

Dynamic two-phase flow modelling of melt segregation in continental crust: batholith emplacement vs. crustal convection, with implications for magmatism in thickened plateaus

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28 **Abstract**

29 Magmatic processes in continental crust such as crustal convection, melt ascent, magma
 30 emplacement, and batholith formation are not well understood. We solve the conservation
 31 equations for mass, momentum and energy for two-phase flow of melt and solid in 2D, for a thick
 32 continental crust heated from below by one or several heat pulses. A simplified binary melting model
 33 is incorporated. We systematically vary: a) the retention number, characterizing melt mobility, b) the
 34 intensity of heat pulses applied at the bottom, and c) the density of the solidified evolved rock. Two
 35 characteristic modes are identified: (a) in the "batholith emplacement mode", segregation is
 36 sufficiently strong allowing melts to separate from the convective flow. This melt freezes to form
 37 buoyant SiO_2 -rich layers. (b) In the "convective recycling mode", melts are formed in the lower crust,
 38 rise together with the hot rock with little segregation, freeze at shallow depth but are partly recycled
 39 back to the lower crust where they remelt. Phase-change-driven convection dominates. Mode (a) is
 40 favored by high heat input, multiple heat pulses, high melt mobility, and low density of the evolved
 41 rock. Mode (b) is favored by less intense heating, less melt mobility, and denser evolved rocks. A
 42 scaling law is derived based on the thermal, melt and compositional Rayleigh numbers and the
 43 retention number. The Altiplano-Puna low-velocity zone could represent the batholith emplacement
 44 mode, with buoyant and voluminous magmas causing intense volcanism. The Tibetan low-velocity
 45 zone is not associated with intense volcanism and might represent the convective recycling mode.

46 **Keywords:** Numerical modelling, Diapirism, Magma genesis and partial melting, Magma migration,
 47 Physics of magma and magma bodies, Batholith emplacement, Phase-change-driven crustal
 48 convection, Altiplano-Puna Plateau, Tibetan Plateau

49 **Keypoints:** Two-phase flow models of crustal magmatic systems identify two modes: batholith
 50 emplacement versus convective recycling of evolved rock. High melt mobility, multiple heating pulses
 51 and low density of solidified evolved rock favor batholith emplacement. The Altiplano-Puna low-
 52 velocity zone (LVZ) is in the batholith emplacement mode, the Tibetan LVZ is in the convective
 53 recycling mode

54 1. Introduction

55 Granite batholiths and smaller intrusions of a wide range of geological ages are common in
 56 continental crust worldwide. There is a long-lasting debate about the origin of granites (e.g. Read,
 57 1948) which has intensified in recent years (Jagoutz & Klein, 2018; Moyen et al., 2021). The two
 58 generic endmembers are granites being a result of fractionation from mantle-derived magmas, or
 59 granites being the product of partial melting and melt segregation of continental crustal rocks. In this
 60 paper, we address the latter process and focus on the impact of pulses of heat at the crust - mantle
 61 boundary on the formation of evolved magmas. In particular, we address the mechanism responsible
 62 for possible emplacement of batholiths or convective recycling of partially molten rocks.

63 Granite batholiths have been proposed to be genetically related to high-grade metamorphic rocks
 64 (e.g. Wells, 1981; Othman et al., 1984). The formation of silicic melt in the lower crust and ascent to
 65 shallower depth is assumed to be a primary process of crustal differentiation (e.g. Wells, 1980).
 66 Granitoid emplacement has been recorded in ancient and active magmatic systems in various regions
 67 of the Earth, such as in the Andes (e.g. Ward et al., 2014), Tibet (e.g. Nelson et al., 1996; Unsworth et
 68 al., 2005) or in Tuscany, Italy (e.g. Puxeddu, 1984) and provide key observations for understanding
 69 the dynamics of crustal magmatic systems and formation of batholiths.

70 Geophysical and geological observations in such active magmatic systems have been reviewed by
 71 Cashman et al. (2017) and include enhanced surface heat flow, decreased electrical resistivity, and, if
 72 enough eruptible melt is generated, ignimbrite surface eruptions. For the Central Andes Springer and
 73 Förster (1998) and Springer (1999) reported heat flux values between 50 mW/m² and 180 mW/m².
 74 Low electrical resistivities of 3 Ωm near 20 km depth in the Altiplano (Comeau et al., 2015) have been
 75 explained by 20 % to 100 % melt depending on the water content (Schmeling et al., 2019). From
 76 surface wave and receiver function data a large low-shear-velocity body between 4 km and 25 km
 77 depth has been found beneath the Altiplano-Puna Volcanic Complex and interpreted to have up to
 78 25 % of partial melt (Ward et al., 2014). Based on a joint interpretation combined with
 79 thermomechanical modelling, Spang et al. (2021) estimated melt fractions between 15 % and 22 %.
 80 Large volumes of ignimbrites have been erupted during the last 11 Ma (Perkins et al., 2016). The
 81 thermal structure of the Altiplano-Puna Magmatic Body has been reviewed, including geophysical
 82 and petrological data, by Gottsman et al. (2017). They inferred a reference geotherm reaching 750 °C
 83 at 25 km depth and an elevated geotherm within the magmatically active plumbing system, which is
 84 hotter than the reference geotherm by 200 K to 350 K.

85 From such observations, a picture of continental magmatic systems has emerged consisting of
 86 vertical sequences of mush bodies with interstitial melts increasing upwards in SiO₂ content. These
 87 mush bodies are sometimes covered by melt-rich caps, connected by magma conduits (Bachmann &

88 Huber, 2016; Cashman et al., 2017; de Silva et al., 2015). While the uppermost mush and associated
 89 melt cap bodies have been constrained by geophysical methods (seismic, magnetotelluric methods)
 90 as mentioned above, deeper mush bodies are difficult to identify.

91 The existence of such deep sequences including stacked bodies of mush and magma with variable
 92 SiO₂ content has been proposed by e.g. Cashman et al. (2017). Such stacking in a dynamic system is
 93 only possible under certain conditions and requires a special thermal, density, and rheological
 94 structure in the crust to be gravitationally stable at least over some 10⁵ years. So far most of the
 95 models investigating the thermal structure of magmatic systems (e.g. Annen et al., 2006; 2015;
 96 Blundy & Annen, 2016; Sparks et al., 2019) have been based on the intrusion of hot magma sills,
 97 experimenting with the rate of intrusion and different initial crustal conditions, like crustal thickness
 98 or thermal structure. They have considered neither the dynamics nor the stability of such stacked
 99 layers of various densities and viscosities, nor the large-scale crustal ductile deformation and flow.
 100 For a single hot mush layer, time scales of a Rayleigh-Taylor instability have been estimated to match
 101 the ranges typical for episodic volcanism (Seporian et al., 2018). van Zalinge et al. (2022) extended
 102 this approach and combined such Rayleigh-Taylor time scales with parameterized magma conduit
 103 models to explain observed repose times of ignimbritic episodic eruptions of order 1 Ma in the Oxaya
 104 formation (Central Andes). With such timescales in mind, we designed our model approach: We
 105 investigate in a dynamically consistent numerical model with an Andean-like crustal structure, the
 106 possible development of granitic batholiths triggered by either a single long-lasting or multiple short-
 107 duration heating events at the bottom of the model (crust-mantle boundary). We will test possible
 108 mechanisms of generating mush bodies in dynamic systems activated by a single or multiple heat
 109 pulses at Moho depth, leading to partial melting and buoyancy that leads to crustal convection and
 110 rising melt capable of segregating to form magmatic bodies or batholiths.

111 A prerequisite for active magmatic systems in continental crust and the generation of silicic melt is a
 112 high heat input or a volatile influx from the mantle. For a thickened crust, such as in collision belts,
 113 radioactive heat at depth might be enough to allow partial melting and weak diapirism (Arnold et al.,
 114 2001). Another possible heat source to initiate partial melting in the lower continental crust is a
 115 sudden heat input from the mantle by underplating or sill intrusion of mantle-derived basaltic
 116 material (Tybo & Artimieva, 2013; Riel et al., 2016; Annen et al., 2006). The nature and temporal
 117 behavior of such heating events control the magmatic evolution of the overlying crust. In a model
 118 setup representing a thick continental crust, heating events associated with underplating may be
 119 realized by temporal variations of thermal bottom boundary conditions. In principle, two end-
 120 members of heating modes can be distinguished: elevated bottom temperature or elevated bottom
 121 heat flux over given durations. If heating is associated with underplating of voluminous basaltic
 122 melts, the overlying crust experiences elevated temperatures of the order of the solidus temperature

123 of basaltic melts. If underplating occurs sporadically and is characterized by episodes of basalt
 124 emplacement followed by complete crystallization, the bottom crustal temperature may be time-
 125 dependent, and the mode of heat source may be better described by an enhanced heat flux
 126 condition at the bottom of the crust of a given duration. The latter heating mode is used here.
 127 Advective heat addition to the crust by mafic melt influx from the mantle is not modelled explicitly.
 128 In subduction related continental magmatism volatile influx may originate from dewatering of
 129 subducted oceanic crust (e.g. Wyllie, 1984) or volatile release due to solidification of underplating of
 130 mafic melts (Annen et al., 2006). While this mechanism is important for lowering the solidus
 131 temperature of lower continental crust (e.g. Weinberg & Hasalova, 2015), we neglect it and subsume
 132 volatile influx under one effective heat influx rate and choosing an appropriate simplified melting
 133 curves as in Schmeling et al. (2019).

134 Zircon U-Pb ages are important time markers that constrain the time-dependence of crustal heating
 135 events and magmatism in subduction-related orogens. They show that magmatism is not a steady
 136 process resulting from constant heat input from the mantle, but it is generally characterized by a
 137 hierarchy of multiscale episodicities. For example, the Tuolumne Intrusive Suite (Sierra Nevada)
 138 shows a flare-up of magmatic activity lasting about 10 Ma modulated by about three maxima of
 139 about 1 Ma to 3 Ma duration (de Silva et al., 2015). Single plutons, such as the 8 km wide Torres del
 140 Paine intrusive complex (Patagonia), have formed during only 0.2 Ma (Leuthold et al., 2012). The
 141 magmatic system of the Oxaya Formation (Central Andes) has lasted at least 7.7 Ma starting with a
 142 period of 4.6 Ma of pluton growth, followed by episodic ignimbrite eruptions on a time scales of
 143 1 Ma (van Zalinge et al., 2022). During the long-term subduction history of the Andes, the Altiplano-
 144 Puna Volcanic Complex (APVC) ignimbrite flare-up represents a waning and then waxing pulse of
 145 about 10 Ma duration and is subdivided into 4 magmatic pulses of about 2 Ma duration, while the
 146 individual ignimbrite zircon spectra define shorter pulses of < 1 Ma (Kern et al., 2016). Lifetimes of
 147 APVC magmas of about 0.4 Ma have been determined by dating the eruption of individual
 148 ignimbrites (Kern et al., 2016). Such a spectrum of episodicities represents surface or middle-to-
 149 upper crustal magmatic pulses and may be the result of crustal modulation of smoother mantle
 150 pulses (de Silva et al., 2015). This is the motivation for our models testing both single, long-lasting
 151 and multiple, shorter mantle heat pulses.

152 As temperatures become elevated, silicic melts on the order of a few volume percent or more are
 153 generated in the lower crust and ascends driven by buoyancy. Consequently, silicic material
 154 accumulates at shallower depths, leaving behind more residual material in the lower crust. The style
 155 of melt ascent at depth will mainly depend on melt fraction, melt viscosity, permeability of the melt
 156 network and solid matrix rheology. Possible scenarios include convective rising of partially molten
 157 mushes driven by thermal and melt buoyancy with or without caps of accumulated melt, or

distributed melt segregation with melt channeling and veining. If melt caps are present, melt carrying heat might infiltrate further into the roof of the melt caps and rise further by a mechanism involving decompaction and sinking of solid roof material with respect to the rising melt (stoping or micro-stoping, Schmeling et al., 2019; Weinberg et al., 2021). Schmeling et al. (2019) called this mechanism "Compaction/decompaction Assisted Two-phase flow Melt Ascent" (CATMA). The extent to which melt accumulates to form stable evolved, granitoid bodies or layers also depends on the density contrast between the solidified felsic material and the original crust. This will be one key parameter for the models shown below.

Once melt caps or mush bodies with high melt fraction reach the transition to the brittle crust, ductile fracturing, compaction instabilities, dyke formation and rapid dyke ascent might occur, possibly supported by an extensional tectonic setting of the crust (Rubin, 1993; Rivalta et al., 2015; Rummel et al., 2020). In this study, we focus on how the deeper processes of mush formation, melt accumulation and batholith formation depend on the heating mode (single or multiple heating event), heating intensity, melt mobility, and density of the solidified enriched material. We neglect dyking and the changes it may bring to the system.

Ascending melts may not always accumulate and be directly extracted, but may instead undergo several cycles of solidification, reworking and remelting. For Himalayan and Karakoram leucogranites, Weinberg (2016) found that typically zircon date ranges in individual samples or multiple samples vary by as much as 10 Ma. They summarized the different hypotheses in the literature used to explain these age ranges: "(i) *multiple events of partial resorption and regrowth of monazite and zircon during anatexis due to evolving metamorphic reactions* (Rubatto et al., 2013), (ii) *physical remobilization of early-formed grains in migmatites and leucogranites by new magma pulses, in an environment of fluctuating melt fraction* (Viskupic et al., 2005; Lederer et al., 2013), (iii) *mixing of ages due to hybridization and magma mixing* (Hasalova et al., 2013), and (iv) *prolonged fractionation and cooling of granites* (Horton & Leech, 2013). Combined, the data suggest that the volumes of melt present in the crust at any time were relatively small, being accumulated, reworked and remobilized several times over the duration of anatexis". This opens the question about the physical conditions and temporal heating modes responsible for such magma accumulations forming evolved granites, possibly being recycled and reworked by crustal convection.

Elaborating this further, rising magmatic bodies may not always be emplaced as mushy zones or batholiths within the crust, instead they may return to the deeper crust in the downwelling limbs of crustal convection where they may undergo remelting. We will call this process recycling. Crustal convection has been described by e.g. Vanderhague et al. (2018), Babeyko et al. (2002), Weinberg (1997). In their dynamic models Louis-Napoléon et al. (2022) approximated partially molten regions

by meso-scale inclusions and identified different flow regimes depending essentially on the choice of viscosities. In the suspension regime the inclusions remained entrained in the convection, while in the layering regime they accumulated beneath a crustal lid. Petrological signatures for recycling could include mixing of primitive magmas with a granitic melt, assimilation of country rock, and age differences between ages of the protolith of a granitic pluton and remelting events (e.g. Jagoutz & Klein, 2018). Trace element variations within individual magma reservoirs of the APVC suggest differences in melt fractions and melt compositions (Kern et al., 2016) which could be a sign of multiple remixing during convection. However, no emphasis has been placed on which physical conditions would control whether rising silicic magma would be emplaced as batholiths or whether felsic magmatic rocks would be recycled back into the lower crust/source zone through convection. One aim of this study is to identify parameter ranges which lead to magma ascent followed by batholith emplacement and those that lead to ongoing convective overturns and recycling. Here the term convection is used in a broad sense, either driven by thermal, phase change (melting-freezing) and compositional (enriched-depleted rock) density variations.

In this paper we will use the following notation: melts or magmatic rocks that have been differentiated from a fertile original lower crust may be called synonymously evolved, silicic, felsic, fractionated, or enriched (in SiO_2), while the residual material may be called residual, restitic or depleted (in SiO_2). Because our two-phase-flow formulation is originally based on mantle melting, we prefer keeping the mantle dynamics notation and mostly use "enriched" and "depleted", referring to the SiO_2 -content, describing the two compositions mentioned above. In our formulation, fully enriched and fully depleted rocks contain finite amounts of SiO_2 , detailed in section 2.2.

In section 2, the numerical model approach is described. In section 3.1, we present a typical model with a single and with pulsed heating events, and investigate models representing the two modes: "*convective recycling*" versus "*batholith emplacement*". Sections 3.2, 3.3, and 3.4 explore the effects of melt mobility (i.e. melt viscosity), intensity of heat input and density of enriched rock with focus on convective recycling versus batholith emplacement. These two modes are analysed in detail in Appendix D. In section 3.5, the full parameter space is explored to identify the regimes in which either of these two modes dominates. In section 3.6 an analytical scaling law for the boundary between these two modes is derived. Section 4 discusses the limitations, the role of pulse frequency and duration, and applies the models to observations of natural magmatic systems and concludes the study.

2. Model Approach

We investigate melting and melt segregation in the continental crust by numerical modelling of two-phase flow including de/compaction of the solid matrix. Starting with McKenzie (1984) such flows

have been formulated for small melt fractions, but recently they have been extended to moderate or even high melt fractions (Schmeling et al., 2019; Keller & Suckale, 2019; Wong & Keller, 2022). We build on the formulation of Schmeling et al. (2019) and assume a single heat pulse or a sequence of heat pulses at the base of an initially unmolten fertile continental crust. We assume a thick continental crust, typical of convergence belts as in the Andes or Tibet. We study the generation, ascent and freezing of melt, and the development of batholiths in the middle and upper crust. As discussed in the introduction, we prefer a heat flow boundary condition over a sudden temperature increase, because this seems to us more natural and allows a better control of the energy input into the model. Indeed, switching on and off the anomalous temperature would also switch the direction of heat flow at the base of the crust from upward to an unreasonable downward heat flow at the Moho.

Our modelling approach is based on the thermo-mechanical two-component two-phase flow formulation of a melt/solid system described in Schmeling et al. (2019). The two phases are solid and melt, with φ as the volumetric melt fraction or melt porosity. Low viscosity melt is allowed to move with respect to the highly viscous flowing solid, and the compositions of melt and solid are advected independently and may vary between depleted (SiO_2 -poor) and enriched (SiO_2 -rich) according to a melting law. Here we provide only the basics of the formulation and those assumptions, which differ from Schmeling et al. (2019).

2.1 Mathematical formulation

The formulation includes the conservation equations of mass, momentum, and composition for the melt and solid, and of energy (as in Schmeling et al. (2019)) based originally on McKenzie (1984) and includes viscous compaction and decompaction of the solid matrix. They are given in Appendix A and B. The low melt viscosity approximation is used, implying that viscous stresses within the melt phase are neglected with respect to the viscous stresses in the solid. Melt movement with respect to the solid is controlled by Darcy flow with a Kozeny-Carman type permeability - porosity relation $k = \frac{d^2}{C} \varphi^{n_\varphi}$ with a power of $n_\varphi = 3$, where C is a geometric factor of the order of 100 depending on the actual pore geometry, and d is the mean distance of the pores (often equal to the grain size). Melt viscosity (\ll solid viscosity) is assumed constant and the formulation includes the compaction pressure (second term in eq. A6 (Appendix A), see also Schmeling et al. 2019 and Katz et al., 2007). The densities of the solid ρ_s and melt ρ_f depend on temperature and composition using two linear equations of state

$$\rho_s = \rho_0(1 - \alpha T - \beta_{def}) \quad (1)$$

$$\rho_f = \rho_0(1 - \alpha T - \beta_f) \quad (2)$$

where ρ_0 is the reference solid density, α is the thermal expansivity, T is the temperature, f is the depletion (if $f > 0$) or enrichment (if $f < 0$) in SiO_2 with respect to the initial composition (see section 2.2), $\beta_{d|e} = \frac{\rho_0 - \rho_{dp}}{\rho_0}$ or $\beta_{d|e} = -\frac{\rho_0 - \rho_{enr}}{\rho_0}$ depending on whether $f > 0$ or $f < 0$, respectively, and $\beta_f = \frac{\rho_0 - \rho_{of}}{\rho_0}$ is the scaled reference solid – melt density difference. See also Table 1 for the definitions of symbols. The mixture density is defined as

$$\rho = \varphi \rho_f + (1 - \varphi) \rho_s \quad (3)$$

where φ is the volumetric melt fraction. A kind of Boussinesq approximation is applied, assuming incompressibility of the solid and fluid phases and constant density in all terms except the buoyancy terms in the momentum equation of the mixture and the Darcy equation (as in Schmeling et al. (2019)). This implies that the mass divergence of the mixture is always zero, even during melting and solidification, but divergence of the solid matrix is non-zero due to compaction/decompaction. The Boussinesq approximation is applicable as long as density variations are below about 10 %. The viscosity of the porous matrix depends on temperature, depth, and stress using a visco-plastic power-law rheology (eq. A8 to A10 (Appendix A)), and porosity following Schmeling et al. (2012) (eq. A11 (Appendix A). see also Schmeling et al. (2019)). The energy conservation equation neglects adiabatic and viscous heating but includes depth dependent radiogenic heating and latent heat due to melting and solidification (eq. A5 (Appendix A)). Local thermal equilibrium between melt and solid is assumed, and the advective velocity assumes a melt porosity (i.e. a melt fraction) weighted mean of the solid and melt velocities (eq. A7 (Appendix A)). All equations are non-dimensionalized which results in four Rayleigh numbers for bottom heat flux, melt, enrichment, and depletion, respectively, and a retention number

$$Ra_q = \frac{\rho_0 \alpha g q h^4}{\lambda \kappa \eta_0}, Ra_f = \frac{(\rho_0 - \rho_{of}) g h^3}{\kappa \eta_0}, Ra_{enr} = \frac{(\rho_0 - \rho_{enr}) g h^3}{\kappa \eta_0}, Ra_{dp} = \frac{(\rho_{dp} - \rho_0) g h^3}{\kappa \eta_0}, Rt = \frac{C \eta_f h^2}{\eta_0 d^2} \quad (4)$$

where ρ_0 , ρ_{of} , ρ_{enr} , ρ_{dp} are the densities of the undepleted or reference rock, of the melt, of the fully enriched rock, and of the fully depleted rock, respectively, α is the thermal expansivity, g is the gravitational acceleration, q is the bottom heat flux, h is the model height, λ is the thermal conductivity, κ is the thermal diffusivity, η_0, η_f are the reference and melt viscosities, respectively, C is a geometric factor of the order of 100 depending on the actual pore geometry, and d is the mean distance of the pores (often equal to the grain size). The inverse of the retention number is proportional to d^2/η_f which describes the mobility of the melt with respect to the solid matrix. A high retention number corresponds to immobile melts, i.e. melts which are retained in the matrix. For typical wet granitic melt viscosities ($10^4 - 10^7$ Pa s) and d in the range of 1 mm to 10 cm the retention number ranges between 1 and 100 (see the discussion in Schmeling et al., 2019). In this

study we investigate the effects of the retention number Rt , the bottom heat flux in terms of Ra_q , and the enriched density in terms of Ra_{enr} on the dynamics of crustal melt transport.

2.2 Chemical composition, melting law, and rock physical parameters

As in Schmeling et al. (2019), we assume an initially uniform, thickened fertile continental crust typical of orogenic belts. The composition of meta-greywacke is assumed containing quartz, plagioclase and biotite, regarded as a protolith of granitic melts. With ongoing melting, melt segregation, and solidification, the rock types evolve both into more depleted, SiO_2 -poor granulite facies residual rocks, and more enriched, SiO_2 -rich granitic rocks after the melt solidifies.

