

# Unification of the microcosm and the macrocosm

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## Abstract

In the previous papers in an elegant way we presented the unification of the fundamental interactions. We calculated the unity formulas that connect the coupling constants of the fundamental forces. Also the expression that connects the gravitational fine-structure constant with the four coupling constants. Perhaps the gravitational fine-structure constant is the coupling constant for the fifth force. Also we presented the unification of atomic physics and cosmology. We find the formulas for the cosmological constant and we propose a possible solution for the cosmological parameters. We present the law of the gravitational fine-structure constant followed by ratios of maximum and minimum theoretical values for natural quantities. We will prove the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter, dark matter and dark energy. The sum of the contributions to the total density parameter at the current time is  $\Omega_0=1,0139$ . It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos.

## Keywords

Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant , Cosmological parameters , Cosmological constant , Unification of the microcosm and the macrocosm , Poincaré dodecahedral space

## 1. Introduction

Among ancient Greek and Hellenistic philosophers, notable proponents of the microcosm–macrocosm analogy included Anaximander, Plato, the Hippocratic authors and the Stoics. In later periods, the analogy was especially prominent in the works of those philosophers who were heavily influenced by Platonic and Stoic thought, such as Philo of Alexandria the authors of the early Greek Hermetica and the Neoplatonists. Since time immemorial philosophers, poets, and scientists have pondered the relationship between the microcosm and the macrocosm. This theory was started by Pythagoras who saw the universe and the body as a harmonious unity. The microcosm and the macrocosm. The relevant scale, what counts as the micro- and the mega-, has always been determined by the scientific knowledge of the time. Since Newton, the scales of the largest and the smallest have extended by ten orders of magnitude in both directions. Equally strikingly, the meanings of ‘micro’ and ‘mega’ have changed in the historical development from the unification of celestial and terrestrial mechanics, to the physical study of stars by means of spectral analysis, to the micro-physical explanation of the baryon asymmetry of the universe. It was only in the late 1910s, however, that the first physical fact was discovered that could provide a quantitative clue to the interconnection between the micro- and mega-worlds. It was a famous mathematician, Hermann Weyl, who made this discovery. His discovery later gave rise to such different ideas as the hypothetical variation of the gravitational constant and the anthropic principle. More cautiously, it was referred to as “an unexplained empirical connection between meta-galactic parameters and micro-physical constants”. Although this link between the micro- and mega-worlds is regarded as an empirical fact, its recognition was intertwined with developments in advanced theoretical physics. Before turning to the circumstances of the discovery of this fact, let us look at its contemporary status, which clearly points to its empirical nature. The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. The most fully developed version of the idea in antiquity was made by Plato, but fragmentary evidence indicates that philosophers before him articulated

some version of it. The idea may have begun as an archetypal theme of mythology that the pre-Socratic philosophers reworked into a more systematic form. Unfortunately, it is impossible to reconstruct their thinking in much detail, and clear references attributing the doctrine to Democritus and Pythagoras are quite late, dating to the fifth and ninth centuries C.E., respectively. Some form of the idea seems to have been common among most ancient cultures. Since comparisons of human beings and the universe were made in India and China, the concept may ultimately be of Asian origin but the available sources do not indicate that the theory in Greece was the result of cultural diffusion. Among extant Greek texts, the term first appears in the *Physics* of Aristotle, where it occurs in an incidental remark. Plato did not use the terminology when he developed the idea. The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. These combinations include the electromagnetic and gravitational fine structure, along with the ratios of elementary particles masses. Cosmological measurements clearly depend on the values of these constants in the past and can therefore give information on their time dependence if the effects of time-varying constants can be separated from the effects of cosmological parameters. Of the fifth Platonic solid, the dodecahedron, Plato obscurely remarked, "...the god used for arranging the constellations on the whole heaven". Aristotle added a fifth element, *aithēr* (aether in Latin, "ether" in English) and postulated that the heavens were made of this element, but he had no interest in matching it with Plato's fifth solid. Euclid completely mathematically described the Platonic solids in the *Elements*, the last book (Book XIII) of which is devoted to their properties. Propositions 13–17 in Book XIII describe the construction of the tetrahedron, octahedron, cube, icosahedron, and dodecahedron in that order. For each solid Euclid finds the ratio of the diameter of the circumscribed sphere to the edge length. In Proposition 18 he argues that there are no further convex regular polyhedra. Andreas Speiser has advocated the view that the construction of the five regular solids is the chief goal of the deductive system canonized in the *Elements*. Much of the information in Book XIII is probably derived from the work of Theaetetus.

According to a recent theory the Universe could be a dodecahedron. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos! Plato also stated that time had a beginning; it came together with the universe in one instant of creation. A polyhedron bounded by a number of congruent polygonal faces, so that the same number of faces meet at each vertex, and in each face all the sides and angles are equal (i.e. faces are regular polygons) is called a regular polyhedron. One morning the young Werner Heisenberg discovered reading Plato's *Timaeus* a description of the world with regular polyhedra. Heisenberg could not understand why Plato, being so rational, started to use speculative ideas. But finally he was fascinated by the idea that it could be possible to describe the Universe mathematically. He could not understand why Plato used the Polyhedra as the basic units in his model, but Heisenberg considered that in order to understand the world it is necessary to understand the *Physics* of the atoms. Theaetetus was a member of Plato's Academy. He was a son of Euphronius of Sounion, student of Theodore of Cyrene. Theaetetus died on his return to Athens after he was wounded at the Battle of Corinth. His friend Plato dedicated one of his dialogues to him. Euclid's *elements* chapter X and XIII are based on the work of Theaetetus. Hippasus, from Metapontum in Magna Graecia (south Italy), who wrote around 465 BC about a "sphere of 12 pentagons" refers to the dodecahedron. Hippasus performed acoustics Experiments with vessels filled with different amounts of water and with copper discs of different thicknesses.

## 2. Unification of the fundamental interactions

In [1] we presented exact and approximate expressions between the Archimedes constant  $\pi$ , the golden ratio  $\phi$ , the Euler's number  $e$  and the imaginary number  $i$ . New interpretation and very accurate values of the fine-structure constant has been discovered in terms of the Archimedes constant and the golden ratio. We propose in [2] , [3] and [4] the exact formula for the fine-structure constant  $\alpha$  with the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \phi^{-2} - 2 \cdot \phi^{-3} + (3 \cdot \phi)^{-5} = 137,035999164... \quad (1)$$

Also we propose in [4] , [5] and [6] a simple and accurate expression for the fine-structure constant  $\alpha$  in terms of the Archimedes constant  $\pi$ :

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2 = 137,035999078... \quad (2)$$

We propose in [7] the exact mathematical expression for the proton to electron mass ratio:

$$\mu^{32} = \phi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19} \Rightarrow \mu = 1836,15267343... \quad (3)$$

$$7 \cdot \mu^3 = 165^3 \cdot \ln^{11} 10 \Rightarrow \mu = 1836,15267392... \quad (4)$$

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15} = 1836,15267343... \quad (5)$$

Also in [7] was presented the exact mathematical expressions that connects the proton to electron mass ratio  $\mu$  and the fine-structure constant  $\alpha$ :

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\varphi + 42) \quad (6)$$

$$\mu - 6 \cdot \alpha^{-1} = 360 \cdot \varphi - 165 \cdot \pi + 345 \cdot e + 12 \quad (7)$$

$$\mu - 182 \cdot \alpha = 141 \cdot \varphi + 495 \cdot \pi - 66 \cdot e + 231 \quad (8)$$

$$\mu - 807 \cdot \alpha = 1205 \cdot \pi - 518 \cdot \varphi - 411 \cdot e \quad (9)$$

In [8] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that  $\mu \cdot \alpha^{-1}$  is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1}) + 13^2 = 0 \quad (10)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \quad (11)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i \quad (12)$$

It was presented in [9] the mathematical formulas that connects the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_G(p)$ :

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (13)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (14)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \quad (15)$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (16)$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (17)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (18)$$

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (19)$$

In [10] we presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Euler' number}}{\text{Gerford's constant}} = \frac{e}{e^\pi} = e^{1-\pi} = 0,11748.. \quad (20)$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. In the papers [11],[12],[13] and [14] was presented the unification of the fundamental interactions. We found the unity formulas that connect the strong coupling constant  $\alpha_s$  and the weak coupling constant  $\alpha_w$ . We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w \quad (21)$$

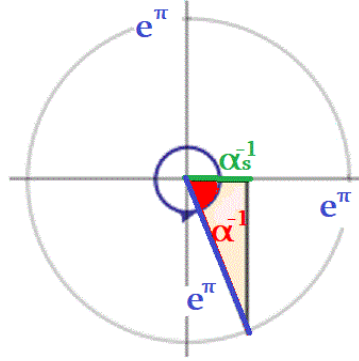
$$\alpha_s^2 = i^{2i} \cdot 10^7 \cdot \alpha_w \quad (22)$$

Resulting the unity formulas that connects the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$\alpha_s \cdot \cos \alpha^{-1} = i^{2i} \quad (23)$$

$$\cos \alpha^{-1} = \frac{\alpha_s^{-1}}{e^\pi} \quad (24)$$

The figure 1 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^\pi$ .



