A Generalization of the Burridge and Knopoff Model

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Abstract

The paper is dedicated to the classical glissade's modeling in order to explain the earthquake mechanisms. The B&K model has been rethought generalizing the uniform movement of the largest stratum, corresponds to any continually acting force. In addition, for the resistance function has been used an expression obtained by the authors, based on the Elementary Catastrophes Theory that fully satisfies the conditions of the existence and uniqueness theorem regarding the solutions of the systems of ODE. The generalized B&K model equations have been solved numerically for a particular case through the respective standard software package MATLAB. The analysis of the results highlights a periodicity at the seismic activity (obtained at constant force) and the earthquake's appearances it is shown as a consequence of the different system's resonance points. All of the above would allow through the reengineering methods obtaining valuable information related to the structure and behavior of local plates.

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Key Points:

- Formulation and source code elaboration of the respective ODE system according to the Classical Mechanics laws.
- Fitting after estimations of the following magnitudes: the blocks' masses, spring and leaf constants and the resistance function parameters.
- Evaluation of the results and, if so not, repeating the process until obtaining realistic seismograms (allowing the usual interpretation).

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Plain Language Summary

We all know that there are mediums that "amplify" the mechanical vibrations: for example, some bridges have fallen due to the force of relatively weak winds or by soldiers who have marched on them. Some parts of the tectonic plates (the shells we live on) also behave like this. The said behavior we have modeled using solid blocks joined by springs. The article explains how virtual seismograms are obtained (clearly identifying their characteristics) from a constant push force which is basically due to tectonic plate collisions. In principle the software allows us to change the different parameters and constants and, in this way, to fit the computerized model to the real seismograms.

1 Introduction

For the first time in an article by (Burridge & Knopoff, 1967), an explanation of telluric movements is proposed through the modeling of some glissade processes. We, after having detected an error in the deduction of the total resistance function $F^{\#}(\nu)$, have proposed (Garzón & NETCHEV, 2022) a new way, based on the liquefaction phenomena, to complete the missing information in said $B\&K$ article regarding the low-velocity dominium. In the present circumstances, after analyzing the corresponding impact, it can be seen as more appropriate to generalize the initial model, in order to be able to use the new analytical properties of the improved by us function $F^{\#}(\nu)$ as well as to specify some moments solving the differential equations system that describes the telluric movements. The model of (Burridge & Knopoff, 1967), has been maybe for a long time a reference that single one accounts for the causes of an earthquake. In that model, a system of differential equations is used, which clearly are the equations of uniform motion. But such a equations system can not describe seismic processes because it does not generate it.

Since our considerations consist of completely including the B&K model as a particular case, here we are not going to make a review of the art state of the subject that comprises later ideas to date assuming that the initial model has not lost importance moreover, the second author exposed it during the SIAM event (Ceballos Garzón & Neytchev Netchev, 2022). We have analyzed additionally the simplest particular case which allows simulating the processes related to the emergence (due to resonance effects) and propagation of seismic waves is the maximum that can be achieved with this type of model.

In the second section, we consider our Mechanical glissade's model and the third section is dedicated to the numerical solution of the respective equations. The article ends with certain conclusions.

2 Mechanical glissade's modeling

Let us consider the following model of a one-dimensional glissade (for the case of the equilibrium position, see Fig.1). The biggest stratum slides against another one through a three small body system linked among the neighbors by springs and by leafs with the big body.

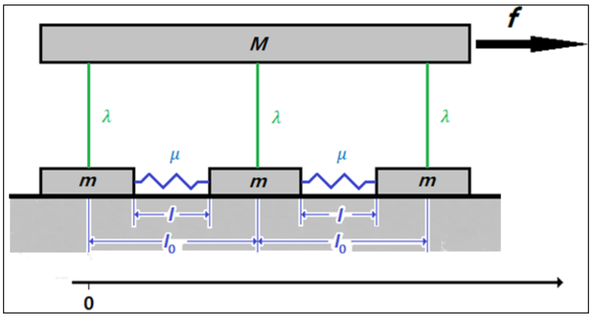

Figure 1. Springs-leafs body configuration.