We use an approximative melting law which is based on the physics of a binary solid solution phase diagram with the components A representing "evolved, differentiated, silicic, felsic rock or melt" and B representing "residual, restitic rock". As noted in section 1, these terms are used synonymously, and we will use the term "enriched" (in SiO_2) for compositions near A and "depleted" (in SiO_2) for compositions near B. Compared to Schmeling et al. (2019), the melting law is improved by prescribing curved ("cigar shaped") solidus and liquidus curves (Fig. 1). Pressure dependence of melting is included by assuming depth-dependent solidus and liquidus curves as in Schmeling et al. (2019). Melt and solid are always in thermodynamic equilibrium (e.g. Ribe, 1985). While natural examples of such a binary solid solution phase diagram include olivine (fayalite to forsterite) as a constituent of mantle rocks, or feldspar (albite to anorthite) as a constituent of continental crust, our approximate phase diagram is used to mimic SiO_2 variations during melting of a typical meta-greywacke. We neglect complications due to multi-component melting which would also include eutectic melting. The composition varies between 0, representing full enrichment (granite), and 1, representing full depletion (granulites). The initial (meta-greywacke) composition c_0 is assumed as 0.5. The approximated solidus and liquidus temperature in Fig. 1 and their pressure dependence (Fig. 2) were chosen in a way to generate a melting curve (degree of melt versus temperature) for the initial $c_0 = 0.5$, which roughly resembles experimental curves for meta-greywacke (Clemens & Vielzeuf, 1987; Clemens, 2005; Li et al., 2022). The eutectic jump in melt fraction near the solidus temperature from a few % up to 20 %, depending on water content, is neglected. It should be noted that our formulation could also be formulated in terms of an approximate eutectic phase diagram (Bittner & Schmeling, 1995), however, in the present formulation this would be numerically too expensive (see Appendix B).

The depletion f of the solid is defined as

$$f = \frac{c_s - c_0}{1 - c_0} \quad (5)$$

where c_s is the solid composition. Depletion is 0 as long as the solid has the initial composition, and it reaches 1 (= 100 %) if the solid has the composition B, i.e. $c_s = 1$. Similarly, the enrichment can be defined by $e = \frac{c_s - c_0}{-c_0}$ ensuring that $e = 1$ if the solid has the composition A, i.e. $c_s = 0$. In other words, depletion is the relative deviation of c from the initial value 0.5 towards composition B, and enrichment towards composition A. The equilibrium melt fraction is given by the lever rule

$$\varphi_{eq} = \frac{c_s - c}{c_s - c_f}, \quad (6)$$

where c is the local composition of the solid-melt mixture,

$$c = (1 - \varphi)c_s + \varphi c_f, \quad (7)$$

and c_f is the melt composition. In the present formulation the lever rule is used for the non-linear liquidus and solidus curves, while in Schmeling et al. (2019) the dashed linear curves have been used (Fig. 1). Melting and solidification are calculated by iteratively adjusting the solid and liquid compositions and the melt fraction to their equilibrium values including latent heat. At the same time the resulting melt and solid compositions are advected with the appropriate flow velocities of the melt and solid, respectively. The numerical formulation of this melting law is presented in Appendix B.

Following the arguments of Schmeling et al. (2019), we identify the composition $c = c_0 = 0.5$ (i.e. enrichment $e = 0$) with a SiO_2 content of 57.5 % representative of the fertile meta-greywacke. Fully depleted rock becomes silica poor and granulitic. We assign 100 % depletion (i.e. $c = 1$) to ~ 40 % SiO_2 . To quantify the formation of granitic batholiths, we follow the classification for S-type granites with a SiO_2 content ranging from 63 % to 75 % (Chappell & White, 2001; Frost et al., 2001). Thus, identifying full enrichment (i.e. $c = 0, e = 1$) with 75 % SiO_2 , the lower bound 63 % SiO_2 for granitic rock correlates with an enrichment of $e_{granite} = 0.314$. Fig. 2b shows the resulting temperature-, porosity-, and composition-dependent density map at zero pressure. From these considerations we can use the 2D-enrichment fields of our models to determine the size of the evolving and final SiO_2 -rich bodies which we call batholiths. The mean thickness of these bodies averaged over the width of the model ($h = 50$ km) is calculated by integration over the full area F of the model

$$h_{bath} = \frac{1}{h} \iint_F H(e(x, z) - e_{granite}) dx dz \quad (8)$$

where $H(\vartheta)$ is the Heaviside step function ($H(\vartheta) = 1$ if $\vartheta > 0$ or $H(\vartheta) = 0$ if $\vartheta \leq 0$).

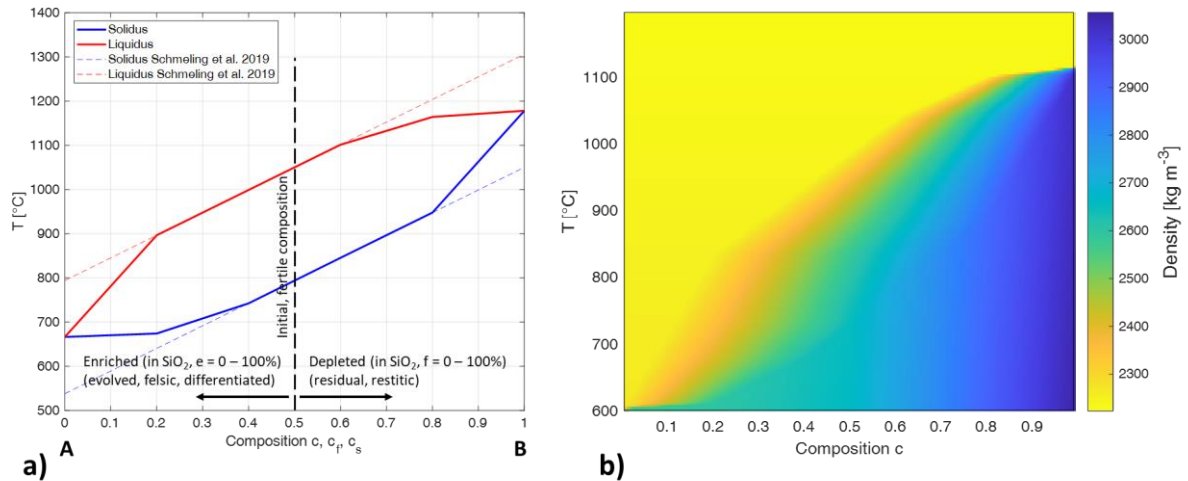


Figure 1: a) Simplified phase diagram of a binary solid solution system with components **A** and **B** at pressure 0. **A** can represent a rock or magma with 75 % SiO₂ and **B** a highly depleted residuum of meta-greywacke with 40 % SiO₂. For the initial fertile meta-greywacke, a SiO₂ component of 57.5 % was assumed, which corresponds to $c = c_0 = 0.5$. The composition of melt and residuum was calculated by the lever-rule (see text and Schmeling et al. (2019)). Pressure dependence is included by assuming depth-dependent solidus and liquidus curves with a constant gradient of 2.334 K/km as in Schmeling et al. (2019). b) Density map based on eq. 1 - 3 including the effect of temperature, melt fraction, and composition at zero pressure.

The rock rheology is assumed independent of chemical composition, and represents wet quartz (Kirby & Kronenberg, 1987). See Appendix A for the equations and Table 1 for the chosen parameters. The porosity dependence of shear and bulk viscosity is taken as in Schmeling et al. (2019). Porosity impacts on the viscosities causing a drop from the intrinsic values of wet quartz to almost zero at porosities close to 0.5, and at higher porosities an exponential lower truncation viscosity is assumed (see Schmeling et al. (2019) for more details and the assumed values). The melt viscosity is assumed constant, its actual value enters indirectly by the parameter combination d^2/η_f (see previous section) which is prescribed by the retention number. To recalculate d^2/η_f from a given retention number the assumed model height h of 50 km and the scaling viscosity η_0 of 10^{20} Pa s have to be used. The retention number in most models is 3, but for exploring the parameter space it is varied between 1 and 100. Given this range, values of the melt viscosity range between $0.4 \cdot 10^3$ Pa s for $d = 1$ mm and $Rt = 1$, to $0.4 \cdot 10^{13}$ Pa s for $d = 10$ m and $Rt = 100$ (see also the discussion in Schmeling et al. (2019)). Spacings of anastomosing leucogranitic sheets forming pathways through partially molten rocks of 5 m have been observed by Symington et al. (2014).

The density of undepleted rock at surface temperature is assumed as $\rho_0 = 2700$ kg/m³, it varies linearly to $\rho_{dp} = 3200$ kg/m³ for fully depleted residual rock (granulite) or to $\rho_{enr} = 2600$ kg/m³ for fully enriched rock (solid granite). When exploring the parameter space, ρ_{enr} is varied to values as low as 2300 kg/m³, while the melt density is fixed to $\rho_{0f} = 2300$ kg/m³ throughout the paper. With

these densities, we note that the limiting 10 % density variations required for the Boussinesq approximation are exceeded when locally the melt fractions, depletion or enrichment exceed 54 %. In other words, local volumes of magma bodies or granitic batholiths with more than 54 % melt fraction or enrichment, respectively, are underestimated by about 10 %, while volumes of highly depleted material are overestimated by about 10 %. When defining batholith or depleted volumes we integrated over larger regions which minimizes such errors to only a few %.

The pre-factor of the permeability-porosity relation, d^2/C , is chosen implicitly by the retention number. Other physical parameters such as the thermal conductivity, heat capacity c_p , thermal expansivity, depth dependent radiogenic heat generation rate, latent heat L , exponent of the porosity-permeability relation n_ϕ are the same as in Schmeling et al. (2019). Most of them are given in Table 1.

Table 1. Symbols, their definition, numerical values, and physical units used in this study.

Symbol	Definition	Value	Units
A_{duc}	Pre-factor in solid ductile viscosity	0.001	MPa ⁿ s ⁻¹
A	Byerlee law constant	10 ⁴	Pa m ⁻¹
B	Byerlee law constant	20	MPa
c_p	Specific heat capacity, equal for all rock types	1000	J kg ⁻¹ K ⁻¹
c, c_0, c_f, c_s	Mass fractions of component B (see Fig. 1): mixture, initial, fluid phase, solid phase	-	-
C	Geometric factor relating permeability to grain size	100	-
d	Mean distance between pores (grain size)	See Rt	m
e	Enrichment field, $e = -f$ if $f < 0$	-	-
E	Activation energy	167 · 10 ³	J mol ⁻¹
e_b	Batholith emplacement parameter	0 to 1	-
\dot{e}_{II}	Second invariant of solid strain rate tensor	variable	s ⁻¹
f	Depletion (if > 0) or enrichment (if < 0) field	-	-
\vec{g}	Gravity acceleration	10	m s ⁻²
h	Scaling length (here height of the model)	50 000	m
H	Radioactive heat production rate in the crust, $H(z)$	variable	W kg ⁻¹
k	Permeability depending on porosity (i.e. melt fraction)	variable	m ²
L	Latent heat	300	kJ kg ⁻¹
n_ϕ	Exponent of the porosity-permeability relation	3	-
n	Stress exponent	2	-
P	Pressure	variable	Pa
P_{lith}	Lithostatic pressure = $\rho_0 g(h - z)$	variable	Pa
q	Mantle or bottom heat flux	variable	W m ⁻²
q_0	Initial or background bottom heat flux	0.0295	W m ⁻²
q_{pulse}	Elevated bottom heat flux during a heat pulse	variable	W m ⁻²
R	Gas constant	8.31	J mol ⁻¹ K ⁻¹
$Ra_q, Ra_f, Ra_{enr}, Ra_{dp}$	Rayleigh number for heat (q), melt phase (f), enriched rock (enr), depleted rock (dp)	-	-
Rt	Retention number	1 to 100	-
t	Time	-	s
T, T_{abs}	Temperature, absolute temperature	variable	°C

\vec{v}	Mean velocity of solid and fluid	variable	m s^{-1}
\vec{v}_f	Fluid (i.e. melt) velocity	variable	m s^{-1}
\vec{v}_s	Solid velocity	variable	m s^{-1}
\vec{v}_{sgr}	Segregation velocity, $\vec{v}_{sgr} = \vec{v}_f - \vec{v}_s$	variable	m s^{-1}
V	Activation volume	10^{-5}	$\text{m}^3 \text{mol}^{-1}$
x	Horizontal coordinate		m
z	Vertical coordinate (positive upward)		m
\tilde{z}	Depth		m
α	Thermal expansivity (equal for all rock types)	$2.4 \cdot 10^{-5}$	K^{-1}
$\beta_f, \beta_d, \beta_e$	Density contrast factors for fluid phase (f), depleted rock (d), enriched rock (e)	0.1481, - 0.1481, - 0.037	-
Γ	Rate of melt generation or solidification	variable	$\text{kg m}^{-3} \text{s}^{-1}$
η_0	Reference (scaling) viscosity	10^{20}	Pa s
η_b	Effective bulk viscosity of the porous matrix	variable	Pa s
η_f	Dynamic viscosity of the fluid phase (i.e. melt), spatially constant	varied	Pa s
η_s	Effective shear viscosity of the porous matrix	variable	Pa s
κ	Thermal diffusivity (equal for all rock types)	$0.923 \cdot 10^{-6}$	$\text{m}^2 \text{s}^{-1}$
λ	Thermal conductivity (equal for all rock types)	2.5	$\text{W m}^{-1} \text{K}^{-1}$
ρ	Mean density of the crustal rocks including melt	variable	kg m^{-3}
ρ_f	Density of the fluid phase (i.e. melt)	variable	kg m^{-3}
ρ_s	Density of the solid phase	variable	kg m^{-3}
ρ_0	Reference density at reference temperature and pressure	2700	kg m^{-3}
ρ_{enr}	Reference density of enriched solid	2600	kg m^{-3}
ρ_{0f}	Reference density of granitic melt	2300	kg m^{-3}
ρ_{dp}	Reference density of the depleted rock	3100	kg m^{-3}
τ_{ij}	Viscous stress tensor in the solid phase	variable	Pa
τ_{II}	Second invariant of the viscous stress tensor	variable	Pa
τ_{max}	Depth dependent Byerlee-type maximum shear stress	variable	Pa
φ	Volumetric melt porosity (i.e. melt fraction)	variable	-
φ_{eq}	Equilibrium melt fraction		-

391

392 2.3 Model layout

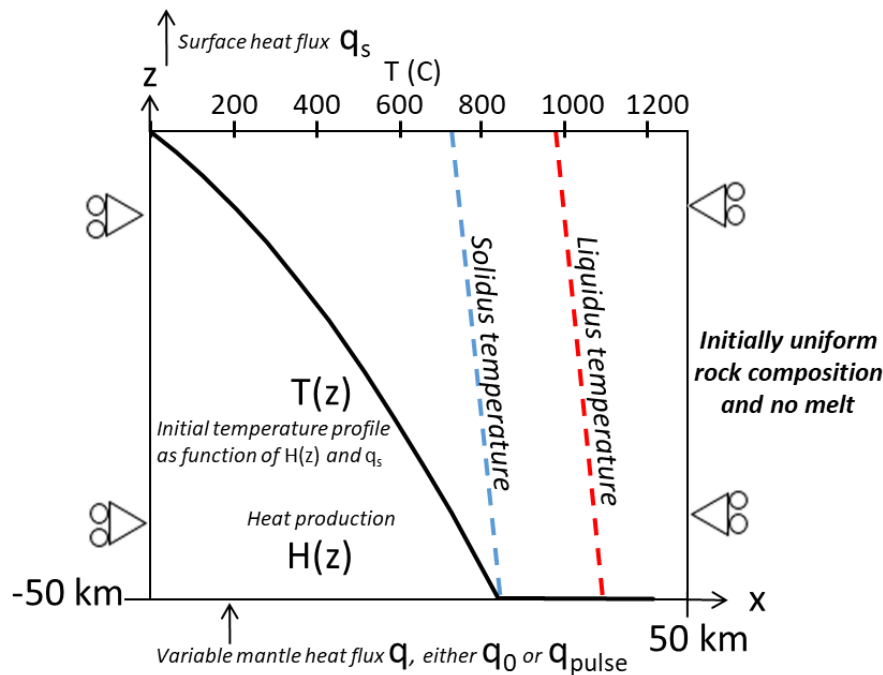
393 Our 2D model aims to represent a magma system in thickened continental crust with a crustal
394 thickness in excess of 50 km (Zandt et al., 1994). The bottom of the model (Fig. 2) represents a depth
395 at which heat is added by magmatic underplating from the mantle, thus the Moho is assumed to be
396 below 50 km. The horizontal size of 50 km with reflective mechanical and thermal boundary
397 conditions allows a total horizontal extent of 100 km, which is about half the width of Altiplano. At
398 top and bottom, non-flexible free slip boundary conditions are assumed, representing low viscosity
399 material above (air) and below (magmatic underplating) the model box. At the top, the temperature
400 is fixed to 0 °C, at the bottom, a laterally constant heat flux is assumed which may vary with time.
401 Boundary conditions for the solid and melt phases are no influx or outflux at all boundaries. The
402 initial temperature profile has been chosen to represent an unmolten crust, i.e. it is subsolidus down

to 50 km depth where the depth-dependent solidus temperature is exactly reached. This is done by calculating a conductive temperature profile with depth-dependent radiogenic heating with the parameters used in Schmeling et al. (2019) but adjusting the bottom heat flux to a value of $q_0 = 29.5 \text{ mW/m}^2$ resulting in a bottom temperature just at the solidus temperature (845°C). A sinusoidal lateral temperature perturbation of 10 K is added to the initial temperature field to initiate dynamic behavior in the model. To mimic magmatic underplating, a bottom heat flux condition is assumed with a single elevated pulse $q_{pulse} = 84 \text{ mW/m}^2$ over a time period of t_{max} of 10 Ma, or a series of three successive heat flux pulses with values of 120 mW/m^2 during time intervals of t_{pulse} of 2 Ma length with intervals of 2 Ma of normal conditions in between. The total (time integrated) heat energy input at the base of the crust in J/m^2 is given by $Q_{tot} = t_{max}q_0 + Nt_{pulse}(q_{pulse} - q_0)$, where N is the number of pulses. It is identical for both models. To explore the parameter space, the values of the heat pulses are varied between 48 mW/m^2 to 96 mW/m^2 for the single pulse models and 60 mW/m^2 to 140 mW/m^2 for the multiple pulse models, always keeping the total heat energy the same for single and multiple pulse experiments. The governing equations (Appendix A and B) are solved by finite differences with the code FDCON (e.g. Schmeling et al. 2019 and references therein).

Given the excess heat flow over the duration of each heat pulse, the equivalent thickness of a molten basalt layer can be calculated by accounting for the latent and sensible heat of such a layer:

$$h_{bas} = \frac{(q_{pulse} - q_0) t_{pulse}}{\rho_{bas}(L_{bas} + c_{pbas}(T_{bas} - T_0))} \quad (9)$$

Here q_{pulse} is the heat flux of the heat pulse of duration t_{pulse} , q_0 is the initial or background heat flux, ρ_{bas} , L_{bas} , c_{pbas} , and T_{bas} are the density, latent heat, heat capacity and temperature of the underplating basalt, respectively, and T_0 is the initial bottom temperature (equivalent to the steady state bottom temperature for a bottom heat flux of q_0). Using typical values for molten basalt, 2 Ma long heat pulses from 60 mW/m^2 to 140 mW/m^2 , results in underplating layers equivalent to 0.7 km to 2.6 km thickness, respectively. These thicknesses must be multiplied by 3 for the total of 3 pulses over 10 Ma or the equivalent single 10 Ma long pulse, resulting in total underplating thicknesses of 2.1 km to 7.8 km for the heat fluxes mentioned above.



430

431 **Figure 2:** Model setup indicating the mechanical and thermal boundary conditions, the initial temperature profile and the
 432 depth-dependent solidus and liquidus temperature curves. The triangles with the circles near the box sides symbolize free
 433 slip boundary conditions.

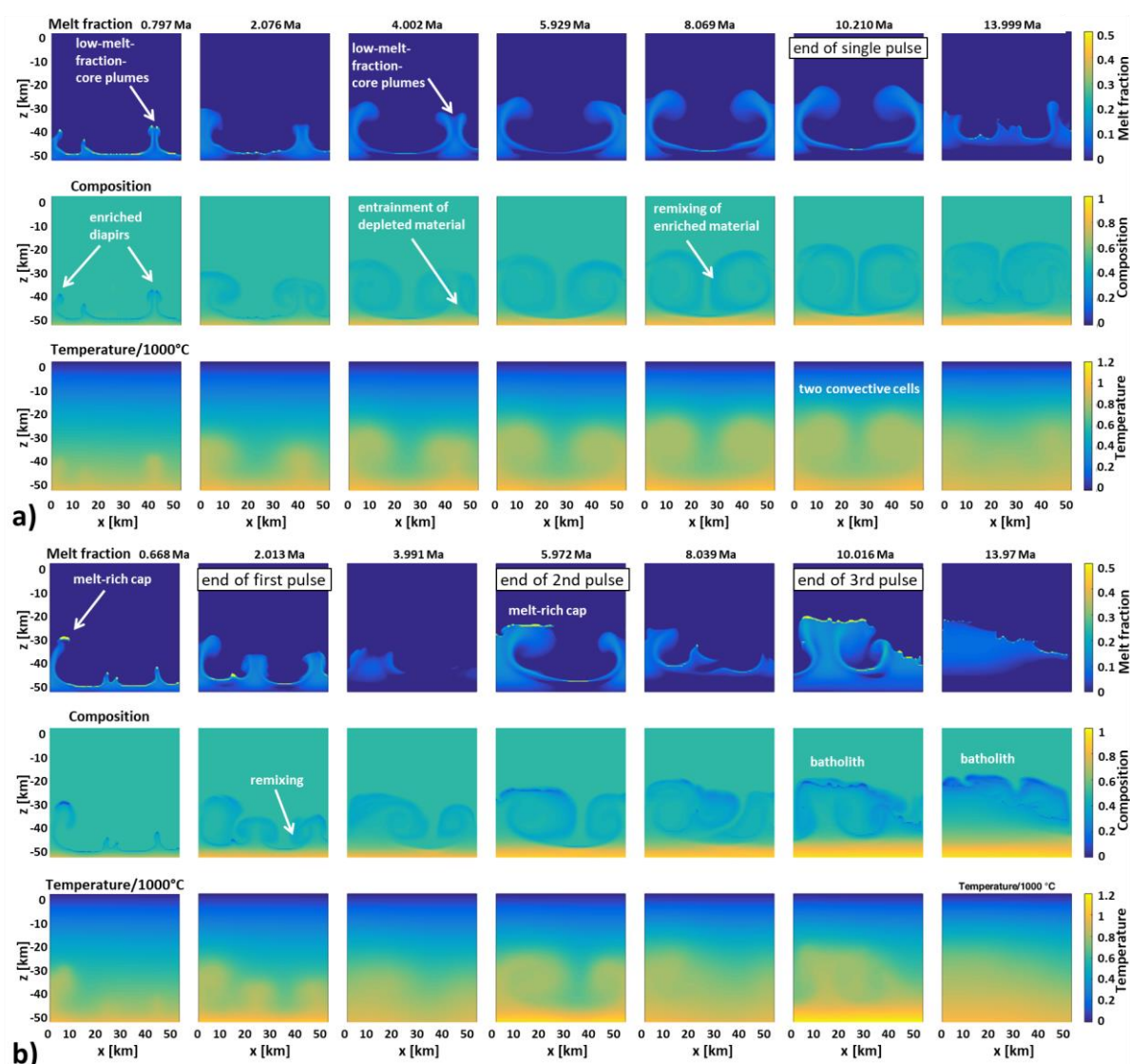
434

435 3. Results

436 3.1. Two typical models showing batholith emplacement and convective recycling modes

437 First, we show the general behavior of melt differentiation and ascent in our models and identify the
 438 two main modes batholith emplacement and convective recycling. We define a reference model
 439 setting with a retention number of 3. We assume either a single heat pulse of 84 mW/m^2 over a time
 440 period of 10 Ma or multiple heat pulses of 120 mW/m^2 of 2 Ma duration each. We ran these models
 441 (and all others) for 14 Ma years. Fig. 3 shows a temporal series of the results (see also the movies in
 442 the supplementary material).

