use C7, C9, C11, C12, or C13, choosing the one that provides the sharpest signal
for a given time and location.

For the interpretation of the observed wave patterns, it is important to have at least a qualitative understanding of the altitude ranges the brightness temperatures represent. Out of the 10 ABI IR channels, six are window channels (C7, C11, C13, C14, C15, C16) with a clear-sky vertical weighting (or contribution) function that peaks at the surface (Schmit et al., 2017). In these channels, a large portion of the signal originates from the surface in clear air, although water vapor, SO₂, and CO₂ also modulate the *BT* to various degrees. In the presence of opaque

- 194 meteorological or volcanic clouds, however, the signal originates largely from the cloud altitude.
- 195 In semitransparent clouds, there is still noticeable contribution from the surface and the BT is
- characteristic of an apparent height somewhere between the surface and cloud level.
- 197 The clear-sky weighting function of the lower-, mid-, and upper-level water vapor
- channels (C10, C9, C8) peak at ~4 km, ~6.5 km, and ~8.2 km altitude, respectively. Therefore, these channels are more representative of the mid to upper troposphere, although high-level
- clouds also affect the signal. Finally, the 'ozone' channel (C12) has a clear-sky weighting
- function with a peak at the surface as well as at ~ 22 km altitude and, thus, characterizes the
- 202 lower stratosphere (albeit water vapor absorbs in this channel too).
- The vertical weighting functions are relatively broad and, hence, the clear-sky *BT* represents a weighted layer mean rather than a single level. Overall, the presence of a wave front in several IR channels and across different scene types is indicative of a vertically extensive and coherent wave.
- 207 We also note that the GOES-17 ABI suffered a loop heat pipe (LHP) anomaly, which prevents
- 208 maintaining the IR detectors at their required temperatures during parts of the night under certain
- 209 orbital conditions. After performance recovery steps, 97 % of imaging capability in the thermal
- 210 infrared bands was regained (McCorkel et al., 2019); however, image degradation such as
- 211 increased noise and striping are occasionally noticeable.
- 212



Figure 1. Geographic location of GOES-17 (G17, blue) and GOES-16 (G16, red) mesoscale 229 domains (M1, M2) used for wave tracking on 15–16 January 2022: (a) GOES-17 fixed grid 230 view, (b) GOES-16 fixed grid view, and (c) equirectangular projection. The black triangle is 231 HTHH. The blue and red stars mark the location of surface air pressure measurements at the 232 National Park of American Samoa and Midland International Air and Space Port, respectively. 233 The orange and magenta lines indicate isodistances (×1,000 km) from the volcano and its 234 antipode, respectively. Arrival times at the dashed isodistances (orange - 15 January, magenta -235 16 January) are derived using a nominal propagation speed of 315 m s⁻¹ and the emission time of 236 the first detectable waves at 04:07 UTC on 15 January 2022. The temporal coverage of the 237 mesoscale domains is summarized in Table S1 in the Supporting Information. 238

240 **3 Visualization of atmospheric waves**

Differencing brightness temperature image sequences has been the main tool for the visualization of the atmospheric waves generated by the eruption. In this section, we investigate which part of the wave spectrum is in fact captured by such difference imagery.

244 3.1 The surface pressure waveform

Atmospheric pressure anomalies caused by the HTHH eruption were recorded all around 245 the globe. Laboratory measurements and numerical simulations show that acoustic waveforms 246 propagating through the atmosphere can get strongly distorted mostly by thermal 247 inhomogeneities and turbulent wind velocity fluctuations (Averiyanov et al., 2011; Stout, 2018; 248 Yuldashev, et al., 2017). Unlike the classic N-wave sonic boom, distorted waveforms can have 249 rounded waves, several spikes, multiple shock fronts, and oscillations or even a maximum 250 overpressure in the tail part. As a result, the observed HTHH Lamb wave packets are quite 251 complex; nevertheless, their far-field envelopes can be roughly approximated by an N-wave or a 252 positive triangular pulse (Matoza et al., 2022; Vergoz et al., 2022; Watada et al., 2023). 253

The amplitude of the pressure wave decreases with distance from the eruption, varying 254 from ~30 hPa in Tonga (64 km from HTHH) to ~0.5 hPa in the far field (10k km from HTHH). 255 The estimated duration (or 'period') of the enveloping pulse is between 20–50 min, which 256 257 corresponds to a horizontal width (or 'wavelength') of 400-900 km for a phase velocity at the sound speed. The global propagation of this broad far-field atmospheric pressure pulse was well 258 259 reproduced by Amores et al. (2022), who introduced an instantaneous sea level perturbation in a shallow water ocean model as well as by Watanabe et al. (2022), who imposed an instantaneous 260 hot anomaly over the volcano in an atmospheric general circulation model. The atmospheric 261 Lamb wave triggered a meteotsunami, the accurate numerical simulation of which also required 262 263 the imposition of such long-period and long-wavelength pressure pulses as forcing (Heinrich et al., 2023; Kubota et al., 2022; Suzuki et al., 2023; Winn et al., 2023; Yamada et al., 2022). 264

265 For acoustic waves such as the Lamb wave, the temperature perturbations are in phase with and proportional to the pressure perturbations (Vadas, 2013). Therefore, the waveform of 266 surface pressure anomalies approximates, to first order, the waveform of the brightness 267 temperature anomalies embedded in the satellite images. The black curve in Figure 2a shows the 268 surface pressure anomaly measured at the National Park of American Samoa (Pago Pago, Tutuila 269 Island) between 04:00–06:00 UTC on 15 January (PurpleAir, 2023). The barometer provides 270 271 measurements at a temporal resolution of $\Delta t = 2 \text{ min}$ and is located ~850 km from HTHH near 272 the center of our G17 M2 Samoa domain (14.27°S, 170.70°W; blue star in Figure 1). This distance translates to a wave travel time of \sim 45 min at the propagation speed of \sim 315 m s⁻¹ 273 derived in Section 4.1. The anomalies are given relative to the mean value of the 04:00-04:30 274 UTC period (1006.5 hPa), when the pressure was very stable. 275

The first small pressure peak (~0.3 hPa) occurs at 04:52 UTC, corresponding to an emission time of ~04:07 UTC at HTHH. Although this pressure anomaly is barely discernable, it likely is a real signal, because its emission time agrees with that of the very first visually detectable wave in our mesoscale *BT* imagery. Seismo-acoustic data (Matoza et al., 2022) and tsunami runup measurements in the Tonga Archipelago (Purkis et al., 2023) also indicate an explosive event at that emission time. In addition, Van Eaton et al. (2023) found a sudden increase in the plume's width at 04:07 UTC using GOES-17 full disk visible images.

283



Figure 2. Atmospheric surface pressure (P, black) and channel 13 (10.3 µm) brightness 302 temperature (BT_{13} , orange) waveforms and their time derivatives at $\Delta t = 2$ min sampling in 303 Pago Pago, American Samoa between 04:00-06:00 UTC on 15 January 2022. (a) Pressure 304 anomaly relative to the mean of the 04:00–04:30 period. The arrival times and the emission times 305 306 (pink, in brackets) of individual pressure peaks are also indicated, assuming a propagation velocity of 315 m s⁻¹. The BT_{13} anomaly was reconstructed from the observed $BT_{13}^{\prime\prime}$. (b) The first 307 time derivatives P' and BT'_{13} . (c) The second time derivatives P'' and BT''_{13} . The time derivatives 308 of BT_{13} were averaged along the isodistance of Pago Pago to reduce noise. Panels (d), (e), and 309 (f) are the same as (a), (b), and (c), respectively, but for $\Delta t = 10$ min. 310

This minor anomaly is followed by two larger anomalies at 05:00 UTC (~0.8 hPa, emission time 04:15 UTC) and 05:07 UTC (~2.1 hPa, emission time 04:22 UTC). The peak overpressure of ~6.1 hPa is recorded in Pago Pago at 05:14 UTC. The maximum HTHH plumetop height of 55–57 km was measured ~6 min after the 04:29 UTC emission time of this main pressure pulse (Carr et al., 2022). Matoza et al. (2022) also report seismo-acoustic events with ~04:15 UTC and ~04:30 UTC emission times, while Purkis et al. (2023) claim an explosive blast with a ~04:18 UTC emission time (in between the 2nd and 3rd peak in our pressure data).

The pressure quickly drops in the next 10 min, reaching its lowest value, a -3.2 hPa anomaly, at 05:24 UTC. Thus, the amplitude of the main explosion is ~9.3 hPa in Samoa. The pressure then recovers with some fluctuations by 05:42 UTC and shows a smaller trailing peak (~1.7 hPa) at 05:46 UTC.

323 The full wave packet is a combination of four consecutive explosions of increasing 324 magnitude. A range of characteristic time spans can be derived, depending on whether one focuses on the main blast or the full packet. Extrapolating backward and forward in time from 325 326 the primary pressure peak and trough, the main blast can be approximated by an N-wave starting at ~05:08 UTC and ending at ~05:32 UTC. This *N*-wave has a rise time of 6–8 min and a total 327 duration of ~24 min. The time difference between the primary peak and the first trailing peak 328 yields a slightly longer period of ~32 min. The duration of the full wave packet is ~50 min 329 (04:52–05:42 UTC), comprising a ~28-min positive phase and a ~22-min negative phase. This 330 range of time span estimates agrees with that reported in previous studies. 331

Figure S1 in the Supporting Information plots 1-min barometer readings (Joe LaPlante, 332 personal communication) collected at nearby Coconut Point (Nu'uuli, Tutuila Island), which 333 show essentially the same pressure waveform comprising of the main pulse and at least two 334 preceding smaller pulses. Although this data has better temporal resolution, its quantization 335 $(\Delta p = 0.34 \text{ hPa})$ is significantly coarser than that of the 2-min pressure measurements ($\Delta p =$ 336 0.01 hPa); hence, we use the latter to calculate waveform derivatives in the next section. Figure 337 S1 also plots 1-min surface pressures at Midland International Air and Space Port (close to the 338 center of the G16 M2 Texas domain; red star in Figure 1) obtained from the Automated Surface 339 Observing System (ASOS, 2023). This far-field waveform (~9,716 km or ~8.5 hr from HTHH) 340 is very similar to the near-field ones in Samoa, demonstrating the stability of the shock wave as it 341 traversed the globe, but its amplitude is an order of magnitude smaller, ~0.9 hPa. 342

The black curve in Figure 2d demonstrates how the observed waveform changes when the $\Delta t = 10$ min cadence of full disk imagery is used. Here the 2-min pressure time series is subsampled backward and forward in time from 05:14 UTC to preserve the main peak. The smaller blasts and other fine details are indistinguishable and all that remains is the ~50-min envelope of the full wave packet. As chance would have it, the rise time and the time span of the peak-to-trough drop of the central *N*-wave are both close to 10 min; hence, the primary Lamb pulse is well represented even at the reduced FD sampling.