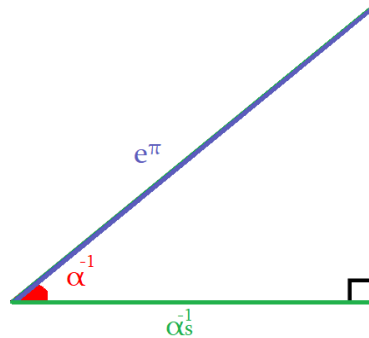
**Figure 1.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^\pi$ .

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^\pi \cdot \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \quad (25)$$

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i} \quad (26)$$

The figure 2 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

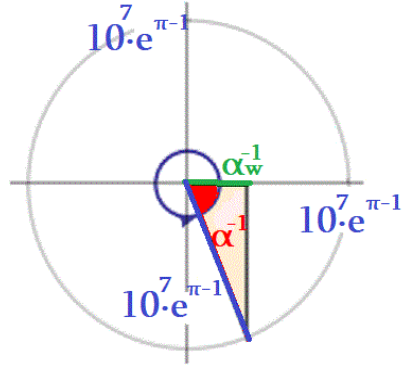


**Figure 2.** Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

The electroweak theory, in physics, is the theory that describes both the electromagnetic force and the weak force. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

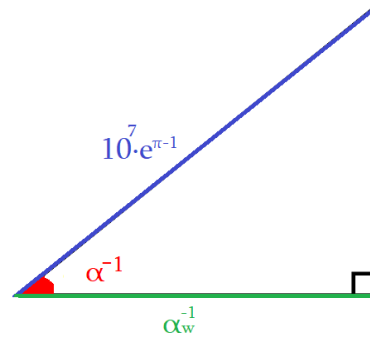
$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e \cdot i^{2i} \quad (27)$$

The figure 3 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{\pi-1}$ .



**Figure 3.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{\pi-1}$ .

The figure 4 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.



**Figure 4.** Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions

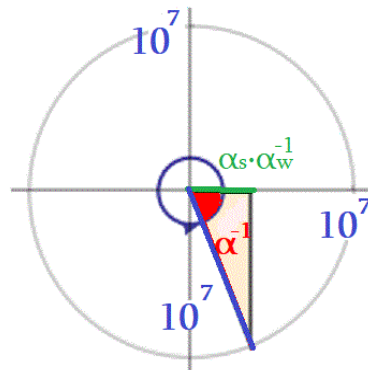
Resulting the unity formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$ :

$$10^7 \cdot \alpha_w \cdot \cos \alpha^{-1} = \alpha_s \quad (28)$$

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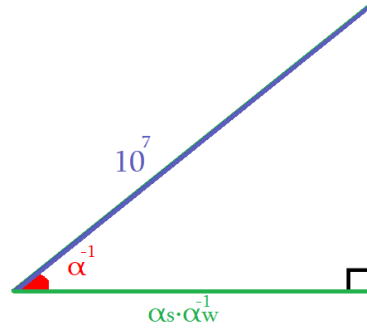
$$\cos \alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7} \quad (29)$$

The figure 5 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ .



**Figure 5.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ .

The figure 6 below shows the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.



**Figure 6.** Geometric representat

strong nuclear,the weak nuclear and the

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic forces:

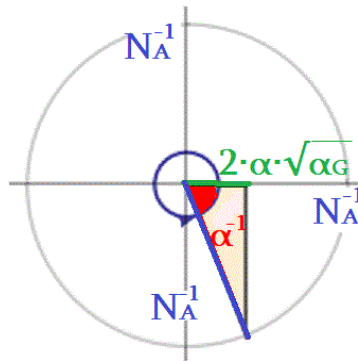
$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot a_s \quad (30)$$

Resulting the unity formula that connects the fine-structure constant  $\alpha$ ,the gravitational coupling constant  $\alpha_G$  and the Avogadro's number  $N_A$ :

$$4 \cdot e^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (31)$$

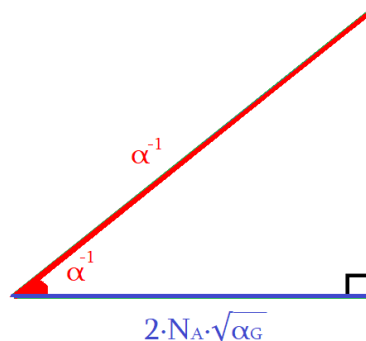
$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot \alpha_G \cdot N_A^2 \quad (32)$$

The figure 7 below shows the angle in  $a^{-1}$  radians. The rotation vector moves in a circle of radius  $N_A^{-1}$ .

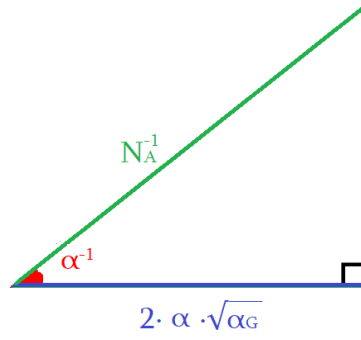


**Figure 7.** The angle in  $a^{-1}$  radians. The rotation vector moves in a circle of radius  $N_A^{-1}$ .

The figures 8 and 9 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.



**Figure 8.** First geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions



**Figure 9.** Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (34)$$

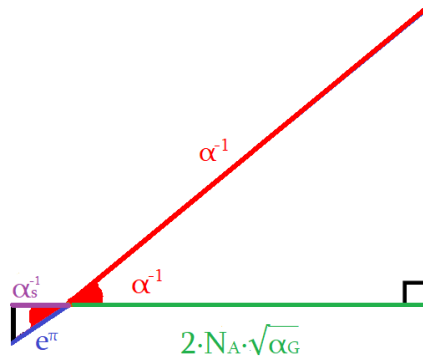
$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2 \quad (35)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \quad (36)$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i} \quad (37)$$

The figure 10 below shows the geometric representation of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions.



**Figure 10.** Geometric representation of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions

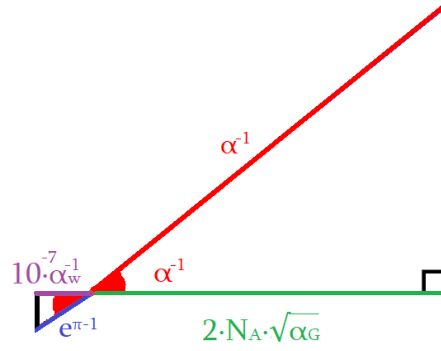
We reached the conclusion of the dimensionless unification of the weak nuclear, the gravitational and electromagnetic forces:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \cdot e^2 \quad (38)$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot N_A^2 = i^{8i} \quad (39)$$

The figure 11 below shows the geometric representation of the dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions.





**Figure 11.** Geometric representation of the dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions

Resulting the unity formula that connect the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G(p)$  for the proton:

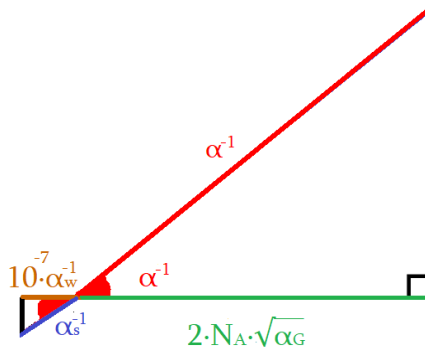
$$4 \cdot 10^{14} \cdot N_A^2 \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G(p) = \mu^2 \cdot \alpha_s^2 \quad (40)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 \quad (41)$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G = \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) \quad (42)$$

The figure 12 below shows the geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions.