Assuming that the forces exerted by the springs and leafs depend linearly on the balance deviations, projecting on the shown in Fig.1 abscissa that determines the grades of freedom, we can write the following system of differential equations (the points above the coordinates or other symbols mark the time derivatives order):

$$
\begin{aligned}
M\ddot{X} &= f + \lambda(x_1 - X) + \lambda(x_2 - X - l_0) + \lambda(x_3 - X - 2l_0)\\
m\ddot{x}_1 &= -\lambda(x_1 - X) + \mu(x_2 - x_1 - l_0) + cF^{\#}(\dot{x}_1),\\
m\ddot{x}_2 &= -\lambda(x_2 - X - l_0) - \mu(x_2 - x_1 - l_0) + \mu(x_3 - x_2 - l_0) + cF^{\#}(\dot{x}_2),\\
m\ddot{x}_3 &= -\lambda(x_3 - X - 2l_0) - \mu(x_3 - x_2 - l_0) + cF^{\#}(\dot{x}_3).
\end{aligned} \tag{1}
$$

Here, f is the pushing force, λ and μ are the coefficients of the linear elasticity according to Fig.1, being M and m the respective mass of the biggest stratum and the glissade blocks. l_0 facilitates the geometric description of the processes and c agrees with the physical dimensions of the respective related magnitudes (if this is necessary). All these are some empiric constants, the symbols X, x_1, x_2, and x_3 are the blocks coordinates regarding the axis of the Fig.1.

Introducing the unknowns Y, y_1, y_2, and y_3 we can in a standard way rewrite this as a normal system of eight ordinary differential equations:

$$
\begin{aligned}
\dot{X} &= Y,\\
\dot{x_1} &= y_1,\\
\dot{x_2} &= y_2,\\
\dot{x_3} &= y_3,\\
\dot{Y} &= A + \Lambda(x_1 - X) + \Lambda(x_2 - X - l_0) + \Lambda(x_3 - X - 2l_0) = A + \Lambda[\Sigma_{i=1}^{3} x_i - 3(X + l_0)],\\
\dot{y_1} &= -\kappa(x_1 - X) + \nu(x_2 - x_1 - l_0) + \iota F^{\#}(y_1),\\
\dot{y_2} &= -\kappa(x_2 - X - l_0) - \nu(x_2 - x_1 - l_0) + \nu(x_3 - x_2 - l_0) + \iota F^{\#}(y_2),\\
\dot{y_3} &= -\kappa(x_3 - X - 2l_0) - \nu(x_3 - x_2 - l_0) + \iota F^{\#}(y_3).
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where, obviously $A = \frac{f}{M}$, $\Lambda = \frac{\lambda}{M}$, $\kappa = \frac{\lambda}{m}$, $\nu = \frac{\mu}{m}$ and $\iota = \frac{c}{m}$. Unlike the equations system used by B&K (and by other authors too), in our case we do have not the imposition of a constant velocity movement of the biggest layer (body) but any continually acting force $f = f(t)$ that pushes it. Let us consider two particular cases. The **first**

case is when the force $f = -\lambda(x_1 - X) - \lambda(x_2 - X - l_0) - \lambda(x_3 - X - 2l_0)$, (i.e. constant velocity for the biggest block) and coincides with the B&K model if one sets $l_0 = 0$. There are another trivial alterations of the used symbols, which the reader can easily see by comparing the respective formulae and taking into account the used configuration geometries. In these particular circumstances all the corroborations that are logically flawless, done by (Burridge & Knopoff, 1967) for its already became classical model, are valid for our generalization too including the affirmation that when all blocks are moving at the same speed, the state is unstable. The **second particular case** will be considered a little later. In a similar way the ODE corresponding to N blocks can be written:

$$\begin{aligned} \dot{X} &= Y \\ \dot{x_j} &= y_j \\ \dot{Y} &= A + \Lambda\left[\Sigma_{i=1}^{N} x_i - N\left(X + \frac{N-1}{2} l_0\right)\right], \\ \dot{y_j} &= -\kappa(\mathbf{x_j} - \mathbf{X} - (\mathbf{j}-\mathbf{1})\mathbf{l_0}) - \nu(x_j - x_{j-1} - l_0) + \nu(x_{j+1} - x_j - l_0) + \iota F^{\#}(y_j) = \\ &\quad - \kappa(x_j - X - (j-1)l_0) + \nu(\mathbf{x_{j+1}} - \mathbf{x_{j-1}}) + \iota F^{\#}(y_j). \end{aligned} \tag{3}$$