350 3.2 Waveform time derivatives

Visualizing the HTHH-induced temperature perturbations in raw *BT* imagery is rather 351 352 problematic. The amplitude of the temperature variations rapidly decreases with distance and at most represents a signal of a couple of percent. Pressure perturbations have a similar relative 353 magnitude, but they are superimposed on a fairly homogeneous and steady background. The 354 temperature perturbations, in contrast, are superimposed on a heterogeneous and rapidly varying 355 background. Spatiotemporal variations in clouds (advection, growth, dissipation) and to a lesser 356 degree in water vapor and trace gases lead to a challenging background scene for the 357 identification and tracking of faint wavelike features. The large dynamic range of BT makes it 358 difficult to achieve enough local contrast both in cold (bright, cloudy) and warm (dark, clear sky) 359 areas when tone mapping the image. 360

361 Waves are easier to detect in the time derivatives of the brightness temperature. For each 362 pixel, we approximate the first time derivative of BT at time t using a backward finite-difference 363 scheme

$$BT' = \frac{BT(t) - BT(t - \Delta t)}{\Delta t}$$

DT(t) $DT(t \Lambda t)$

366

- and the second time derivative using a second-order central finite-difference scheme
 - $BT'' = \frac{BT(t+\Delta t) 2BT(t) + BT(t-\Delta t)}{\Delta t^2},$ (2)

370

369

where Δt is the time difference between the images. Apart from a scaling factor, the first time derivative is just the difference between two subsequent images, while the second time derivative is the difference between the mean of three subsequent images and the central image of the triplet.

The mesoscale images are available at a time resolution of $\Delta t = 1$ min. However, with a subsampling rate of n > 1, that is, selecting only every *n*th image from the sequence, we can investigate the spatiotemporal aliasing effect of longer sampling periods on the appearance of wave patterns. For example, a sampling period of $\Delta t = 10$ min mimics the GOES-R and Himawari-8 full-disk images that were used in previous studies.

It is important to note that these (discrete) time derivatives act as high pass filters, where 380 higher frequencies get larger weights than lower frequencies. Before considering BT images, we 381 first demonstrate the effect of the time differentiators on the pressure waveform, which is a proxy 382 for the temperature waveform. The black curves in Figure 2b and Figure 2c plot the first 383 derivative (P') and second derivative (P'') of the Samoa pressure time series ($\Delta t = 2 \min$), 384 respectively. These can be thought of as the pressure (or *BT*) waveform derivatives at the image 385 pixel which corresponds to the barometer's location. As shown, the differentiators emphasize the 386 shorter period variations within the waveform rather than the long-period envelope of the full 387 wave packet. The first time derivative is the pressure tendency, which characterizes the local 388 steepness of the waveform. The second derivative describes the local concavity of the graph: the 389 waveform is concave up (or convex) if P'' > 0 and concave down if P'' < 0. 390

The derivative of the waveform is frequently used in laboratory investigations of N-391 waves, especially ones with multiple shocks similar to the HTHH pressure wave. For example, 392 the peak overpressure and the rise time are difficult to evaluate in distorted waveforms found in 393 394 turbulent flow. These important acoustic wave parameters can be better defined based on the first derivative of the pressure waveform (Averiyanov et al., 2011; Yuldashev, et al., 2017). 395 Analyzing P' allows ranking shocks by strength (steepness), separating close shocks that 396 otherwise might be considered as one long shock, and enables detecting shocks in the tail part of 397 the packet. 398

The first and second derivatives of the pressure waveform sampled at the reduced FD rate of $\Delta t = 10$ min are given by the black curves in Figures 2e and 2f, respectively. In both cases, the maxima of the derivatives are separated by 20 min. By chance, this time span agrees well

(1)

with the \sim 24 min duration of the *N*-wave that can be fitted to the primary Lamb pulse of the full wave packet.

404 3.3 Image processing

405 3.3.1 Gray scaling, smoothing, contrast enhancement

Although it high pass filters the data, the major advantage of taking the time derivatives 406 of BT is that it largely eliminates the high dynamic-range background image comprising a 407 mixture of clear (warm) and cloudy (cold) areas. In the significantly reduced dynamic range BT 408 derivatives, it is easier to achieve good local contrast throughout the entire scene. The time 409 derivatives are first scaled to 256 gray levels using an appropriate initial range, e.g., ± 1 K min⁻¹ 410 for BT' and ± 0.25 K min⁻² for BT''. The images are then histogram equalized. Because this 411 means a palette change for each image, we omit the colorbar in the figures and animations. . For 412 the visualization of wave fronts, however, spatial coherence is more important than pixel 413 intensity (the magnitude of the time derivative). Histogram equalization readjusts the intensities 414 and allows areas of lower local contrast to gain a higher contrast. In the resulting images, the 415 bright wave crests and dark troughs respectively correspond to the maxima and minima of the 416 waveform derivatives plotted in Figure 2. 417

The images are also smoothed with a moving average boxcar filter. The smallest 418 averaging window of 3×3 pixels is already adequate, but in this study, we opt for a slightly larger 419 5×5 -pixel window, which in our experience leads to a clearer separation of wave fronts. Mean 420 filtering is applied in all cases, except in the analysis of the shortest wavelength (~15 km) gravity 421 waves, where the crests and throughs already show good contrast in the unfiltered images and 422 smoothing would unnecessarily bias the results (see Section 4.2.1). As an example, Figure 3 423 compares the highest temporal resolution ($\Delta t = 1 \text{ min}$) first and second time derivatives of 424 425 channel 11 (8.4 µm) BT₁₁ for a sunset scene in the GOES-17 M2 Samoa domain. The channel 2 $(0.65 \,\mu\text{m})$ visible radiances and BT_{11} itself are also plotted for context in Figures 3a and 3b, 426 respectively. Most of the domain is covered with clouds, ranging from low-level ones to deep 427 convection. In the bottom left corner of Figure 3a, the stratospheric plume from the main 428 429 eruption intrudes into the domain and casts a shadow on lower-level clouds. The right and bottom third of the domain are warm ocean peppered only with small popcorn cumuli and, 430

- 431 hence, show high BT_{11} (black areas in Figure 3b).
- 432
- 433
- 434
- 435
- 436
- 437
- 438
- 439
- 440
- 441



Figure 3. (a) Channel 2 (0.65 µm) visible radiances, (b) channel 11 (8.4 µm) brightness 455 temperatures BT_{11} , (c) first time derivative BT'_{11} , and (d) second time derivative BT''_{11} in 456 G17 M2 (Samoa) at 05:35 UTC on 15 January 2022. The visible image was enhanced by 457 adaptive histogram equalization and BT_{11} was gray scaled between 190K (white) and 290K 458 (black). The interval used for calculating the time derivatives is $\Delta t = 1$ min. The time derivative 459 images were gray scaled over the respective ranges of ± 1 K min⁻¹ and ± 0.25 K min⁻², histogram 460 equalized, and mean filtered using a 5×5-pixel window. The yellow star marks the location of the 461 Pago Pago pressure time series plotted in Figure 2. The yellow arc in the bottom left are clear 462 pixels ~341±2 km from HTHH with a backazimuth of 320°–345°, the mean brightness 463 temperature of which is plotted in Figure 5. 464

The concentric arcs of waves emanating from HTHH are partially visible in BT'_{11} , 466 especially over mostly clear ocean (Figure 3c). However, the waves are still difficult to discern 467 in thicker clouds, where large temperature fluctuations due to horizontal or vertical cloud motion 468 mask the small eruptive temperature anomalies. This complicating background is greatly 469 diminished in $BT_{11}^{\prime\prime}$, which enhances the spatially coherent signal from fast propagating waves 470 (Figure 3d). The second time derivative collates information from three images and, thus, works 471 better than the first time derivate, which is based on two images. The $BT_{11}^{\prime\prime}$ field reveals a 472 multitude of closely spaced wave crests and troughs, even in cloudy areas. Due to its obvious 473 superiority, we use the second time derivative of brightness temperatures for most subsequent 474 analysis. 475

The emergent pattern of alternating brighter and darker arcs is insensitive to the type of
finite difference scheme. Replacing the central differences in Equation 2 with forward or
backward differences leads to a pattern that is shifted slightly forward or backward in the radial
direction, but otherwise shows the same fine-scale band structure.

The concentric waves in Figure 3d can be easily followed from clear ocean over to highlevel clouds without a noticeable dislocation in the arcs at the clear-cloudy interface and are also present in all IR channels, pointing to Lamb waves that span the full depth of the troposphere. Channel differences are further demonstrated in the Supporting Information (Figures S2 and S3).

484 Of the earlier studies, Wright et al. (2022) used the first time derivative, while Amores et 485 al. (2022), Matoza et al. (2022), Otsuka (2022), and Winn et al. (2023) used the second time 486 derivative of brightness temperatures to identify waves. However, all previous studies were 487 based on full-disk imagery with a significantly longer sampling period of $\Delta t = 10$ min for 488 GOES-R and Himawari-8 and $\Delta t = 15$ min for Meteosat. The consequences of such reduced 489 temporal sampling are discussed in Section 3.6.

490 3.3.2 Spatial Fourier filtering

491 Although the used boxcar averaging already constitutes a low-pass spatial filter, 492 additional spatial filtering can be applied to the data to improve the detection of the dominant 493 wave signature. The BT'' images are converted by a fast Fourier transform (FFT) into the 494 frequency domain, in which an ellipsoidal filter removes certain high-wavenumber components 495 and then the result is transformed back into the spatial domain by the inverse FFT.

Figure 4 demonstrates FFT filters for the two main wave pattern types encountered in the mesoscale observations. In near-field domains, the dominant pattern consists of concentric circles or arcs (Figure 4a). Here we note that the vertical near-side perspective view (fixed grid image) is a non-conformal projection in which angles and shapes are distorted; thus, circles appear as ellipses away from the subsatellite point. In far-field domains, the characteristic pattern is a train of bands, whose orientation (slope) depends on the domain's geographic location relative to HTHH (Figure 4d).

The 2D spatial power spectrum (Figures 4b and 4e) is zeroed outside of an ellipsoidal mask to reduce high-frequency variations, noise, and striping. Instead of using a fixed mask, we subjectively adjust the size and rotation of the ellipse per domain, such that it emphasizes the dominant wave patterns (Figures 4c and 4f). The animations given in the Supporting Information demonstrate the filter adjustment.

In our experience, low-pass FFT filtering might not significantly enhance the visual perception of waves in individual static images. Nevertheless, it always helps detecting the passage of waves and estimating their phase speed in the time–distance plots (or keograms) we introduce in Section 4.1.