**Figure 12.** Geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions

From these expressions resulting the unity formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_G(p)$ :

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 \quad (43)$$

$$\mu^2 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (44)$$

$$\mu \cdot N_1 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \quad (45)$$

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 \quad (46)$$

$$\mu^3 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (47)$$

$$\mu \cdot \alpha_s = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (48)$$

$$\mu \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (49)$$

These equations are applicable for all energy scales. The expressions for the gravitational constant are:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (50)$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (51)$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (52)$$

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (53)$$

### 3. Unification of atomic physics and cosmology

In [15] and [16] resulting in the dimensionless unification of atomic physics and cosmology. The relevant constant in atomic physics is the fine-structure constant  $\alpha$ , which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant  $\alpha_g$ . It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant  $\alpha_g$  is an equivalent way to express the biggest issue in theoretical physics. The gravitational fine structure constant  $\alpha_g$  is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3} \quad (54)$$

$$\alpha_g = \frac{\sqrt{\alpha_G^3}}{\alpha^3} \quad (55)$$

$$\alpha_g = \sqrt{\frac{\alpha_G^3}{\alpha^6}} \quad (56)$$

with numerical value:

$$\alpha_g = 1,886837 \times 10^{-61}$$

Also equals:

$$\begin{aligned} \alpha_g^2 \cdot \alpha^6 &= \alpha_G^3 \\ \alpha_g^2 &= \alpha_G^3 \cdot \alpha^{-6} \\ \alpha_g^2 &= \left( \frac{\alpha_G}{\alpha^2} \right)^3 \end{aligned} \quad (57)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the golden ratio  $\phi$  and the Euler's number  $e$  is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\phi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (58)$$

Resulting the unity formula for the gravitational fine-structure constant  $\alpha_g$ :

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3} \quad (59)$$

$$\alpha_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3} \quad (60)$$

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-3} \quad (61)$$

$$\alpha_g = (10^7 \cdot a_w \cdot a_G^{1/2} \cdot e^{-1} \cdot a_s^{-1} \cdot a^{-1})^3 \quad (62)$$

$$\alpha_g^2 = (10^{14} \cdot a_w^2 \cdot a_G \cdot e^{-2} \cdot a_s^{-2} \cdot a^{-2})^3 \quad (63)$$

$$\alpha_g = 10^{21} \cdot i^{6i} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-6} \cdot a^{-3} \quad (64)$$

So the unity formulas for the gravitational fine-structure constant  $\alpha_g$  are:

$$\alpha_g^2 = 10^{42} \cdot i^{12i} \cdot a_w^6 \cdot a_G^3 \cdot a_s^{-12} \cdot a^{-6} \quad (65)$$

The cosmological constant  $\Lambda$  is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of the universe. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$|p|^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (66)$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (67)$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6} \quad (68)$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot a_G^3 \cdot a_w^6 \quad (69)$$

$$a_s^{12} \cdot a^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot a_G^3 \cdot a_w^6 \quad (70)$$

For the cosmological constant equals:

$$\Lambda = \left( 2e\alpha^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (71)$$

$$\Lambda = i^{12i} (2\alpha_s a^2 N_A)^{-6} \frac{c^3}{G\hbar} \quad (72)$$

$$\Lambda = i^{12i} e^6 (2 \cdot 10^7 \alpha_w a^3 N_A)^{-6} \frac{c^3}{G\hbar} \quad (73)$$

$$\Lambda = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \frac{c^3}{G\hbar} \quad (74)$$

$$\Lambda = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{a^2 \alpha_s^4} \right)^3 \frac{c^3}{G\hbar} \quad (75)$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{a^2 \alpha_s^4} \right)^3 \quad (76)$$

We proposed a possible solution for the cosmological parameters. From the dimensionless unification of the fundamental interactions the density parameter for normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0,043214 = 4,32\% \quad (77)$$

The density parameter for dark matter is:

$$\Omega_D = 6 \cdot e^{-n} = 6 \cdot i^{2i} = 0,2592835 = 25,92\% \quad (78)$$

The density parameter for the dark energy is:

$$\Omega_\Lambda = 17 \cdot e^{-n} = 17 \cdot i^{2i} = 0,73463661 = 73,46\% \quad (79)$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = 24 \cdot e^{-n} = 24 \cdot i^{2i} = 1,037134 \quad (80)$$

A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold, such as the Poincaré dodecahedral space, all of which are quotients of the 3-sphere. The state equation  $w$  has value:

$$w = -24 \cdot e^{-n} = -24 \cdot i^{2i} = -1,037134 \quad (81)$$

#### 4. Maximum and minimum values for natural quantities

In [17] we presented the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length  $l$ , time  $t$ , speed  $v$  and temperature  $T$  have the same max/min ratio which is.

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}} \quad (82)$$

Energy  $E$ , mass  $M$ , action  $A$ , momentum  $P$  and entropy  $S$  have another max/min ratio, which is the square of  $\alpha_g$ .

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}} \quad (83)$$

Force  $F$  has max/min ratio which is  $\alpha_g^4$ :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \quad (84)$$

Mass density has max/min ratio which is  $\alpha_g^5$ :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \quad (85)$$

#### 5. Length scales

A Planck length  $l_p$  is about  $10^{-20}$  times the diameter of a proton, meaning it is so small that immediate observation at this scale would be impossible in the near future. The length Planck  $l_p$  defined as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl}c} = \frac{h}{2\pi m_{pl}c} = \frac{m_p r_p}{4m_{pl}}$$

The classical electron radius is a combination of fundamental physical quantities that define a length scale for problems involving an electron interacting with electromagnetic radiation. The classical electron radius is given as:

$$r_e = \alpha^2 \alpha_0 = \frac{\hbar \alpha}{m_e c} = \frac{\lambda_c \alpha}{m_e c^2} = \frac{\mu_0}{4\pi} \frac{q_e^2}{m_e} = \frac{k_e q_e^2}{m_e c^2} = \frac{\alpha^3}{4\pi R_\infty}$$

The Bohr radius  $\alpha_0$  is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius  $\alpha_0$  is defined as:

$$\alpha_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi \alpha}$$

The proton radius  $r_p$  is the distance from the center of the proton to the tip of the proton. The proton radius  $r_p$  is an unanswered physics problem related to the size of the proton. In atomic physics, there are two common and “natural” scales of length. The first scale of length is given by Compton's wavelength of electrons. Using the de Broglie equation, we get that Compton's wavelength is the wavelength of a photon whose energy is the same as the rest mass of the particle, or mathematically speaking: The Compton wavelength of a particle is equal to the wavelength of a photon whose energy is the same as the mass of that particle. It was introduced by Arthur Compton in his explanation of the scattering of photons by electrons. The standard Compton wavelength  $\lambda_c$  of a particle is given by  $\lambda_c = h/m \cdot c$ . Thus respectively the Compton wavelength  $\lambda_c$  of the electron with mass  $m_e$  is given by the formula:

$$\lambda_c = \frac{2\pi r_e}{\alpha} = \frac{h}{m_e c}$$

Sometimes the Compton wavelength is expressed by the reduced Compton  $\tilde{\lambda}_c$  wavelength. When the Compton  $\lambda_c$  wavelength is divided by  $2 \cdot \pi$ , we obtain the reduced Compton  $\tilde{\lambda}_c$  wavelength, i.e. the Compton wavelength for 1 radius instead of  $2 \cdot \pi$  rad  $\tilde{\lambda}_c = \lambda_c / 2 \cdot \pi$ . The fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi \alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left( \frac{2\pi l_{pl}}{\lambda_e} \right)^2 = \left( \alpha \frac{l_{pl}}{r_e} \right)^2 = \left( \frac{l_{pl}}{\alpha \alpha_0} \right)^2$$

The Planck constant, or Planck's constant, is a fundamental physical constant of foundational importance in quantum mechanics. The constant gives the relationship between the energy of a photon and its frequency, and by the mass-energy equivalence, the relationship between mass and frequency. Specifically, a photon's energy is equal to its frequency multiplied by the Planck constant. The constant is generally denoted by  $h$ . The reduced Planck constant, equal to the constant divided by  $2 \cdot \pi$ , is denoted by  $\hbar$ . For the reduced Planck constant  $\hbar$  apply:

$$\hbar = \alpha \cdot m_e \cdot \alpha_0 \cdot c$$

So from these expressions we have:

$$\hbar^2 = \alpha^2 \cdot m_e^2 \cdot \alpha_0^2 \cdot c^2$$

$$(\hbar \cdot G / c^3) = \alpha^2 \cdot m_e^2 \cdot \alpha_0^2 \cdot (G / \hbar \cdot c)$$

$$(\hbar \cdot G / c^3) = \alpha^2 \cdot \alpha_0^2 \cdot (G \cdot m_e^2 / \hbar \cdot c)$$

$$l_{pl}^2 = \alpha^2 \cdot \alpha_G \cdot \alpha_0^2$$

So the new formula for the Planck length  $l_{pl}$  is:

$$l_{pl} = a\sqrt{a_G}\alpha_0 \quad (86)$$

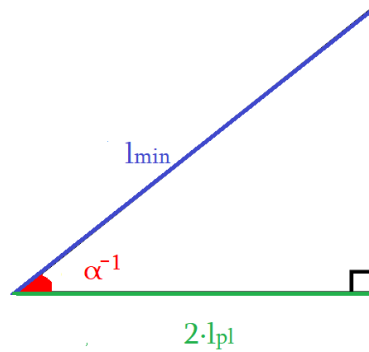
A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Max Planck proposed natural units that indirectly discovered the lowest-level properties of free space, all born from equations that simplified the mathematics of physics equations. The fundamental unit of length in this unit system is the Planck length  $l_{pl}$ . The smallest components of spacetime will never be seen with the human eye as it is orders of magnitudes smaller than an atom. Thus, it will never be directly observed but it can be deduced by mathematics. We proposed to be a lattice structure, in which its unit cells have sides of length  $2 \cdot e \cdot l_{pl}$ . Perhaps for the minimum distance  $l_{min}$  apply:

$$l_{min} = 2 \cdot e \cdot l_{pl} \quad (87)$$

From expressions apply:

$$\begin{aligned} \cos \alpha^{-1} &= e^{-1} \\ \cos \alpha^{-1} \cdot l_{min} &= 2 \cdot l_{pl} \\ \cos \alpha^{-1} &= \frac{2l_{pl}}{l_{min}} \end{aligned} \quad (88)$$

The figures 13 below show the geometric representation of the fundamental unit of length.