Here, $x_1 = x_0 + l_0$ and $x_{N+1} = X_N + l_0$ while $j = 1, 2 \ldots, N$. Another way to generalize the results, is including different types of viscosity, block sizes, etc; it will be working with different values for the constants, which appears in the Eq.3. A little more technical effort is necessary to widespread our model in two dimensions (Rundle & Brown, 1991) but, that does not lead to new considerable interesting conceptualizations and it can be yes indeed important only for its use in concrete real situations.

Let now us consider with more attention the **second particular case** i.e. the force $f = const$. The reason is that if the large block of Fig.1 is a stratum, the experience shows that within an interval of thousand, maybe several million years, its movement is with good (let us call it if necessary "zero") approximation under uniform conditions. That is why, if we suppose that the small blocks number (which move (quasi) randomly i.e. without a strong correlation) is sufficiently large, one can write:

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} X_{MC} + x_i^{'} = N x_{MC} + \sum_{i=1}^{n} x_i^{'} = N x_{MC} = N\langle Y \rangle t = NVt, \tag{4}$$

being the last sum equal to zero because for each deviation of a small block forward with respect to the mass center there would be another one lagging (almost) in the same measure (the coordinates $x_i^{'}$ are with respect to the mass center). Replacing in the system (3) the value of the sum (4) by NVt, one can see that this is the nearest resemblance form of our system of $2N+2$ equations regarding the $2N$ B&K equations (considered by us for this reason as a rough calculation where the consecutive description of one of the stratums is missing). The thus obtained "leftover equations" give us an estimation of the relative stratum motion:

$$\begin{aligned} \dot{X} &= Y \\ \dot{Y} &= A + \Lambda N\left[Vt - \left(X + \frac{N-1}{2} l_0\right)\right]. \end{aligned} \tag{5}$$

Here t is the elapsed time from a fixed initial instant. The real solutions of the system (5) represent the instantaneous velocity of the large block as a sinus function around a constant velocity level:

$$Y = Y_0 \sin(\sqrt{\Lambda N} t + \varphi_0) + V. \tag{6}$$

The integration constants Y_0 and φ_0 can be obtained by introducing (when required) initial conditions something which now and here is not interesting to us. The above result reaffirms the conclusion by (Burridge & Knopoff, 1967) that the system is unstable (even when it starts from "ideal" initial conditions) but clearly, this type of movement as that provided by the simplified equations also could not generate earthquakes.

Now we will proceed to certain numerical solutions of the equations (2). The cases for $N > 3$ can be considered in the same way but they would require the use of more powerful computing resources.

3 Numerical solutions and their analysis

The first think that we want to note is that the Eq.(2) form an autonomous normal system of ordinary differential equations whose right parts have continuous partial derivatives with respect to all the symbols/magnitudes which participate in its constructions. This allows us to affirm the existence and uniqueness of the respective solutions for any reasonable set of initial conditions. But finding analytical solutions in the general case is an impossible task. However presently, there exists software that permits solving this type of systems. For convenience, we have selected the software MATLAB (More specifically, all the calculations have been made with Matlab 2022a, and all three programs are attached to this article).