- 512
- 513
- 514
- 515
- 516
- 517
- 518
- 519



Figure 4. Examples of low-pass FFT filtering of BT_{12}'' (9.6 µm): (top row) concentric circles in the near-field G17_M1 HTHH domain (HTHH marked by the yellow triangle) at 07:15 UTC on 15 January 2022 and (bottom row) meridional bands in the far-field G16_M2 domain (Texas) at 13:35 UTC on 15 January 2022. The interval used for calculating the time derivative is $\Delta t = 1$ min. (a, d) Raw grayscale image. (b, e) Log of the 2D spatial power spectrum. The filter only keeps frequencies within the white dashed circle/ellipse. (c, f) Spatial frequency-filtered image.

539

540 3.4 The primary Lamb wave in *BT*

As discussed earlier, detecting the HTHH-induced absolute temperature perturbations on 541 a pixel-by-pixel basis is difficult, because the typical spatiotemporal variations in BT due to 542 tropospheric dynamics usually mask the signal. However, it is possible to extract the mean 543 signature of the long-period primary wave in certain cloud-free areas, where the background 544 temperature remains relatively steady without large fluctuations. The yellow arc in Figure 3 545 marks such a set of mostly clear pixels, which are ~341±2 km from HTHH with a backazimuth 546 of 320°-345°. Assuming a wave velocity of ~315 m s⁻¹, these pixels are ~18 min downstream 547 548 from HTHH and ~26 min upstream from Pago Pago (indicated by the yellow star).

Figure 5a plots the 1-min resolution temporal variation of the mean BT_{12} over the arc between 03:58–05:58 UTC. We use the ozone channel 12, because it is less sensitive to clouds due to its weighting function peaking in the stratosphere. A wave with an amplitude of ~5K and period of ~30 min is apparent in the mean temperature. The 04:51 UTC and 05:20 UTC maxima in the mean BT_{12} correspond to the 05:14 UTC and 05:46 UTC pressure peaks measured downstream in Pago Pago (see Figure 2a).

A similar ~30 min-period temperature variation corresponding to the main Lamb pulse is also detectable in the far-field G16_M2 Texas domain. Figure 5b depicts the temporal variation of BT_{12} averaged over a subset of clear pixels (~9,974±2 km from HTHH with a backazimuth of 49°–54°) between 12:00–14:00 UTC. Here the HTHH-induced temperature perturbations are superimposed on a generally decreasing background *BT* and have an order of magnitude smaller amplitude of only ~0.3K.



571

Figure 5. Time series of mean BT_{12} calculated for a set of clear pixels in (a) G17_M2 Samoa domain (yellow arc in Figure 3) and (b) G16_M2 Texas domain (pixels ~9,974±2 km from HTHH with a backazimuth of 49°–54°). The orange cubic spline fit is plotted merely as visual guide.

576

577 3.5 A simple model of the temperature wave packet

As discussed previously, the pressure waveform can be used as a first-order model of the 578 temperature waveform, especially in the nearest Samoa domain, where the pressure signal is 579 robust. We devise temperature anomalies by linearly scaling the 2-min pressure anomalies in 580 581 Figure 2a such that the peak-to-trough amplitude becomes 2K (+1.3K at 05:14 UTC and -0.7K at 05:24 UTC). The amplitude of the temperature anomaly generally decreases with distance from 582 HTHH and also depends on altitude and channel. We show later that the amplitude of the mean 583 temperature anomaly reconstructed from the observed window channel BT time derivatives is 584 \sim 2K at the isodistance of Pago Pago. Such a *BT* perturbation is representative of the cloudy 585 regions dominating our scene, but it is an overestimate in clear sky regions. In the latter, the 586 587 peak-to-trough temperature amplitude is only 0.5–1.0K, as estimated by Otsuka (2022) for an isothermal atmosphere and adiabatic pressure change. 588

This synthetic cloud-level temperature waveform T_P is propagated through the G17_M2 Samoa domain assuming isotropic spreading with a speed of 315 m s⁻¹. The waveform derivatives are then calculated and processed as described in Sections 3.2 and 3.3.

592 Movie S1 in the Supporting Information shows the propagation of the modeled 593 temperature anomaly waveform and its derivatives between 03:58–05:58 UTC. Snapshots 594 corresponding to the passage of the maximum positive and negative temperature anomaly over 595 Pago Pago are plotted in Figure 6.

At 05:14 UTC, the broad bright band in the T_P image along Pago Pago's isodistance (850 km) represents the primary peak of the wave packet (Figure 6a). The two fainter bands downstream correspond to the two smaller preceding peaks of the waveform. The darkest band ~200 km upstream (650 km isodistance) is the trough of the main Lamb pulse, which reaches Pago Pago at 05:24 UTC (Figure 6d). The full ~50 min wave packet is confined between the 250–1,250 km isodistances in Figure 6a and between the 450–1,450 km isodistances in Figure 6d

- 602 (the packet moves \sim 190 km per 10 min).
- 603



616

Figure 6. Temperature anomaly waveform and its time derivatives modeled on the Pago Pago pressure anomaly waveform and propagated through the G17_M2 Samoa domain, at (top row) 05:14 UTC and (bottom row) 05:24 UTC on 15 January 2022. (a, d) Temperature anomaly T_P gray scaled between ±1K (white to black). (b, e) First derivative T'_P . (c, f) Second derivative T''_P . The star marks Pago Pago and the orange or yellow lines are isodistances (×1,000 km) from HTHH drawn at 200 km intervals.

623

The first derivative of the temperature perturbations T'_P shows a larger number of thinner bands, as it highlights the temperature tendency within the packet. The bright band along Pago Pago's isodistance at 05:14 UTC marks the largest temperature increase at the peak of the main pulse (Figure 6b). In contrast, the dark band passing Pago Pago 10 min later represents the steepest temperature drop at the trough of the main pulse (Figure 6e). The broad long-period variation in temperature, however, cannot be identified in this representation.

630 The second derivative of the temperature perturbations T_P'' visualizes the fine-scale 631 variations the sharpest, as it represents the local concavity of the waveform. Its sign is usually 632 opposite of T_P' for our particular waveform. Along Pago Pago's isodistance, T_P'' is negative at

- 633 05:14 UTC (dark band in Figure 6c) and positive at 05:24 UTC (bright band in Figure 6f), 634 because the waveform is concave down near the peak and concave up near the trough of the 635 main Lamb pulse. The long-period temperature variation is not visually encoded in T_P'' either, 636 because concavity is generally uncorrelated with the magnitude of the temperature anomaly.
- In real world data, the temperature waveform is superimposed on a spatiotemporally highly variable background. To mimic this, we simply add the modeled temperature perturbation T_P to actual BT_{11} imagery obtained before the eruption. Recall that mesoscale imagery in the G17_M2 Samoa domain is available between 02:41–05:59 UTC on 15 January and the first visually detectable wave only appears at 04:16 UTC. Thus, imagery prior to 04:16 UTC do not contain the HTHH wave signature and can be safely combined with the T_P shown in Figure 6.

Movie S2 in the Supporting Information shows such 'simulated' cloudy BT_{11} imagery, its second time derivative, as well as the second time derivative of the actually observed BT_{11} . The 05:14 UTC and 05:24 UTC snapshots of the simulated and observed waveform derivatives are plotted in Figure 7. Note that the cloud patterns are at slightly different locations in the simulated and observed images, because the former depicts the cloud field at an earlier time than the latter.



649

650

651

652

653

- 654
- 655
- 656

657

658

659

660

661

Figure 7. Second time derivative BT_{11}'' visualized against a cloudy background: **(a, c)** simulated and **(b, d)** observed in the G17_M2 Samoa domain at **(top row)** 05:14 UTC and **(bottom row)** 05:24 UTC on 15 January 2022. The simulated images combine the modeled temperature anomalies in Figure 6 with pre-eruption BT_{11} images that were obtained 77 min earlier than the actual observation time of the waves indicated in the panels. The star marks Pago Pago and the yellow lines are isodistances (×1,000 km) from HTHH drawn at 200 km intervals.

668

Unlike against a uniform background as in Figure 6, the passage of the temperature perturbations cannot be discerned against a natural cloudy background (see Movie S2). The



second derivative of the simulated brightness temperatures, however, well describes that of the

- observed ones. The sequence, width, and location of the bright and dark arcs of the observed and
- 673 simulated waveform derivatives are in excellent agreement—the contrast variations of the
- 674 simulated patterns are, however, overemphasized in clear sky areas, because the imposed cloud-
- level temperature anomalies are excessive in cloud-free regions. The only exception is the
 bottom left corner of the domain, where the observations show a larger number of shorter period
- wave fronts. It is here where the stratospheric umbrella of the HTHH plume intrudes the domain
- and hence the satellite observations are likely dominated by gravity waves. Unlike Lamb waves,
- 679 which extend through the entire troposphere, these high-altitude gravity waves cause no
- measurable anomalies in the Pago Pago surface pressure on which our assumed temperature
- 681 perturbations are based; thus, gravity waves are missing from the simulated images.

As demonstrated, the pressure perturbations explain the salient features of the 682 temperature waveform derivatives visualized in the satellite imagery. The relationship between 683 the pressure and temperature perturbations, however, can be further quantified. The orange 684 curves in Figures 2b and 2c depict the observed (real) temporal variation of the first and second 685 derivative of BT_{13} at Pago Pago-here the temperature derivatives were averaged along the 850-686 km isodistance to reduce noise. The temperature and pressure waveform derivatives match up 687 excellently. The correlation between P' and BT'_{13} is 0.9 and that between P'' and BT''_{13} is 0.94 for 688 the entire two-hour period. The slightly lower correlation between the first derivatives is due to 689 natural temperature variations in the 4–5 UTC period before the arrival of the eruption's pressure 690 wave. In the post-arrival period of 5–6 UTC, both the first and second derivatives correlate at 691 0.94. 692

Finally, the temperature perturbations can be approximately reconstructed from the measured BT time derivatives by recursively applying the inverse of the finite difference operators in Equations 1 and 2 (antidifference operator). The reconstruction is unique up to an additive constant when using the first derivatives and up to an additive linear trend when using the second derivatives.

The orange curve in Figure 2a depicts the BT_{13} anomalies reconstructed from the mean B T_{13}'' plotted in Figure 2c. Here, we started the recursion at 5 UTC and removed a linear trend from the reconstructed temperature anomalies—the observed mean BT_{13} for Pago Pago's isodistance also increased between 5–6 UTC, unrelated to the eruption. As shown, the reconstructed mean temperature anomaly waveform has a peak-to-trough amplitude of ~2K and correlates well with the observed surface pressure waveform with a coefficient of 0.94.