443



444

445 **Figure 3:** Temporal evolution of melt fraction, composition, and temperature for a model of a fertile, 50 km thick crust
 446 where a sudden heat input from below leads to melting and melt ascent with chemical differentiation. (a) Single heat pulse
 447 of 84 mW/m² over a time period of 10 Ma, (b) multiple (three) heat pulses each with a duration of 2 Ma, a time interval of
 448 2 Ma between the pulses and an amplitude of 120 mW/m²; retention number, Rt , is 3 and enrichment solid rock density is
 449 2600 kg/m³. High resolution movies of the temporal evolution are also available in the Supplementary material as
 450 [movie_Fig3a_meltfraction](#), [movie_Fig3a_composition](#), [movie_Fig3a_temperature](#), [movie_Fig3b_meltfraction](#),
 451 [movie_Fig3b_composition](#), [movie_Fig3b_temperature](#).

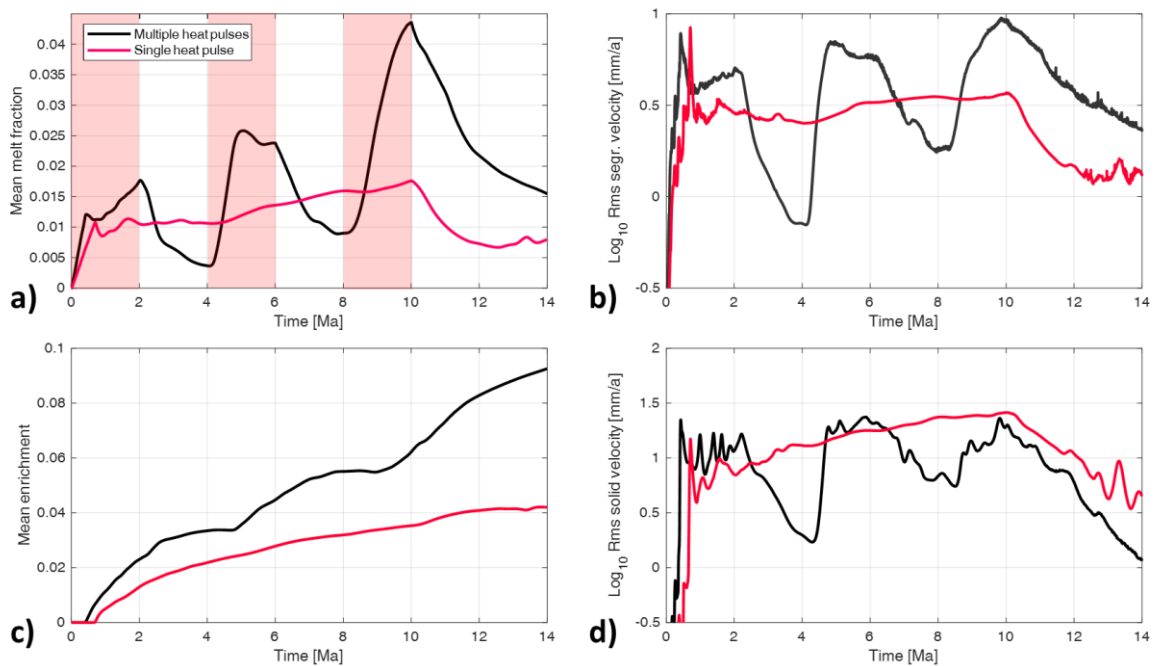
452

453 In the single heat pulse model, a thin partially molten layer forms near the bottom shortly after onset
 454 of heating. As the melt fraction increases with time, two-phase flow allows for melt segregation and
 455 a thin melt accumulation layer (> 50 % melt) forms on top of the progressively thickening partially
 456 molten layer. This layer breaks up into diapirs after 0.797 Ma (Fig. 3a, uppermost row, 0.797 Ma), or
 457 more precisely, into partially molten rising plumes or mush bodies which start freezing at their top. It
 458 is noteworthy that there are two processes running in parallel: the *en masse* mush rise in the diapirs
 459 and the relative motion between melt and solid through the interstices. In this example at 6 Ma the

460 velocity of the melt within the core of the diapir is about 30 % faster than the rise of the surrounding
 461 solid mass. Two well developed convection cells form with two mature partially molten hot plumes in
 462 the upwelling parts which rise to 22 km in 10 Ma and one cold unmolten downwelling limb in the
 463 center of the figure near $x = 25$ km. See third row of Fig. 3a shows the temperature distribution of
 464 this convection pattern. This convection is essentially driven by melt and thermal buoyancy and can
 465 be referred to as phase-change-driven convection. After switching off the heating at 10 Ma,
 466 convection continues, persisting as partially molten mush bodies for at least past 14 Ma, but as the
 467 melt gradually freezes the convective vigor decreases. Focusing on composition c (second row),
 468 results show the evolution of large bodies with $c < 0.5$ indicating that they are enriched in silica (light
 469 blue), while a depleted layer with $c > 0.5$ (yellow) grows to a final thickness of ~ 4 km at the bottom.
 470 The tops of melt-rich diapirs are clearly enriched (yellow structures at 0.797 Ma). Upon freezing, the
 471 enriched regions lose some of their buoyancy and are sheared and incorporated into the
 472 downwellings (faint bluish rims around the wide green cells at 2 Ma and later). This remixing of
 473 enriched material into the downwellings will be referred to as **mode of convective recycling**. Parts
 474 of the depleted bottom layer are entrained into the rising plumes (e.g. light yellow central cores in
 475 the plumes at 4 Ma) (e.g. Chappell et al., 1987; Cruden et al., 1995). Due to this entrainment, the
 476 melting temperature locally increases within the rising plumes accompanied by reduced melt
 477 fractions in the center of plumes. Furthermore, the increased depletion within the plumes leads to
 478 reduced buoyancy. This leads to splitting of the plumes, which is most pronounced when the plumes
 479 rise at the sides of the model box where symmetrical boundary conditions fix the position of the
 480 plumes. We may call such plumes "low melt-fraction-core plumes", labeled in Fig. 3a (see also the
 481 [movies in the Supplementary material](#)). Such plumes transport depleted material upwards into the
 482 shallower regions reducing the average enrichment there. Thus, despite a general trend towards a
 483 crustal stratification comprising depleted material below and enriched above, we observe both,
 484 convective downward recycling of enriched material and upwards recycling of depleted material.

485 The evolution of the model with three heat pulses shows a stark contrast (Fig. 3b). The higher heat
 486 pulses (120 mW/m^2) lead to stronger partially molten diapirs/plumes, larger segregation velocities
 487 \vec{v}_{sgr} , i.e. larger differences between melt and solid velocity. Melt accumulates in melt-rich caps
 488 reaching further up (0.668 Ma). During the first heat pulse these caps widen and freeze, and enriched
 489 material is remixed back into the lower crust (wide areas of light blueish material within the
 490 downwellings). Switching off the heat pulse at 2 Ma leads to the melt fraction dropping to almost
 491 zero. However, plumes are still warm and attract the new pulses of rising melt generated during the
 492 2nd and 3rd pulse. At the end of the second pulse the melt within the mushy diapir rises ~ 40 %
 493 faster than the surrounding solid mass. Melt accumulates (melt fraction > 50 %) at the top of the
 494 mushy plumes, forming potentially eruptible melt-rich caps. These caps form on top of plumes which

do not contain as much depleted material as those in Fig. 3a, i.e. the plumes in Fig. 3b evolve into less pronounced "low melt-fraction-core plumes". This is because the more intense bursts of melting lead to more intense differentiation and density contrasts: more enriched material is more buoyant resisting recycling, while more strongly depleted material at the base are denser resisting being dragged up. During the second and third pulses, less enriched material (both melt and frozen material) is recycled back than during the first pulse, and less depleted material is entrained into the rising plumes. This leads to the accumulation of melt-rich caps at the top of the plumes that merge into batholiths of enriched material (blue layers in the composition field at 10 Ma and 14 Ma). This enriched silicic body is irregular in shape and distribution because it is a result of the gradual aggregation of several small melt-rich diapirs that enter previously enriched regions and that may solidify at different depths. This accumulation into an enriched layer will be referred to as the **mode of batholith emplacement**. Another example clearly depicting the two modes is shown and discussed in Appendix D, Fig. D3



508

Figure 4: Evolution of the models in Fig. 3. a) Mean melt fraction, b) root mean square of segregation velocity (i.e. the difference between melt and solid velocity), c) mean enrichment, and d) root mean square of solid velocity as a function of time. Pink columns in (a) indicate the duration of heat pulses for the model with multiple pulses.

512

Fig. 4 compares global key quantities of the single and multiple pulse models. These parameters include the spatially averaged melt fraction, enrichment, solid velocity and segregation velocity \vec{v}_{sgr} i.e. the difference between melt and solid velocity. The means of the velocity vector fields was obtained by taking the *square root* of the spatial *mean* of all *squared* vector components, i.e. the *root mean square* (RMS) velocities. Although the total heat input is the same, on average three pulses

517

518 generate more melt than a single pulse (Fig. 4a). The mean enrichment increases to twice the value
 519 of the single pulse model (Fig. 4c) because each pulse leads to more intense melting (Fig. 4a)
 520 facilitating faster melt migration (Fig. 4b), forming enriched caps, which are less effectively recycled
 521 back into the lower crust. The root mean square (RMS) velocity of segregation of the multiple pulse
 522 model is higher on average and particularly during and after the 3rd pulse (Fig. 4b). This leads to the
 523 formation of a batholith instead of convective recycling. The RMS of the solid velocity of the single
 524 pulse model is essentially faster (Fig. 4d). This is typical for the convective recycling mode in Fig. 3a in
 525 contrast to the batholith emplacement mode of Fig. 3b and will be discussed in more detail in
 526 section 4.1.

527 We now focus on the evolution of the thermal structure of the two models in Fig. 3a and 3b (bottom
 528 rows). In the single pulse model, after 4 Ma two spatially persistent convection cells with growing hot
 529 plumes and cold downwellings develop. In contrast, in the multiple pulse model each new pulse
 530 reorganizes the temperature field and the number and shapes of plumes and after 10 Ma essentially
 531 one plume dominates the thermal field. Lateral temperature variations reach up to 220 K at 10 Ma in
 532 both models. Fig. 5a and 5d show the characteristic temperature profiles at 10 Ma for the two
 533 models. The profiles are taken at the x-positions where the vertically averaged temperatures are
 534 largest (red curves) or smallest (blue curves), respectively, and the positions are indicated in the
 535 legends. Comparison of the hot profiles for the single and the multiple pulse cases shows that the hot
 536 plume of the multiple pulse model reaches shallower crust and melt is present to 20 km – 25 km
 537 depth compared to 25 km – 30 km in the single pulse case. However, this is only an apparent
 538 difference because inspection of the full 2D temperature field (Fig. 3a at 10 Ma) reveals that the
 539 single pulse plume also reaches the same shallower depths 20 km - 25 km but this is displaced
 540 horizontally in relation to the hottest vertical position in Fig. 5d. The most important difference
 541 between the single- and multiple-pulse models is the thickness of the lower thermal boundary layer:
 542 about 7 km in the single pulse model and 10 km in the multiple pulse model. The thicker thermal
 543 boundary layer results from the thicker compositional depleted layer (Fig. 5c, 5f), which is stagnant
 544 and does not take part in the convection. This difference in thermal boundary layer thickness results
 545 in a thinner central convection layer in the multiple pulse model with smaller convection velocities
 546 (and smaller RMS flow velocities, black curve in Fig. 4d) resulting in a less effective vertical heat
 547 transport. Consequently, the surface heat flux in the single pulse model exceeds that of the multiple
 548 pulse model at times > 9 Ma (Fig. 6a).

549 In Fig. 5c and 5f the composition profiles are shown at different times. Since the single pulse model is
 550 dominated by convective recycling, the composition within the convective zone is reduced only
 551 slightly from 0.5 to between 0.4 and 0.45 (Fig. 5c). The corresponding melt fractions are small
 552 (mostly less than 10%) or even zero in the uppermost enriched part (Fig. 5b). In contrast, the multiple

553 pulse model is dominated by compositional differentiation, focusing melt and enriched rocks into a
 554 highly enriched batholith (c close to 0) fed by melt segregation from a wide melt-rich mush zone
 555 between 23 km and 45 km depth (Fig. 5e). This mush zone is responsible for providing a pathway and
 556 a source of melt out of the progressively depleted layer (c up to 0.9) underneath (Fig. 5f). The end
 557 result is that the multiple pulse model has a much more strongly differentiated crustal profile.

558 This compositional differentiation leads to spatial variations of the solidus and liquidus temperatures
 559 that depend on c , the rock composition (c.f. Fig. 1 for the dependence of the solidus temperature on
 560 composition). In Fig. 5a and 5d, we have also plotted the depth- and c -dependent solidus
 561 temperatures (dashed lines). Interestingly the actual temperatures within the hot plumes are sub-
 562 parallel to the solidus temperatures, exceeding them by a few 10s K. In the multiple pulse model,
 563 both in the cold and hot columns the temperatures within the depleted stagnant conductive layer
 564 are also above solidus (Fig. 5d) (note that within the depleted layer the solidus is approximately the
 565 same in the hot and cold regions). In contrast, in the convective recycling model the temperatures in
 566 the bottom thermal boundary layer are colder and lie below the solidus.

567 For a better understanding of the higher melt fractions and crustal differentiation of the multiple
 568 pulse case compared to the single pulse model, Fig. 6b shows the evolution of the heat content of
 569 both models. During the first half of the heating time, i.e. until 5 Ma, the multiple pulse model gains
 570 heat earlier and more rapidly than the single pulse model. Thus, more melt is created increasing
 571 crustal differentiation even though the total heat content is equal in both cases. During the second
 572 half (5 Ma to 10 Ma) the heat supply of the multiple pulse model is delayed becoming steep at the
 573 end. Until 8 Ma the heat supply of the single pulse model is higher and because of its less
 574 differentiated crust, the convection is stronger, as well as the vertical heat transfer and the
 575 convective remixing. As expected after 10 Ma the total heat content of the two models is
 576 approximately the same, but for later times the stronger convection in the single pulse models
 577 increases the total cooling rate associated with a higher surface heat loss (Fig. 6a, 6b). Because the
 578 total amount of melt is always smaller than 4 % (c.f. Fig. 4a), the fraction of energy stored in latent
 579 heat is always small compared to the contribution of total sensible heat (Fig. 6b, dash-dotted versus
 580 dashed curves). In the case of the multiple pulse model, the maximum amount of melt and stored
 581 latent heat is reached at 10 Ma, its subsequent release at shallow depths contributes to the surface
 582 heat flux by $\sim 6 \text{ mW/m}^2$, marked by an increase after 10 Ma (Fig. 6a, black curve).

583

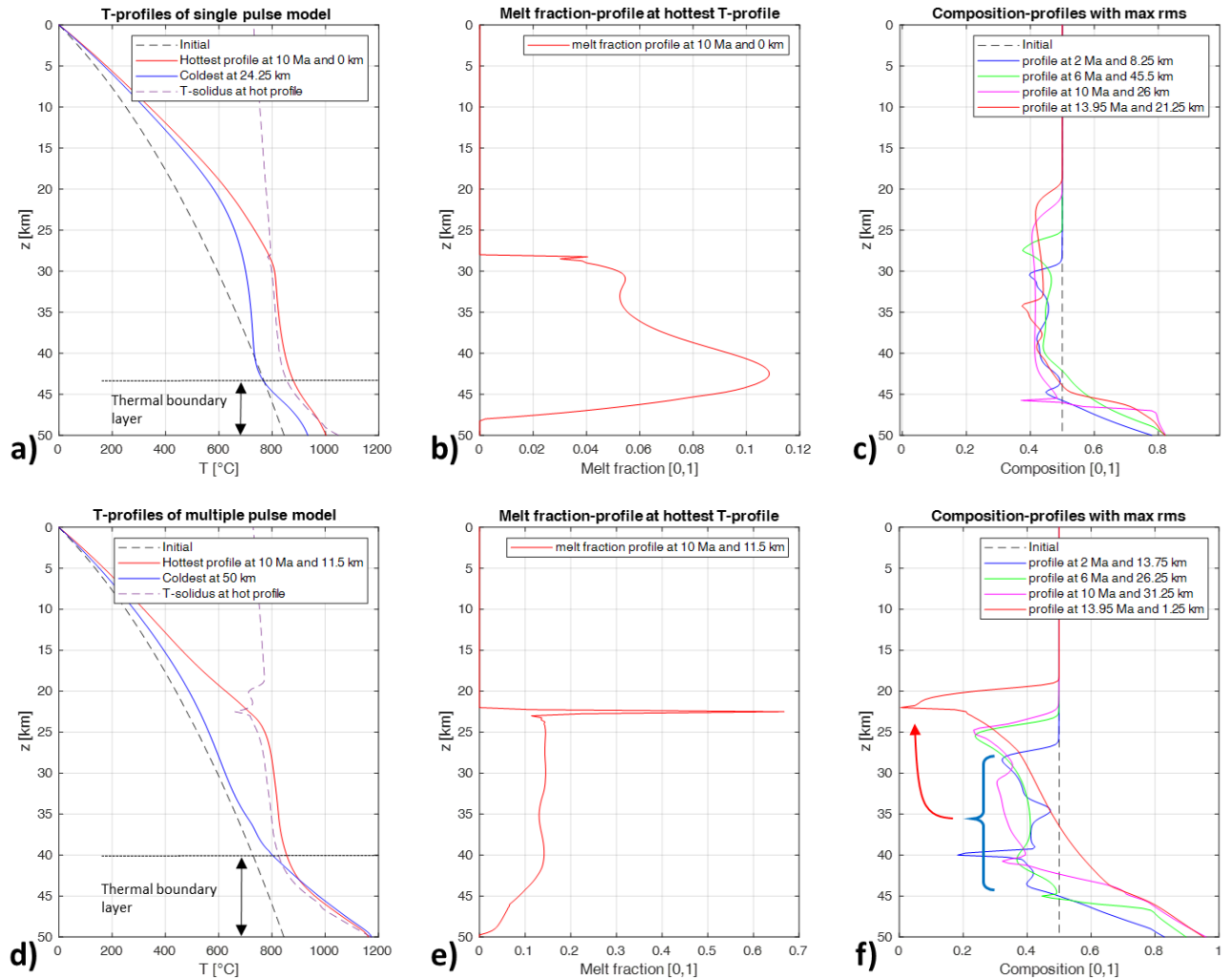


Figure 5: Vertical profiles of temperature and solidus temperature (a, d), melt fraction (b, e) and composition (c, f) at specific times and position indicated in the legends. "Hottest" and "Coldest" profiles are defined by taking the vertical average of temperature at all x-position and choosing those x-positions with the highest or lowest average temperatures, respectively. Upper row: single pulse model, lower row: multiple pulse model. In f), the red arrow indicates that, with progressing time, enriched compositions are transported upwards and accumulate in an enriched batholithic layer at 22 km depth. Results are for the models in Fig. 3.

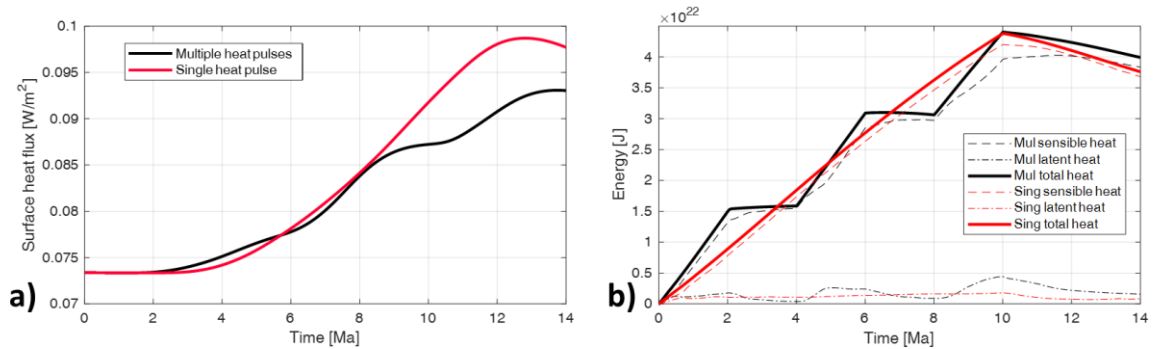


Figure 6: Thermal evolution with time of the two models. a) Horizontally averaged surface heat flux. b) Evolution of total heat content of the models. Total heat (bold curves) and the contributions due to sensible (dashed) and latent (dash-dotted) are shown. All energies are calculated assuming a 50 km x 50 km x 50 km crustal volume.

597 The single or multiple pulse models show that for the same average heat input the results are
 598 completely different. We thus tested three-pulsed cases with the same total heat input but with
 599 decreasing, identical or increasing heat flux amplitudes (Fig. 7). These three cases show significantly
 600 different behaviors: the decreasing heat pulse case shows important recycling, while constant or
 601 increasing heat pulses result in batholith formation. To explain the results, we conjecture that the
 602 first strong heat pulse (140 mW/m^2) of the decreasing heat pulse model already generates small
 603 melt-rich caps leading to a localized thin layer of enriched material at $\sim 25 \text{ km}$ depth. The subsequent
 604 weaker pulses (120 mW/m^2 , 100 mW/m^2) generate plumes with smaller melt fractions than the first
 605 pulse, but entrain more depleted material from the bottom, left over after the first pulse. As a result,
 606 these later plumes are pronounced "melt-poor core plumes" with only small amounts of melt within
 607 melt-rich caps. Thus, almost no accumulation of enriched material to form batholiths happens. The
 608 remnant of a thin enriched layer formed at 10 Ma is visible at $x = 32 \text{ km}$, $z = -20 \text{ km}$ in Fig. 7a
 609 whereas most of the enriched material has been recycled and forms a contorted blue band at depth.
 610 In contrast, the other two models generate more melt from pulse to pulse (Fig. 7c) and entrain less
 611 depleted material within the plumes, i.e. there are no "melt-poor core plumes". As a result, they
 612 contain higher fractions of melt allowing for faster segregation during the second and third pulses
 613 (Fig. 7c) while the solid plume velocities remain the same (Fig. 7e). Melt accumulates within melt-rich
 614 caps, forming significant bodies of enriched material when freezing.