The initial condition assignments regarding the movements of the small blocks can be more or less random ones safeguarding, clear, the physical meaning of each of these (the runnings of the program shows the results depend little or nothing on these). Unfortunately, there are no measurements made on purpose, but we have selected an example where one of the most interesting research results easy can be observed. In the following figure (Fig.2), which is a frame of the video added as an application, it can be seen

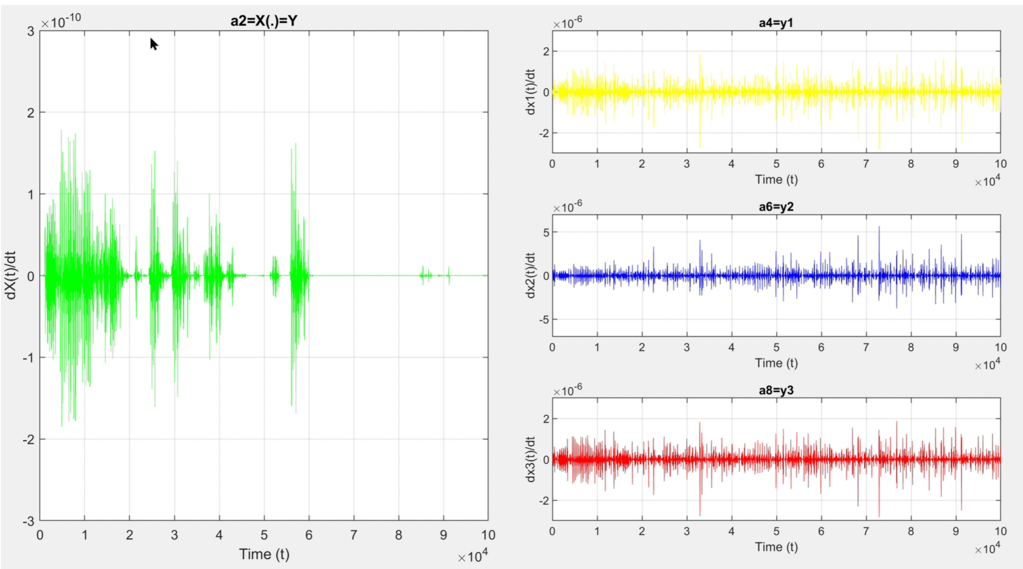

Figure 2. Dependence on time (in relative units) of the oscillations' speeds of the large body (left-hand side) and consecutively of the three small ones (right-hand side).

how the occurrence of earthquakes is explained as the passing of the present modeled system through its resonance points (it is common knowledge that a system of springs under certain very general circumstances would pass through their resonance points (Moulton, 1952)). The foregoing suggests interpreting the graph of Fig.2 as follows: the maximum oscillations around time equal to 1 as the main tremor, the remaining three or four ones as aftershocks, and the small seismic activity around the abscissa at 9 as a partial rupture of local plates being the geophonic noise created by the oscillations of the small blocks. It is clear that in the long run, the average velocity can be considered constant as indicated by the observations because obviously, the average drag force is opposite to the push force. The advance of the large block is very small compared to the back and forth vibrations and therefore it would not be relevant to show it explicitly (during an earthquake the epicentre is practically immobile) although it is calculated by the program.

Let us note assuming that there are no losses and big distortions along the seismic waves way, the respective real seismograms will largely repeat the behavior of the velocity oscillations in situ because the contemporary seismographs record the speed of vibrations (Kramer, 1996).

4 Conclusions

From the above considerations we can make some conclusions that show why this generalized model is an improvement. **The first one** is related to the fact that our mechanical model is more consistent: the movement of the biggest stratum is not assumed to be always uniform but obeys any motion under the influence of a force (due to the interaction of the tectonic plates). **The second** advantage is that the resistance function used by us allows us to use the standard methods for solving systems of ordinary differential equations with no extra precautions and in this way the laws of classical mechanics are from the methodological point of view correctly applied too. As a result, a very realistic behavior of the stratum speed stands out, a thing that matches with the records of the observations completed by the seismologists – for any historically known epicenter, the mini earthquakes appear periodically although the time lapses are not exactly the same. On the other hand, if it is possible to measure the pushing force of the moving stratum, there will be feasible to compare the model with the actual observational data. In the end, let us use technological language: our model (if available an appropiate HPC cluster) is like an unknown prototype, which works well linked to tectonic movements then, to obtain results it is convenient to do reengineering consisting of making a fit with the real seismograms lead to assume certain substantiated things regarding the structure of the ground (due to that, in general, the same consequences lead to the same causes) that, completed with an appropriate instrumentation research, could give valuable information trustworthy.

