# Moulin density impacts the effect of subglacial hydrology on ice dynamics

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## Abstract

The current increase in temperature over Greenland and other glaciated regions allows for more surface melt, which poses the question of the impact of this extra amount of meltwater on ice dynamics. As subglacial hydrology models evolve they are now easier to apply to realistic scenarios to quantify the effect of an increase in melt on the dynamics of glaciers. However, a number of processes linking the surface melt to the water pressure at the base of glaciers are still overlooked in models due to a lack of knowledge or an excess of complexity. Here, we apply a subglacial hydrology model coupled to an ice dynamics model to a synthetic geometry to investigate the impact of moulins distribution on the dynamics of the glacier. Our results show that a sparser distribution of moulins leads to the faster development of the efficient drainage system and greatly slows down the glacier.

# Moulin density impacts the effect of subglacial hydrology on ice dynamics

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## Key Points:

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•	The density	of moulins	changes	the	impact	of sub	glacial	water	drainage	on	ice	dy-
	namics.											

• Localised water inputs to the subglacial hydrological system helps with the stability of the system.

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## 12 Abstract

The current increase in temperature over Greenland and other glaciated regions allows 13 for more surface melt, which poses the question of the impact of this extra amount of 14 meltwater on ice dynamics. As subglacial hydrology models evolve they are now easier 15 to apply to realistic scenarios to quantify the effect of an increase in melt on the dynam-16 ics of glaciers. However, a number of processes linking the surface melt to the water pres-17 sure at the base of glaciers are still overlooked in models due to a lack of knowledge or 18 an excess of complexity. Here, we apply a subglacial hydrology model coupled to an ice 19 dynamics model to a synthetic geometry to investigate the impact of moulins distribu-20 tion on the dynamics of the glacier. Our results show that a sparser distribution of moulins 21 leads to the faster development of the efficient drainage system and greatly slows down 22 the glacier. 23

## <sup>24</sup> Plain Language Summary

As climate warms, a larger amount of meltwater is produced at the surface of ice 25 sheets and glaciers. Most of this water makes its way through cracks and other passage-26 ways of the ice to end up at the interface between the glacier itself and the underlying 27 rock. Once at the base of the glacier this water acts as a lubricant and as its pressure 28 increase it has the potential to speed-up the overlying ice. The relationship between ice 20 velocity and water amount is however not straightforward as the drainage system at the 30 base of the glacier can reconfigure and potentially has a large impact on water pressure. 31 Due to this complex interactions, numerical models of subglacial drainage system are needed 32 to get a better idea of the effect of an increase in meltwater production on ice dynam-33 ics. Using such a model in a conceptual set-up, we show that reducing the number of in-34 jection points into the subglacial drainage system in models can lead to a substantial de-35 celeration of the glacier above. 36

## 37 1 Introduction

Since the first measurement of the impact of meltwater on the dynamics of glaciers 38 in Greenland by Zwally et al. (2002) the question of the long term impact of an increase 39 of Greenland surface melt on its dynamics has been debated within the community. Fur-40 ther observations (e.g., Sole et al., 2013; Meierbachtol et al., 2013; Doyle et al., 2014; 41 Tedstone et al., 2015) have revealed a complex interaction between the amount of runoff 42 at the ice surface and the observed accelerations or lack thereof. A large part of the com-43 plexity of the system resides in the way the subglacial water drainage system operates, 44 with the capacity to enhance its efficiency when the water volume injected into the sys-45 tem increases (e.g., Chandler et al., 2013; Andrews et al., 2014). These complex inter-46 actions lead to a decoupling between the volume of available runoff water and the sub-47 glacial water pressure that drives glacier sliding (Bartholomew et al., 2010; Schoof, 2010; 48 Fitzpatrick et al., 2013; Sole et al., 2013; van de Wal et al., 2015). The complexity of this system warrants the use of fully coupled subglacial hydrology ice dynamics models to 50 evaluate if the increase of meltwater production will have a notable effect on ice dynam-51 ics as the temperatures continue to rise (M. Hoffman & Price, 2014; Stevens et al., 2018; 52 Davison et al., 2019). 53