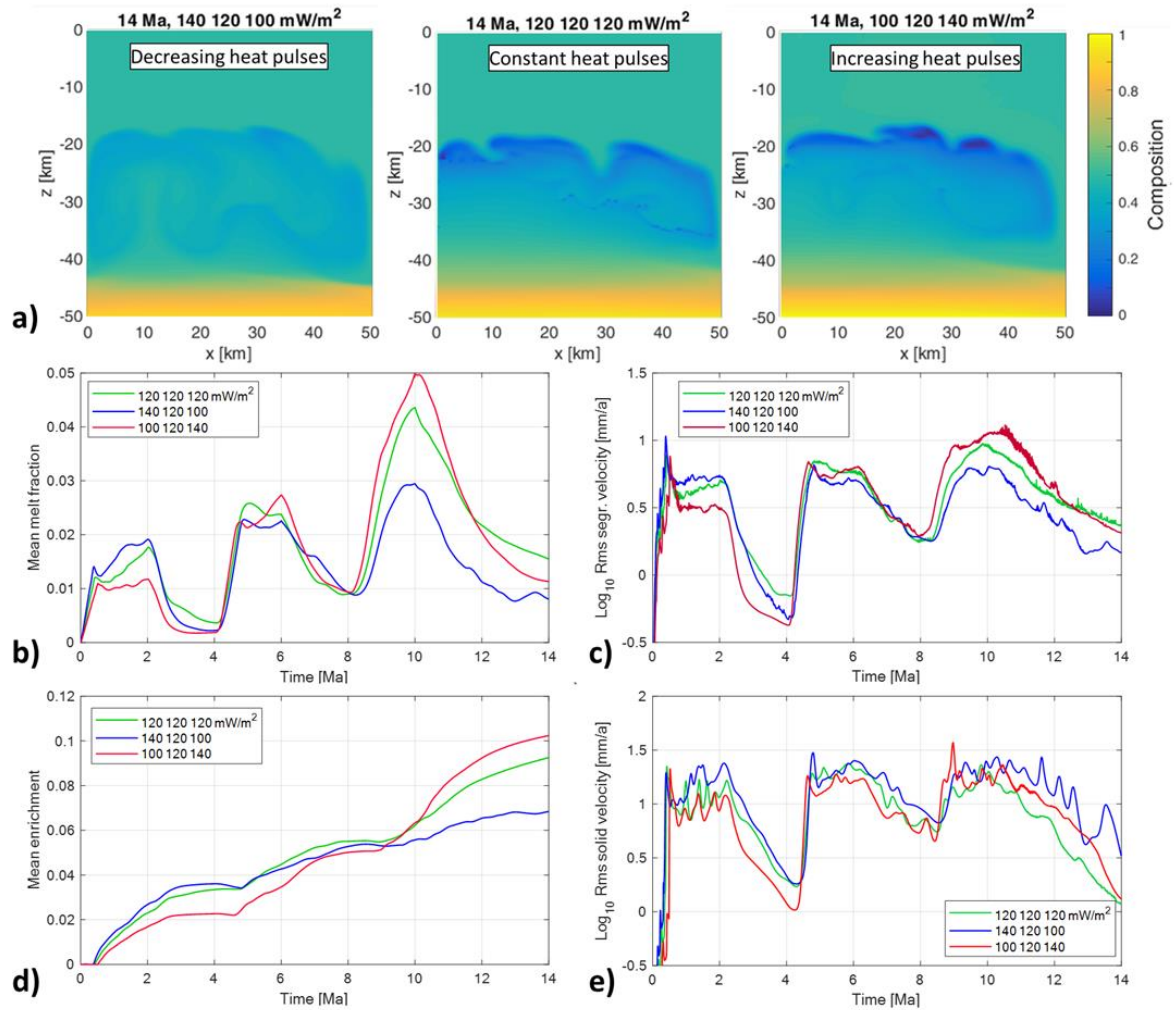


Figure 7: Effect of changing heat flux amplitudes systematically. a) Final stage (after 14 Ma) of the composition fields for decreasing heat pulses (140 mW/m², 120 mW/m², 100 mW/m²) (left), three consistent heat pulses of 120 mW/m² (center); and increasing heat pulses (100 mW/m², 120 mW/m², 140 mW/m²) (right). b) mean melt fraction, c) root mean square of segregation velocity, d) mean enrichment, and e) root mean square of solid velocity as a function of time.

3.2 Effect of varying melt mobility

Melt segregation strongly depends on the viscosity of the melt and on the permeability of the solid rock, i.e. on the retention number Rt . A high retention number corresponds to immobile melts, which are retained in the matrix. To investigate its effect, we use the same model layout as above (Fig. 3, section 3.1). Retention number is set to 1, 3, 10, 30, and 100. From section 2.2, such retention numbers are related to melt viscosities between 10^3 Pa s and 10^{13} Pa s, depending on the porosity length scale. Fig. 8 shows the late-stage composition fields of the single- and multiple-pulsed models, **but see also the full time evolution in the movies available in the Supplementary material**. For both series, the models with the highest melt mobility, $Rt = 1$, show strong segregation with extensive formation of enriched batholiths at around 20 km depth (blue regions) and strongly depleted lower crusts (yellow layers). Increasing Rt to 10 or higher, in both series, melt segregation becomes less

important and the results are dominated by recycling. At $Rt = 100$, high melt viscosity inhibits segregation to such extent that batch melting dominates the phase-change-driven convection. This means that the upwelling limbs of the convection cells consist of partially molten mushes with relatively high melt fractions ($O(10\% \text{ to } 30\%)$) contributing to the plume buoyancy while the cold downwellings are essentially solid, due to melt crystallization without its segregation. This means that convecting rock volumes undergo several melting – solidification – remelting cycles without differentiation. At high Rt , melt segregation near the bottom is slow but still strong enough over 14 Ma to form a thin depleted layer (Fig. 8, cases on the right). For the multiple pulse case, the change in mode from batholith accumulation to recycling occurs at $Rt > 3$ whereas it is between 1 and 3 for the single pulse case.

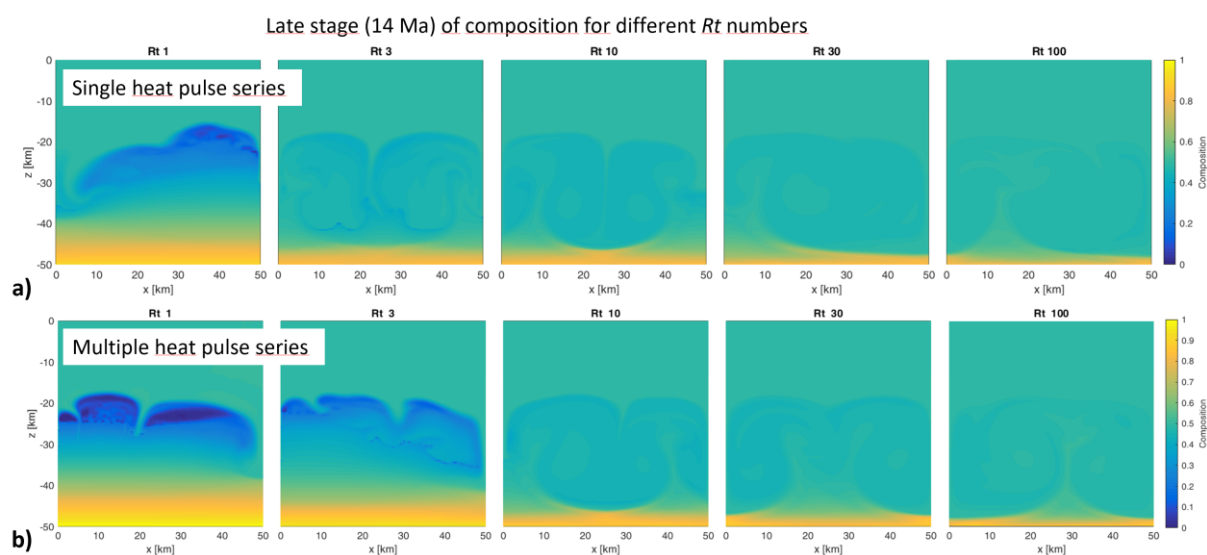
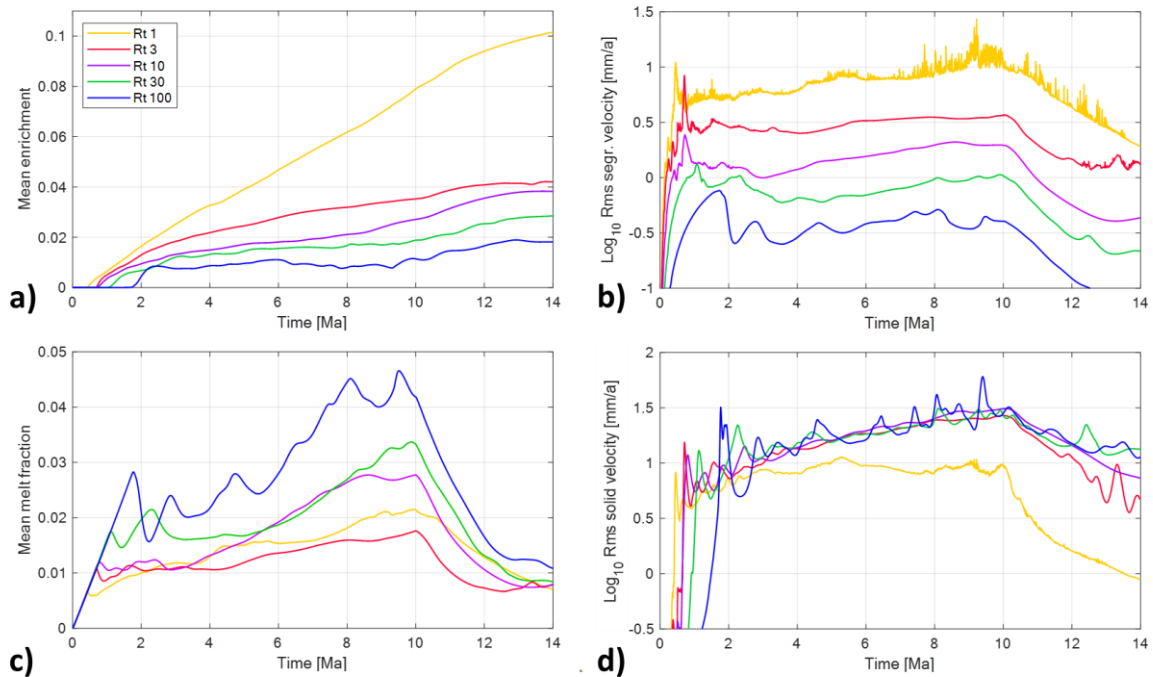


Figure 8: Composition at late stage (14 Ma) for different retention numbers Rt . a) Upper row shows the models with a single heat pulse of 10 Ma duration, b) lower row shows the multiple heat pulse models with three equal heat pulses, and with equivalent total heat input. High resolution movies of the full time evolution of the composition for models $Rt = 1, 3, 10$ are available in the Supplementary material (*movie_Fig8a_Rt1*, *movie_Fig8a_Rt3*, etc.).

To study this transition in more detail, Figs. 9 and 10 show the time-dependent evolution of a few key parameters of the single and multiple pulse models, respectively. Fig. 9a and 10a show the mean enrichment. The batholith emplacement mode ($Rt = 1$ for both heating cases, $Rt = 3$ only for the multiple case) correlates with pronounced increases of mean enrichment up to 0.10, while the recycling mode is characterized by weak increases of mean enrichment up to 0.04. Fig. 9c shows that first the amount of melt increases linearly and equally for all Rt values. During this stage, melt may already segregate within the thin layer near the bottom or stay within the matrix in the models with high Rt . In both cases the same amount of melt is generated and freezing has not yet started as long as the enrichment curves in Fig. 9a are still equal to zero. Only after the rise of plumes and

658 subsequent melt segregation and accumulation within melt-rich caps, melt starts to freeze in the
 659 upper crust. At this time the mean melt fraction curves depart from the initial linear trends. Due to
 660 high melt mobility, this happens much earlier for the low Rt models. As a result of reduced
 661 segregation, most melt is retained in the $Rt = 100$ model. The $Rt = 1$ model (Fig. 9c) does not follow
 662 this trend of melt fraction decreasing with decreasing Rt . This deviation can be explained by the
 663 continuous formation of an enriched region between 15 km and 30 km (Fig. 8a, $Rt = 1$) which
 664 effectively reduces the solidus curve in that region and leads to melts reaching 20 km depth (at
 665 10 Ma). In contrast, the $Rt = 3$ model (Fig. 8a, or Fig. 3a) is associated with pronounced remixing
 666 convection and thus less enrichment and higher solidus temperatures at > 20 km depth. Thus, the
 667 $Rt = 3$ model has smaller melt fractions. The effect of reduction of melt mobility is clearly seen in the
 668 RMS melt segregation velocity (Fig. 9b) that increases with decreasing Rt . However, the RMS solid
 669 velocity of all models with $Rt > 1$ (Fig. 9d) evolves similarly (except for the times of onset of diapiric
 670 rise). This is noteworthy in view of the mean melt fraction which differ by up to a factor 3. Checking
 671 the models in detail (not shown) reveals that the different sources of positive and negative buoyancy
 672 (thermal, melt, entrained enriched or depleted material within downwelling or upwelling,
 673 respectively) combined with the lower viscosity of the warmer $Rt = 100$ model, indeed compensate
 674 each other and result in similar overall solid velocities (Fig. 9d). In the model with batholith formation
 675 ($Rt = 1$) the thickness of the convective layer becomes smaller with time reducing the RMS solid
 676 velocity.



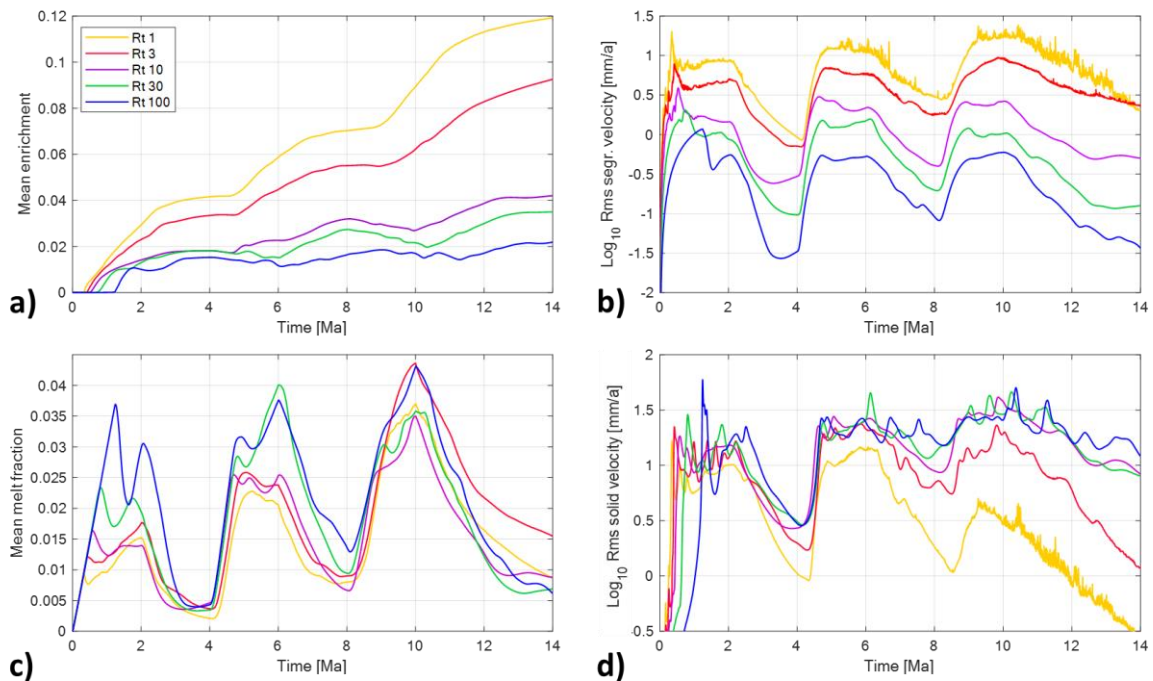
677

678 **Figure 9:** Effect of different Rt numbers ($Rt = 1, 3, 10, 30, 100$) for the single heat pulse case on enrichment (a), RMS melt
 679 segregation velocity (b), melt fraction (c) and solid RMS-velocity (d) versus time. Bottom heat flux is increased to 84 mW/m²
 680 for 10 Ma.

681

682 Fig. 10 shows similar curves for the model with the three heat pulses. Clearly, the mean enrichment
 683 curves for $Rt = 1$ and $Rt = 3$ (Fig. 10a) significantly depart from the other curves indicating effective
 684 formation of enriched magmatic bodies. The correlation between mean melt fraction and Rt is only
 685 clearly observed during the first two pulses (Fig. 10c). When the third pulse occurs, in case of low Rt
 686 larger amounts of enriched material have already accumulated at shallow depth and have solidified.
 687 This enriched material has a lower melting temperature and melts when the front of the third heat
 688 pulse reaches shallow depth, increasing the melt fraction to values comparable or above the values
 689 of the retained melt in high Rt cases.

690 Switching off the heat pulse leads to a clear decrease in RMS solid velocity only after the first pulse,
 691 while after the second and third pulses the decrease is much less pronounced (Fig. 10d). We explain
 692 this by some transient behavior in establishing phase-change-driven convection: while after the first
 693 pulse not enough heat and melt is available to drive convection by phase change, after the second
 694 and third pulses enough melt and heat is present so that continuous phase-change-driven convection
 695 takes over. Similar observation can be made for the segregation velocity. Both models with batholith
 696 formation ($Rt = 1, 3$) keep high segregation velocities during and after the third pulse, while solid
 697 velocity (diapiric velocity) drops steeply.



698

699 **Figure 10:** Effect of different Rt numbers ($Rt = 1, 3, 10, 30, 100$) for the multiple heat pulse case on the enrichment (a),
 700 RMS-melt segregation velocity (b), melt fraction (c) and solid RMS-velocity (d) versus time. Bottom heat flux is increased to
 701 120 mW/m^2 for three successive heat pulses with a duration of 2 Ma each and a time interval of 2 Ma between pulses.

702

703 3.3. Effect of varying heat flow

704 In this section the effect of the magnitude of heat input is studied both for a single and for multiple
 705 heat pulses with retention number of 3. Again, the total amount of heat input in the single and
 706 multiple pulse models is the same for 10 Ma. Fig. 11 shows the composition fields after 14 Ma. A
 707 weak single heat pulse of 48 mW/m^2 leads to some melting and weak diapiric rise with some
 708 segregation up to a depth of 30 km. Convection is only weakly supercritical, both the RMS solid and
 709 segregation velocities are slow, of order 1 mm/a to 2 mm/a (Figs. 12b and 12d). The actual velocities
 710 are so slow ($O(4 \text{ mm/a})$) within the lower partially molten crust (30 km to 45 km) that during 10 Ma
 711 no multiple overturns are possible to remix enriched material back into the crust. As a result,
 712 batholith emplacement of enriched material is visible near the top of the melting zone at 30 km
 713 depth (Fig. 11a, left).

714 Increasing the heat flux from 60 mW/m^2 to 84 mW/m^2 (for the single pulse heating case) intensifies
 715 convection and recycling. RMS velocities of the solid increase from about 6 mm/a to 25 mm/a,
 716 segregation velocities from 2 mm/a to $\sim 5 \text{ mm/a}$ (Fig. 12). As will be discussed below, such ratios of
 717 segregation to solid velocities of less than 30 % are indicative of the recycling mode, with multiple
 718 convective overturns, where solid velocities dominate with respect to melt velocities. During the late
 719 stage of the hottest model with 96 mW/m^2 (Fig. 11a, right) RMS segregation velocities reach
 720 6.3 mm/a (Fig. 12b) (locally significantly higher) leading to some differentiation and emplacement of
 721 enriched material within the top of the partially molten layer near 15 km depth. Furthermore, the
 722 total amount of melt within the crust increases by about a factor three to a mean melt fraction of
 723 0.03 (Fig. 12c), equivalent to 75 km^2 of melt in 2D, or 3750 km^3 melt within an equivalent $50 \text{ km} \times$
 724 $50 \text{ km} \times 50 \text{ km}$ crustal volume in 3D.

725 In summary, for the single pulse heating case increasing the heat flux leads to a shallowing of the top
 726 of the partially molten zone from 30 km to 15 km depth (Fig. 13). Due to lower temperatures and
 727 high viscosity of the upper crust, the depth range between $\sim 20 \text{ km}$ to 15 km is a strong boundary for
 728 magmas rising by convection and segregation. This implies that this is an accumulation zone for
 729 strongly evolved, enriched melts, and that upwards from here, other modes of melt or melt-mush
 730 transport are necessary.

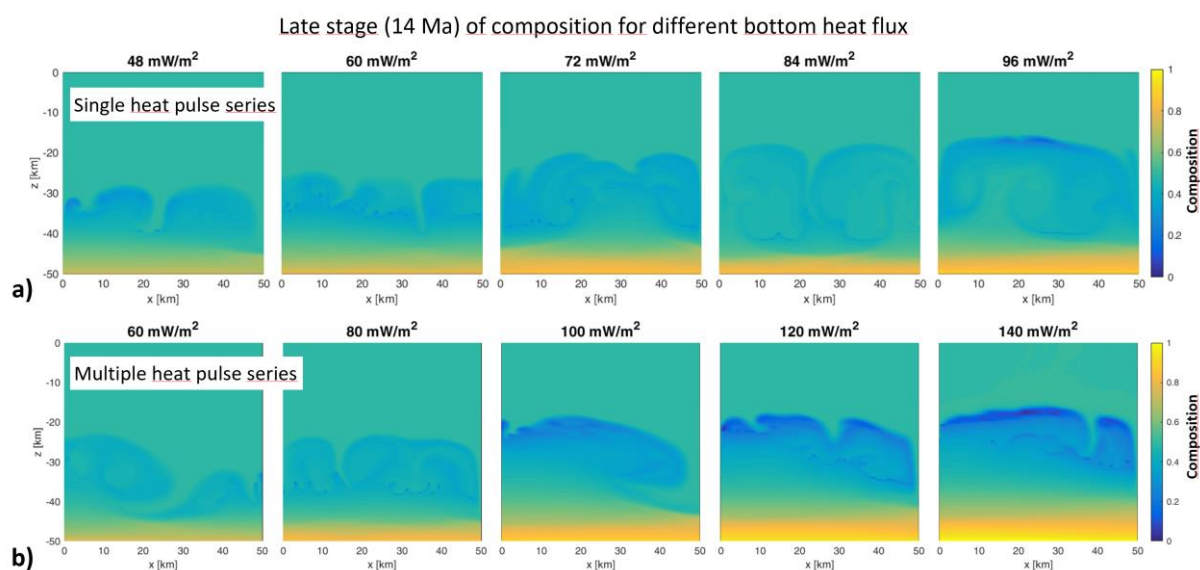


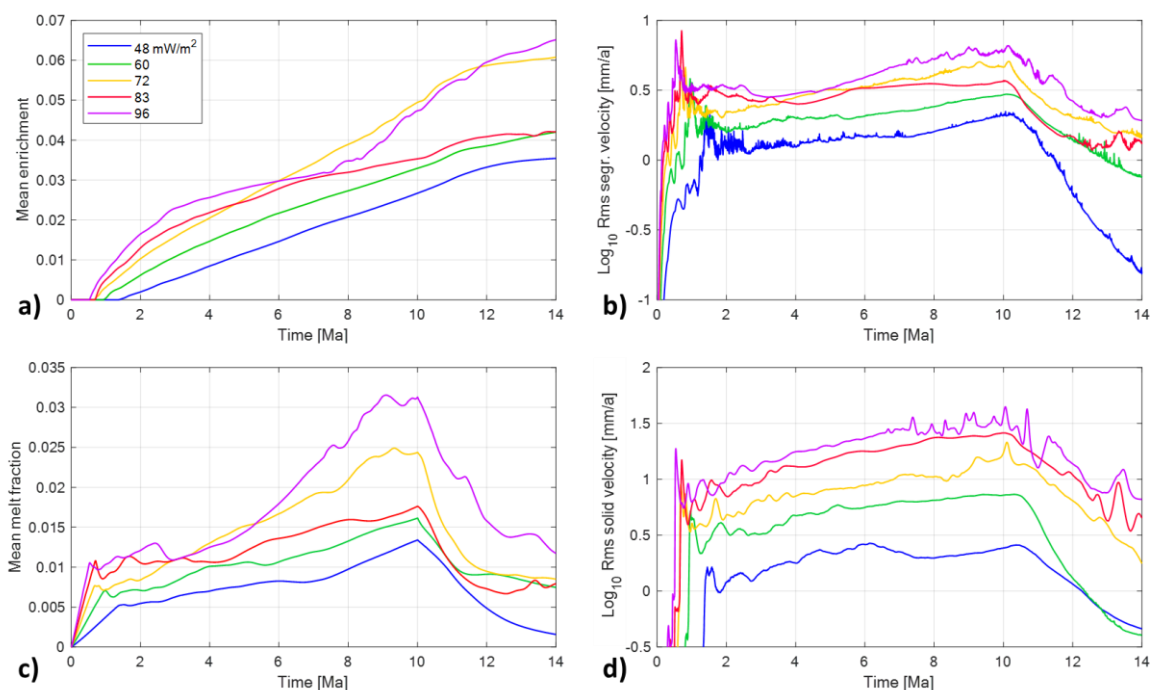
Figure 11: Composition at late stage (14 Ma) for different bottom heat fluxes. a) Models with a single heat pulse of 10 Ma duration with heat fluxes indicated by the subplot titles, b) Models with multiple heat pulse of 2 Ma duration each with values indicated in the subplot titles. The time-integrated total heat input in the two models in each vertical column is equal.