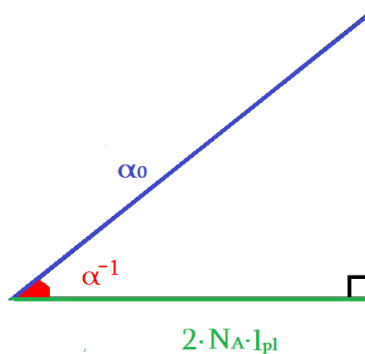


**Figure 13.** Geometric representation of the fundamental unit of length.

For the Bohr radius  $\alpha_0$  apply:

$$\begin{aligned} \alpha_0 &= N_A \cdot l_{min} \\ \alpha_0 &= 2 \cdot e \cdot N_A \cdot l_{pl} \end{aligned} \quad (89)$$

The figures 14 below show the geometric representation of the relationship between the Bohr radius and the Planck length.



**Figure 14.** Geometric representation of the relationship between the Bohr radius and the Planck length.

The cosmological constant  $\Lambda$  has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length  $L$ :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H = c \cdot H_0^{-1}$$

the speed of light multiplied by the Hubble time. It is equivalent to 4,420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting  $D = c \cdot H_0^{-1}$  into the equation for Hubble's law,  $v = H_0^{-1} \cdot D$ . For the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{L_H^2}{R_d^2}$$

$$\Omega_\Lambda = \left( \frac{L_H}{R_d} \right)^2$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0,73576 = 73,57\%$$

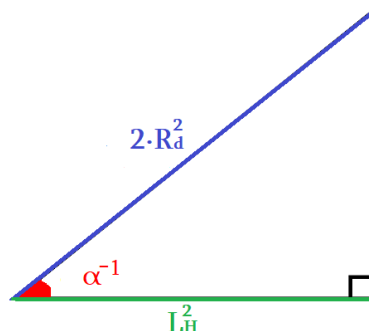
So from this expression apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (90)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (91)$$

The figure 15 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.



**Figure 15.** Geometric representation of the relationship between the de Sitter radius and the Hubble length.

The maximum distance  $\ell_{\max}$  corresponds to the distance of the universe  $\ell_U = c \cdot H_0^{-1}$ . Therefore:

$$\ell_{\max} = \ell_U = c \cdot H_0^{-1}$$

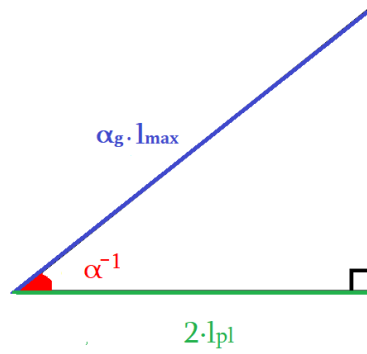
Length  $\ell$  has the max/min ratio which is:

$$\alpha_g = \frac{\ell_{\min}}{\ell_{\max}} \quad (92)$$

The maximum distance  $\ell_{\max}$  corresponds to the distance of the universe:

$$\ell_{\max} = L_H = c \cdot H_0^{-1} = \alpha_g^{-1} \cdot \ell_{\min} \quad (93)$$

The figure 16 shows the geometric representation of the relationship between the maximum distance and the Planck length.



**Figure 16.** Geometric representation of the relationship between the maximum distance and the Planck length.

The value of the maximum distance  $\ell_{\max}$  is:

$$\ell_{\max} = 4,656933 \times 10^{26} \text{ m}$$

## 6. Mass scales

The Planck mass  $m_{pl}$  appears everywhere in astrophysics, cosmology, quantum gravity, string theory, etc. Its mass is enormous compared to any subatomic particle and even the mass of heavier atoms. The mass Planck  $m_{pl}$  can be defined by three fundamental natural constants, the speed of light in vacuum  $c$ , the reduced Planck constant  $\hbar$  and the gravity constant  $G$  as:

$$m_{pl} = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{l_{pl} c} = \frac{\mu_0 q_{pl}^2}{4\pi l_{pl}}$$

In [18] J. Forsythe and T. Valev found an extended mass relation for seven fundamental masses. Six of these masses are successfully identified as mass of the observable universe, Eddington mass limit of the most massive stars, mass of hypothetical quantum “Gravity Atom” whose gravitational potential is equal to electrostatic potential, Planck mass, Hubble mass and mass dimension constant relating masses of stable particles with coupling constants of fundamental interactions. The seventh mass is unidentified and could be considered as a prediction of the suggested mass relation for an unknown fundamental mass, potentially a yet unobserved light particle. First triad of these masses describes macro objects, the other three masses belong to particle physics masses, and the Planck mass appears intermediate in relation to these two groups. We found a similar mass relation for seven fundamental masses:

$$M_n = a^{-1} \cdot \alpha_g^{(2-n)/3} \cdot m_e \quad (94)$$

For  $n=0$   $M_0$  is the minimum mass:

$$M_0 = a^{-1} \cdot \alpha_g^{2/3} \cdot m_e \quad (95)$$



For n=1 M1 is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass,most likely a yet unobserved light particle:

$$M_1 = a^{-1} \cdot a_g^{1/3} \cdot m_e \quad (96)$$

For n=2 M2 is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass:

$$M_2 = a^{-1} \cdot m_e \quad (97)$$

For n=3 M3 is the Planck mass:

$$M_3 = a^{-1} \cdot a_g^{-1/3} \cdot m_e \quad (98)$$

For n=4 is the central mass of a hypothetical quantum “Gravity Atom”.

$$M_4 = a^{-1} \cdot a_g^{-2/3} \cdot m_e \quad (99)$$

For n=5 is of the order of the Eddington mass limit of the most massive stars:

$$M_5 = a^{-1} \cdot a_g^{-1} \cdot m_e \quad (100)$$

For n=6 is the mass of the Hubble sphere and most probably appears to be the mass of the observable universe.

$$M_6 = a^{-1} \cdot a_g^{-4/3} \cdot m_e \quad (101)$$

Gravitons do indeed have mass, and their motions generate kinetic energy. Thus, they have both energy and mass, and they obey the law of conservation of energy and matter. If gravitons did not have mass there would be no physics that we could understand. Other particles have mass, but they are much larger, much less numerous, and cannot substitute for the gravitational effects which generate space curvature. The great mystery of so-called force at a distance is explained by the mass of gravitons. What escapes our logical eyes is the incredibly small dimensionality of gravitons, expressed on a logarithmic scale. Where things get really interesting is in the smallest dimensions. Even an incredibly small and nearly massless particle can have great adjacent gravitational powers, as long as the centers of two attracted particles are sufficiently close. The reality of dark energy is the reality of gravity flows, seen from different perspectives. Dark energy and dark matter are thus aspects of a phenomenon. That phenomenon is the flow of gravitons on a Planck scale, expressed as spacetime foam. Gravitons flow on a massive scale among universe bubbles and the matter between. Given enough flowing gravitons in the spacetime foam, on a scale the human mind can hardly comprehend, there is apparent force at a distance, expressed as the bending of space. Where gravitons gravitate, there is an accelerating flow of gravitons to their collective core, along with a release of energy to help avoid quickly forming singularities. The following applies to the minimum mass  $M_{\min}$ :

$$M_{\min} c^2 = \frac{\hbar}{t_{\max}}$$

$$M_{\min} c^2 = \hbar H_0$$

$$M_{\min} = \frac{\hbar H_0}{c^2} \quad (102)$$

$$M_{\min} = \frac{\hbar}{c l_{\max}} \quad (103)$$

So apply the expressions:

$$M_{\min} = \frac{\hbar}{c} \sqrt{\Lambda} \quad (104)$$

$$M_{\min} = \frac{m_{pl}^2}{M_{\max}} \quad (105)$$

$$M_{\min} = \frac{m_{pl}^2}{M_{\max}} \quad (106)$$

Therefore for the minimum mass  $M_{\min}$  apply:

$$M_{\min} = \alpha_g m_{pl} \quad (107)$$

$$M_{\min} = \frac{\alpha_G}{\alpha^3} m_e \quad (108)$$

$$M_{\min} = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \quad (109)$$

$$M_{\min} = (2 \cdot e \cdot N_A)^{-2} \cdot \alpha^{-1} \cdot m_e \quad (110)$$