Important efforts have been made to improve the representation of the subglacial 54 hydrological drainage system in models This lead to the development of a new gener-55 ation of multi component models able to compute the water pressure at the base of glaciers 56 (e.g. Pimentel et al., 2010; Werder et al., 2013; de Fleurian et al., 2014; M. J. Hoffman 57 et al., 2018). These models have recently been coupled to ice dynamics models in order 58 to investigate the meltwater lubrication feedback on various timescales (Gagliardini & 59 Werder, 2018; de Fleurian et al., 2022), Given the complexity of the system these stud-60 ies have focused on synthetic designs to isolate the effect of a specific component of the 61

system. Here, we continue on this trend and investigate the impact of the water supply distribution on the subglacial water pressure and its impact on ice dynamics. This question have been under the scope of different studies so far (A. Banwell et al., 2016; Scholzen et al., 2021) but those studies focused on the effect on subglacial water pressure without taking the final step of assessing the effect on ice dynamics.

The distribution intensity and timing of meltwater input to the subglacial drainage 67 system is controlled by the intensity of the surface melt and the efficiency of the supraglacial 68 and intraglacial drainage systems. Models are emerging to represent these components 69 (e.g. A. F. Banwell et al., 2012; Clason et al., 2015; Yang et al., 2022) but the complex-70 ity of each subsystem means that there is still no simulations that can introduce in a re-71 alistic manner all the component of the system. This is a drawback for the modelling 72 of subglacial water pressure as the intensity of water input at the base of glaciers has a 73 large impact on the development of an efficient subglacial drainage system (Colgan et 74 al., 2011; Bartholomew et al., 2012; Tedesco et al., 2013; de Fleurian et al., 2022) and 75 hence on the subglacial water pressure. A recent study by Yang et al. (2020) showed that 76 changing the supraglacial drainage model that was feeding into the subglacial drainage 77 system did not yield large changes in pressure on timescales longer than a day. Mejia 78 et al. (2022) drew similar conclusion after monitoring supraglacial drainage catchment 79 with different characteristics showing again that a change in the supraglacial drainage 80 can lead to lags in subglacial water peak pressure on the order of a few hours. Here we 81 focus on longer timescales and investigate the impact of a changes in the distribution of 82 meltwater input into the subglacial drainage system on a yearly timescale. 83

## $\mathbf{^{84}}$ 2 Methods

## 2.1 Model Description

In order to investigate the influence of moulins density on the effective pressure and 86 velocity evolution of glaciers we carry out coupled ice dynamic, subglacial hydrology sim-87 ulations within the Ice-sheet and Sea-level System Model (ISSM, Larour et al., 2012). 88 Within ISSM we use the Double Continuum (DoCo) approximation for the hydrology 89 model as described in de Fleurian et al. (2014, 2016). The ice flow is then resolved with 90 a Shallow Shelf Approximation (SSA, Morland & Zainuddin, 1987; MacAyeal, 1989) and 91 the coupling is achieved through a friction law in which the effective pressure (N), de-92 fined as the difference between the water pressure at the bed and the ice overlying pres-93 sure, is a key parameter. We elected to use a non-linear friction law described by Schoof 94 (2005) and Gagliardini et al. (2007) which ties sliding velocities  $(\vec{u_b})$  and basal shear stress 95  $(\vec{\tau_b})$ : 96

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$$\vec{\tau_b} + \frac{CN|\vec{u_b}|^{(1/n-1)}}{(|\vec{u_b}| + C^n N^n A_s)^{(1/n)}} \vec{u_b} = 0,$$
(1)

where C is Iken's bound (Iken, 1981) which sets the maximum value taken by  $\tau_b/N$  while A<sub>s</sub> is the sliding parameter without cavitation and n is the rheological exponent in Glen's flow law (Glen, 1958) taken here as (n = 3).

As for most of the model set-up, the subglacial hydrology model version and parameters used in the present study are exactly the same as the one described in de Fleurian et al. (2022). As in de Fleurian et al. (2022) the model is initialised with a parabolic function which resembles a west Greenland land terminating glacier surface elevation. The glacier is initially 150 km long and 20 km wide with a flat bedrock at an elevation  $z_b = 465$ m The surface elevation (z) then follows:

$$z_s(x,y) = 4.5 \times \sqrt{x} + 4000 + 186 \tag{2}$$

The geometry is then relaxed within the coupled model framework to achieve a pseudo steady-state geometry.