Results for the multiple heat pulse series (Fig. 11b) shows that already for the coolest model with three 60 mW/m² pulses, equivalent to one 48 mW/m² single pulse, results in significantly shallower rise of the magmatic system, to about 23 km compared to 30 km. During each pulse the RMS velocities of the solid are about a factor 3 higher than in the single pulse model, forcing some recycling of enriched material compared to the weak single pulse model. Increasing the strength of the pulses to 100 mW/m² and above clearly shows that emplacement of enriched material is intensified with an extensive batholith between 15 km to 25 km.

Comparing Fig. 14b and 14d shows that the models with batholith emplacement rather than convective recycling are characterized by RMS segregation velocities reaching 11 mm/a or 30 % to 50 % of the solid RMS velocities. Compared to the single pulse models the total amount of melt and enrichment are higher at 10 Ma or 14 Ma, respectively (Fig. 14a, 14c). The highest mean melt fraction of 0.05 is equivalent to 125 km² melt or 6250 km³ in 3D in a 50 km x 50 km x 50 km section of the crust. A typical feature of the multiple pulse models with batholith emplacement is the heterogeneous spatial distribution of the batholiths (Fig. 11b; 100, 120 and 140 mW/m²), sometimes three subsequent emplacement events are still visible as enrichment maxima at different depths.

Comparing the depths of the top of the partially melted mush columns and of the enriched batholith bodies (Fig. 13) shows that for low to intermediate heat fluxes the multiple pulse models predict shallower depths. Thus, for the same time-integrated heat inputs, the pulsed models are more

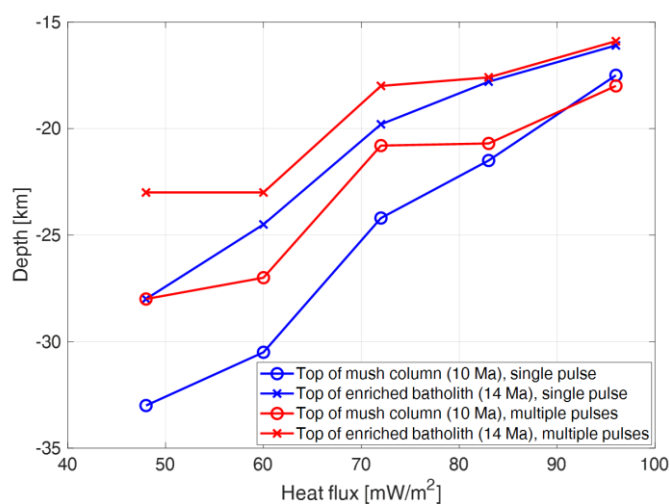
756 effective regarding melt ascent and batholith formation. By comparison, the melt and enriched
 757 material only ascend to similar depths of 16 km to 17 km when the heat flux is highest in the single
 758 pulse case, but even in this case the batholith of enriched material is less voluminous (Fig. 13).



759

760 **Figure 12:** Effect of bottom heat flux pulse amplitudes (48, 60, 72, 84, 96 mW/m²) for a single bottom heat flux pulse of
 761 10 Ma duration. Temporal evolution of a) mean enrichment, b) RMS segregation velocity, c) mean melt fraction, and d) RMS
 762 solid velocity.

763



764

765 **Figure 13:** Depth of the top of mush columns and felsic batholith bodies as a function of heat flux. For the multiple pulse
 766 models the heat fluxes are equal to the time-integrated fluxes from 0 Ma to 10 Ma of the single pulse models.

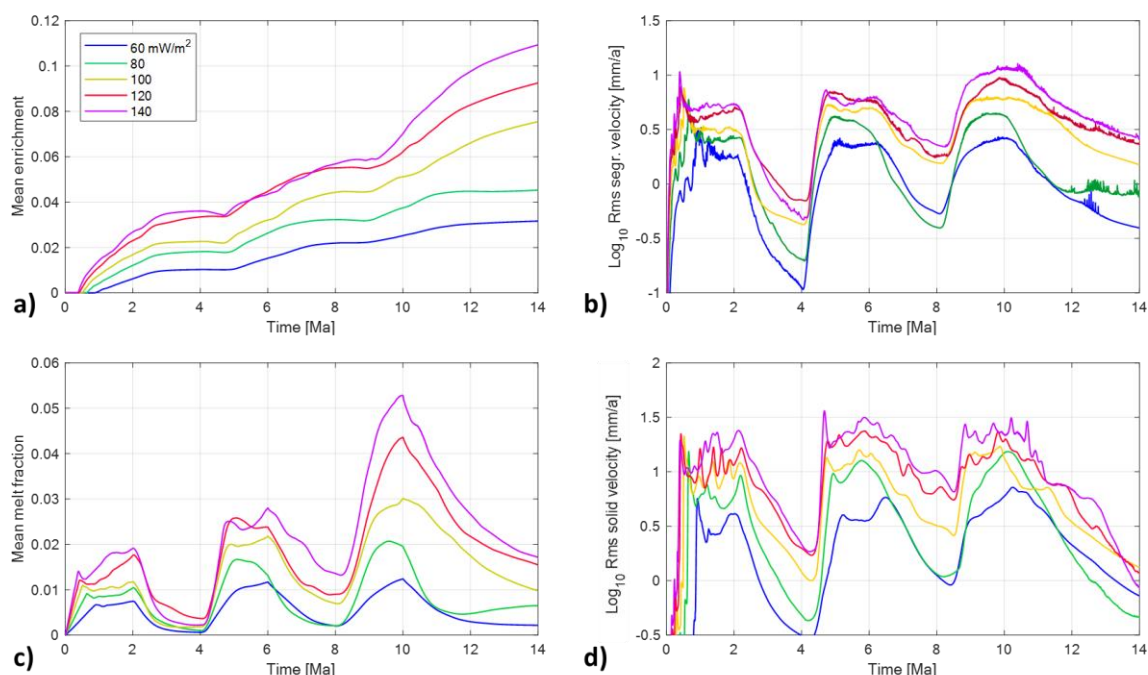


Figure 14: Effect of different heat pulse amplitude for the multiple heat pulse model with heat pulses lasting 2 Ma each. Temporal evolution of a) mean enrichment, b) RMS segregation velocity, c) mean melt fraction, and d) RMS solid velocity. The used heat pulses are indicated in the legend in (a). The time-integrated total heat input of the corresponding single and pulsed models are equal and represented by lines with the same color in Fig. 12.

3.4 Effect of density of the enriched felsic rock

In this section we study the effect of the density of the enriched solid material generated by melt solidification where the density of 100 % enriched solid material corresponds to a density of felsic granite with 75 % SiO_2 (section 2.2). In the previous models, this density was 2600 kg/m^3 compared to the reference density of fertile rock of 2700 kg/m^3 , the density of melt of 2300 kg/m^3 and the density of the residual, depleted rock of 3100 kg/m^3 . Remember, that the thermal expansivity of all solid compositions and the melt is equal. Note that per definition the enrichment and depletion densities are for 100 % enrichment and depletion, respectively. As discussed in section 2.2, zero to 100 % enrichment correspond to a solid composition of c_s varying from 0.5 to zero, while zero to 100 % depletion correspond to c_s varying from 0.5 to 1. For further details about the effective densities used in the calculations see Schmeling et al. (2019).

The density of the enriched solid material is varied between 2600 kg/m^3 and 2300 kg/m^3 keeping the melt density at 2300 kg/m^3 . The solid value of 2300 kg/m^3 is too low for igneous felsic rocks such as granites, which generally vary between 2500 kg/m^3 and 2800 kg/m^3 , only rhyolites are lighter, with densities between 2300 kg/m^3 and 2500 kg/m^3 (e.g. Schön, 2015). The very low values used are meant for testing the sensitivity of the magmatic systems to the density of the enriched solid material.

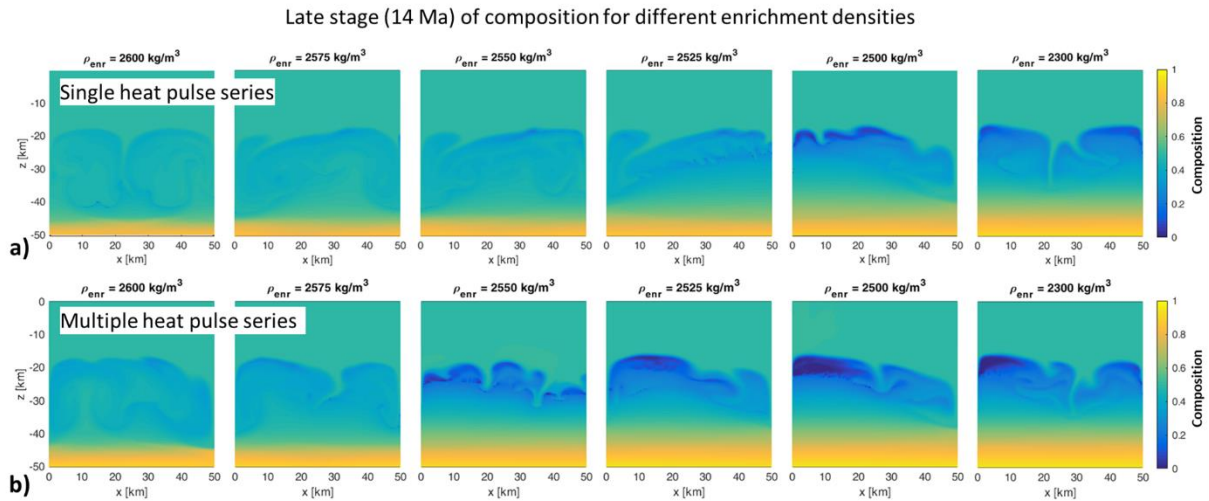


Figure 15: Late stage (14 Ma) of composition for different enrichment densities of the enriched solid material given above each figure. Previous models used 2600 kg/m^3 , the crustal reference density is 2700 kg/m^3 , and the melt density is 2300 kg/m^3 . In row a) one 10 Ma long heat pulse of 84 mW/m^2 has been imposed. In row b) three 2 Ma long single heat pulses of 140, 120, and 100 mW/m^2 have been imposed with the same total equivalent heat as in a). The background bottom heat flux was 29.5 mW/m^2 .

Fig. 15 shows the late stages of models with a single and multiple heat pulses. A clear bifurcation exists between models with convective recycling of enriched material and models with batholith emplacement. This bifurcation takes place when the density of the enriched solid material becomes $< 2525 \text{ kg/m}^3$ for the single or $< 2575 \text{ kg/m}^3$ for the multiple pulse models. Once the density of the enriched solid material is below these values, batholith emplacement occurs and their overall appearance does not vary significantly anymore. Following the discussion in sections 2.1 and 2.2, the Boussinesq approximation underestimates the volumes of enriched batholiths for the extremely low density values of the enriched material, such as 2300 kg/m^3 : the dark blue batholiths in the rightmost models in Fig. 15 may thus be about 10 % smaller than without this approximation.

In Appendix D, we detail the effects of different densities of enriched rocks in mean enrichment, mean melt fraction, RMS segregation and solid velocities. These models can be used to elucidate the physics responsible for convective recycling and batholith emplacement, respectively. We discuss this in detail in Appendix D. The patterns observed indicate that in the recycling case, the segregation and solid velocities are relatively similar and the melt does not have time to segregate significantly from the solid matrix before freezing at the top of the convection cell, and is recycled back down where it melts preferentially because of its enriched nature. In contrast, for the batholith emplacement cases, the segregation velocities are about twice that of solid velocities. In this case, melt segregates forming a buoyant solid batholith that is displaced horizontally at emplacement level and only small volumes get recycled.

In conclusion, batholith emplacement is favored if the segregation velocity is significantly higher than the solid flow velocity, and that convective recycling is favored if segregation is slow so that buoyant enriched material can be entrained into the downwelling flow to be remelted in the lower part.

3.5 Parameter space for batholith emplacement vs. convective recycling

We have run 116 models with different retention numbers Rt , bottom heat fluxes (i.e. thermal Rayleigh numbers Ra_q , c.f. eq. 4), and enriched densities (i.e. enrichment Rayleigh numbers Ra_{enr} , c.f. eq. 4) both for multiple and single bottom heat pulses, including some cases with multiple pulses either decreasing or increasing with time (Fig. 7). All models have been evaluated on whether they lead to batholith emplacement bodies after 14 Ma or convective recycling of enriched material. By calculating the area in which enrichment is larger than the lower bound defining granites (local enrichment of 0.314 corresponding to 63 % SiO_2 , see section 2.2) and normalizing this size by the width of the model (50 km), we obtain the mean thickness h_{bath} of the batholiths (eq. 8). Fig. 16a shows the mean batholith thicknesses of all models with the same total bottom heat flux (single and multiple pulses) but different enrichment densities and retention numbers as a function of mean enrichment at 14 Ma. Up to a mean enrichment of about 0.06 no batholith emplacement occurs (i.e. local enrichment lies below 0.314 everywhere). Any SiO_2 rich material is recycled back into the lower crust. Within a narrow enrichment range of 0.06 to 0.08, finite batholith thicknesses develop (grey transition range) and rapidly increase up to 10 km. Thus, we can use the mean enrichment as a proxy for distinguishing between batholith emplacement or convective recycling modes. The ratio of the RMS of segregation velocity and solid flow velocity at ~ 10 Ma versus mean enrichment (Fig. 16b) confirms that effective batholith emplacement correlates with RMS segregation velocities larger than 0.3 to 0.5 times the RMS solid velocity. This relation has been verified in detail for one example (Appendix D, Fig. D3). It should be noted that taking the average or RMS of a quantity like enrichment or velocity, respectively, depends on the model geometry, particularly on the size of the regions with a non-zero quantity with respect to the total model domain including the unmolten and stagnant regions. Therefore, the actual values discussed above may not be easily generalized, but allow consistent and meaningful comparisons among the models. As a rule of thumb, inspection of the models indicates that in all cases where the RMS segregation to solid velocity ratios exceed 0.3 to 0.5, locally the segregation velocity exceeds the solid velocity by a factor 3 or more, especially in regions of small solid velocity. Such high segregation velocities effectively reinforce the formation of batholiths.

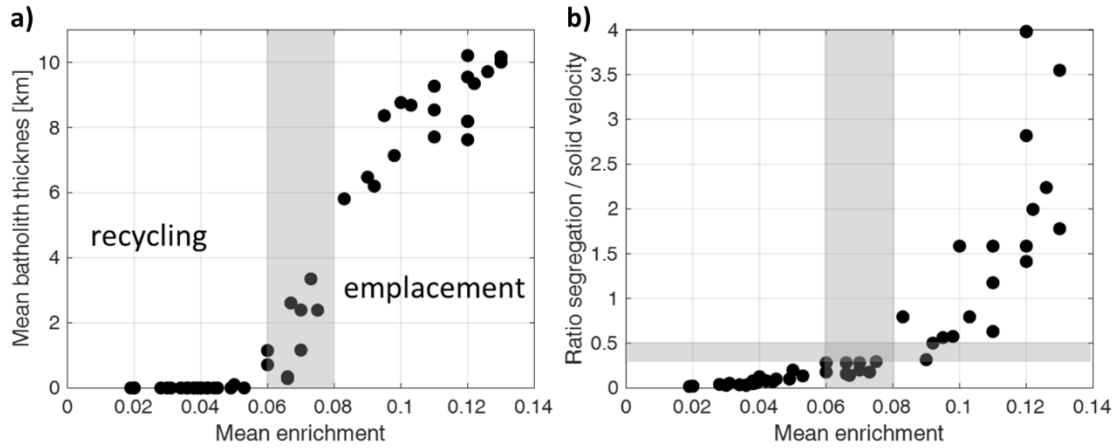
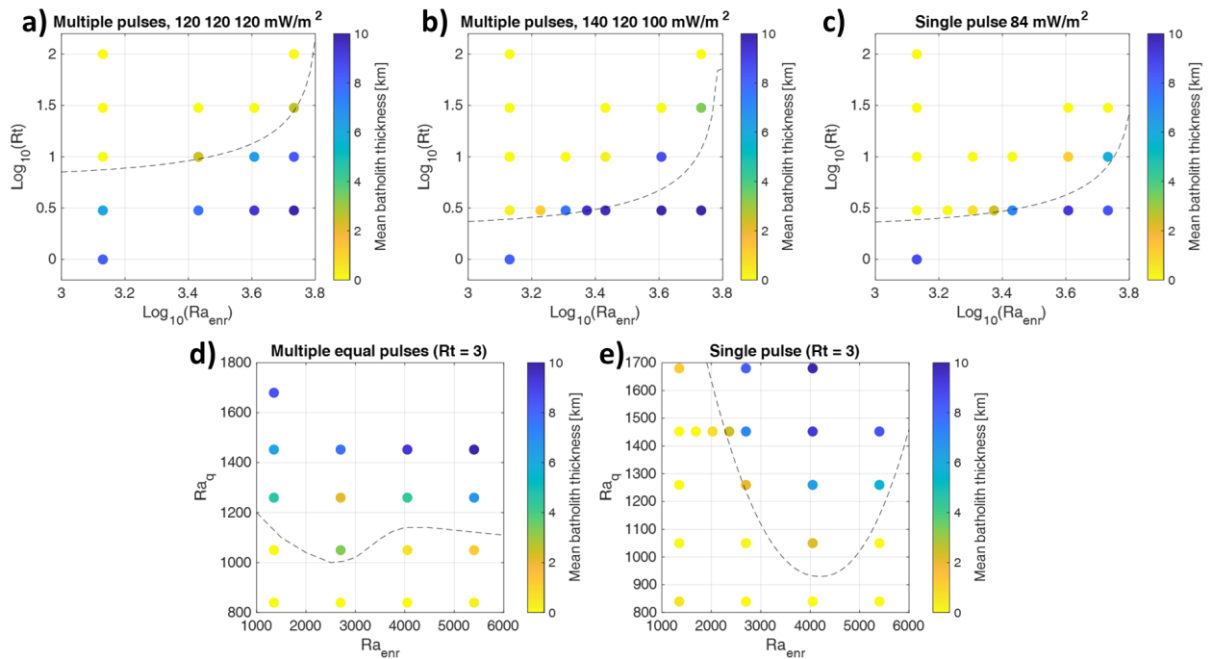


Figure 16: a) Mean batholith thickness h_{bath} versus mean enrichment at 14 Ma, b) ratio of the root mean square segregation velocity to the root mean square solid velocity versus mean enrichment at ~ 10 Ma. All models have the same total bottom heating (either three pulses or one single pulse with an equivalent heat flux of 84 mW/m^2). The grey bands indicate the transitional ranges between emplacement and recycling.

In Fig. 17 we explore the parameter spaces $Ra_{enr} - Rt$ and $Ra_{enr} - Ra_q$ by plotting the mean batholith thicknesses after 14 Ma of all models as functions of Ra_{enr} , Rt , and Ra_q . Low Rt means high mobility of the melt, and high Ra_{enr} means low density of enriched solid material with respect to fertile crust, so that segregation is enhanced in models with low Rt and high Ra_{enr} . The yellow dots indicate models without batholiths, i.e. where the enriched material has been recycled back into the lower crust by thermal and phase-change-driven convection. The blue dots indicate models in which significant batholiths with thicknesses of several km have developed. In all model sets a relatively sharp boundary exists (approximated by dashed curves) separating the recycling domain from the batholith emplacement domain. In the $Ra_{enr} - Rt$ parameter space, high melt mobility (low Rt) and lower density of enriched solid material (higher Ra_{enr}) favor emplacement while lower melt mobility and higher enrichment densities favor recycling. Comparing multiple equal pulse models to models with three pulses of decreasing amplitude with time and with one single pulse clearly shows different behaviors (Fig. 17a to 17c). Three equal pulses (Fig. 17a) shift the boundary between the regimes to higher Rt , i.e. lead to a more pronounced emplacement behavior compared to the others. This may be explained as follows: switching off the bottom heat pulse several times hampers recycling because convection slows down after each pulse while segregation still takes place during the intervals of paused heat flux.

We now fix the melt mobility to $Rt = 3$, which is near the boundary between recycling and batholith emplacement in Figs. 17a to 17c, and explore the parameter space $Ra_{enr} - Ra_q$. Increasing the convective vigor by increasing the bottom heat flux (i.e. increasing Ra_q) favors batholith emplacement (blue dots in Fig. 17d to 17e). We explain this by the strong increase of total melt fraction which segregates due to the chosen mobility and stays buoyant after solidification. Only for

875 small Ra_{enr} this buoyancy is not sufficient for batholith emplacement and the felsic material is
 876 recycled (Fig. 17e). The slight preference for recycling also at high Ra_{enr} (e.g. 6000) as suggested by
 877 Fig. 17e needs an explanation. Evaluating the models within the lower right of Fig. 17e in detail (not
 878 shown) suggests that segregation velocities within the slow rising plumes (because of low heat flow)
 879 are reduced due to freezing already within these plumes. Due to freezing, the solid within the plumes
 880 becomes enriched, increasing the total buoyancy within the plumes and thus increasing the
 881 convective vigor. However, the melt is less buoyant or even denser with respect to the buoyant
 882 enriched matrix, which decreases segregation. As a result, convection and associated recycling is
 883 enhanced at very low enrichment densities of 2300 kg/m^3 ($Ra_{enr} > 5000$) at small Ra_q . Comparing
 884 the models with three pulses and a single pulse for low enrichment buoyancy (low Ra_{enr}) supports
 885 the previous observation of favoring batholith formation when multiple pulses are involved instead
 886 of a single one (Figs. 17d, 17e).



887

888 **Figure 17:** Diagrams distinguishing the batholith emplacement (orange to blue dots) versus convective (no batholiths,
 889 yellow dots) regimes in the $Ra_{enr} - Rt$ and $Ra_{enr} - Ra_q$ spaces. Models in a), b) and c) had an integrated total bottom
 890 heat flux corresponding to the heat flux Rayleigh number $Ra_q = 1470$. The models in d) and e) have a retention number
 891 $Rt = 3$. (a) Three equal heat pulses of 120 mW/m^2 with 2 Ma duration, b) three heat pulses of 140, 120, and 100 mW/m^2 ,
 892 and c) single 10 Ma long heat pulse of 84 mW/m^2 . In a) to c) the total time integrated heat input was identical over 10 Ma
 893 and the dashed curves show the boundaries between batholith emplacement and granite recycling cases calculated by the
 894 scaling law (section 3.6). Fits were obtained with the parameters: a) $\phi = 0.09$, $f = 0.15$, $a_0 = 0.5$, b) $\phi = 0.05$, $f = 0.14$,
 895 $a_0 = 0.5$, c) $\phi = 0.05$, $f = 0.13$, $a_0 = 0.5$. d) Three equal heat pulses with magnitudes varied from 60 mW/m^2 to
 896 140 mW/m^2 corresponding to Ra_q from 840 to 1680. e) Single 10 Ma long heat pulses varied from 48 mW/m^2 to
 897 96 mW/m^2 , corresponding to the same Ra_q values as in d). The dashed lines show the boundaries between the
 898 emplacement and recycled models estimated by visual inspection.