Open Research

It is worth noting that no data were used in this study.

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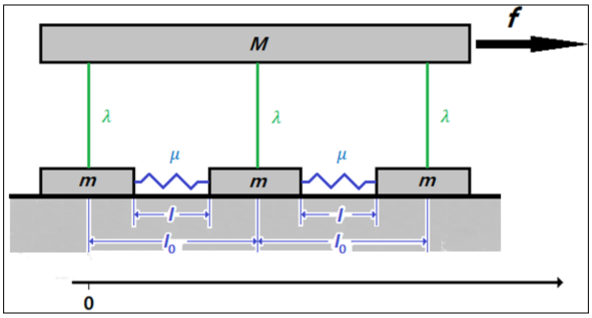

Figure 1. Springs-leafs body configuration.

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where, obviously $A = \frac{f}{M}$, $\Lambda = \frac{\lambda}{M}$, $\kappa = \frac{\lambda}{m}$, $\nu = \frac{\mu}{m}$ and $\iota = \frac{c}{m}$. Unlike the equations system used by B&K (and by other authors too), in our case we do have not the imposition of a constant velocity movement of the biggest layer (body) but any continually acting force $f = f(t)$ that pushes it. Let us consider two particular cases. The **first**

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$$\begin{aligned}
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Here, $x_1 = x_0 + l_0$ and $x_{N+1} = X_N + l_0$ while $j = 1, 2 \ldots, N$. Another way to generalize the results, is including different types of viscosity, block sizes, etc; it will be working with different values for the constants, which appears in the Eq.3. A little more technical effort is necessary to widespread our model in two dimensions (Rundle & Brown, 1991) but, that does not lead to new considerable interesting conceptualizations and it can be yes indeed important only for its use in concrete real situations.

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\dot{X} &= Y \\
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\end{aligned}$$

Here t is the elapsed time from a fixed initial instant. The real solutions of the system (5) represent the instantaneous velocity of the large block as a sinus function around a constant velocity level:

$$Y = Y_0 \sin(\sqrt{\Lambda N} t + \varphi_0) + V. \quad (6)$$

The integration constants Y_0 and φ_0 can be obtained by introducing (when required) initial conditions something which now and here is not interesting to us. The above result reaffirms the conclusion by (Burridge & Knopoff, 1967) that the system is unstable (even when it starts from "ideal" initial conditions) but clearly, this type of movement as that provided by the simplified equations also could not generate earthquakes.

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3 Numerical solutions and their analysis

The first think that we want to note is that the Eq.(2) form an autonomous normal system of ordinary differential equations whose right parts have continuous partial derivatives with respect to all the symbols/magnitudes which participate in its constructions. This allows us to affirm the existence and uniqueness of the respective solutions for any reasonable set of initial conditions. But finding analytical solutions in the general case is an impossible task. However presently, there exists software that permits solving this type of systems. For convenience, we have selected the software MATLAB (More specifically, all the calculations have been made with Matlab 2022a, and all three programs are attached to this article).

The initial condition assignments regarding the movements of the small blocks can be more or less random ones safeguarding, clear, the physical meaning of each of these (the runnings of the program shows the results depend little or nothing on these). Unfortunately, there are no measurements made on purpose, but we have selected an example where one of the most interesting research results easy can be observed. In the following figure (Fig.2), which is a frame of the video added as an application, it can be seen