For the value of the minimum mass  $M_{\min}$  apply:

$$M_{\min} = 4,06578 \times 10^{-69} \text{ kg}$$

Three independent calculations calculate the mass of the universe  $1,46 \times 10^{53} \text{ kg}$ ,  $1,7 \times 10^{53} \text{ kg}$  and  $1,20 \times 10^{53} \text{ kg}$ . In this context, mass refers to ordinary matter and includes the interstellar medium (ISM) and the intergalactic medium (IGM). However, it excludes dark matter and dark energy. This reported value for the mass of ordinary matter in the universe can be estimated on the basis of critical density infinite. So to estimate the mass value of the universe we will calculate the average of the three independent calculations that produce relatively close results. So the  $M_U$  mass of the observable universe is approximately  $M_U = 1,45 \times 10^{53} \text{ kg}$ . Mass  $M$  have max/min ratio, which is the square of  $\alpha_g$ :

$$\alpha_g^2 = \frac{M_{\min}}{M_{\max}} \quad (111)$$

For the maximum mass  $M_{\max}$  applies:

$$M_{\max} = \frac{F_{\max} l_{\max}}{c^2} \quad (112)$$

$$M_{\max} = \frac{m_{pl}^2}{M_{\min}} \quad (113)$$

$$M_{\max} = \alpha^{-1} \cdot \alpha_g^{-4/3} \cdot m_e \quad (114)$$

$$M_{\max} = \alpha^3 \cdot \alpha_G^{-2} \cdot m_e \quad (115)$$

For the value of the maximum mass  $M_{\max}$  apply:

$$M_{\max} = 1,153482 \times 10^{53} \text{ kg}$$

Also apply the expressions:

$$m_{pt} \cdot L_{\max} = m_{\max} \cdot l_{pt} \quad (116)$$

$$l_{\max}^2 \cdot M_{\min} = l_{\min}^2 \cdot M_{\max} \quad (117)$$

## 7. Energy scales

R. Adler in [19] calculated the energy ratio in cosmology, the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies, the rest energy of the electron  $E_e$ , and the binding energy of the hydrogen atom  $E_H$ . The rest energy of the electron  $E_e$  is defined as:

$$E_e = m_e c^2$$

The binding energy of the hydrogen atom  $E_H$  is defined as:

$$E_H = \frac{m_e e^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:

$$\frac{E_H}{E_e} = \frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales, the Planck energy density  $\rho_{pl}$ , and the density of the dark energy  $\rho_\Lambda$ . The Planck energy density is defined as:

$$\rho_{pl} = \frac{E_{pl}}{l_{pl}} = \frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density  $\rho_\Lambda$  is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

So with expression (57) for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120} \quad (118)$$

Some authors consider the small value of the ratio to be arguably one of the most mysterious problems in present day physics. The understanding of atomic structure required the discovery of the fundamental dynamical constant  $\hbar$ . Viewed in this way the cosmological analog of  $\hbar$  is  $\Lambda$ , but any dynamical role it may play is not yet apparent. It is amusing to note that in the presence of two length scales, and their dimensionless ratio, dimensional analysis becomes problematic, a dimensional estimate can contain an arbitrary function of the ratio, for example a power or a logarithm. In the case of cosmology it is clear that dimensional estimates, with two disparate length scales, may be much worse than useless.

## 8. Time scales

All scientific experiments and human experiences occur on time scales that are many orders of magnitude larger than the Planck time  $t_{pl}$ , making events on the Planck time  $t_{pl}$  undetectable with current scientific technology. The Planck time  $t_{pl}$  is defined as:

$$t_{pl} = \frac{l_{pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} = \frac{\hbar}{m_{pl} c^2}$$

The Hubble constant  $H_0$  is one of the most important numbers in cosmology because it is required to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding. The Hubble constant

can be used to determine the inherent brightness and masses of stars in nearby galaxies, examine the same properties in more distant galaxies and galaxy clusters, infer the amount of dark matter in the universe, and obtain the scale size of distant clusters as far as clusters test for theoretical cosmological models. In 1929, American astronomer Edwin Hubble announced his discovery that galaxies, in all directions, seemed to be moving away from us and have greater displacement for attenuated galaxies. The unit of the Hubble constant is 1 km/s/Mpc. The 2018 CODATA recommended value of the Hubble constant is  $H_0 = 67,66 \pm 0,42 \text{ (km/s)/Mpc} = (2,1927664 \pm 0,0136) \times 10^{-18} \text{ s}^{-1}$ . Hubble length or Hubble distance is a unit of distance in cosmology, defined as the speed of light multiplied by Hubble time  $L_H = c \cdot H_0^{-1}$ . This distance is equivalent to 4,550 million parsecs, or 14,4 billion light-years, 13,8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light, as shown by the substitution  $r = c \cdot H_0^{-1}$  in the equation for Hubble's law,  $u = H_0 \cdot r$ .

The maximum time period  $t_{\max}$  is the time from the time of Bing Bang to the present day. This time period corresponds to the time of the universe  $t_U = H_0^{-1}$ . Therefore:

$$t_{\max} = t_U = H_0^{-1}$$

In 1961, Dicke observed that a dimensionless number must necessarily be large to make the lifetime of stars long enough to produce heavy chemical elements such as carbon. Knowing that carbon is the most essential element for biological materials, this is the first claim called "Human Coincidence", which infers that the connection between physical constants is necessary for the existence of life in the universe. Thermonuclear combustion is necessary for the production of elements heavier than hydrogen. Again it takes several billion years for this to occur. type of conversion inside a star. According to the general theory of relativity no universe can provide several billion years of time unless it is several light-years in extent. Serious criticisms and interpretations have been made on the issue of the large number hypothesis and the existence of intelligent beings or life. One of the most difficult issues in understanding consciousness is understanding how information is synthesized to form our subjective experience. The widely accepted hypothesis is that gamma currents ranging from many places in the brain combine to create a unified subjective experience. In this way, neurons performing different tasks in separate areas of the brain are divided into a single instantaneous activity. The gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. Changes in electrical membrane potential generate neuronal action potentials. Oscillatory activity of neurons is connected to these spikes. The oscillation of the single neuron can be observed in fluctuations at the threshold of the membrane potential. The time quantum in the brain  $t_B$ , the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \quad (119)$$

The Planck time depends on the fundamental constants such as the Planck constant  $h$ , the gravitational constant  $G$  and the speed of light  $c$ , while it is not clear whether the shorter time scale in the brain also depends on these fundamental constants. Thus, the observer who can observe a universe tuned to the various fundamental constants must have synchronous activity of gamma oscillations of about 40 Hz in his nervous system. This is what we find from the experimental results in modern neuroscience.

#### 4. Poincaré dodecahedral space

Finite well-proportioned spaces, especially the Poincaré dodecahedron, open something of a Pandora's box for the physics of the early universe. The standard model of cosmology relies in the main on the hypothesis that the early universe underwent a phase of exponential expansion called inflation, which produced density fluctuations on all scales. In the simplest inflationary models, space is supposed to have become immensely larger than the observable universe. Therefore, a positive curvature (i.e.  $\Omega > 1$ ), even if weak, implies a finite space and sets strong constraints on inflationary models. It is possible to build "low scale" inflationary universes in which the inflation phase ends more quickly than it does in general inflationary modes, leading to a detectable space curvature. In other words, even if space is not flat, a multi connected topology does not contradict the general idea of inflation. However, no convincing physical scenario for this has yet been proposed. Perhaps the most fundamental challenge is to link the present-day topology of space to a quantum origin, since general relativity does not allow for topological changes during the course of cosmic evolution. A quantum theory of gravity could allow us to address this problem, but there is currently no indication about how such a unified theory might actually describe the emergence of multiple connected spaces. Data from the European Planck Surveyor, which is scheduled for launch in 2007 will be able to determine  $\Omega$  with a

precision of 1%. A value lower than 1,01 will rule out the Poincaré dodecahedron model, since the size of the corresponding dodecahedron would become greater than the observable universe and would not leave any observable imprint on the microwave background. A value greater than 1,01, on the other hand, would strengthen the models' cosmological pertinence. Whether or not some multiply connected model of space such as the Poincaré dodecahedron is refuted by future astronomical data, cosmic topology will continue to remain at the heart of our understanding about the ultimate structure of our universe.