899

900

901 3.6 An analytical scaling law for batholith emplacement vs. convective recycling

902 From the evaluations of the parameter space in section 3.5, a clear boundary emerges between cases
 903 with batholith emplacement and cases of convective recycling. In Appendix C, we derive an analytical
 904 scaling law for such a boundary by defining a batholith emplacement parameter, e_b . Emplacement of
 905 enriched silicic material takes place predominantly within the upper horizontal limb of the convective
 906 flow by segregation. Relating the characteristic segregation time t_{sgr} of the melt to leave the
 907 horizontal convection limb to the characteristic time t_{hor} of a solid particle to travel along the
 908 horizontal limb, we define the emplacement parameter as:

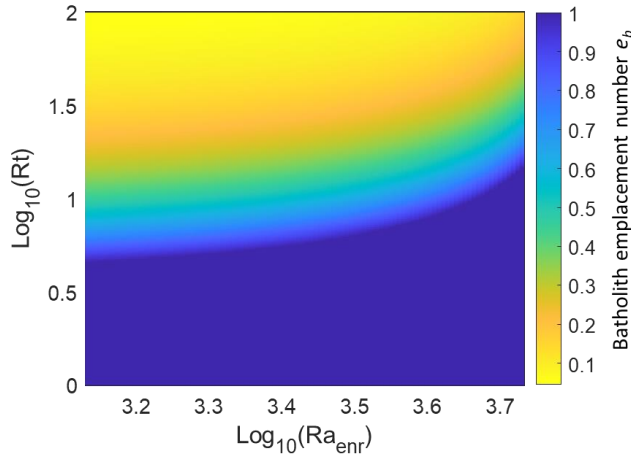
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$$910 \quad e_b \equiv \begin{cases} \frac{t_{hor}}{t_{sgr}} & \text{if } t_{sgr} \geq t_{hor} \\ 1 & \text{if } t_{sgr} < t_{hor} \end{cases} \quad (10)$$

911 If $t_{sgr} \geq t_{hor}$ only a fraction of the available melt contributes to emplacement, and this fraction
 912 scales with the ratio of these times, i.e. e_b . If $t_{sgr} < t_{hor}$ melt has enough time to fully segregate out
 913 of the partially molten horizontally flowing layer to form a batholith, and we truncate e_b at 1 to
 914 define the case of batholith emplacement. Estimating these characteristic times using Darcy's law
 915 and convective scaling laws including effective Rayleigh numbers, we arrive at (for details, see
 916 Appendix C):

$$917 \quad e_b = \frac{Ra_f}{Rt} \frac{\varphi^{n\varphi^{-1}}}{ba_0 \left(1 - \frac{eRa_{enr}}{Ra_{eff}} \right) Ra_{eff}^\gamma} \quad (11)$$

918 truncated at 1. The important unknown parameters controlling e_b are the average melt fraction φ in
 919 the partially molten regions, the typical degree of enrichment e , and the non-dimensional thickness
 920 of the upper layer a_0 in which horizontal flow occurs and segregation takes place (see Appendix C).
 921 Inspecting several models complemented by some tests shows that reasonable guesses may be: $\varphi =$
 922 0.1, $e = 0.15$, $a_0 = 0.5$. Fig. 18 shows e_b as a function of Ra_{enr} and Rt in a similar range as the
 923 observed mean batholith thicknesses from the numerical models. A good general agreement is found
 924 when compared to the empirical results shown in Fig. 17a to 17c. A similar derivation of a scaling law
 925 for emplacement or recycling for constant Rt but various bottom heat fluxes, Ra_q , as shown in the
 926 regime diagrams Fig. 17 d to 17e does not work because for small Rayleigh numbers the scaling law
 927 for the convection velocity breaks down, as the Rayleigh numbers are close to the critical Rayleigh
 928 number.



929

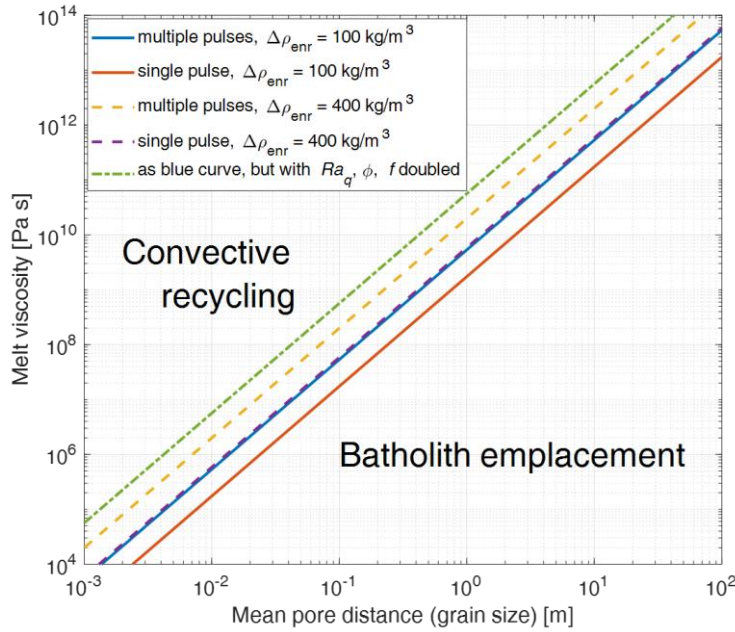
930 **Figure 18:** Regime diagram for the analytical batholith emplacement number e_b (eq. 11 truncated at 1) in the parameter space
 931 $Rt - Ra_{enr}$. Batholith emplacement is favored by low Rt and high Ra_{enr} , because they increase segregation and buoyancy. Used values
 932 are $\varphi = 0.1$, $f = 0.15$, $a_0 = 0.5$. Other assumptions: $Ra_q = 1470$, $Ra_m = 5400$.

933

934 The regime boundaries in Fig. 17a to 17c can be defined by parameter combinations Rt and Ra_{enr} at
 935 which the emplacement parameter e_b is equal to $e_{b05} = 0.5$. Inserting this value into eq. 11 gives the
 936 boundary retention number $Rt_b = Rt(Ra_{enr}, e_b = e_{b05})$. We can rewrite this equation in
 937 dimensional form by solving for the least known parameters: melt viscosity η_f as a function of mean
 938 distances between pores or channels, d

$$939 \quad \eta_f|_{boun} = \frac{\Delta\rho_f g h d^2 \varphi^{n-1}}{\kappa C e_{b05} b a_0 \left(1 - \frac{\Delta\rho_{enr} e}{\frac{\rho_0 \alpha q h}{\lambda} + \Delta\rho_f \varphi - \Delta\rho_{enr} e} \right) Ra_{ef}^\gamma} \quad (12)$$

940 All terms are defined in Table 1 or Appendix C. Assuming typical values for $\Delta\rho_{enr} = 100 \text{ kg/m}^3$ up to
 941 400 kg/m^3 , eq. 12 is plotted in Fig. 19 as a function of grain size or distance d between melt channels.
 942 The resulting lines divide the viscosity - d space into the regimes “batholith emplacement” and
 943 “convective recycling”. While for melt flow on grain scale, low viscosities of 10^5 Pa s or smaller are
 944 needed to generate granitic batholiths, melt channels with distances of meters to 10 meters
 945 (Symington et al, 2014) allow formation of batholiths for viscosities of $< 10^{10} \text{ Pa s}$. The latter
 946 viscosities are in the range of dry granite melts, while the smaller ones require wet melts. Increasing
 947 the total heat input, Ra_q , accelerates both thermal and phase-change-driven convection as well as
 948 melt segregation. This favors batholith emplacement and shifts the boundary upwards to higher melt
 949 viscosities (Fig. 19, dot-dashed curve).



950

951 **Figure 19:** The boundaries between the “batholith emplacement” and the “convective recycling” regimes of Fig. 17a and c
 952 are plotted in a dimensional diagram of melt viscosity versus mean pore distance (or grain size). Further assumptions are
 953 $h = 50 \text{ km}$, $\kappa = 0.9259 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $\eta_0 = 10^{20} \text{ Pa s}$, $C = 100$, $n_\phi = 3$, $b = 0.354$, $\gamma = 0.5$, $\alpha = 2.4 \cdot 10^{-5} \text{ K}^{-1}$,
 954 $\rho_0 = 2700 \text{ kg m}^{-3}$, $\Delta\rho_f = 400 \text{ kg m}^{-3}$. For each curve Rt is constant with values between 4.3 and 141. Solid and dashed
 955 curves are based on an integrated total heat flux of 84 mW/m^2 during a single heat pulse time of 10 Ma. For the green dot-
 956 dashed curve the heat input (i.e. Ra_q) was doubled, as well as the average melt fraction ϕ and enrichment f .

957

958 4 Discussion

959 4.1 Limitations and assumptions

960 One severe simplifying assumption in the models is the constant viscosity of the melt, while for the
 961 solid we used a more realistic visco-plastic rheology. Melt viscosity depends on temperature, SiO_2
 962 content and water. Melts generated from fertile rocks during the early evolution of our models are
 963 expected to be more water-rich and less evolved. Both effects result in low melt viscosities, which
 964 will increase by perhaps three orders of magnitude as the melts become less water-rich and more
 965 evolved during further evolution of the model. At later stages, the depletion of source rock increases
 966 and probably its water content decreases, thus, the melt temperature is higher due to a higher
 967 solidus temperature of the depleted rock. A higher temperature is expected to reduce the viscosity
 968 by perhaps one order of magnitude. Thus, in total we expect that as enrichment increases in the
 969 active parts of the models the melts become stiffer as they evolve. The melt viscosity has only a
 970 minor effect on the dynamics of the mush (partially molten mixture) as long as the melt fraction is
 971 below the disaggregation limit $\sim 20\%$ to $\sim 50\%$ and the two-phase flow is in the low viscosity regime
 972 ($\eta_f \ll \eta_s$). However, melt viscosity strongly controls the segregation velocity. Thus, during early
 973 stages the magmatic system may be characterized by lower retention numbers than during later

974 stages. A time-dependent variation of the effective retention number is a future goal and might also
 975 have some effect on the heterogeneity or stratification of the evolving batholiths.

976 As discussed in Schmeling et al. (2019), once the porosity reaches or exceeds the disaggregation limit,
 977 we use a numerical truncation value for melt viscosity, which mimics highly viscous dry silicic melts.
 978 This is a severe assumption, which neglects short term flow processes within the evolving magma
 979 chambers. Another important neglected factor is the possibility that melt may ascend as dykes and
 980 magma conduits. This limits the applicability of our models to depths > 15 km. First tests with models
 981 in which melt exceeding some critical melt fraction is extracted and transferred to shallower depth
 982 are promising.

983 To our knowledge, all dynamical modelling approaches of magmatic systems assume isotropic
 984 distributions of melts. However, there are indications from magnetotellurics that in the mush zone
 985 the melt may be distributed anisotropically (e.g. beneath the Ceboruco Volcano (Mexico) at mid- to
 986 lower-crustal depths (Hering et al., 2022)). Coupling flow stresses in a mush with the orientation of
 987 melt inclusions to formulate an anisotropic permeability tensor and including it into the Darcy
 988 equations is a challenging topic, which should be addressed in the future.

989 Our initial condition assumes an unmolten crust close to the solidus temperature at the bottom of
 990 the model, and mimics basaltic underplating by a thermal boundary condition. Another endmember
 991 would be an already strongly molten lower crust. This has been studied in detail by Schmeling et
 992 al. (2019). Another possible initial condition could model the crustal intraplate of basaltic magma as
 993 in Annen et al. (2015). However, this would require distinguishing between two types of melts using
 994 two melting curves and compositionally extending our model to three or four components.

995 For all models in this study, single or multiple heat pulses at the bottom have been assumed to mimic
 996 magmatic underplating events. An alternative condition would be prescribing an increased bottom
 997 temperature during such pulses. Tests show several differences in the evolution: in case of sudden
 998 temperature increase, strong plumes (or diapirs) with melt caps develop, while for increased bottom
 999 heat flux, melting is weaker and more spatially distributed. However, for the second and third
 1000 heating events in the case of prescribed heat flow, more fertile material is still present at depth and
 1001 leads to more wide-spread melting for these later pulses. Another difference is that the mean melt
 1002 fraction is much higher for the first heat pulse in the prescribed bottom temperature case and
 1003 decreases to almost zero during the cooling intervals. For the bottom heat flux case, more melt is still
 1004 present during the cooling intervals and melt segregation is continuous leading to enriched silicic
 1005 material being emplaced at shallower depths.

1006 The heat flow boundary condition also allows a better control of the total heat energy input because
 1007 the switching off of the temperature pulse temporarily causes a downward heat loss from the crust
 1008 to the mantle. Thus, the constant temperature boundary condition is unrealistic. In fact, a
 1009 compromise between these two end-member boundary conditions would be a Robin boundary
 1010 condition in which a superposition of bottom temperature and heat flux are prescribed. This could be
 1011 formulated to mimic an initially hot conductive layer of finite thickness below our model whose
 1012 bottom temperature is fixed.

1013 **4.2 Effect of pulse duration and number of pulses**

1014 Our reference models with a single and with three 2 Ma heat pulses resulted in each of the two
 1015 modes of convective recycling and batholith emplacement, respectively. It is interesting to test the
 1016 effect of more than three pulses, i.e. $N > 3$, and the role of the ratio of pulse duration to the
 1017 intervals in between, defined as a . Keeping the total heat input and the total time window of 10 Ma,
 1018 we ran a few extra models with $N = 5, a = 1$ and with $N = 3$ and a varying between 0.22 and 2.
 1019 Variation of a strongly influences the extra heat flux of each of the pulses, while for higher frequency
 1020 the heat input variations are dampened. Evaluating the models with respect to the two modes of
 1021 batholith emplacement and convective recycling shows that shorter but more intense pulses ($a < 1$)
 1022 favor segregation and thus batholith formation, while longer ($a > 1$, corresponding to shorter gaps
 1023 between the pulses) or higher frequency pulses ($N > 3$) favor convective recycling.

1024 **4.3 Comparison to magmatic systems in continental crust: Altiplano-Puna and Tibetan Plateaus**

1025 The models were inspired by the Altiplano-Puna Plateau but the results have implications also to the
 1026 Tibetan plateau and other thickened crusts. In this section, we explore these implications keeping in
 1027 mind the limitations of our models. In both cases, the low-velocity seismic zones define the presence
 1028 of melt at depths >15 km (Nelson et al., 1996; Unsworth et al., 2005; Gottsmann et al., 2017; Fialko &
 1029 Pearce, 2012; del Potro et al., 2013). We argue that Altiplano-Puna Plateau and the Tibetan Plateau
 1030 may represent the two modes of behavior – the batholith emplacement and the convection
 1031 recycling, respectively.

1032 **4.3.1 Altiplano-Puna Plateau: batholith emplacement mode**

1033 The batholith emplacement mode is characterized by mush zones with melt-rich caps that could feed
 1034 intense volcanism. We argue that the Altiplano-Puna Plateau could be an example of this mode with
 1035 intense volcanism in the last 10 Ma (Folkes et al., 2011; de Silva & Gosnold, 2007; de Silva et al.,
 1036 2015), including a large region being currently uplifted, pushed by magmas underneath, centred
 1037 around the Uturuncu volcano (Fialko & Pearce, 2012; del Potro et al., 2013).

1038 After 10 Ma our models reach temperatures at 20 km to 25 km depth well in agreement with the
 1039 reference temperature suggested for the Plateau by Gottsmann et al. (2017). Their higher
 1040 temperatures for the shallower plumbing system are not modelled here. It is interesting to compare
 1041 the melt segregation flux of our models with the current andesitic melt flux of $\sim 2 \cdot 10^7 \text{ m}^3/\text{a}$
 1042 modelled beneath the Uturuncu volcano based on uplift and subsidence rates (Gottsmann et
 1043 al., 2017). The averaged (RMS) segregation velocities of our reference models at 10 Ma model time in
 1044 section 3.1 are $\sim 3 \text{ mm/a}$ or 10 mm/a for continuous or pulsed heating events (Fig. 4b), respectively.
 1045 These values have to be multiplied by a factor of ~ 2 if averaging would include only the lower half of
 1046 the model, namely the partially molten source zone. Multiplying the resulting average segregation
 1047 velocity by a representative area of $50 \text{ km} \times 50 \text{ km}$ we arrive at a melt segregation flux of $1.5 \cdot 10^7$ or
 1048 $5 \cdot 10^7 \text{ m}^3/\text{a}$ for the single or multiple pulse models, respectively. These values are in good
 1049 agreement with Gottsmann et al.'s (2017) values indicating that their estimate could also be
 1050 representative of the long-term segregation rates and not only the current melt segregation flux.

1051 While the depth of the minimum low-shear-velocity anomaly beneath the Altiplano-Puna volcanic
 1052 complex is between 10 km and 20 km depth (Ward et al., 2014), our models give the depth of the top
 1053 of the melt zones between 18 km and 33 km for the various heat inputs (c.f. Fig. 13). Shallower
 1054 depths are expected for: a) an increased bottom heat flux during the pulses, b) a weaker crustal
 1055 rheology, or c) additional melt ascent out of the melt-rich caps by dykes, which is not accounted for
 1056 here. As for a), an extrapolation of the depths of the top of the molten zones (Fig. 13) to 10 km depth
 1057 requires 116 mW/m^2 for a single 10 Ma pulse or 190 mW/m^2 for three 2 Ma pulses. As for b),
 1058 choosing other published wet granite or quartz rheologies would shift these depths by 4 km at most.

1059 Van Zalinge et al. (2022) used geochronology data of the Oxayan Formation (Central Andes) to
 1060 constrain the minimum duration of pluton growth of 4.6 Ma and subsequent ignimbritic eruptions.
 1061 They used Rayleigh-Taylor instability characteristics and a simple conduit flow model to arrive at a
 1062 reasonable transport model of a middle crustal mush zone feeding a shallower magma chamber.
 1063 They used 10^5 Pa s for the silicic melt viscosity and a conduit width of order 5 m corresponding to
 1064 large conduit distances. From Fig. 19, such parameters lie well within the "batholith emplacement"
 1065 regime.

1066 **4.3.2 Tibetan Plateau and others: recycling mode**

1067 Unlike the Altiplano-Puna Plateau, magmatism in Tibet for the last 10 Ma is minor and typically
 1068 concentrated to central and northern Tibet (Guo & Wilson, 2019; Wang et al., 2016). Our convective
 1069 recycling mode requires anomalous high mantle heating and subsequently high surface heat flux (e.g.
 1070 Fig. 6a) but does not lead to significant accumulation of eruptible melt at the top of the rising
 1071 plumes. This is because melt extraction is inefficient because of high melt viscosity (requiring highly

1072 silicic dry melts, c.f. Fig. 19), high melt density or small melt fractions, or relatively rapid phase-
 1073 change-driven convective overturns. In this mode, surface volcanism is expected to be weak or
 1074 absent, explaining the absence of widespread volcanism in Tibet despite the widespread presence of
 1075 shallow magmatic mushes. Another possible candidate for this mode of behavior is the low-seismic
 1076 velocity zone beneath central Taiwan which has been interpreted as partial melting and that is
 1077 unrelated to volcanism (Ma et al., 1996).

1078 **5 Conclusion**

1079 Dynamic models of magmatic systems in continental crust heated from below show a diversity of
 1080 magmatic scenarios depending on the temporal heating modes, melt mobility, and density of evolved
 1081 melts and residual rocks. The heating modes have been designed to represent basaltic underplating
 1082 events on time scales of up to 10 Ma. In all models melting occurs in the lower crust, and melt
 1083 segregates and rises by a combination of porous flow and diapiric flow of the partially molten mass.
 1084 This mass represents the rising mushy limb of lower crustal convection cells driven by the heat input
 1085 and phase change related to melting, melt segregation and solidification of an enriched rock
 1086 extracted from a residual rock. The maximum ascent height of the evolving mush columns, with or
 1087 without melt-rich caps, depends on the total heat input and ranges from 33 km to 18 km depth.
 1088 Above these depths, crustal viscosity becomes too high to allow for significant rise by viscous flow
 1089 and further ascent of magma requires dykes or volcanic conduits not included on the models.

1090 We compared the behavior of two temporal heating modes: a) the single heating event lasting 10 Ma
 1091 and b) the three 2 Ma long pulses of anomalous bottom heat flux, distributed over a total time of
 1092 10 Ma. Despite them having the same integrated extra heat input, the total amount of magma and
 1093 enriched, silicic rocks generated for the pulsed model was larger and the resulting batholiths were
 1094 more heterogeneously distributed in space.

1095 Two main modes of model behavior have been found:

1096 a) Batholith emplacement mode, where silicic melts segregate from the rising and horizontally
 1097 flowing upper part of the convection mush zones and form large bodies of evolved granitic rocks. In
 1098 the lower crust a thick layer of depleted, restitic rock forms.

1099 b) Convective recycling mode, where silicic melts do not segregate and accumulate to form
 1100 batholiths. Instead, melt solidifies within the mush and is recycled back into the lower crust by the
 1101 downwelling limbs of the convection cells where it undergoes remelting. Only thin depleted restitic
 1102 layers form in this case.

1103 Which of these two classes is favored depends on several factors: mode (a) is favored by relatively
 1104 high total amplitude of the heat input and multiple versus single heat pulses, high melt mobility (low

1105 melt viscosity, high rock permeability), and relatively low density of the melt and the evolved, silicic
1106 rock. Mode (b) is favored by the less intense, but not too small heat input, less mobile melt and
1107 denser melts or evolved rocks. The distinction between the two modes can be formulated by a non-
1108 dimensional scaling law and depends on the thermal, melt and compositional Rayleigh number and
1109 the retention number, which controls melt mobility. The magmatic system of the Altiplano-Puna
1110 Plateau is a possible example of mode (a) "batholith emplacement", while the Tibetan Plateau and
1111 Central Taiwan are possible examples of mode (b) "convective recycling".

1112

1113 **Data availability**

1114 The data of this paper consist of modeling results generated by the own developed code FDCON
1115 version 22.1a. The Fortran source code, the input and output data of the reference model (Fig. 3a,
1116 Fig. 4, Fig. 5d,e,f), a few visualizing MATLAB routines and an instruction file "readme" are available at
1117 <https://user.uni-frankfurt.de/~schmelin/G-cubed2023/> (for the review process; will be available on
1118 Zenodo after acceptance).

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1134 References

- 1135 Annen, C., Blundy, D., & Sparks, R. S. J., 2006. The genesis of intermediate and silicic magmas in deep
1136 crustal hot zones, *J. Petrol.*, 47, 505 – 539.
- 1137 Annen, C., Blundy, J. D., Leuthold, J., & Parks, R. S., 2015. Construction and evolution of igneous
1138 bodies: Towards an integrated perspective of crustal magmatism, *Lithos*, 230, 206 – 221,
1139 <http://dx.doi.org/10.1016/j.lithos.2015.05.008>.
- 1140 Arnold, J., Jacoby, W. R., Schmeling, H., & Schott, B., 2001. Continental collision and the dynamic and
1141 thermal evolution of the Variscan orogenic crustal root - numerical models, *J. Geodynamics*, 31, 273 -
1142 291.
- 1143 Babeyko, A. Y., Sobolev, S. V., Trumbull, R. B., Oncken, O., & Lavier, L., 2002. Numerical models of
1144 crustal scale convection and partial melting beneath the Altiplano-Puna plateau, *Earth Planet. Sci.*
1145 *Lett.*, 199, 373 – 388.
- 1146 Bachmann, O. & Huber, C., 2016. Silicic magma reservoirs in the earth's crust, *Am. Mineral.*, 101,
1147 2377 – 2404.
- 1148 Bittner, D. & Schmeling, H., 1995. Numerical modelling of melting processes and induced diapirism in
1149 the lower crust, *Geophys. J. Int.*, 123, 59 – 70, doi.org/10.1111/j.1365-246X.1995.tb06661.x.
- 1150 Blundy, J. D. & Annen, C. J., 2016. Crustal magmatic systems from the perspective of heat transfer,
1151 *Elements*, 12, 115 – 120, DOI: 10.2113/gselements.12.2.115.
- 1152 Cashman, K. V., Sparks, R. S. J., & Blundy, J. D., 2017. Vertically extensive and unstable magmatic
1153 systems: a unified view of igneous processes, *Science*, 355(6331), eaag3055,
1154 [doi:10.1126/science.aag3055](https://doi.org/10.1126/science.aag3055).
- 1155 Chappell, B. W., White, A. J. R., & Wyborn, D., 1987. The Importance of Residual Source Material
1156 (Restite) in Granite Petrogenesis, *Journal of Petrology*, 28 (6), 1111 – 1138,
1157 <https://doi.org/10.1093/petrology/28.6.1111>.
- 1158 Chappell, B. W. & White, A. J. R., 2001. Two contrasting granite types: 25 years later, *Australian J.*
1159 *Earth. Sci.* 48, 489 – 499.
- 1160 Clemens, J. D., 2005. Melting of the continental crust I: fluid regimes, melting reactions and source-
1161 rock fertility, in *Evolution and Differentiations of the Continental Crust*, pp. 297–331, eds Brown, M.
1162 & Rushmer, T., Cambridge University Press.
- 1163 Clemens, D. J. & Vielzeuf, D., 1987. Constraints on melting and magma production in the crust, *Earth*
1164 *Planet. Sci. Lett.*, 86, 287 – 306.