The corresponding results for $\Delta t = 10$ min sampling are given in Figures 2d, 2e, 2f and Figure 8. Both time derivatives of the subsampled temperature waveform show the familiar positive-negative-positive anomaly pattern of the main Lamb pulse found by previous studies using FD imagery. There is a slight asymmetry between the positive peaks. The first peak is larger and narrower than the second one for the first derivative, while the opposite is true for the second derivative where the second peak is somewhat larger.

710 In FD imagery, these subsampled waveform derivatives manifest as bright–dark–bright bands of

- ~190–200 km width (the distance the acoustic wave packet travels in 10 min). Depending on the
- relative magnitude of the positive peaks, the cloud background, as well as the applied spatial
- smoothing and color saturation, sometimes only the single bright band of the larger peak is
- discernable. As before, the differences between the simulated and observed patterns in the

bottom left corner of the domain are likely due to gravity waves in the stratospheric umbrella.



Figure 8. Same as Figure 7, but for $\Delta t = 10$ min sampling.

731

732 3.6 Spatiotemporal aliasing

To summarize our results so far, the wave patterns seen in the imagery are determined by the interplay of two opposing effects. The time derivatives amount to a high pass filter, which highlights the shorter period fluctuations within the wave packet. Conversely, only the longer period variations are preserved when the observation frequency is reduced. In this section, we discuss the latter aliasing effect in more detail.

The minimum wavelength and wave period that can be captured in the satellite 738 observations are determined by the spatiotemporal characteristics of the images and basic 739 740 sampling theorem. The Nyquist-Shannon condition requires at least two samples per period (both spatial and temporal) for successful signal reconstruction. As discussed in Section 2, the 741 typical grid spacing in most of the mesoscale domains is 3–6 km, depending on propagation 742 azimuth and location in the domain. This translates to a minimum observable horizontal 743 wavelength of 6-12 km. In the GOES-17 M2 Alaska domain the minimum observable 744 wavelength is larger, $\sim 16-20$ km, due to the reduced image resolution near the limb. The 745 maximum detectable wavelength, on the other hand, is equal to the domain size or $\sim 1,000$ km. 746

Similarly, the 1-min sampling period of the mesoscale scans corresponds to a minimum
resolved wave period of 2 min, or a maximum resolved frequency of 8.3 mHz. All higher
frequency variations are aliased to lower frequencies. The temporal aliasing (or under-sampling)
artifact is significantly more severe in the lower cadence full disk imagery. The shortest period
and highest frequency that can be resolved in GOES-R and Himawari-8 FD with a sampling

period of 10 min are 20 min and 0.83 mHz, respectively, while these cutoffs are 30 min and 0.56
 mHz for Meteosat FD with a sampling period of 15 min.

Microbarometer measurements reported by Matoza et al. (2022) indicate that the HTHH eruption generated infrasound (>10 mHz) as well as audible sound (>20 Hz), which are above the Nyquist frequency of the mesoscale scans. Johnson et al. (2023) demonstrated that infrasonic waves in volcanic plumes are detectable by high frame-rate video obtained in the immediate vicinity of the vent. The amplitude of these higher-frequency waves, however, is too small to cause noticeable perturbations in satellite observations. The satellite brightness temperatures contain information about Lamb waves and gravity waves instead.

Figure 9 demonstrates the effect of longer sampling periods on the wave patterns that emerge in the near-field *BT*^{''} images. The results for the shortest sampling period of $\Delta t = 1$ min are compared with those for longer periods of $\Delta t = 2$, 5, and 10 min, which cover the typical sampling rates of operational geostationary imagers: Himawari-8 rapid scan, Meteosat rapid scan, and GOES-R and Himawari-8 full disk scan, respectively.

766 The highest temporal resolution data reveal a swarm of densely packed concentric waves with the main pulse near the 1,200 km isodistance (Figure 9a). The waves propagate with a speed 767 768 of \sim 315 m s⁻¹ as we show in Section 4.1. Their horizontal wavelength increases from \sim 40–50 km to ~50–70 km with distance from HTHH. The increase in λ_{H} across the domain might be due to 769 dispersion-longer waves traveling faster-caused by vertical and horizontal variations in wind 770 771 and temperature (Garrett, 1969). Alternatively, it could be due to the emission of several individual wave packets. There were multiple explosions between 4-6 UTC (see Figure 2a), 772 which could have different horizontal extent, leading to different characteristic λ_{H} . 773

 $\begin{array}{c}
774\\
775\\
776\\
777\\
778\\
779\\
780\\
781\\
782\\
783\\
784
\end{array}$



787	min, (c) $\Delta t = 5$ min, and (d) $\Delta t = 10$ min. The images were histogram equalized and mean
788	filtered using a 5×5-pixel window. The yellow lines are isodistances (×1,000 km) from HTHH.

As the sampling period gets longer, the fine detail in the *BT* time derivatives is gradually lost (Figures 9b-9c-9d). This is the manifestation of spatiotemporal aliasing, where frequencies above the Nyquist frequency of the given sampling rate become aliased to lower frequencies and longer wavelengths, resulting in an effect resembling 'motion blur'. Some loss of detail is already noticeable for $\Delta t = 2$ min, but for full disk sampling with $\Delta t = 10$ min, the observed pattern is reduced to ~200 km wide dark and bright bands as shown previously in Figure 8.

The aliasing in a far-field domain is depicted in Figure 10. At high frequency sampling 796 $(\Delta t = 1 \text{ or } 2 \text{ min})$, the main Lamb pulse is represented by the dark band near the 9,720 km 797 isodistance in the 12:50 UTC image. There is a fainter dark band ~120-140 km ahead (east) of 798 the main pulse, which corresponds to the smaller pressure peak emitted \sim 7 min earlier (see 799 Figures 2a and S1). Here we note that Vadas et al. (2023b) analyzed 4.3-um BT perturbations in 800 an Atmospheric Infrared Sounder (AIRS) granule over Antarctica (7,500–8,000 km from 801 HTHH). They found a Lamb amplitude peak at a wavelength of ~146 km, which agrees with our 802 finding for the distance between the main pressure peak and the preceding one. 803

The main pulse is followed by a long train of waves with wavelengths decreasing in time from ~65–75 km to ~40–50 km, which again suggests dispersion. Full disk sampling ($\Delta t = 10$ min) only reveals the primary pulse, with the dark and bright bands having an increased width and wavelength of ~250 km and ~500 km, respectively, compared to the near field. The waves travel with a slightly larger observed propagation velocity of ~330 m s⁻¹, which likely is due to the added effect of westerly winds in this domain.



Figure 10. Far-field spatiotemporal aliasing in $BT_{12}^{\prime\prime}$ in G16_M2 (Texas) at (top row) 12:50 UTC and (bottom row) 13:00 UTC on 15 January 2022. The interval used for calculating the

- time derivative is (a, d) $\Delta t = 1 \min$, (b, e) $\Delta t = 2 \min$, and (c, f) $\Delta t = 10 \min$. The images were
- histogram equalized and mean filtered using a 5×5 -pixel window. The yellow lines are
- isodistances (×1000 km) from HTHH.

825 4 Far-field Lamb waves and near-field gravity waves

4.1 Minor arc and major arc passage of Lamb waves

827 In this section, we track the first minor arc (A1, from west to east towards the antipode) and major arc (A2, from east to west towards HTHH) passage of the waves across the mesoscale 828 domains on 15 and 16 January 2022, respectively. The third and fourth passages are also weakly 829 detectable in the data, but we focus on the stronger and more informative signature of the first 830 two passages. The ground-based propagation speed of the waves is estimated with the help of 831 time-distance plots introduced in Figure 11. A time-distance plot is constructed by stacking 1D 832 slices of the BT" images taken along a path marked by the yellow arrow. The path is aligned 833 with a fixed back-azimuth and indicates the overall propagation direction. In such a stack plot, 834 835 wave crests and troughs traveling along the path are represented by bright and dark stripes, whose slope encodes the propagation speed. With distance on the x-axis and time on the y-axis, 836 837 the slope decreases with increasing speed.

For simplicity, we use line segments as paths, which, nevertheless, are well aligned with geodesics over the mesoscale distances considered here. Due to the spatial (pixel) discretization, the speed estimates are virtually insensitive to a 5° - 10° variation in the back-azimuth of the segment.

The longer the line segment, however, the better the separation of stripes corresponding 842 to different speeds. The longest possible segment is the domain's diagonal, which is ~2,000 km 843 for the near-field and ~ 1.500 km for the far-field domains. In practice, however, the length of the 844 line segment available for tracking is significantly shorter, because the waves are visible only in 845 certain parts of the domain. In our case, the length of segments used for speed estimation ranges 846 between ~300–1,200 km. Over such distances, the travel time difference between speeds of 285 847 m s⁻¹ and 315 m s⁻¹ varies from 1.7 min to 6.7 min. These travel time differences correspond to a 848 mere 2–6 or 1–3 tick mark separation along the y-axis for $\Delta t = 1 \min \text{ and } \Delta t = 2 \min \text{ sampling}$, 849 respectively. Thus, a variation of 30 m s⁻¹ (or 10%) around the sound speed is slightly or just 850 barely distinguishable in the mesoscale data, depending on the length of the segment and the 851 sampling period. This uncertainty should be kept in mind when interpreting the stack plots. 852

The initial passage of A1 waves across the GOES-17 M2 Samoa domain between 04:00-853 854 06:00 UTC on 15 January is shown in Movie S3 and Figures 11a-b. The first visually detectable wave appears at the bottom of the domain ~170 km from HTHH at 04:16 UTC. The time-855 distance plot reveals the passage of more than a dozen wave crests during this 2-hour period. The 856 observed speed clearly exceeds the GW speed limit of ~285 m s⁻¹ (see Extended Data Figure 6 in 857 858 Wright et al., 2022) and is estimated at \sim 315 m s⁻¹. This velocity combined with the arrival time puts the emission time of the first wave at ~04:07 UTC, which agrees with the time of the first 859 eruptive event in the Samoa surface pressure (Figure 2a) as well as in distant infrasound and 860 seismo-acoustic records (Matoza et al., 2022). The wave fronts follow isodistances very well, 861 indicating isotropic propagation in the near field. 862



Figure 11. Passage of A1 waves across the mesoscale domains on 15 January 2022. From top to 883 bottom: (a, b) G17 M2 (Samoa) 03:58–05:58 UTC, C11, (c, d) G17 M2 (Alaska) 11:30–13:30 884 UTC, C11, (e, f) G16 M2 (Texas) 12:30–13:30 UTC, C12, and (g, h) G16 M1 (Alabama) 885 15:01–16:01 UTC, C09 (6.9 µm). The left panels are snapshots of BT" at the indicated times and 886 the right panels are time-distance plots over 1- or 2-hour periods with guides of constant phase 887 speed drawn (dashed yellow, in m s⁻¹). The yellow arrows are aligned with fixed back-azimuths 888 and indicate the overall propagation direction. Distance is measured from the circle (0 km) 889 towards the arrowhead. The interval used for calculating BT'' is $\Delta t = 2 \min$ for G17 M2 890 (Alaska) and $\Delta t = 1$ min otherwise. Except for the top row, the BT'' images were FFT-filtered. 891

The horizontal wavelength tends to increase from ~40 km to ~70 km with increasing distance from HTHH (period 2.1–3.7 min, frequency 4.5–7.9 mHz), suggesting some dispersion. The corresponding relatively small variations in speed, however, cannot be clearly detected given the uncertainty of our estimation method. The steeper stripes in the top-left corner of the stack plot represent slower GWs traveling with a speed of ≤ 85 m s⁻¹, which appear in the stratospheric umbrella cloud as it intrudes into the domain after ~05:00 UTC.