Surprisingly, not all small-volume universes suppress the large-scale fluctuations. In 2003 J.-P. Luminet in [20] proved that the long-wavelength modes tend to be relatively lowered only in a special family of finite, multi connected spaces that are called "well-proportioned spaces" because they have a similar extent in all three dimensions. More specifically, we discovered that the best candidate to fit the observed power spectrum is a well-proportioned space called the Poincaré dodecahedral space. This space may be represented by a polyhedron with 12 pentagonal faces, with opposite faces being "glued" together after a twist of  $36^\circ$ . This is the only consistent way to obtain a spherical (i.e. positively curved) space from a dodecahedron: if the twist was  $108^\circ$ , for example, we would end up with a radically different hyperbolic space. The Poincaré dodecahedral space is essentially a multiply connected variant of a simply connected hypersphere, although its volume is 120 times smaller. A rocket leaving the dodecahedron through a given face immediately re-enters through the opposite face, and light propagates such that any observer whose line-of-sight intercepts one face has the illusion of seeing a slightly rotated copy of their own dodecahedron. This means that some photons from the cosmic microwave background, for example, would appear twice in the sky. The power spectrum associated with the Poincaré dodecahedral space is different from that of a flat space because the fluctuations in the cosmic microwave background will change as a function of their wavelengths. In other words, due to a cut-off in space corresponding to the size of the dodecahedron, one expects fewer fluctuations at large angular scales than in an infinite flat space, but at small angular scales one must recover the same pattern as in the flat infinite space. In order to calculate the power spectrum we varied the mass-energy density of the dodecahedral universe and computed the quadrupole and the octopole modes relative to the WMAP data. To our delight, we found a small interval of values over which both these modes matched the observations perfectly. Moreover, the best fit occurred in the range  $1,01 < \Omega < 1,02$ , which sits comfortably with the observed value.

The Poincaré dodecahedral space therefore accounts for the lack of large-scale fluctuations in the microwave background and also for the slight positive curvature of space inferred from WMAP and other observations. Moreover, given the observed values of the mass-energy densities and of the expansion rate of the universe, the size of the dodecahedral universe can be calculated. We found that the smallest dimension of the Poincaré dodecahedron space is 43 billion light-years, compared with 53 billion light-years for the "horizon radius" of the observable universe. Moreover, the volume of this universe is about 20% smaller than the volume of the observable universe. (There is a common misconception that the horizon radius of a flat universe is 13,7 billion light-years, since that is the age of the universe multiplied by the speed of light. However, the horizon radius is actually much larger because photons from the horizon that are reaching us now have had to cross a much larger distance due to the expansion of the universe.) If physical space is indeed smaller than the observable universe, some points on the map of the cosmic microwave background will have several copies. As first shown by Neil Cornish of Montana State University and co-workers in 1998, these ghost images would appear as pairs of so-called matched circles in the cosmic microwave background where the temperature fluctuations should be the same. This "lensing" effect, which can be precisely calculated, is thus purely attributable to the topology of the universe. Due to its 12-sided regular shape, the Poincaré dodecahedral model actually predicts six pairs of diametrically opposite matched circles with an angular radius of  $10\text{--}50^\circ$ , depending on the precise values of cosmological parameters such as the mass-energy density.

The critical density is the average density of matter required for the Universe to just halt its expansion, but only after an infinite time. A Universe with a critical density is said to be flat. In his theory of general relativity, Einstein demonstrated that the gravitational effect of matter is to curve the surrounding space. In a Universe full of matter, both its overall geometry and its fate are controlled by the density of the matter within it. If the density of matter in the Universe is high (a closed Universe), self-gravity slows the expansion until it halts, and ultimately re-collapses. In a closed Universe, locally parallel light rays converge at some extremely distant point. This is referred to as spherical geometry. If the density of matter in the Universe is low (an open Universe), self-gravity is insufficient to stop the expansion, and the Universe continues to expand forever (albeit at an ever decreasing rate). In an open Universe, locally parallel light rays ultimately diverge. This is referred to as hyperbolic geometry. Balanced on a knife edge between Universes with high and low densities of matter, there exists a Universe where parallel light rays remain parallel. This is referred to as a flat geometry, and the density is called the critical density. In a critical density Universe, the expansion is halted only after an infinite time. To date, the critical density is estimated to be approximately five atoms per cubic meter, whereas the average density of ordinary matter in the Universe is believed to be 0,2–0,25 atoms per cubic meter. A much greater density comes from the unidentified dark matter; both ordinary and dark matter contribute in favor of contraction of the universe. However, the largest part comes from so-called dark

energy, which accounts for the cosmological constant term. Although the total density is equal to the critical density the dark energy does not lead to contraction of the universe but rather may accelerate its expansion. Therefore, the universe will likely expand forever. An expression for the critical density is found by assuming  $\Lambda$  to be zero and setting the normalized spatial curvature,  $k$ , equal to zero. When the substitutions are applied to the first of the Friedmann equations we find:

$$\rho_c = \frac{3H_0^3}{8\pi G}$$

It should be noted that this value changes over time. The critical density changes with cosmological time, but the energy density due to the cosmological constant remains unchanged throughout the history of the universe. The amount of dark energy increases as the universe grows, while the amount of matter does not. The density parameter  $\Omega$  is defined as the ratio of the actual density  $\rho$  to the critical density  $\rho_c$  of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe; when they are equal, the geometry of the universe is flat (Euclidean). The galaxies we see in all directions are moving away from the Earth, as evidenced by their red shifts. Hubble's law describes this expansion. Remarkably, study of the expansion rate has shown that the universe is very close to the critical density that would cause it to expand forever. The density parameter  $\Omega$  is defined as the ratio of the average density of matter and energy in the Universe  $\rho$  to the critical density  $\rho_c$  of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe; when they are equal, the geometry of the universe is flat (Euclidean). In earlier models, which did not include a cosmological constant term, critical density was initially defined as the watershed point between an expanding and a contracting Universe. The density parameter is given by:

$$\Omega_0 = \frac{\rho}{\rho_c}$$

where  $\rho$  is the actual density of the Universe and  $\rho_c$  the critical density. Although current research suggests that  $\Omega_0$  is very close to 1, it is still of great importance to know whether  $\Omega_0$  is slightly greater than 1, less than 1, or exactly equal to 1, as this reveals the ultimate fate of the Universe. If  $\Omega_0$  is less than 1, the Universe is open and will continue to expand forever. If  $\Omega_0$  is greater than 1, the Universe is closed and this will eventually halt its expansion and recollapse. If  $\Omega_0$  is exactly equal to 1 then the Universe is flat and contains enough matter to halt the expansion but not enough to recollapse it. It is important to note that the  $\rho$  used in the calculation of  $\Omega_0$  is the total mass/energy density of the Universe. In other words, it is the sum of a number of different components including both normal and dark matter as well as the dark energy suggested by recent observations. We can therefore write:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda$$

$$\Omega_0 = \Omega_m + \Omega_\Lambda$$

$$\Omega_0 = \Omega_B + \Omega_{D+\Lambda}$$

where:

$\Omega_B$  is the density parameter for normal baryonic matter,

$\Omega_D$  is the density parameter for dark matter,

$\Omega_\Lambda$  is the density parameter for dark energy,

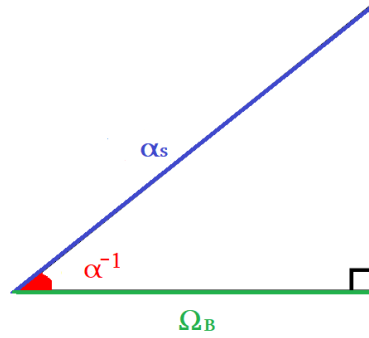
$\Omega_m$  is the sum of the density parameter for normal baryonic matter and the density parameter for dark matter,

$\Omega_{D+\Lambda}$  is the sum of the density parameter for the density parameter for dark matter and the density parameter for dark energy. The sum of the contributions to the total density parameter  $\Omega_0$  at the current time is:

$$\Omega_0 = 1.02 \pm 0.02$$

Current observations suggest that we live in a dark energy dominated Universe with  $\Omega_\Lambda = 0.73$ ,  $\Omega_D = 0.23$  and  $\Omega_B = 0.04$ . To the accuracy of current cosmological observations, this means that we live in a flat,  $\Omega_0 = 1$  Universe. Instead of the cosmological constant  $\Lambda$  itself, cosmologists often refer to the ratio between the energy density due to the cosmological constant and the critical density of the universe, the peak point of a density sufficient to prevent the universe from expanding forever, at one level of the universe is the ratio between the energy of the universe due to the cosmological constant  $\Lambda$  and the critical density of the universe, that is what we would call the fraction of the universe consisting of dark energy [21]. By definition, baryonic matter should only include matter composed of baryons. In

other words, it should include protons, neutrons and all the objects composed of them (i.e. atomic nuclei), but exclude things such as electrons and neutrinos which are actually leptons. In astronomy, however, the term ‘baryonic matter’ is used more loosely, since on astronomical scales, protons and neutrons are always accompanied by electrons. Astronomers therefore use the term ‘baryonic’ to refer to all objects made of normal atomic matter, essentially ignoring the presence of electrons which, after all, represent only  $\sim 0,0005$  of the mass. Neutrinos, on the other hand, are considered non-baryonic by astronomers. Another slight oddity in the usage of the term baryonic matter in astronomy is that black holes are included as baryonic matter. While the matter from which black holes form is mainly baryonic matter, once swallowed by the black hole, this distinction is lost. For example, a theoretical black hole constructed purely out of photons is indistinguishable from one made from normal baryonic matter. This is often referred to as the ‘black holes have no hair’ theorem which simply states that black holes do not have properties such as baryonic or non-baryonic. Objects in the Universe composed of baryonic matter include Clouds of cold gas, Planets, Comets and asteroids, Stars, Neutron stars and Black holes.