- 1165 Comeau, M. J., Unsworth, M. J., Ticona, F., & Sunagua, M., 2015. Magnetotelluric images of magma
1166 distribution beneath Volcan Uturuncu, Bolivia: implications for magma dynamics, *Geology*, 43, 243 –
1167 246.
- 1168 Cruden, A. R., Koyi, H. & Schmeling, H., 1995. Diapiric basal entrainment of mafic into felsic magma,
1169 *Earth Planet. Sci. Lett.*, 131, 321 - 340.
- 1170 de Silva, S. L. & Gosnold, W. D., 2007. Episodic construction of batholiths: Insights from the
1171 spatiotemporal development of an ignimbrite flare-up, *J. Volcan. Geotherm. Res.*, 167, 320 – 335,
1172 doi:10.1016/j.jvolgeores.2007.07.015.
- 1173 de Silva, S. L., Riggs, N. R., & Barth, A. P., 2015. Quickening the pulse: fractal tempos in continental
1174 arc magmatism, *Elements*, 11, 113 – 118.
- 1175 del Potro, R., Diez, M., Blundy, J., & Camacho, A. G., 2013. Diapiric ascent of silicic magma beneath
1176 the Bolivian Altiplano, *Geophys. Res. Lett.*, 40, 2044 – 2048, doi:10.1002/grl.50493.
- 1177 Fialko, Y. & Pearce, J., 2012. Sombrero uplift above the Altiplano-Puna Magma Body: Evidence of a
1178 ballooning mid-crustal diapir, *Science*, 338, 12 October 2012, 250 - 252.
- 1179 Folkes, C. B., de Silva, S. L., Schmitt, A. K., & Cas, R. A. F., 2011. A reconnaissance of U-Pb zircon ages
1180 in the Cerro Galán system, NW Argentina: Prolonged magma residence, crystal recycling, and crustal
1181 assimilation, *J. Volcan. Geotherm. Res.*, 206, 136 - 147, doi:10.1016/j.jvolgeores.2011.06.001.
- 1182 Frost, B. R., Barnes, C. G., Collins, W. J., Arculus, R. J., Ellis, D. J., & Frost, C. D., 2001. A geochemical
1183 classification for granitic rocks, *J. Petrol.*, 42, 2033 – 2048.
- 1184 Gottsmann, J., Blundy, J., Henderson, S., Pritchard, M. E., & Sparks, R. S. J., 2017. Thermomechanical
1185 modeling of the Altiplano-Puna deformation anomaly: Multiparameter insights into magma mush
1186 reorganization, *Geosphere*, 13 (4), 1042 – 1065, doi:10.1130/GES01420.1.
- 1187 Guo, Z. & Wilson, M., 2019. Late Oligocene–early Miocene transformation of postcollisional
1188 magmatism in Tibet, *Geology*, 47 (8), 776–780, <https://doi.org/10.1130/G46147.1>.
- 1189 Hasalová, P., Weinberg, R., Ward, L., & Fanning, C. M., 2013. Diatexite Deformation and Magma
1190 Extraction on Kangaroo Island, South Australia, *Geol. Soc. Am. Bull.*, 125, 1282 – 1300,
1191 DOI:10.1130/B30781.1.
- 1192 Hering, P., González-Castillo, L., Castro, C., Junge, A., Brown, C., Márquez-Ramírez, V. H., Pinzón
1193 López, J. I., & Gutiérrez, Q. J., 2022. Tectonic controls on magmatic systems: Evidence from a three-
1194 dimensional anisotropic electrical resistivity model of Ceboruco Volcano, *J. Volcan. Geoth. Res.*, 428,
1195 107382, <https://doi.org/10.1016/j.jvolgeores.2021.107382>.

- 1196 Horton, F. & Leech, M. L., 2013. Age and origin of granites in the Karakoram shear zone and Greater
1197 Himalaya Sequence, NW India, *Lithosphere*, 5 (3), 300 – 320, doi.org/10.1130/L213.1.
- 1198 Jagoutz, O. & Klein, B., 2018. On the importance of crystallization – differentiation for the generation
1199 of SO_2 -rich melts and the compositional build-up of arc (and continental) crust, *Am. J. Sci.*, 318, 29 –
1200 63, DOI 10.2475/01.2018.03.
- 1201 Katz, R. F., Knepley, M. G., Smith, N., Spiegelman, M., & Coon, E. T., 2007. Numerical simulation of
1202 geodynamic processes with the Portable Extensible Toolkit for Scientific Computation, *Phys. Earth
1203 Planet. Int.*, 163, 52 – 68, doi.org/10.1016/j.pepi.2007.04.016.
- 1204 Keller, T. & Suckale, J., 2019. A continuum model of multi-phase reactive transport in igneous
1205 systems, *Geophys. J. Int.*, 219, 185 – 222, doi: 10.1093/gji/ggz287.
- 1206 Kern, J. M., de Silva, S. L., Schmitt, A. K., Kaiser, J. F., Iriarte, A. R., & Economos, R., 2016.
1207 Geochronological imaging of an episodically constructed subvolcanic batholith: U-Pb in zircon
1208 chronochemistry of the Altiplano-Puna Volcanic Complex of the Central Andes, *Geosphere*, 12 (4),
1209 1054 – 1077, doi:10.1130/GES01258.1.
- 1210 Kirby, S. H. & Kronenberg, A. K., 1987. Rheology of the lithosphere: selected topics, *Rev. Geophys.*,
1211 25(6), 1219 – 1244, doi:10.1029/RG025i006p01219.
- 1212 Lederer, G. W., Cottle, J. M., Jessup, M. J., Langille, J. M., & Ahmad, T., 2013. Time-scales of partial
1213 melting in the Himalayan middle crust: Insight from the Leo Pargil dome, northwest India,
1214 *Contributions to Mineralogy and Petrology*, 166, 1415 – 1441, doi:10.1007/s00410-013-0935-9.
- 1215 LePape, F., Jones, A. G., Vozar, J., & Wenbo, W., 2012. Penetration of crustal melt beyond the Kunlun
1216 Fault into northern Tibet, *Nature Geoscience*, 5, May 2012, 330 - 335, DOI: 10.1038/NGEO1449.
- 1217 Leuthold, J., Müntener, O., Baumgartner, L. P., Putlitz, B., Ovtcharove, M., & Schaltegger, U., 2012.
1218 Time resolved construction of a bimodal laccolith (Torres del Paine, Patagonia), *Earth Planet. Sci.
1219 Lett.*, 325 – 326, 85 – 92, Doi: 10.1016/j.epsl.2012.01.032.
- 1220 Li, J., Ding, X., & Liu, J., 2022. The role of fluids in melting the continental crust and generating
1221 granitoids: An overview, *Geosciences*, 12, 285, https://doi.org/10.3390/geosciences12080285.
- 1222 Louis-Napoléon, A., Bonometti, T., Gerbault, M., Martin, R., & Vanderhaeghe, O., 2022. Models of
1223 convection and segregation in heterogeneous partially molten crustal roots with a VOF method – I:
1224 flow regimes, *Geophys. J. Int.*, 229, 2047 – 2080, doi.org/10.1093/gji/ggab510.
- 1225 Ma, K.-F., Wang, J.-H., & Zhao, D., 1996. Three dimensional seismic velocity structure of the crust and
1226 uppermost mantle beneath Taiwan, *J. Phys. Earth*, 44, 85 – 105.

- 1227 McKenzie, D., 1984. The generation and compaction of partially molten rock, *J. Petrol.*, 25(3), 713 –
1228 765.
- 1229 Moyen, J.-F., Janoušek, V., Laurent, O., Bachmann, O., Jacob, J.-B., Farina, P., Fiannacca, P., & Villaros,
1230 A., 2021. Crustal melting vs. fractionation of basaltic magmas: Part 1, The bipolar disorder of granite
1231 petrogenetic models, *Lithos*, 402, 106291, doi.org/10.1016/j.lithos.2021.106291.
- 1232 Nelson, K. D., Zhao, W., Brown, L. D., Kuo, J., Che, J., Liu, X., Klemperer, S.L., Makovsky, Y., Meissner,
1233 R., Mechie, J., Kind, R., Wenzel, F., Ni, J., Nabelek, J., Leshou, C., Tan, H., Wei, W., Jones, A. G., Booker,
1234 J., Unsworth, M., Kidd, W. S. F., Hauck, M., Alsdorf, D., Ross, A., Cogan, M., Wu, C., Sandvol, E., &
1235 Edwards, M., 1996. Partially Molten Middle Crust Beneath Southern Tibet: Synthesis of Project
1236 INDEPTH Results, *Science*, 274, 1684 – 1688.
- 1237 Othman, D. B., Polvé, M., & Allègre, C. J., 1984. Nd—Sr isotopic composition of granulites and
1238 constraints on the evolution of the lower continental crust, *Nature*, 307, 510 – 515.
- 1239 Perkins, J. P., Ward, K. M., de Silva, S. L., Zandt, G., Beck, S. L., & Finnegan, N. J., 2016. Surface uplift
1240 in the central Andes driven by growth of the Altiplano Puna magma body, *Nat. Commun.*, 7, 13185,
1241 doi:10.1038/ncomms13185.
- 1242 Puxeddu, M., 1984. Structure and late Cenozoic evolution of the upper lithosphere in Southwest
1243 Tuscany (Italy), *Tectonophysics*, 101, (3–4), 357–382, [https://doi.org/10.1016/0040-1951\(84\)90121-5](https://doi.org/10.1016/0040-1951(84)90121-5).
- 1244 Read, H. H., 1948. Granites and granites, in: Gilluly, J. (Ed.), *Origin of Granite*, Geological Society of
1245 America Memoirs, 28, 1 – 19.
- 1246 Ribe, N. M., 1985. The generation and composition of partial melts in the earth's mantle, *Earth*
1247 *planet. Sci. Lett.*, 73, 361 – 376.
- 1248 Riel, N., Mercier, J., & Weinberg, R., 2016. Convection in a partially molten metasedimentary crust?
1249 Insights from the El Oro complex (Ecuador), *Geology*, 44, 31 – 34, <https://doi.org/10.1130/G37208.1>.
- 1250 Rivalta, E., Taisne, B., Buger, A. P., & Katz, R. F., 2015. A review of mechanical models of dike
1251 propagation: schools of thought, results and future directions, *Tectonophysics*, 638, 1–42.
- 1252 Rubatto, D., Chakraborty, S., & Dasgupta, S., 2013. Timescales of crustal melting in the Higher
1253 Himalayan Crystallines (Sikkim, Eastern Himalaya) inferred from trace element-constrained monazite
1254 and zircon chronology, *Contrib. Mineral. Petrol.*, 165, 349 – 372. DOI 10.1007/s00410-012-0812-y.
- 1255 Rubin, A. M., 1993. Tensile fracture of rock at high confining pressure Implication for dike
1256 propagation, *J. Geophys. Res.*, 98, 15 919 – 15 935.

- 1257 Rummel, L., Kaus, B. J. P., Baumann, T. S., White, R. W., & Riel, N., 2020. Insights into the
1258 Compositional Evolution of Crustal Magmatic Systems from Coupled Petrological-Geodynamical
1259 Models, *J. Petrol.*, 61, No. 2: egaa029, doi: 10.1093/petrology/egaa02.
- 1260 Schmelting, H., Marquart, G., Weinberg, R., & Wallner, H., 2019. Modelling melting and melt
1261 segregation by two-phase flow: new insights into the dynamics of magmatic systems in the
1262 continental crust, *Geophys. J. Int.*, 217, 422 – 450, doi: 10.1093/gji/ggz029.
- 1263 Schön, J. H., 2015. *Physical properties of rocks: Fundamentals and principles of petrophysics*, Elsevier,
1264 Amsterdam, 512 pp, ISBN 0081004230, 9780081004234.
- 1265 Seporian, G., Rust, A. C., & Sparks, R. S. J., 2018. The gravitational stability of lenses in magma
1266 mushes: confined Rayleigh-Taylor instabilities, *J. Geophys. Res.*, 123, 3593 – 3607,
1267 doi.org/10.1029/2018JB015523.
- 1268 Spang, A., Baumann, T. S., & Kaus, B. J. P., 2021. A Multiphysics Approach to Constrain the Dynamics
1269 of the Altiplano-Puna Magmatic System, *J. Geophys. Res.*, 126, e2021JB021725,
1270 <https://doi.org/10.1029/2021JB021725>.
- 1271 Sparks, R. S. J., Annen, C., Blundy, J. D., Cashman, K. V., Rust, A. C., & Jackson, M. D., 2019. Formation
1272 and dynamics of magma reservoirs, *Phil. Trans. R. Soc. A*, 377: 20180019,
1273 <http://dx.doi.org/10.1098/rsta.2018.0019>.
- 1274 Springer, M., 1999. Interpretation of heat-flow density in the Central Andes, *Tectonophysics*, 306,
1275 377 – 395.
- 1276 Springer, M. & Förster, A., 1998. Heat-flow density across the Central Andean subduction zone,
1277 *Tectonophysics*, 291, 123 – 139.
- 1278 Symington, N. J., Weinberg, R. F., Hasalová, P., Wolfram, L. C., Raveggi, M., & Armstrong, R. A., 2014.
1279 Multiple intrusions and remelting-remobilization events in a magmatic arc: The St. Peter Suite, South
1280 Australia, *Geol. Soc. Am. Bull.*, 126, 1200 – 1218, doi: 10.1130/B30975.1.
- 1281 Thybo, H. & Artemieva, I. M., 2013. Moho and magmatic underplating in continental lithosphere,
1282 *Tectonophysics*, 609, 605 – 619.
- 1283 Unsworth, M., Jones, A., & Wei, W., 2005. Crustal rheology of the Himalaya and Southern Tibet
1284 inferred from magnetotelluric data, *Nature*, 438, 78 – 81, <https://doi.org/10.1038/nature04154>.
- 1285 Vanderhaeghe, O., Kruckenberg, S. C., Gerbault, M., Martin, L., Duchêne, S., & Deloule, E., 2018.
1286 Crustal-scale convection and diapiric upwelling of a partially molten orogenic root (Naxos dome,
1287 Greece), *Tectonophysics*, 746, 459 – 469. <https://doi.org/10.1016/j.tecto.2018.03.007>.

- 1288 Van Zalinge, M. E., Mark, D. F., Sparks, R. S. J., Tremblay, M. M., Keller, C. B., Cooper, F. J., & Rust, A.,
 1289 2022. Timescales for pluton growth, magma-chamber formation and super-eruptions, *Nature*, 608,
 1290 87 – 92.
- 1291 Viskupic, K., Hodges, K. V., & Bowring, S. A., 2005. Timescales of melt generation and the thermal
 1292 evolution of the Himalayan metamorphic core, Everest region, eastern Nepal, *Contrib. Mineral.*
 1293 *Petrol.*, 149, 1 – 21, doi.org/10.1007/s00410-004-0628-5.
- 1294 Ward, K. M., Zandt, G., Beck, S. L., Christensen, D. H., & McFarlin, H., 2014. Seismic imaging of the
 1295 magmatic underpinnings beneath the Altiplano-Puna volcanic complex from the joint inversion of
 1296 surface wave dispersion and receiver functions, *Earth Planet. Sci. Lett.*, 404, 43 – 53,
 1297 <https://doi.org/10.1016/j.epsl.2014.07.022>.
- 1298 Wang, Q., Hawkesworth, C., Wyman, D., Chung, S. L., Wu, F. Y., Li, X. H., Li, Z. X., Gou, G. N., Zhang, X.
 1299 Z., Tang, G. J., Dan, W., Ma, L., Dong, Y. H., 2016. Pliocene-Quaternary crustal melting in central and
 1300 northern Tibet and insights into crustal flow, *Nat Commun* 7, 11888,
 1301 <https://doi.org/10.1038/ncomms11888>.
- 1302 Weinberg, R. F., 1997. Diapir-driven crustal convection: decompression melting, renewal of the
 1303 magma source and the origin of nested plutons, *Tectonophysics*, 271, 217 – 229,
 1304 doi.org/10.1016/S0040-1951(96)00269-7.
- 1305 Weinberg, R., 2016. Himalayan leucogranites and migmatites: nature, timing and duration of
 1306 anatexis, *J. Metam. Geol.*, 34, 821 – 843, doi:10.1111/jmg.12204.
- 1307 Weinberg, R. F. & Hasalova, P., 2015. Water-flux melting of continental crust: A review, *Lithos*, 212 –
 1308 215, 158 – 188, <http://dx.doi.org/10.1016/j.lithos.2014.08.0>.
- 1309 Weinberg, R. F., Vernon, R. H., & Schmeling, H., 2021. Processes in mushes and their role in the
 1310 differentiation of granitic rocks, *Earth Sci. Rev.*, 220, 103665,
 1311 <https://doi.org/10.1016/j.earscirev.2021.103665>
- 1312 Wells, P. R. A., 1980. Thermal models for the magmatic accretion and subsequent metamorphism of
 1313 continental crust, *Earth planet. Sci. Lett.*, 46, 253 – 265.
- 1314 Wells, P. R. A., 1981. Accretion of continental crust: thermal and geochemical consequences, *Philos.*
 1315 *Trans. R. Soc. London, Ser. A*, 301, 347 – 357.
- 1316 Wong, Y.-Q. & Keller, T., 2022. A unified numerical model for two-phase porous, mush and
 1317 suspension flow dynamics in magmatic systems, *Geophys. J. Int.*, ggac481,
 1318 <https://doi.org/10.1093/gji/ggac481>.

1319 Wyllie, P. J., 1984. Constraints imposed by experimental petrology on possible and impossible
1320 magma sources and products, *Phil. Trans. R. Soc. Lond. A*310, 439 – 456.

1321 Zandt, G., Velasco, A. A., & Beck, S. L., 1994. Composition and thickness of the southern Altiplano
1322 crust, Bolivia, *Geology*, 22, 1003 – 1006.

1323

1324

1325 Appendix A: The governing equations

1326 In this Appendix the governing equations of our two-phase flow approach are listed. They are taken
1327 from Schmeling et al. (2019) where more detailed discussion can be found. For the definition of
1328 quantities see Table 1.

1329 The equations of conservation of mass, momentum and energy are given as

$$1330 \quad \rho_f \left(\frac{\partial \varphi}{\partial t} + \vec{\nabla} \cdot (\varphi \vec{v}_f) \right) = \Gamma, \quad (\text{A1})$$

$$1331 \quad \rho_s \left(\frac{\partial (1-\varphi)}{\partial t} + \vec{\nabla} \cdot ((1-\varphi) \vec{v}_s) \right) = -\Gamma, \quad (\text{A2})$$

$$1332 \quad \vec{v}_f - \vec{v}_s = -\frac{k}{\eta_f \varphi} (\vec{\nabla} P - \rho_f \vec{g}), \quad (\text{A3})$$

$$1333 \quad \rho \vec{g} - \vec{\nabla} P + \frac{\partial \tau_{ij}}{\partial x_j} = 0, \quad (\text{A4})$$

$$1334 \quad \rho c_P \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T \right) = \vec{\nabla} \cdot (\lambda \vec{\nabla} T) + \rho H - L\Gamma, \quad (\text{A5})$$

1335 with the viscous stress tensor and averaged flow velocity, respectively

$$1336 \quad \tau_{ij} = \eta_s \left(\frac{\partial v_{si}}{\partial x_j} + \frac{\partial v_{sj}}{\partial x_i} \right) + \left(\eta_b - \frac{2}{3} \eta_s \right) \delta_{ij} \vec{\nabla} \cdot \vec{v}_s, \quad (\text{A6})$$

$$1337 \quad \vec{v} = \varphi \cdot \vec{v}_f + (1 - \varphi) \cdot \vec{v}_s. \quad (\text{A7})$$

1338 The visco-plastic solid viscosity is given as

$$1339 \quad \frac{1}{\eta_s} = \frac{1}{\eta'_\varphi} \left(\frac{1}{\eta_{MC}} + \frac{1}{\eta_{duc}} \right) \quad (\text{A8})$$

$$1340 \quad \eta_{MC} = \frac{\tau_{max}}{2\dot{\epsilon}_{II}}, \quad \eta_{duc} = A_{duc} \cdot \exp \left(\frac{E+VP_{Lith}}{RT_{abs}} \right) \cdot \dot{\epsilon}_{II}^{\frac{1}{n}-1}, \quad (\text{A9})$$

$$1341 \quad \tau_{max} = A \cdot \tilde{z} + B \quad (\text{A10})$$

1342 and the non-dimensional porosity dependent parts of the shear and bulk viscosities in eq. (A8) are

1343 defined as

$$1344 \quad \eta'_\varphi = \left(1 - \frac{\varphi}{c_1} \right)^{k_1}, \quad \eta'_b = c_2 \frac{(c_1 - \varphi)^{k_2}}{\varphi} \quad \text{for } \varphi < c_1, \quad (\text{A11})$$

1345 respectively. For values used in the approximation (A11) and the finite viscosity stabilization
 1346 formulation used at porosities above the disaggregation porosity, $c_1 = 0.505$, see Schmeling et al.
 1347 (2019).

1348 The actual parameter values (Table 1) are taken from Table 2 of Schmeling et al. (2019), which
 1349 contains a misprint: the activation volume should be $10^{-5} \text{ m}^3 \text{ Mole}^{-1}$. We note that even when taking
 1350 a zero activation volume, the results in Schmeling et al. (2019) are almost identical to those that they
 1351 obtained. Another misprint in Schmeling et al. (2019) concerns eq. (14) in that paper, which should
 1352 be replaced by eq. (A9).

1353 Non-dimensionalizing the equations, eliminating the pressure by combining eq. A3 and A4 and
 1354 replacing the solid velocities by the stream function and irrotational velocity potential, the equations
 1355 are solved by the code FDCON (see Schmeling et al. (2019) and references therein) on a grid with
 1356 resolution of 101×101 for the momentum equation of the solid phase, while 201×201 is assumed
 1357 for the melt momentum and mass equations, the composition equations and the energy equation.
 1358 The solid velocities and viscosities are taken as arithmetic, geometric or harmonic means at the sides
 1359 or cell interiors to go into the FD-equations of the temperature and melt-transport equations.

1360

1361 **Appendix B: The melting law**

1362 Numerically, melting is calculated in the following way: During one time step the solid and melt
 1363 compositions are advected separately using an advection equation of the type

$$1364 \quad \frac{\partial c_{f,s}}{\partial t} + v_{f,s} \cdot \nabla c_{f,s} = 0, \quad (\text{B1})$$

1365 where $v_{f,s}$ is the melt and solid velocity, respectively (eq. A11 in Schmeling et al., 2019). During the
 1366 same time step of segregating the melt, a new φ -field is calculated by solving the melt momentum
 1367 equation, and after advection on different paths the solid and melt compositions c_s, c_f as well as the
 1368 temperature T are obtained. At this stage of calculation latent heat is not yet included in the
 1369 temperature field. With these variables the mean composition at any point is given by eq. 7. From T
 1370 and c the melting increment to reach an equilibrium melt fraction (eq. 6) is calculated by iteration.