Tracking the waves in the near-limb GOES-17 M2 Alaska domain is more difficult due to 899 the coarse pixel size and considerable horizontal striping. To increase the signal to noise ratio 900 (SNR), we used $\Delta t = 2$ min sampling and applied FFT filtering to the images when constructing 901 the stack plot (Movie S4 and Figures 6c-d). The waves are first visible near the bottom left 902 (southwest) corner of the domain at ~11:15 UTC. Waves with longer $\lambda_H \approx 90-110$ km enter 903 first, followed by waves with somewhat shorter $\lambda_H \approx 70-90$ km. The wavelength of subsequent 904 waves cannot be estimated with confidence. Here, a propagation speed of \sim 330 m s⁻¹ is a 905 marginally better fit than the previously estimated \sim 315 m s⁻¹, at least for the longer wavelength 906 waves that produce the clearest signal. Note that audible sound was reported at Kenai, Alaska 907 between 13:15–13:45 UTC (Matoza et al., 2022), which is at the end of the 2-hour period when 908 909 the passage of the wave train was detectable in the mesoscale data.

As the waves travel eastward, they enter the GOES-16 M2 (Texas) domain at \sim 12:20 910 UTC (Movie S5 and Figures 11e-f). The main Lamb pulse is detected first, followed by waves 911 with a λ_{H} that generally decreases in time from ~75 km to ~40 km (frequency 4.2–7.9 mHz). 912 Waves can be observed in this domain at least until 15:00 UTC. Here, wave dispersion is perhaps 913 also borne out by the stack plot speed estimates. The main pulse appears to propagate at a speed 914 of \sim 330 m s⁻¹, while the traces of later wave fronts are better explained by a reduced speed of 915 \sim 315 m s⁻¹. These waves are likely Lamb waves, because they travel faster than the absolute 916 maximum GW speed limit of ~ 285 m s⁻¹, which is too slow to fit the data. 917

Finally, the waves are observed in the bottom half of our easternmost GOES-16 M1 (Alabama) domain between 15:01–16:20 UTC (Movie S6 and Figures 11g-h). This domain misses the main pulse and only captures the subsequent shorter waves with $\lambda_H \approx 40-55$ km. Because the waves can be best tracked over a short line segment in the bottom right corner, speed estimation from the stack plot is relatively uncertain. The wave traces indicate a propagation speed between 315–330 m s⁻¹.

924 By the time the waves reach the continental US, the wave fronts exhibit small but noticeable deviations from the isodistances. As shown in Figure 12 for the GOES-16 M2 Texas 925 domain, the wave fronts travel faster in the middle than at the top or bottom of the domain (see 926 also Figures 10 and 11e). The maximum deviation is \sim 120 km for the main Lamb pulse and \sim 50 927 km for the shorter-wavelength trailing waves. If we assume that the main pulse was emitted at 928 \sim 04:29 UTC, the acquisition time and distance from HTHH lead to a mean propagation speed in 929 the middle and at the bottom of the domain of \sim 323 m s⁻¹ and \sim 319 m s⁻¹, respectively—in good 930 agreement with the stack plot estimate of \sim 315 m s⁻¹. This \sim 4 m s⁻¹ mean speed difference 931 amounts to a remarkably small, $\sim 1.3\%$ anisotropy in propagation along paths that go over the 932 US. In full-disk data, Wright et al. (2022) found larger deviations from isodistances in the 933 portion of the main pulse which passed over the northern half of South America and slowed. The 934 observed propagation anisotropy can likely be explained by temperature and wind variations as 935 well as topography effects encountered along the travel path as argued by Sepúlveda et al. (2023) 936 and Watada et al. (2023). 937

939

940

941 942

. . .



Figure 12. Anisotropic wave propagation in G16_M2 (Texas): (a) raw grayscale image and (b) FFT-filtered image of BT_{12}'' at 13:11 UTC on 15 January 2022. The interval used for calculating the time derivative is $\Delta t = 1$ min. The yellow lines are isodistances (×1,000 km) from HTHH, which were fitted to the bottom (southernmost) part of the thickest dark band (at 10k km) and one of the thinner white bands (at 9.59k km). The width of the dark band is ~50 km, while that of the white band is ~25 km. The maximum deviation of the midline of the wave bands from the fitted isodistances is also indicated (~120 km and ~50 km, respectively).

951

The A2 antipodal passage of the waves is shown in Figure 13 and Movies S7–S10. Because the signal becomes weaker during the longer major arc paths, we used $\Delta t = 2$ min sampling to increase the SNR. As before, the observed wavelength tends to decrease in time, suggesting wave dispersion. However, the visually detectable wavelengths shift towards larger values compared to the A1 passage and are typically within 95–120 km. The disappearance of shorter wavelengths is likely caused by the increased (acoustic) attenuation at higher frequencies.

The estimated propagation speed is in the range of $300-330 \text{ m s}^{-1}$. In the US domains, 958 there is a hint of a slightly faster A1 passage (maximum \sim 330 m s⁻¹) than A2 passage (maximum 959 \sim 315 m s⁻¹), which likely is due to the waves traveling with the midlatitude westerlies in the 960 former case but traveling against them in the latter case. This observed speed reduction agrees 961 with the modeling by Sepúlveda et al. (2023), which accounts for wind effects: the apparent 962 phase speed decreases when the Lamb wave propagates against the wind and vice versa. Also 963 note the complex interference patterns as the returning waves enter from different azimuths at 964 different times and converge in the HTHH domain after circling the planet (Movie S10). Such 965 complex far-field waveforms are produced and shaped by global wind variations. 966





Figure 13. Passage of A2 waves across the mesoscale domains on 16 January 2022. From top to 986 bottom: (a, b) G16 M2 (Indiana) 06:02-08:02 UTC, C09, (c, d) G16 M1 (Alabama) 06:02-987 08:02 UTC, C09, (e, f) G17 M2 (Alaska) 08:00–10:00 UTC, C11, and (g, h) G17 M1 (HTHH) 988 15:45–17:45 UTC, C07. The left panels are snapshots of BT" at the indicated times and the right 989 panels are time-distance plots over 2-hour periods with guides of constant phase speed drawn 990 (dashed yellow, in m s⁻¹). The yellow arrows are aligned with fixed back-azimuths and indicate 991 the overall propagation direction. Distance is measured from the circle (0 km) towards the base 992 of the arrow. The interval used for calculating BT'' is $\Delta t = 1 \text{ min for G17 M1 (HTHH)}$ and 993 $\Delta t = 2$ min otherwise. The snapshots in the left column are unfiltered images, while the time-994 995 distance plots in the right column are based on low pass-filtered images. The yellow triangle in panel (g) marks the location of HTHH. 996

998 4.2 Wave phenomena observed over the plume

So far, we have focused on far-field domains which did not observe the plume itself. The GOES-17 M1 domain, however, was moved to HTHH ~3 hours after the main eruption at 07:05 UTC on 15 January and remained there for 2 days. This domain allows us to study the generation and propagation of waves directly over the plume. Below we present three examples that demonstrate the potential of this data source. We focus on the first two hours of the data, because later imagery shows increased noise and striping, although waves are detectable at least until 14:00 UTC.

1006 4.2.1 GWs obscured by the stratospheric umbrella

1007 A case when GWs are observed in the tropospheric umbrella but not in the stratospheric 1008 umbrella was captured during the first hour of the GOES-17 M1 HTHH data between 07:06– 08:06 UTC (Movie S11). At that time the plume consists of two distinct layers, as shown by the 1009 10.3 µm (band 13, 'clean' longwave window) brightness temperatures in Figure 14a. The 1010 1011 warmer stratospheric umbrella (~ 230 K, light gray) is located between 30–35 km, while the 1012 colder near-tropopause umbrella (~195K, white) is spreading between 16-19 km. The eastwardpropagating concentric arcs of short-wavelength GWs are clearly visible in the near-tropopause 1013 1014 umbrella; however, their westward-propagating counterparts are absent from the stratospheric umbrella, as depicted by the BT_{13}'' images in Figure 14b. The stack plot in Figure 14c indicates 1015 horizontal wavelengths of ~13–18 km and phase speeds of ~45–50 m s⁻¹ (periods of ~4.8–6.0 1016 min and frequencies of ~2.8–3.5 mHz). Note that at full-disk sampling ($\Delta t = 10$ min), these 1017 1018 eastward-propagating short GWs seemingly move westward due to the wagon-wheel temporal 1019 aliasing effect (Movie S12).

1020 If the source of these waves is below the tropopause, a potential explanation is critical 1021 level filtering. When the horizontal phase speed of upward propagating gravity waves equals the 1022 projection of the background wind speed along the GW propagation direction at a given altitude, 1023 the vertical component of group velocity approaches zero. At that critical level, the GWs are 1024 eliminated as their energy is absorbed and transferred to the background flow. This leads to the 1025 blocking of wave propagation in certain directions, depending on the wind profile.

1026 A reanalysis mean wind profile is available from ERA5 (Hersbach et al., 2017; 1027 Supporting Information Figure S4), while actual plume motion and height retrievals were obtained by Carr et al. (2022) using stereo observations from Himawari-8 FD and GOES-17 M1 1028 HTHH imagery. Both wind datasets show weak meridional winds typically between $\pm 5 \text{ m s}^{-1}$ 1029 1030 throughout the entire troposphere and stratosphere. The zonal winds were eastwardly and relatively weak in the troposphere and near the tropopause; for the 16–19 km altitude range 1031 varying between 0–10 m s⁻¹ in ERA5 and between 0–15 m s⁻¹ with a mean of \sim 7 m s⁻¹ in the 1032 1033 stereo retrievals. Such weak winds have little effect on the eastward-propagating GWs detected in the lower umbrella. 1034

- 1035
- 1036
- 1037
- 1038
- 1039



Figure 14. GW obscuration in G17_M1 (HTHH) on 15 January 2022: (a) channel 13 (10.3 μ m) brightness temperatures BT_{I3} at 07:42 UTC, (b) BT''_{13} at 07:42 UTC, and (c) time-distance plot for the period 07:06–08:06 UTC along the yellow arrow in panel (b). The interval used for calculating the time derivatives is $\Delta t = 1 \text{ min. } BT_{I3}$ was gray scaled between 190K (white) and 290K (black), but no filtering was applied to BT''_{13} . The yellow triangle marks the location of HTHH. The height range and typical zonal velocity of the stratospheric and near-tropopause plumes are also indicated in panel (a).