**Figure 17.** Geometric representation of the the density parameter for the baryonic matter

The assessment of baryonic matter at the current time was assessed by WMAP to be  $\Omega_B = 0,044 \pm 0,004$ . From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0,0432 = 4,32\% \quad (120)$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^i + 1 = 0 \quad (121)$$

$$\Omega_B^i = i^2 \quad (122)$$

$$\Omega_B^{2i} = 1 \quad (123)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B = e^{-1} \cdot a_s \quad (124)$$

$$\Omega_B = a_w^{-1} \cdot a_s^2 \cdot 10^{-7} \quad (125)$$

$$\Omega_B = 2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a}) \quad (126)$$

$$\Omega_B = 2 \cdot N_A \cdot a_s \cdot a \cdot a_G^{1/2} \quad (127)$$

$$\Omega_B = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (128)$$

$$\Omega_B = 2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2} \quad (129)$$

$$\Omega_B = 10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} \quad (130)$$

In physical cosmology and astronomy, dark energy is an unknown form of energy that affects the universe on the largest scales. The first observational evidence for its existence came from measurements of supernovas, which

showed that the universe does not expand at a constant rate; rather, the universe's expansion is accelerating. Understanding the universe's evolution requires knowledge of its starting conditions and composition. Before these observations, scientists thought that all forms of matter and energy in the universe would only cause the expansion to slow down over time. Measurements of the cosmic microwave background (CMB) suggest the universe began in a hot Big Bang, from which general relativity explains its evolution and the subsequent large-scale motion. Without introducing a new form of energy, there was no way to explain how scientists could measure an accelerating universe. Since the 1990s, dark energy has been the most accepted premise to account for the accelerated expansion. As of 2021, there are active areas of cosmology research to understand the fundamental nature of dark energy.

The fraction of the effective mass of the universe attributed to dark energy or the cosmological constant is  $\Omega_\Lambda = 0,73 \pm 0,04$ . With 73% of the influence on the expansion of the universe in this era, dark energy is viewed as the dominant influence on that expansion. The previous history of the big bang is viewed as being at first radiation dominated, then matter dominated, and now having passed into the era where dark energy is the dominant influence. The density parameter for dark energy is defined as:

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$

The cosmological constant is the inverse of the square of a length  $L$ :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So for the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{c^2}{R_d^2 H_0^2}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H = c \cdot H_0^{-1}$$

the speed of light multiplied by the Hubble time. It is equivalent to 4,420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting  $D = c \cdot H_0^{-1}$  into the equation for Hubble's law,  $v = H_0^{-1} \cdot D$ . So for the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{L_H^2}{R_d^2}$$

$$\Omega_\Lambda = \left( \frac{L_H}{R_d} \right)^2$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0,73576 = 73,57\% \quad (131)$$

So from expression apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (132)$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:



$$\Omega_{\Lambda}=2 \cdot \cos \alpha^{-1} \quad (133)$$

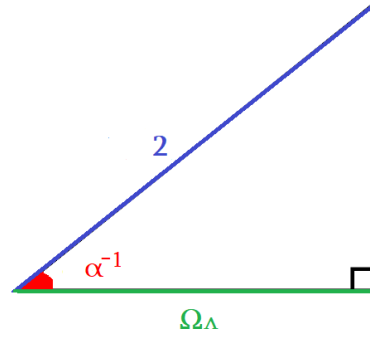
So apply the expression:

$$\cos \alpha^{-1} = \frac{\Omega_{\Lambda}}{2} \quad (134)$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_{\Lambda}=e^{i/a}+e^{-i/a} \quad (135)$$

The figure 2 shows the geometric representation of the density parameter for dark energy.

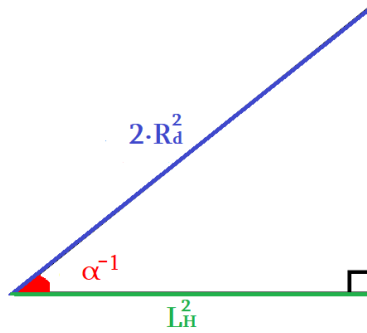


**Figure 18.** Geometric representation of the the density parameter for the dark energy

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (136)$$

The figure 3 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.



**Figure 19.** Geometric representation of the relationship between the de Sitter radius and the Hubble length

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_{\Lambda}=2 \cdot 10^{-7} a_s \cdot a_w^{-1} \quad (137)$$

$$\Omega_{\Lambda}=2 \cdot i^{2i} \cdot a_s^{-1} \quad (138)$$

$$\Omega_{\Lambda}=2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1} \quad (139)$$

$$\Omega_{\Lambda}=2 \cdot 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (140)$$

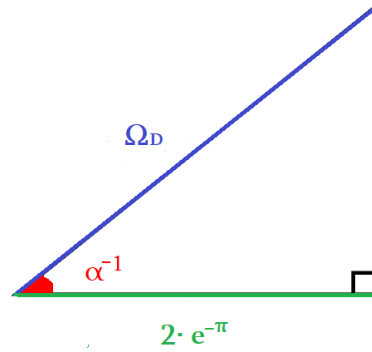
$$\Omega_{\Lambda}=4 \cdot a \cdot a_G^{1/2} \cdot N_A \quad (141)$$

$$\Omega_{\Lambda} = i^{8i} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot N_A^{-2} \quad (142)$$

$$\Omega_{\Lambda} = 10^7 \cdot i^{4i} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot N_A^{-1} \quad (143)$$

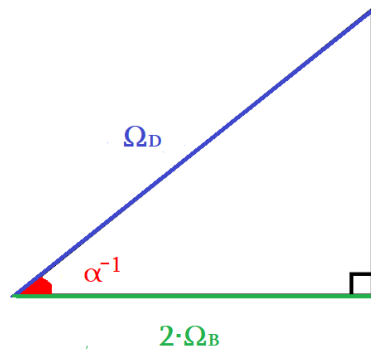
$$\Omega_{\Lambda} = 8 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G \cdot a_s^{-1} \quad (144)$$

Dark matter is a hypothetical form of matter thought to account for approximately 85% of the matter in the universe. Dark matter is called "dark" because it does not appear to interact with the electromagnetic field, which means it does not absorb, reflect, or emit electromagnetic radiation and is, therefore, difficult to detect. Various astrophysical observations – including gravitational effects which cannot be explained by currently accepted theories of gravity unless more matter is present than can be seen – imply dark matter's presence. For this reason, most experts think that dark matter is abundant in the universe and has had a strong influence on its structure and evolution.



**Figure 20.** Geometric representation of the density parameter of dark matter.

The primary evidence for dark matter comes from calculations showing that many galaxies would behave quite differently if they did not contain a large amount of unseen matter. Some galaxies would not have formed at all and others would not move as they currently do. Other lines of evidence include observations in gravitational lensing and the cosmic microwave background, along with astronomical observations of the observable universe's current structure, the formation and evolution of galaxies, mass location during galactic collisions, and the motion of galaxies within galaxy clusters. The figure 5 shows the geometric representation of the relationship between the density parameter of dark and baryonic matter.