1371 We start the iteration $i = 1$ at any position with the present melt fraction $\varphi_i = \varphi$ and temperature
 1372 $T_i = T$ and determine φ_{equ_i} of the i -th iteration by the lever rule

$$1373 \quad \varphi_{equ_i} = \frac{c - c_s(T_i)}{c_f(T_i) - c_s(T_i)}. \quad (B2)$$

1374 Note that c remains fixed during the iteration and $c_f(T_i), c_s(T_i)$ are the c_f and c_s – values on the
 1375 liquidus and solidus curves, respectively, i.e. they are different from the previously advected values.
 1376 The melting increment needed to reach the equilibrium value would be $\delta\varphi_i = \varphi_{equ} - \varphi_i$ if no latent
 1377 heat would be released or consumed. Thus, to account for this, we underrelax the increment by
 1378 *delay* and get the melt fraction of the next iteration

$$1379 \quad \varphi_{i+1} = \varphi_i + \delta\varphi_i (1 - \text{delay}) = \varphi_i + \delta\varphi_i' \quad (B3)$$

1380 This increment $\delta\varphi_i'$ contributes cumulatively to the melt generation rate

$$1381 \quad \Gamma_{i+1} = \Gamma_i + \delta\varphi_i'/dt, \quad (B4)$$

1382 where the starting melt generation rate was set to $\Gamma_1 = 0$. The melting increment is also associated
 1383 with a simultaneous local change of temperature. The temperature of the next iteration is

$$1384 \quad T_{i+1} = T_i - L\delta\varphi_i'. \quad (B5)$$

1385 With this new temperature a new equilibrium melt fraction can be determined by setting $i = i + 1$
 1386 and jumping to eq. B2. The iteration converges if the change in melt generation rate $\delta\varphi_i'/dt$ falls
 1387 below a small threshold (or more precisely, the last n increments fall below a threshold). Note that
 1388 the iteration does not change c , only c_f, c_s . Near the compositions $c = 0$ or $c = 1$ the T -dependence
 1389 of melt fraction is very strong and requires a strong damping with $\text{delay} > 0.9$. Thus, an automatic
 1390 c – dependent *delay* is used with the formula

$$1391 \quad \text{delay} = (1 - [1 - (\cos(2\pi c) + 1)/2]^{pow}) \cdot (1 - \text{delay}0) + \text{delay}0 \quad (B6)$$

1392 Good results are achieved with $\text{delay}0 = 0.6$ and $pow = 2$. As *delay* can approach 1 very closely its
 1393 maximum value is truncated at 0.995.

1394 To mimic a binary eutectic system with compositions A and B the composition $c = 0$ is assumed to
 1395 represent the eutectic composition and the composition $c = 1$ represents composition B. The solidus
 1396 curve is assumed to be almost horizontal at the eutectic temperature with a numerically necessary
 1397 non-zero slope and a sharp kink near $c = 1$ rising to the melting temperature of pure composition B.
 1398 There it merges with the liquidus curve. Applying the lever rule as in the solid solution case above
 1399 gives a sub-vertical melt fraction curve near the solidus temperature (due to the sub-horizontal
 1400 solidus curve) followed by a moderate slope depending on the shape of the liquidus temperature. In

1401 this approach the eutectic melt fraction jump is approximated by a melting curve with a steep but
 1402 finite slope. This numerically strong temperature dependence requires strong damping with
 1403 *delay* close to 1.

1404 **Appendix C: Derivation of the analytical scaling law for batholith emplacement vs. convective** 1405 **recycling**

1406 We now derive an analytical scaling law for a boundary in the parameter space in section 3.5 by
 1407 defining a batholith emplacement parameter e_b . Emplacement of enriched silicic material takes place
 1408 predominantly within the upper horizontal limb of the convective flow by segregation, i.e. vertical
 1409 porous flow of melt with a characteristic segregation velocity v_{sgr} out of solid material with
 1410 horizontal viscous flow with a characteristic velocity u . This horizontal flow region represents the
 1411 upper partially molten layer of convection, whose thickness can be denoted by $(a \cdot h)$, where h is the
 1412 total thickness of the convecting layer, and a is the non-dimensional thickness of this upper layer. For
 1413 sufficiently high Rayleigh number u scales with the Rayleigh number for bottom heat flux (Turcotte et
 1414 al., 2001)

$$1415 \quad u = \frac{\kappa}{h} b Ra_q^\gamma \quad (C1)$$

1416 where κ is the thermal diffusivity, b is a non-dimensional number (= 0.632 for constant viscosity
 1417 convection with constant bottom heat flux), and γ is a power exponent (= 1/2). In our case
 1418 convection is driven by both thermal and melt buoyancy but is hampered by lighter enrichment
 1419 material formed due to solidification. We modify the above scaling law by using an effective Rayleigh
 1420 number

$$1421 \quad Ra_{eff} = Ra_q + \varphi Ra_f - e Ra_{enr} \quad (C2)$$

1422 where the Rayleigh numbers are defined in eq. 4, φ is an average melt fraction in the partially molten
 1423 region, and e gives the typical degree of enrichment within the upper part of the convection cell. The
 1424 segregation velocity can be obtained from Darcy's law as

$$1425 \quad v_{sgr} = \frac{d^2 \varphi^{n-1} \Delta \rho_f g}{C \eta_f} \quad (C3)$$

1426 where η_f is the melt viscosity, C is a geometric constant of the order of 100 depending on the actual
 1427 pore geometry, d is the mean distance between the pores (often equal to the grain size), n_φ is the
 1428 power exponent of the permeability – porosity relation, $\Delta \rho_f = \rho_0 - \rho_{0f}$ is the difference between
 1429 melt and reference density, and g is the gravitational acceleration. We define the degree of batholith
 1430 emplacement by the ratio of the time t_{hor} needed for a partially molten material volume to travel

1431 the horizontal distance within the upper part of the convection cell which scales with h , and the time
 1432 t_{sgr} needed for the melt to segregate vertically across the distance ($a \cdot h$):

$$1433 \quad e_b \equiv \begin{cases} \frac{t_{hor}}{t_{sgr}} = \frac{v_{sgr}}{u a} & \text{if } t_{sgr} \geq t_{hor} \\ 1 & \text{if } t_{sgr} < t_{hor} \end{cases} \quad (C4)$$

1434 If $t_{sgr} \geq t_{hor}$ only a fraction of the available melt contributes to emplacement, and this fraction
 1435 scales with the ratio of these times, i.e. e_b . If $t_{sgr} < t_{hor}$ melt has enough time to fully segregate out
 1436 of the partially molten horizontally flowing layer to form a batholith emplacement layer, and we
 1437 truncate e_b at 1 to define the case of full emplacement. The non-dimensional layer thickness a will
 1438 depend on how much melt solidifies at its top, i.e. it depends on the buoyancy of the enriched
 1439 solidified material. We describe this behavior by

$$1440 \quad a = a_0 \left(1 - \frac{eRa_{enr}}{Ra_{eff}} \right) \quad (C5)$$

1441 with a_0 as thickness of this layer in the absence of solidification. The idea behind the definition (C5)
 1442 is: Thinning of this layer due to batholith emplacement near its top scales with the contrast of
 1443 buoyant density of the enriched solid material $\Delta\rho_{enr}$ times the typical enrichment e relative to the
 1444 total buoyant density driving convection. This ratio can be written in terms of Rayleigh numbers, i.e.
 1445 $\frac{eRa_{enr}}{Ra_{eff}}$, and the effective layer a can be defined by eq. C5. We now use eq. C1, replace the Rayleigh
 1446 number by the effective Rayleigh number of eq. C2, and insert the two velocities and a from eq. C5
 1447 into eq. C4 to obtain, after some rearranging

$$1448 \quad e_b = \frac{Ra_f}{Rt} \frac{\varphi^{n-1}}{ba_0 \left(1 - \frac{eRa_{enr}}{Ra_{eff}} \right) Ra_{eff}^\gamma} \quad (C6)$$

1449 truncated at 1.

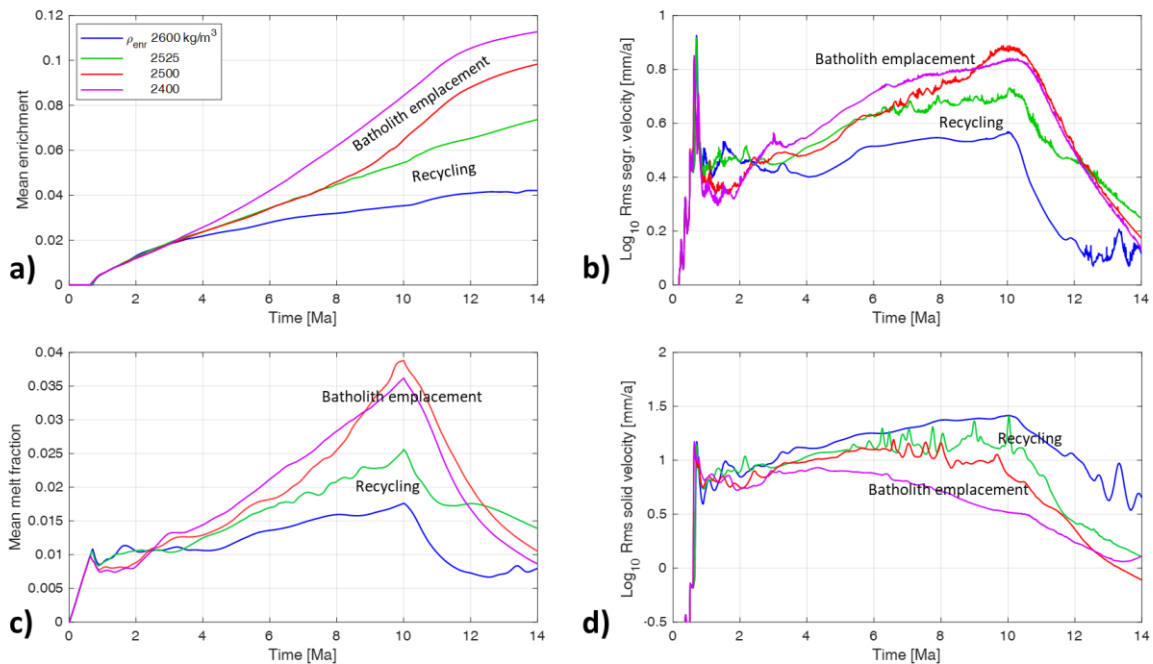
1450 The regime boundaries found in Fig. 17a - c can be described by defining the boundary as those
 1451 parameter combinations Rt , and Ra_{enr} at which the emplacement parameter e_b is equal to $e_{b05} =$
 1452 0.5. Inserting this value into eq. C6 gives the boundary retention number $Rt_b = Rt(Ra_{enr}, e_b =$
 1453 $e_{b05})$. We can rewrite this equation in dimensional form by solving for the least known parameters
 1454 melt viscosity as a function of mean distances between pores or channels

$$1455 \quad \eta_f|_{boun} = \frac{\Delta\rho_f g h a^2 \varphi^{n-1}}{\kappa C e_{b05} b a_0 \left(1 - \frac{\Delta\rho_{enr} e}{\frac{\rho_0 \alpha q h}{\lambda} + \Delta\rho_f \varphi - \Delta\rho_{enr} e} \right) Ra_{eff}^\gamma} \quad (C7)$$

1456

Appendix D: More details on the effect of density of the enriched felsic rock and the physics of convective recycling and batholith emplacement

In section 3.2 models have been shown in which the density of the enriched felsic rock has been systematically varied. Here some more details are discussed. Fig. D1a and D1c show that the models with batholith emplacement (red and pink curves) are characterized by an acceleration of enrichment and melt fraction after about 4 Ma to 6 Ma, while the convective recycling models show a nearly steady increase. Batholith emplacement is accompanied by RMS segregation velocities and solid velocities that are of the same order of magnitude at around 10 Ma (Fig. D1b and D1d). In contrast, the recycling models show significantly larger RMS solid velocities than segregation velocities. The solid velocities of the batholith emplacement models first increase with time as the models heat up. After 4 Ma (pink curve) or 6 Ma (red curve) the RMS solid velocities decrease with time because the active part of the convecting crust progressively becomes thinner beneath the accumulated enriched layer, which does not participate in the flow anymore. The green and blue curves (Fig. D1d), however, increase over the full 10 Ma duration of the heat pulse, because the convecting layer progressively thickens as hot material reaches increasingly shallower regions, but is continuously recycled.



1473

Figure D1: Effect of different densities of enriched material. Temporal evolution of a) mean enrichment, b) RMS segregation velocity, c) mean melt fraction, and d) RMS solid velocity of four selected models covering the bifurcation density of 2525 kg/m³ between recycling and batholith emplacement of the single pulse models.

1477

A similar behavior is observed for the pulsed models (Fig. D2): Starting from the first pulse, the lowest density model (2500 kg/m³, pink curve) departs from the others as enriched silicic material

1479

underplates the upper crust and is not dragged back into the lower crust. Already during the first pulse, the segregation and solid velocities of the pink model are of similar order, and during the later pulses, segregation velocities exceed the solid velocities reinforcing strong batholith emplacement. The 2550 kg/m³ model (red curve) is transitional between batholith emplacement and recycling: until the end of the second pulse, it follows the same trend as the two denser models (blue and green lines in Fig. D2), with recycling. At that time (around 8 Ma) the 2550 kg/m³ model deviates from the other two and batholiths of enriched material form as in the case of 2500 kg/m³ density model.

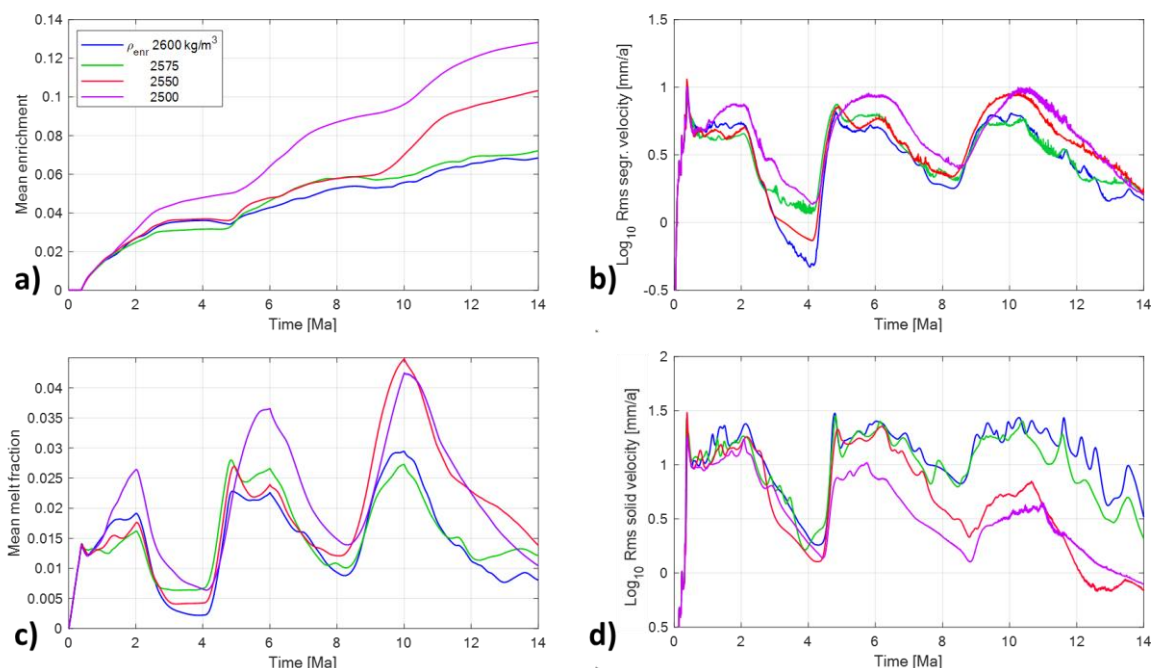
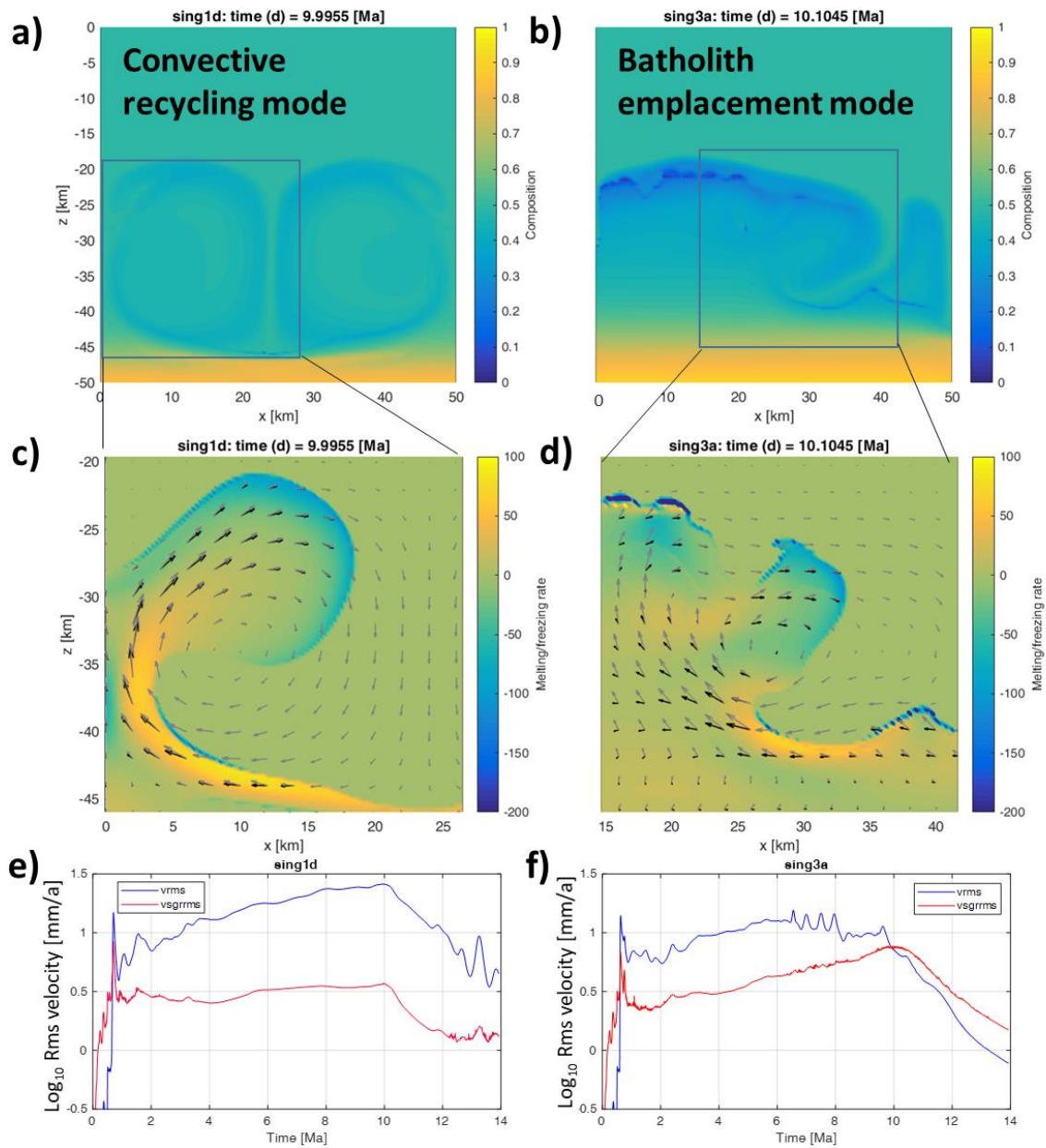


Figure D2: Effect of different densities of enriched rocks. Temporal evolution of a) mean enrichment, b) RMS segregation velocity, c) mean melt fraction, and d) RMS solid velocity of four selected models covering the bifurcation density of 2555 kg/m³ between recycling and batholith emplacement of the multiple pulse models.

The models in Fig. 15a with $\rho_{enr} = 2600$ kg/m³ (model sing1d in Fig. D3) and with $\rho_{enr} = 2500$ kg/m³ (model sing3a) can be used to elucidate the physics responsible for convective recycling and batholith emplacement, respectively. Fig. D3 shows these models at 10 Ma. In the recycling case of model sing1d (Figs. D3a, D3c, D3e), the convection cycle undergoes melting in the lower crust and in the rising limb of the flow, while freezing occurs mostly in the upper horizontal part and early in the downwelling limb. This is typical of phase-change-driven convection: most of the enriched material forming at the freezing front is incorporated into the downwelling flow and this solidified material remelts preferentially because of its lower solidus temperature as it reaches the base of the convection cell. In the batholith emplacement model sing3a (Fig. D3b, D3d, D3f), freezing dominates in the upper part of the rising limb, and the frozen material is displaced horizontally with only some material is being recycled into the downwelling flow (blue middle part in Fig. D3b, d). The reason for freezing in the top part of the rising flow is the enhanced segregation: The fluid has a considerably

1504 faster upwelling velocity compared to the solid (about two times). This can be seen in the divergence
1505 of the black and grey arrows in Fig. D3d and contributes to the differentiation and emplacement of
1506 enriched, buoyant material in the uppermost region of the diapir/plume. Compared to the strong
1507 segregation in Fig. D3b, the segregation velocity in Fig. D3a is slower, only about 20 % of the solid
1508 velocity. In this case, melt does not have enough time to leave its host rock and to freeze before it is
1509 recycled. Note that the local ratio of segregation to solid velocity may be much larger than the RMS
1510 averaged ratio: locally the ratio can reach a factor 3 (e.g. Fig. D3d, depth range 25 km to 30 km),
1511 while RMS averaged ratio is just about unit at 10 Ma. Only during the late stage (10 Ma to 14 Ma) of
1512 this model the RMS segregation velocity continuously exceeds the solid velocity and builds up a well-
1513 developed enriched body (see Fig. 15a, model with 2500 kg/m³). In contrast, even at late stages, the
1514 RMS segregation velocity of the recycling model (Fig. D3e) stays well below the solid velocity and no
1515 enriched body forms (see Fig. 15a, left model with 2600 kg/m³).

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Figure D3: Comparison of models representing the convective recycling mode (left, panels a, c, and e) and the batholith emplacement mode (right, panels b, d, and f). All parameters are the same except the enrichment densities, which correspond to $Ra_{enr} = 1350$ or $\rho_{enr} = 2600 \text{ kg/m}^3$ (left model) and $Ra_{enr} = 2700$ or $\rho_{enr} = 2500 \text{ kg/m}^3$ (right model). For both models, a single heat pulse of 10 Ma duration was assumed, $Ra_q = 1450$ (corresponding to 84 mW/m^2 , and $Rt = 3$ (similar to Fig. 3). The resulting mean enrichment at the end of the model run (14 Ma) is 0.04 (left model) and 0.098 (more than double on right model). In panels a) and b) the colors show the composition at 10 Ma. In c) and d) the colors show the non-dimensional volumetric melting- or solidification rates, positive values in orange-red are melting rates and negative values in blue tones are solidification rates. To dimensionalize these rates they must be multiplied by $0.01168 / \text{Ma}$ or $1.1168 \% / \text{Ma}$. The black arrows show the solid velocity with a maximum vector length of c) 81 mm/a and d) 23 mm/a , the grey arrows show the fluid velocity with a maximum vector length of c) 99 mm/a and d) 41 mm/a . In panels e) and f) the logarithmic root mean square segregation (red, "vsgrms") and solid (blue, "vrms") velocities are shown as a function of time.

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Inspecting the full-time evolution of the batholith emplacement model sing3a reveals that recycling of solidified enriched material continues until 5.5 Ma, as long as the RMS segregation to solid velocity

ratio is well below $\sim 1/3$ (see Fig. D3f, where the difference of the \log_{10} velocities is -0.5). Between 7.2 Ma and 10 Ma the segregation velocity approaches the solid velocity and small melt rich bodies rise to the growing emplacement zone where they solidify without being incorporated into the downwelling flow. The mean enrichment of the model accelerates at this time (Fig. D1a, red curve with 2500 kg/m^3). At 10 Ma the two velocities become equal and the differentiation of enriched material strongly intensifies once more, which is indicated by the sharp drop of the solid velocity because no more thermal and melt buoyancy contributes to the convective circulation, while segregation dominates and continues to increase the size of the enriched batholith. This is also seen in the mean enrichment curve (Fig. D1a, red curve) which has its strongest increasing rate between 10 Ma and 12 Ma.