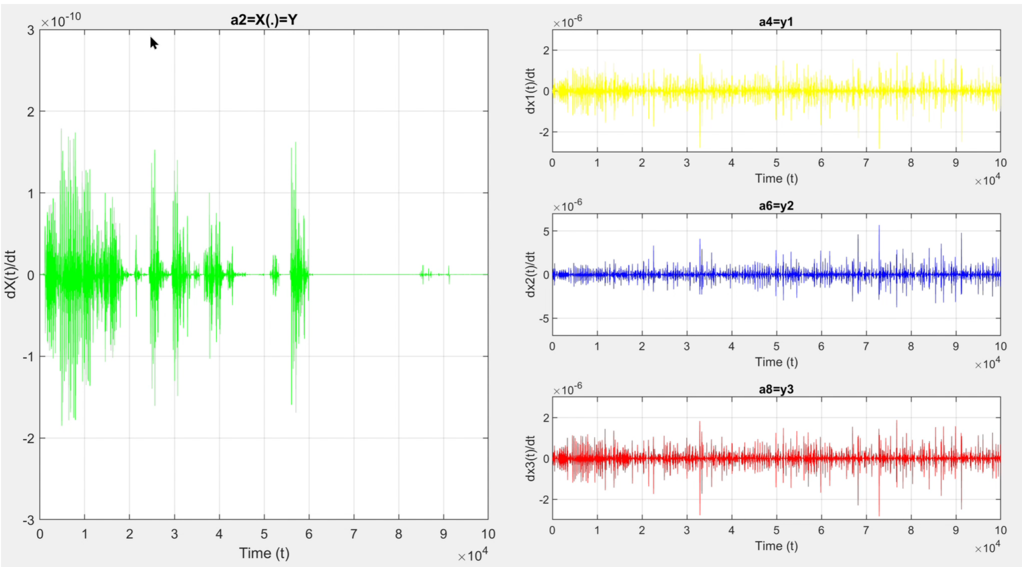

Figure 2. Dependence on time (in relative units) of the oscillations' speeds of the large body (left-hand side) and consecutively of the three small ones (right-hand side).

how the occurrence of earthquakes is explained as the passing of the present modeled system through its resonance points (it is common knowledge that a system of springs under certain very general circumstances would pass through their resonance points (Moulton, 1952)). The foregoing suggests interpreting the graph of Fig.2 as follows: the maximum oscillations around time equal to 1 as the main tremor, the remaining three or four ones as aftershocks, and the small seismic activity around the abscissa at 9 as a partial rupture of local plates being the geophonic noise created by the oscillations of the small blocks. It is clear that in the long run, the average velocity can be considered constant as indicated by the observations because obviously, the average drag force is opposite to the push force. The advance of the large block is very small compared to the back and forth vibrations and therefore it would not be relevant to show it explicitly (during an earthquake the epicentre is practically immobile) although it is calculated by the program.

Let us note assuming that there are no losses and big distortions along the seismic waves way, the respective real seismograms will largely repeat the behavior of the velocity oscillations in situ because the contemporary seismographs record the speed of vibrations (Kramer, 1996).

4 Conclusions

From the above considerations we can make some conclusions that show why this generalized model is an improvement. **The first one** is related to the fact that our mechanical model is more consistent: the movement of the biggest stratum is not assumed to be always uniform but obeys any motion under the influence of a force (due to the interaction of the tectonic plates). **The second** advantage is that the resistance function used by us allows us to use the standard methods for solving systems of ordinary differential equations with no extra precautions and in this way the laws of classical mechanics are from the methodological point of view correctly applied too. As a result, a very realistic behavior of the stratum speed stands out, a thing that matches with the records of the observations completed by the seismologists – for any historically known epicenter, the mini earthquakes appear periodically although the time lapses are not exactly the same. On the other hand, if it is possible to measure the pushing force of the moving stratum, there will be feasible to compare the model with the actual observational data. In the end, let us use technological language: our model (if available an appropiate HPC cluster) is like an unknown prototype, which works well linked to tectonic movements then, to obtain results it is convenient to do reengineering consisting of making a fit with the real seismograms lead to assume certain substantiated things regarding the structure of the ground (due to that, in general, the same consequences lead to the same causes) that, completed with an appropriate instrumentation research, could give valuable information trustworthy.

Open Research

It is worth noting that no data were used in this study.

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