**Figure 21.** Geometric representation of the relationship between the density parameter of dark and baryonic matter.

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter  $\Omega_D = 0,23$ . From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D = 2 \cdot e^{1-n} = 2 \cdot e \cdot i^{2i} = 0,2349 = 23,49\% \quad (145)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D = 2 \cdot a_s \quad (146)$$

$$\Omega_D = 2 \cdot 10^7 \cdot e^{-1} \cdot a_w \quad (147)$$

$$\Omega_D = 2 \cdot (i^{2i} \cdot 10^7 \cdot a_w)^{1/2} \quad (148)$$

$$\Omega_D = 4 \cdot i^{2i} \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (149)$$

$$\Omega_D = 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (150)$$

$$\Omega_D = 4 \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot N_A \quad (151)$$

$$\Omega_D = 16 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (152)$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D = 2 \cdot e \cdot \Omega_B \quad (153)$$

The relationship between the density parameter of dark energy, dark matter and baryonic matter is:

$$\Omega_D \cdot \Omega_\Lambda = 4 \cdot \Omega_B \quad (154)$$

The shape of the universe, in physical cosmology, is the local and global geometry of the universe. The local features of the geometry of the universe are primarily described by its curvature, whereas the topology of the universe describes general global properties of its shape as a continuous object. The spatial curvature is defined by general relativity, which describes how spacetime is curved due to the effect of gravity. The spatial topology cannot be determined from its curvature, due to the fact that there exist locally indistinguishable spaces that may be endowed with different topological invariants. Cosmologists distinguish between the observable universe and the entire universe, the former being a ball-shaped portion of the latter that can, in principle, be accessible by astronomical observations. Assuming the cosmological principle, the observable universe is similar from all contemporary vantage points, which allows cosmologists to discuss properties of the entire universe with only information from studying their observable universe. The main discussion in this context is whether the universe is finite, like the observable universe, or infinite. Several potential topological and geometric properties of the universe need to be identified. Its topological characterization remains an open problem. Some of these properties are Boundedness (whether the universe is finite or infinite), Flatness (zero curvature), hyperbolic (negative curvature), or spherical (positive curvature) and Connectivity: how the universe is put together as a manifold, i.e., a simply connected space or a multiply connected space. There are certain logical connections among these properties. For example, a universe with positive curvature is necessarily finite. Although it is usually assumed in the literature that a flat or negatively curved universe is infinite, this need not be the case if the topology is not the trivial one. For example, a multiply connected space may be flat and finite, as illustrated by the three-torus. Yet, in the case of simply connected spaces, flatness implies infinitude. From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter  $\Omega_0$  at the current time is:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda = e^{-n} + 2 \cdot e^{1-n} + 2 \cdot e^{-1} = 1,0139 \quad (155)$$

A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold (such as the Poincaré dodecahedral space), all of which are quotients of the 3-sphere. Poincaré dodecahedral space is a positively curved space, colloquially described as "soccer ball-shaped", as it is the quotient of the 3-sphere by the binary icosahedral group, which is very close to icosahedral symmetry, the symmetry of a soccer ball. This was proposed by Jean-Pierre Luminet and colleagues in 2003 and an optimal orientation on the sky for the model was estimated in 2008. When the universe expands sufficiently, the cosmological constant  $\Lambda$  becomes more important than the energy density of matter in determining the fate of the universe. If  $\Lambda > 0$  there will be an approximately exponential expansion. This seems to be happening now in our universe.

In [22] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J.-P. Uzan presents a simple geometrical model of a finite, positively curved space, the Poincaré dodecahedral space – which accounts for WMAP's observations with no fine-tuning required. Circle searching (Cornish, Spergel and Starkman, 1998) may confirm the model's topological predictions, while upcoming Planck Surveyor data may confirm its predicted density of:

$$\Omega_0 = 1,013 > 1$$

If confirmed, the model will answer the ancient question of whether space is finite or infinite, while retaining the standard Friedmann-Lemaître foundation for local physics. The Poincaré dodecahedral space is a dodecahedral block of space with opposite faces abstractly glued together, so objects passing out of the dodecahedron across any face return from the opposite face. Light travels across the faces in the same way, so if we sit inside the dodecahedron and look outward across a face, our line of sight re-enters the dodecahedron from the opposite face. We have the illusion of looking into an adjacent copy of the dodecahedron. If we take the original dodecahedral block of space not as a Euclidean dodecahedron (with edge angles  $\approx 117^\circ$ ) but as a spherical dodecahedron (with edge angles exactly  $120^\circ$ ), then adjacent images of the dodecahedron fit together snugly to tile the hypersphere, analogously to the way adjacent images of spherical pentagons (with perfect  $120^\circ$  angles) fit snugly to tile an ordinary sphere. Thus the Poincaré space is a positively curved space, with a multiply connected topology whose volume is 120 times smaller than that of the simply connected hypersphere (for a given curvature radius).

The Poincaré dodecahedral space's power spectrum depends strongly on the assumed mass-energy density parameter  $\Omega_0$ . The octopole term ( $\ell=3$ ) matches WMAP's octupole best when  $1,010 < \Omega_0 < 1,014$ . Encouragingly, in the subinterval  $1,012 < \Omega_0 < 1,014$  the quadrupole ( $\ell=2$ ) also matches the WMAP value. More encouragingly still, this subinterval agrees well with observations, falling comfortably within WMAP's best fit range of  $\Omega_0 = 1,02 \pm 0,02$ . The excellent agreement with WMAP's results is all the more striking because the Poincaré dodecahedral space offers no free parameters in its construction. The Poincaré space is rigid, meaning that geometrical considerations require a completely regular dodecahedron. By contrast, a 3-torus, which is nominally made by gluing opposite faces of a cube but may be freely deformed to any parallelepiped, has six degrees of freedom in its geometrical construction. Furthermore, the Poincaré space is globally homogeneous, meaning that its geometry-and therefore its power spectrum - looks statistically the same to all observers within it. By contrast a typical finite space looks different to observers sitting at different locations. Confirmation of a positively curved universe ( $\Omega_0 > 1$ ) would require revisions to current theories of inflation, but the jury is still out on how severe those changes would be. Some researchers argue that positive curvature would not disrupt the overall mechanism and effects of inflation, but only limit the factor by which space expands during the inflationary epoch to about a factor of ten. Others claim that such models require fine-tuning and are less natural than the infinite flat space model. Having accounted for the weak observed quadrupole, the Poincaré dodecahedral space will face two more experimental tests in the next few years:

The Cornish-Spergel-Starkman circles-in-the-sky method predicts temperature correlations along matching circles in small multi connected spaces such as this one. When  $\Omega_0 = 1,013$  the horizon radius is about 0,38 in units of the curvature radius, while the dodecahedron's inradius and outradius are 0,31 and 0,39, respectively, in the same units; as a result, the volume of the physical space is only 83% the volume of the horizon sphere. In this case the horizon sphere self intersects in six pairs of circles of angular radius about  $35^\circ$ , making the dodecahedral space a good candidate for circle detection if technical problems (galactic foreground removal, integrated Sachs-Wolfe effect, Doppler effect of plasma motion) can be overcome. Indeed the Poincaré dodecahedral space makes circle searching easier than in the general case, because the six pairs of matching circles must a priori lie in a symmetrical pattern like the faces of a dodecahedron, thus allowing the searcher to slightly relax the noise tolerances without increasing the danger of a false positive. The Poincaré dodecahedral space predicts  $\Omega_0 = 1,013 > 1$ . The upcoming Planck surveyor data (or possibly even the existing WMAP data in conjunction with other data sets) should determine  $\Omega_0$  to within 1%. Finding  $\Omega_0 < 1,01$  would refute the Poincaré space as a cosmological model, while  $\Omega_0 > 1,01$  would provide strong evidence in its favor.

## 9. Conclusions

It presented the dimensionless unification of the fundamental interactions. We reached the conclusion of the simple unification of the nuclear and the atomic physics:

$$10 \cdot (e^{i\mu/a} + e^{-i\mu/a})^{1/2} = 13 \cdot i$$

We calculated the unity formulas that connect the coupling constants of the fundamental forces. The dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e \cdot a_s = 10^7 \cdot a_w$$

$$a_s^2 = i^2 \cdot 10^7 \cdot a_w$$

The dimensionless dimensionless unification of the strong nuclear and electromagnetic interactions:

$$\alpha_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

The dimensionless dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7 \cdot \alpha_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i}$$

The dimensionless unification of the strong nuclear, the weak nuclear and electromagnetic interactions:

$$10^7 \cdot \alpha_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot \alpha_s$$

The dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2$$

The dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i}$$

$$\alpha^2 \cdot (e^{i/a} + e^{-i/a}) \cdot \alpha_s^4 \cdot \alpha_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of of the weak nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$10^{14} \cdot \alpha^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G = \alpha_s \cdot (e^{i/a} + e^{-i/a})$$

We found the formula for the Gravitational constant:

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

We calculated the expression that connects the gravitational fine structure constant with the four coupling constants:

$$\alpha_g^2 = 10^{42} i^{2i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. It presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. The conclusion of the dimensionless unification of atomic physics and cosmology:

$$\alpha_s^{12} \cdot \alpha^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6$$

We found the formula for the cosmological constant:

$$\Lambda = 10^{42} i^{12} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar}$$

The Equation of the Universe is:

$$\frac{\Lambda G \hbar}{c^3} = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

We presented the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. Perhaps for the minimum distance  $l_{\min}$  apply:

$$l_{\min} = 2 \cdot e \cdot |p|$$

We proved the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions propose a possible solution for the density parameters of baryonic matter, dark matter and dark energy:

$$\Omega_B = e^{-n} = i^{2i} = 0,0432 = 4,32\%$$

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0,73576 = 73,57\%$$

$$\Omega_D = 2 \cdot e^{1-n} = 2 \cdot e \cdot i^{2i} = 0,2349 = 23,49\%$$

The sum of the contributions to the total density parameter at the current time is  $\Omega_0 = 1,0139$ . It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. These results prove that the weather space is finite.

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