1056 In the mid stratosphere, however, the zonal winds were westwardly and much faster. In 1057 the 30–35 km altitude range of the upper umbrella, ERA5 indicates westward winds of 25–35 m 1058 s⁻¹. The stereo wind retrievals are significantly stronger, showing westward plume motion of 30– 1059 55 m s⁻¹. Thus, stratospheric plume motion is fast enough for critical level filtering to eliminate 1060 westward-propagating GWs emitted below the tropopause.

1061 AIRS observations, however, show similar waves to the west of the volcano a few hours 1062 later (Wright et al., 2022). If the GWs are emitted in a region extending from the tropopause to 1063 the lower stratosphere, critical level filtering is unlikely to play a major role. The GWs might 1064 simply be obscured by the optically thick mid-stratospheric plume, which masks the *BT* 1065 perturbations.

1066 4.2.2 Mean flow advection of GWs

Strong winds can horizontally shift the apparent center of concentric gravity waves, 1067 which was clearly demonstrated by the modeling study of Vadas et al. (2009). This effect is also 1068 1069 observed in the first hour of the GOES-17 M1 HTHH data stream. Here we focus on faster GWs 1070 that are already noticeable in the northwestern (top left) corner of Figure 14b. These GWs, 1071 however, can be detected more clearly in the 9.6-µm 'ozone' channel (C12), especially after FFT-filtering, as shown in Figure 15a (see also Movie S13). The vertical weighting function of 1072 1073 C12 peaks at ~22 km, thus, the GWs are captured at a higher altitude, near the mid stratosphere, compared to the 10.3-µm window channel. The stack plot in Figure 15b indicates horizontal 1074 wavelengths of $\sim 32-42$ km and phase speeds of $\sim 130-135$ m s⁻¹ (typical period of ~ 4.7 min and 1075 frequency of ~3.5 mHz). Note that this speed is essentially the theoretical maximum GW phase 1076 speed at this particular wavelength (see Extended Data Figure 6 in Wright et al., 2022). 1077 It is evident in Figure 15a that the concentric rings cannot be fitted with ellipses centered 1078

1079 on HTHH. A caveat to note is that the emission center of GWs in the satellite images is always 1080 shifted from HTHH, even in windless conditions, due to parallax. Because GOES-17 views HTHH from the east-northeast (azimuth of 66°) at a zenith angle of 50° , the apparent emission 1081 center is shifted to west-southwest (left and below) relative to HTHH by a distance depending on 1082 1083 the emission height. For an emission height of 17 km (lower umbrella) and 32 km (upper umbrella) the shift is ~ 20 km and ~ 38 km, respectively, or $\mathcal{O}(10$ pixels). 1084

1085



Figure 15. Mean flow advection in G17 M1 (HTHH) on 15 January 2022: (a) FFT-filtered BT'_{12} 1093 at 07:06 UTC and (b) time-distance plot for the period 07:06-08:06 UTC along the vellow 1094 arrow in panel (a). The interval used for calculating the time derivatives is $\Delta t = 1$ min. The 1095 1096 yellow dashed curve is an ellipse fitted to one of the concentric rings with its center marked by 1097 the plus sign. The distance between the ellipse center and HTHH (yellow triangle) is also indicated. 1098

1099

The center of the ellipse fitted to one of the concentric rings in Figure 15a is located ~ 192 1100 km west of an assumed emission source at ~22 km (peak of C12 weighting function). Note that 1101 1102 the uncertainty of ellipse fitting can easily be on the order of 10 km, because the concentric rings are not perfect circles (or ellipses in the satellite perspective) due to anisotropic propagation and 1103 they might only be partially observed. Here we focused on fitting the northern and western part 1104 of the rings, which are the clearest in the images. 1105

The semi major axis of the fitted ellipse is ~461 km, which corresponds to a travel time 1106 of \sim 59 min from emission assuming a propagation velocity of 130 m s⁻¹. This, in turn, 1107 corresponds to a mean westward advection speed of the ellipse center of \sim 54 m s⁻¹. By fitting 1108 several concentric rings, we estimate a typical westward advection speed of 50–60 m s⁻¹, which 1109 1110 is within the range of the stereo plume motion retrievals by Carr et al. (2022). The largest ring in Figure 15a indicates a travel time of \sim 75 min; therefore, these GWs were likely triggered by a 1111 later explosion which started between 05:44–05:51 UTC according to surface pressure records 1112 1113 (Purkis et al., 2023; Wright et al., 2022).

4.2.3 Lamb waves from the last major explosion 1114

Our final example using the GOES-17 M1 HTHH imagery demonstrates the detection of 1115 Lamb waves triggered by the last major eruption ~4.0–4.5 hours after the primary climactic

- between 8–9 UTC (Movie S11; Carr et al., 2022), with the two most prominent ones occurring at
- 1119 08:41 UTC (~168K) and 08:46 UTC (~172K). Stereo motion retrievals by Carr et al. (2022)
- 1120 reveal strong plume-top divergence as these bubbles reach the upper umbrella and spread out.
- 1121 The global infrasound and seismo-acoustic network detected an explosive event at 08:31 UTC
- 1122 (Matoza et al., 2022), while the Tonga surface pressure data indicate the last major signal at
- 1123 08:46 UTC (Wright et al., 2022).
- This series of explosions generated Lamb waves as well as GWs, as depicted in Figure 16 1124 (see also Movie S14). The animations show a fast-moving packet of waves in the top left 1125 (northwestern) quadrant of the domain between ~08:55–09:30 UTC. The stack plot in Figure 16b 1126 yields a propagation speed of \sim 315 m s⁻¹ and horizontal wavelength of \sim 60–70 km, similar to the 1127 Lamb waves generated by the primary explosion. Unlike the primary Lamb waves, however 1128 1129 these weaker secondary Lamb waves could not be detected in the far-field domains. Note that the concentric rings in Figure 16a can be well fitted with circles centered on HTHH. This indicates 1130 negligible parallax in the projected location of the emission center, which is further proof of 1131 surface-triggered Lamb waves that coherently move through the troposphere guided by weak 1132 near-surface winds. 1133

In contrast, the smaller concentric rings visible in the upper umbrella can only be fitted 1134 with circles whose center is shifted west-southwest relative to HTHH due mostly to parallax and 1135 to a lesser degree wind. This indicates gravity waves emitted at 30+ km altitude. A time-distance 1136 1137 analysis (not shown) reveals that the spectral characteristics of these GWs (wavelength of ~40 km and speed of ~ 135 m s⁻¹) are similar to those of the earlier-emitted GWs plotted in Figure 15. 1138 Figure 16b also hints at the presence of slower GWs propagating at a speed of ~40–50 m s⁻¹. 1139 These small-scale GWs detected in the plume are consistent with the dense concentric wave 1140 patterns observed by AIRS at a somewhat larger distance from the volcano (Adam, 2022). 1141

1142



- Figure 16. Lamb and gravity waves in G17_M1 (HTHH) triggered by the last major explosion on 15 January 2022: (a) BT_{11}'' at 09:16 UTC and (b) time-distance plot for the period 08:45– 09:45 UTC along the yellow arrow in panel (a). The interval used for calculating the time derivatives is $\Delta t = 1$ min and the stack plot is based on FFT-filtered images. The larger ellipse
- fitted to a Lamb wave ring is a circle with a radius of 585 km centered on HTHH (triangle),
- 1155 while the smaller ellipse fitted to a GW ring is a circle of 175 km radius with a shifted center
- 1156 (plus sign).

1158 **5 Discussion and Summary**

We analyzed GOES-R high-cadence mesoscale imagery to estimate the spectral 1159 1160 properties of Lamb and gravity waves emitted by the 15 January 2022 HTHH eruption. In a \sim 1,000 \times 1,000 km² domain, the 1-min mesoscale brightness temperatures provide an order of 1161 magnitude better temporal sampling than the 10-min or 15-min full-disk data used in previous 1162 studies. The absolute temperature perturbations produced by the eruption are generally difficult 1163 1164 to extract, because they are superimposed on a large dynamic range and highly variable background. Instead, we used the common visualization method of differencing BT image 1165 sequences, which amounts to taking the time derivatives of the waveform. The appearance of 1166 1167 wave patterns in such difference imagery is controlled by two opposing factors. Waveform derivatives represent a high-pass filter, which eliminates the long period variation. Contrarily, 1168 reducing the observation frequency, as in full-disk data, leads to the temporal aliasing of short-1169 period fluctuations to longer periods and wavelengths. 1170

The surface pressure signature of the HTHH Lamb waves contains multiple pulses of 1171 1172 varying magnitude, but its broad envelope can be approximated by an N-wave or positive triangular pulse of 20–50 min duration and 400–900 km width, depending on location. We 1173 1174 showed that imposing temperature anomalies that trace the measured surface pressure anomalies explains the salient features of the BT" second derivative patterns seen in the high-cadence 1175 1176 mesoscale imagery. These patterns visualize the short-period variations and individual shocks 1177 within the wave packet rather than the broad envelope. However, the full temperature anomaly 1178 waveform can be reconstructed reasonably well from the observed mesoscale BT''. The reconstructed temperature anomalies highly correlate with the measured pressure anomalies. 1179

In contrast, the temporally aliased full-disk data only captures the primary Lamb pulse, which can be approximated by an *N*-wave with a duration of 30-40 min. In the low-cadence fulldisk *BT*^{''} images, the primary Lamb pulse appears as a bright–dark–bright triplet of ~200 km wide bands (the distance an acoustic packet travels in 10 min).

1184 The mesoscale imagery indicates that the primary Lamb wave is trailed by long-1185 continuing waves of ~40–80 km wavelength. With the estimated horizontal phase velocity of 1186 \sim 315±15 m s⁻¹, the trailing waves have typical periods of 2.1–4.2 min and frequencies of 4–8 1187 mHz. The mesoscale data also suggest dispersion, because wavelength tends to systematically 1188 decrease with time as the wave train traverses the domains.