**Table 1.** Catchment area characteristics for every simulation, the area taken into account for the average catchment area is the area that experiences melt at anytime during the year. The number of moulins corresponds to the number of catchment areas as there is a single moulin per catchment area.

Simulation name	number of moulins	Average catchment area	smallest catchment area
Uniform	844	$0.7 \ \mathrm{km^2}$	$0.1 \ \mathrm{km}^2$
Fine	467	$1.4 \text{ km}^2$	$0.3 \ \mathrm{km^2}$
Mid	99	$6.5 \ \mathrm{km^2}$	$1.2 \ \mathrm{km^2}$
Coarse	50	$13 \ \mathrm{km^2}$	$2.1 \ \mathrm{km^2}$

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## 2.2 Meltwater Forcing

The surface mass balance is applied through an idealised method following Hewitt (2013), where the surface temperature is described at a reference elevation through the length of the melt season  $(\Delta_m)$ , a positive degree day  $(r_m)$  at the reference elevations, the day of the year when the melt season starts  $(t_{spr})$  and the duration that the temperature takes to reach its maximum value  $(\Delta_t \text{ in days})$ . The reference temperature  $(T_{ref})$ then reads :

$$T_{ref}(t) = \frac{r_m}{\Delta_m} \times \left(\frac{1}{2} \tanh\left(\frac{t - t_{spr}}{\Delta_t}\right) - \frac{1}{2} \tanh\left(\frac{t - (t_{spr} + \Delta_m)}{\Delta_t}\right)\right)$$
(3)

From that temperature, the runoff (r) at the surface of our synthetic geometry is computed through a lapse rate  $(r_s)$  and a given degree day factor (ddf):

$$r(s,t) = \max\left\{0, T_{ref}(t) \times (z_s - 465) \times r_s\right\} \times ddf \tag{4}$$

This setup for the mass balance computation uses the parameters of the reference 121 simulation in de Fleurian et al. (2022) with  $\Delta_t=141$  days, a maximum temperature at 122 the reference elevation  $(r_m/\Delta_m)$  of 5.85 °C and  $\Delta_t=10$  days. The major difference here 123 is that the injection in the subglacial hydrology model is slightly different. Our reference 124 simulation (Uniform in Table 1) uses the set-up that was previously described where ev-125 ery model node is an injection point in the subglacial hydrology model. However we also 126 use some setup where a smaller number of injection point is used. For simplicity we will 127 further call those injection points moulins even if we do not try to actually model the 128 supraglacial and intraglacial components of the drainage system, implying a direct trans-129 fer of surface melt to the base of the glacier at each moulin. The procedure to define those 130 moulins is as follow. First, a given number of nodes are drawn randomly within the nodes 131 of the model that fall within the region that experiences runoff at any time during the 132 year. From this initial draw, we define a Voronoi diagram in which each polygon asso-133 ciated to a given model node represents one drainage basin at the surface of the ice. The 134 moulin for each of those basins is then placed on the lowest elevation node contained in 135 the catchment. From there on, the runoff is integrated over the whole catchment area 136 and the given water discharge is then injected at the location of the moulin through a 137 vertical shaft into the subglacial hydrology model. This procedure insures that for all 138 the simulations the same volume of water is injected in the subglacial drainage system 139 and only the location of the injection changes. Table 1 gives an overview of the differ-140 ent simulations with a few statistics on the catchment areas and number of moulins. 141

## 142 2.3 Ensemble Design

The experiments of de Fleurian et al. (2022) showed that the model presented a physical instability that needed to perform an ensemble of simulations in order to inves-