1189 These transient volcanic Lamb frequencies are well within the 0.2–10 mHz frequency range of turbulence-induced background Lamb waves, which are generally present in the 1190 1191 atmosphere (Nishida et al., 2014). The short acoustic periods observed in our data are also in agreement with early findings on atmospheric nuclear explosions and other major volcanic 1192 eruptions. We note that most of the early works analyzed data from microbarovariographs, which 1193 are high-pass instruments. They measure the rate of change of pressure (the first time derivative) 1194 1195 rather than the absolute pressure; thus, they are readily comparable with our observations of BT waveform derivatives. 1196

The oscillatory part of microbarovariograms recorded after nuclear tests contains dominant wavelengths less than 100 km and periods of 1–10 min (Donn & Ewing, 1962; Donn et al., 1963; Garrett, 1969; Pierce & Posey, 1971; Posey & Pierce, 1971). Waves from nuclear explosions are also dispersive, such that in the far field the initial impulse becomes resolved into a train of waves of decreasing period. Concerning volcanic eruptions, Bolt & Tanimoto (1981) and Mikumo & Bolt (1985) found atmospheric pressure waves from the 1980 Mount St. Helens
eruption with dominant periods of 5–8 min, while Kanamori et al. (1994) report atmospheric
oscillations with periods of 3–5 min for the 1883 Krakatoa, the 1982 El Chicón, and the 1991
Pinatubo eruptions as well.

Acoustic waves with a period of ~ 5 min are thus often generated by energetic 1206 1207 atmospheric blasts. A recent analysis of HTHH air and seafloor pressure spectrograms by Tonegawa & Fukao (2023) also identified long-continuing waves at frequencies near 3.6 mHz 1208 (period of ~4.6 min), which bear a close resemblance to the trailing waves observed in our 1209 1210 satellite imagery. They argue that these waves represent the resonant coupling of the main Lamb wave and thermospheric gravity waves that propagate horizontally at the sound speed. The long-1211 period primary Lamb wave can be adequately generated by a sea level or mid-tropospheric 1212 pressure source (Amores et al., 2022; Watanabe et al., 2022). The excitation of the primary Lamb 1213 wave together with the shorter period Lamb waves, thermospheric waves, and the resonant 1214 waves, however, requires a more energetic pressure source located in the mesosphere. 1215

1216 Besides the climatic explosion at ~04:29 UTC, Lamb waves were generated by the last 1217 major explosion at ~08:40–08:45 UTC too. These weaker waves, however, could only be 1218 observed in the near-field but not in the far-field domains. The mesoscale data also proved useful 1219 to detect wind effects such as mean flow advection in the propagation of gravity waves and 1220 captured gravity waves propagating near their theoretical maximum speed (~130 m s⁻¹ for a 1221 wavelength of ~42 km).

In conclusion, the GOES-R mesoscale observations represent a rich data source for further work. More sophisticated analysis techniques (e.g., wavelet transform) could be used to extract additional spectral information, confirm Lamb wave dispersion, and exploit IR channel differences in order to characterize the 3D nature and vertical propagation of waves. The data could also be used to constrain the HTHH eruption source parameters and validate the detailed numerical modeling of global wave propagation.

1228

1229 Acknowledgments

1230 Á.H. was supported by the Deutsche Forschungsgemeinschaft (DFG) as part of the Research

1231 Unit VolImpact, subproject VolPlume (FOR2820, DFG Grant 398006378). This study also

1232 contributes to the Center for Earth System Research and Sustainability (CEN) of Universität

1233 Hamburg. S.L.V. was supported by NSF Grant AGS-1832988. C.C.S. received funding from the

- 1234 Minerva Fast Track Program of the Max Planck Society. The suggestions of Neil P. Hindley,
- 1235 University of Bath, UK and two anonymous reviewers greatly improved the paper.

1237 Data Availability Statement

- 1238 GOES-16 and GOES-17 datasets are publicly accessible through Amazon Web Services (AWS,
- 1239 2023). ERA5 data are available from Hersbach et al. (2017). The Pago Pago, American Samoa
- surface pressure data (sensor index 85435) are freely available for non-commercial use after
- 1241 registration at PurpleAir (2023). The Coconut Point, American Samoa surface pressure data are
- 1242 from the personal weather station of Joe LaPlante (joseph.laplante@noaa.gov) and are included
- in the Supporting Information submitted with this paper. The publicly available ASOS Page 2
- 1244 station data are provided by ASOS (2023).

1245 References

- Adam, D. (2022). Tonga volcano created puzzling atmospheric ripples. *Nature*, 602, 497.
- 1247 https://doi.org/10.1038/d41586-022-00127-1
- 1248 Amores, A., Monserrat, S., Marcos, M., Argüeso, D., Villalonga, J., Jordà, G., & Gomis, D.
- 1249 (2022). Numerical simulation of atmospheric Lamb waves generated by the 2022 Hunga-Tonga
- 1250 volcanic eruption. *Geophysical Research Letters*, 49, e2022GL098240.
- 1251 https://doi.org/10.1029/2022GL098240
- 1252 ASOS. (2023). National Centers for Environmental Information (NCEI) [Dataset]. Retrieved
- 1253 from https://www.ncei.noaa.gov/data/automated-surface-observing-system-one-minute-pg2/
- 1254 Astafyeva, E., Maletckii, B., Mikesell, T. D., Munaibari, E., Ravanelli, M., Coisson, P., et al.
- 1255 (2022). The 15 January 2022 Hunga Tonga eruption history as inferred from ionospheric
- 1256 observations. *Geophysical Research Letters*, 49, e2022GL098827.
- 1257 https://doi.org/10.1029/2022GL098827

- 1258 Averiyanov, M., Ollivier, S., Khokhlova, V., & Blanc-Benon, P. (2011). Random focusing of
- 1259 nonlinear acoustic *N*-waves in fully developed turbulence: Laboratory scale experiment. *Journal*
- 1260 of the Acoustical Society of America, 130(6), 3595–3607. https://doi.org/10.1121/1.3652869
- 1261 AWS. (2023). Amazon Web Services (AWS) [Dataset]. Retrieved from
- 1262 https://registry.opendata.aws/noaa-goes/
- Bolt, B. A., & Tanimoto, T. (1981). Atmospheric oscillations after the May 18, 1980 eruption of
- 1264 Mount St. Helens. *Eos Transactions American Geophysical Union*, 62(23), 529–530.
- 1265 https://doi.org/10.1029/EO062i023p00529
- 1266 Bór, J., Bozóki, T., Sátori, G., Williams, E., Behnke, S. A., Rycroft, M. J., et al. (2023).
- 1267 Responses of the AC/DC global electric circuit to volcanic electrical activity in the Hunga
- 1268 Tonga-Hunga Ha'apai eruption on 15 January 2022. *Journal of Geophysical Research:*
- 1269 Atmospheres, 128, e2022JD038238. https://doi.org/10.1029/2022JD038238
- 1270 Bretherton, F. P. (1969). Lamb waves in a nearly isothermal atmosphere. *Quarterly Journal of*
- 1271 *the Royal Meteorological Society*, *95*(406), 754–757. https://doi.org/10.1002/qj.49709540608
- 1272 Carr, J. L., Horváth, Á., Wu, D. L., & Friberg, M. D. (2022). Stereo plume height and motion
- retrievals for the record-setting Hunga Tonga-Hunga Ha'apai eruption of 15 January 2022.
- 1274 Geophysical Research Letters, 49, e2022GL098131. https://doi.org/10.1029/2022GL098131
- 1275 Donn, W. L., & Ewing, M. (1962). Atmospheric waves from nuclear explosions—Part II: The
- 1276 Soviet test of 30 October 1961. *Journal of the Atmospheric Sciences*, 19(3), 264–273.
- 1277 https://doi.org/10.1175/1520-0469(1962)019<0264:AWFNEI>2.0.CO;2

- 1278 Donn, W. L., Pfeffer, R. L., & Ewing, M. (1963). Propagation of air waves from nuclear
- 1279 explosions. Science, 139(3552), 307-317. https://doi.org/10.1126/science.139.3552.307
- 1280 Francis, S. H. (1973). Acoustic-gravity modes and large-scale traveling ionospheric disturbances
- 1281 of a realistic, dissipative atmosphere. Journal of Geophysical Research: Space Physics, 78(13),
- 1282 2278–2301. https://doi.org/10.1029/JA078i013p02278
- 1283 Garrett, C. J. R. (1969). Atmospheric edge waves. *Quarterly Journal of the Royal*
- 1284 Meteorological Society, 95(406), 731–753. https://doi.org/10.1002/qj.49709540607
- 1285 GOES R. (2019). GOES R Series Product Definition and User's Guide (PUG), 416-R-PUG-L1B-
- 1286 0347 Vol. 3 Revision 2.2. Retrieved from https://www.goes-r.gov/users/docs/PUG-L1b-vol3.pdf
- 1287 Harding, B. J., Wu, Y.-J. J., Alken, P., Yamazaki, Y., Triplett, C. C., Immel, T. J., et al. (2022).
- 1288 Impacts of the January 2022 Tonga volcanic eruption on the ionospheric dynamo: ICON-
- 1289 MIGHTI and Swarm observations of extreme neutral winds and currents. *Geophysical Research*
- 1290 Letters, 49(9), e2022GL098577. https://doi.org/10.1029/2022GL098577
- 1291 Heinrich, P., Gailler, A., Dupont, A., Rey, V., Hébert, H., & Listowski, C. (2023). Observations
- and simulations of the meteotsunami generated by the Tonga eruption on 15 January 2022 in the
- 1293 Mediterranean Sea. *Geophysical Journal International*, 234(2), 903–914.
- 1294 https://doi.org/10.1093/gji/ggad092
- 1295 Heki, K. (2022). Ionospheric signatures of repeated passages of atmospheric waves by the 2022
- 1296 Jan. 15 Hunga Tonga-Hunga Ha'apai eruption detected by QZSS-TEC observations in Japan.
- 1297 Earth Planets Space, 74, 112. https://doi.org/10.1186/s40623-022-01674-7

- 1298 Hersbach, H., Bell, B., Berrisford, P., Hirahara, S., Horányi, A., Muñoz-Sabater, J., et al. (2017).
- 1299 Complete ERA5 from 1940: Fifth generation of ECMWF atmospheric reanalyses of the global
- 1300 climate [Dataset]. Copernicus Climate Change Service (C3S) Climate Data Store (CDS).
- 1301 https://doi.org/10.24381/cds.143582cf
- 1302 Johnson, J. B., Boyer, T., Watson, L. M., & Anderson, J. F. (2023). Volcano opto-acoustics:
- 1303 Mapping the infrasound wavefield at Yasur Volcano (Vanuatu). *Geophysical Research Letters*,
- 1304 50, e2022GL102029. https://doi.org/10.1029/2022GL102029
- 1305 Kanamori, H., Mori, J., & Harkrider, D. G. (1994). Excitation of atmospheric oscillations by
- 1306 volcanic eruptions. Journal of Geophysical Research: Solid Earth, 99(B11), 21947–21961.
- 1307 https://doi.org/10.1029/94JB01475
- 1308 Khaykin, S., Podglajen, A., Ploeger, F., Grooß, J.-U., Tence, F., Bekki, S., et al. (2022). Global
- 1309 perturbation of stratospheric water and aerosol burden by Hunga eruption. *Communications*
- 1310 Earth & Environment, 3, 316. https://doi.org/10.1038/s43247-022-00652-x
- 1311 Kubota, T., Saito, T., & Nishida, K. (2022). Global fast-traveling tsunamis driven by
- atmospheric Lamb waves on the 2022 Tonga eruption. *Science*, *377*(6601), 91–94.
- 1313 https://doi.org/10.1126/science.abo4364
- 1314 Matoza, R. S., Fee, D., Assink, J. D., Iezzi, A. M., Green, D. N., Kim, K., et al. (2022).
- 1315 Atmospheric waves and global seismoacoustic observations of the January 2022 Hunga eruption,
- 1316 Tonga. Science, 377(6601), 95–100. https://doi.org/10.1126/science.abo7063
- 1317 McCorkel, J., Van Naarden, J., Lindsey, D., Efremova, B., Coakley, M., Black, M., &
- 1318 Krimchansky, A. (2019). GOES-17 Advanced Baseline Imager performance recovery summary.

- 1319 In Proceedings of IGARSS 2019 IEEE International Geoscience and Remote Sensing
- 1320 Symposium. Retrieved from https://ntrs.nasa.gov/search.jsp?R=20190028689
- 1321 Mikumo, T., & Bolt, B. A. (1985). Excitation mechanism of atmospheric pressure waves from
- the 1980 Mount St Helens eruption. *Geophysical Journal International*, 81(2), 445–461.
- 1323 https://doi.org/10.1111/j.1365-246X.1985.tb06412.x
- 1324 Millán, L., Santee, M. L., Lambert, A., Livesey, N. J., Werner, F., Schwartz, M. J., et al. (2022).
- 1325 The Hunga Tonga-Hunga Ha'apai hydration of the stratosphere. *Geophysical Research Letters*,
- 1326 49, e2022GL099381. https://doi.org/10.1029/2022GL099381
- 1327 Nishida, K., Kobayashi, N., & Fukao, Y. (2014). Background Lamb waves in the Earth's
- 1328 atmosphere. *Geophysical Journal International*, 196(1), 312–316.
- 1329 https://doi.org/10.1093/gji/ggt413
- 1330 Otsuka, S. (2022). Visualizing Lamb waves from a volcanic eruption using meteorological
- 1331 satellite Himawari-8. *Geophysical Research Letters*, 49, e2022GL098324.
- 1332 https://doi.org/10.1029/2022GL098324
- 1333 Pierce, A. D., & Posey, J. W. (1971). Theory of the excitation and propagation of Lamb's
- 1334 atmospheric edge mode from nuclear explosions. *Geophysical Journal International*, 26(1-4),
- 1335 341–368. https://doi.org/10.1111/j.1365-246X.1971.tb03406.x
- 1336 Posey, J. W., & Pierce, A. D. (1971). Estimation of nuclear explosion energies from
- 1337 microbarograph records. *Nature*, 232, 253. https://doi.org/10.1038/232253a0

- 1338 Proud, S. R., Prata, A. T., & Schmauss, S. (2022). The January 2022 eruption of Hunga Tonga-
- Hunga Ha'apai volcano reached the mesosphere. *Science*, *378*(6619), 554–557.
- 1340 https://doi.org/10.1126/science.abo4076
- 1341 Purkis, S. J., Ward, S. N., Fitzpatrick, N. M., Garvin, J. B., Slayback, D., Cronin, S. J., et al.
- 1342 (2023). The 2022 Hunga-Tonga megatsunami: Near-field simulation of a once-in-a-century
- event. Science Advances, 9, eadf5493. https://doi.org/10.1126/sciadv.adf5493
- 1344 PurpleAir. (2023). PurpleAir, Inc. [Dataset]. Retrieved from https://www2.purpleair.com/
- 1345 Randel, W. J., Johnston, B. R., Braun, J. J., Sokolovskiy, S., Vömel, H., Podglajen, A., et al.
- 1346 (2023). Stratospheric water vapor from the Hunga Tonga–Hunga Ha'apai volcanic eruption
- 1347 deduced from COSMIC-2 radio occultation. *Remote Sensing*, 15(8), 2167.
- 1348 https://doi.org/10.3390/rs15082167
- 1349 Schmit, T. J., Griffith, P., Gunshor, M. M., Daniels, J. M., Goodman, S. J., & Lebair, W. J.
- 1350 (2017). A closer look at the ABI on the GOES-R Series. Bulletin of the American Meteorological
- 1351 Society, 98(4), 681–698. https://doi.org/10.1175/BAMS-D-15-00230.1
- 1352 Sepúlveda, I., Carvajal, M., & Agnew, D. C. (2023). Global winds shape planetary-scale Lamb
- 1353 waves. *Geophysical Research Letters*, *50*(19), e2023GL106097.
- 1354 https://doi.org/10.1029/2023GL106097
- 1355 Stout, T. A. (2018). Simulation of N-wave and shaped supersonic signature turbulent variations,
- 1356 (Doctoral dissertation). Retrieved from Electronic Theses and Dissertations for Graduate School
- 1357 (ETDA). (https://etda.libraries.psu.edu/catalog/15959tqs5346). State College, PA: The
- 1358 Pennsylvania State University.

- 1359 Suzuki, T., Nakano, M., Watanabe, S., Tatebe, H., & Takano, Y. (2023). Mechanism of a
- 1360 meteorological tsunami reaching the Japanese coast caused by Lamb and Pekeris waves
- 1361 generated by the 2022 Tonga eruption. *Ocean Modelling*, *181*, 102153.
- 1362 https://doi.org/10.1016/j.ocemod.2022.102153
- 1363 Tonegawa, T., & Fukao, Y. (2023). Mesospheric pressure source from the 2022 Hunga, Tonga
- 1364 eruption excites 3.6-mHz air-sea coupled waves. *Science Advances*, 9(26), eadg8036.
- 1365 https://doi.org/10.1126/sciadv.adg8036
- 1366 Vadas, S. L. (2013). Compressible *f*-plane solutions to body forces, heatings, and coolings, and
- 1367 application to the primary and secondary gravity waves generated by a deep convective plume.
- 1368 Journal of Geophysical Research: Space Physics, 118(5), 2377–2397.
- 1369 https://doi.org/10.1002/jgra.50163
- 1370 Vadas, S. L., Becker, E., Figueiredo, C., Bossert, K., Harding, B. J., & Gasque, L. C. (2023a).
- 1371 Primary and secondary gravity waves and large-scale wind changes generated by the Tonga
- 1372 volcanic eruption on 15 January 2022: Modeling and comparison with ICON-MIGHTI winds.
- 1373 *Journal of Geophysical Research: Space Physics, 128,* e2022JA031138.
- 1374 https://doi.org/10.1029/2022JA031138
- 1375 Vadas, S. L., Figueiredo, C., Becker, E., Huba, J. D., Themens, D. R., Hindley, N. P., et al.
- 1376 (2023b). Traveling ionospheric disturbances induced by the secondary gravity waves from the
- 1377 Tonga eruption on 15 January 2022: Modeling with MESORAC/HIAMCM/SAMI3 and
- 1378 comparison with GPS/TEC and ionosonde data. Journal of Geophysical Research: Space
- 1379 *Physics*, *128*, e2023JA031408. https://doi.org/10.1029/2023JA031408

- 1380 Vadas, S. L., Yue, J., She, C.-Y., Stamus, P. A., & Liu, A. Z. (2009). A model study of the
- 1381 effects of winds on concentric rings of gravity waves from a convective plume near Fort Collins
- 1382 on 11 May 2004. Journal of Geophysical Research: Atmospheres, 114, D06103.
- 1383 https://doi.org/10.1029/2008JD010753
- 1384 Van Eaton, A. R., Lapierre, J., Behnke, S. A., Vagasky, C., Schultz, C. J., Pavolonis, M., et al.
- 1385 (2023). Lightning rings and gravity waves: Insights into the giant eruption plume from Tonga's
- Hunga Volcano on 15 January 2022. *Geophysical Research Letters*, 50, e2022GL102341.
- 1387 https://doi.org/10.1029/2022GL102341
- 1388 Vergoz, J., Hupe, P., Listowski. C., Le Pichon, A., Garcés, M. A., Marchetti, E., et al. (2022).
- 1389 IMS observations of infrasound and acoustic-gravity waves produced by the January 2022
- 1390 volcanic eruption of Hunga, Tonga: A global analysis. Earth and Planetary Science Letters, 591,
- 1391 117639. https://doi.org/10.1016/j.epsl.2022.117639
- 1392 Watada, S., Imanishi, Y., & Tanaka, K. (2023). Detection of air temperature and wind changes
- 1393 synchronized with the Lamb wave from the 2022 Tonga volcanic eruption. *Geophysical*
- 1394 *Research Letters*, 50, e2022GL100884. https://doi.org/10.1029/2022GL100884
- 1395 Watanabe, S., Hamilton, K., Sakazaki, T., & Nakano, M. (2022). First detection of the Pekeris
- 1396 internal global atmospheric resonance: evidence from the 2022 Tonga eruption and from global
- reanalysis data. *Journal of the Atmospheric Sciences*, 79(11), 3027–3043.
- 1398 https://doi.org/10.1175/JAS-D-22-0078.1

- 1399 Winn, S., Sarmiento, A., Alferez, N., & Touber, E. (2023). Two-way coupled long-wave
- 1400 isentropic ocean-atmosphere dynamics. *Journal of Fluid Mechanics*, 959, A22.
- 1401 https://doi.org/10.1017/jfm.2023.131
- 1402 Wright, C. J., Hindley, N. P., Alexander, M. J., Barlow, M., Hoffmann, L., Mitchell, C. N., et al.
- 1403 (2022). Surface-to-space atmospheric waves from Hunga Tonga–Hunga Ha'apai eruption.
- 1404 *Nature*, 609, 741–746. https://doi.org/10.1038/s41586-022-05012-5
- 1405 Yamada, M., Ho, T.-C., Mori, J., Nishikawa, Y., & Yamamoto, M.-Y. (2022). Tsunami triggered
- 1406 by the Lamb wave from the 2022 Tonga volcanic eruption and transition in the offshore Japan
- 1407 region. *Geophysical Research Letters*, 49, e2022GL098752.
- 1408 https://doi.org/10.1029/2022GL098752
- 1409 Yuldashev, P. V., Ollivier, S., Karzova, M. M., Khokhlova, V. A., & Blanc-Benon, P. (2017).
- 1410 Statistics of peak overpressure and shock steepness for linear and nonlinear *N*-wave propagation
- 1411 in a kinematic turbulence. Journal of the Acoustical Society of America, 142(6), 3402–3415.
- 1412 https://doi.org/10.1121/1.5015991