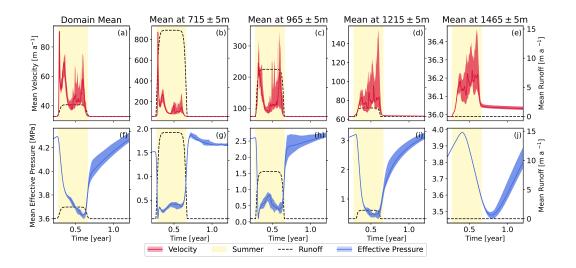


Figure 1. Evolution of velocities (a to e) and effective pressures (f to j) presented as a mean value for the whole domain (a and f) or a given elevation band (b-e and g-j) for the *Uniform* ensemble. The red line and shading show the mean ensemble and spread of the velocity respectively. In the same way, the blue colour represents the effective pressure while the dashed black line is the runoff presented on the right axis. Note that the runoff axis is the same for every plot but that this is not true for the velocity and effective pressure axes. The yellow shading represents the summer period from day 100 to 241.

tigate the impact of the different forcings. Here we expect those instability to be similar for the simulations with a large number of moulins (*Uniform* and *Fine* in Table 1), to cope with this issue we will again perform a ensemble of simulations with this time only 20 members per ensembles. We use the same method as was used in de Fleurian et al. (2022) to produce the ensemble with all parameters of the simulations identical but the starting time which is delayed by one extra second for every simulation.

## 151 **3 Results**

We take as a reference the *Uniform* simulation in which each model node is an injection point for the subglacial drainage system. In this case, the models counts 844 moulins that are active at one time or an other during the simulation. That translates to a mean drainage basin area of 0.7 km<sup>2</sup> (Table 1). The results of this ensemble of simulations are presented in Figure 1 where we show the evolution of the surface velocity and effective pressure as a mean value over the whole domain and at given altitudes.

The mean velocity over the domain (Figure 1a) presents a typical pattern for a glacier 158 with a marked and short-lived spring speed-up event followed by a second acceleration 159 event before the velocities drop down to a lower level. At the end of the melt season we 160 see a large spread in the evolution of the velocities with some of the ensemble members 161 showing a strong re-acceleration while other tend to stay at a more reasonable summer 162 velocity level. The velocity patterns are driven by the evolution of the effective pressure, 163 as shown on Figure 1f the initial speed up is related to a sharp drop in effective pres-164 sure at the beginning of the melt season. Then the activation of the efficient drainage 165 system leads to a gentler slope in the decrease of the effective pressure which in turn al-166 lows the velocity to slow down to their summer level. Finally at the end of the season, 167 the end of the melt leads to a fast increase of the effective pressure which is mediated 168 by the pace at which the efficient drainage system collapses and explains why some mem-169

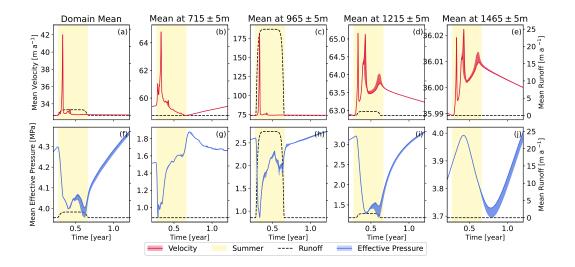


Figure 2. Same as Figure 1 but for the *Coarse* ensemble. Note that the runoff is null at 715 m as there is no moulins in this elevation band

bers have a strong end of summer acceleration while other don't. The velocity and ef-170 fective pressure pattern is quite different depending on the altitude that we consider. At 171 low elevations (bellow 1000m, Figures 1b-c and 1g-h) the velocity and effective pressure 172 evolution are very similar to the one described for the whole domain with a slight dif-173 ference in the fact that the effective pressure rebounds to slightly higher values once the 174 efficient drainage system is activated. At higher elevation but below the no melt zone 175 (Figures 1d and i) the most obvious change is the absence of a spring speed-up event. 176 At this elevation, the velocity evolution is characterised by a gradual increase in veloc-177 ities from the beginning of the melt season all the way to its end. This acceleration is 178 driven by a decrease in effective pressure that first drops at the beginning of the melt 179 season and then levels out but without the rebound that was observed at lower eleva-180 tions. Above the melt region (Figures 1e and j) the velocity pattern is similar as described 181 above but with a much smaller amplitude (note the changes in y-axis range in Figure 1). 182 Here the small amplitude of the velocity changes, the decoupling with the effective pres-183 sure evolution, and the absence of meltwater input points towards a velocity change that 184 is due to the downstream evolution of velocities. 185

Changing the number of moulins that are used to inject water into the subglacial drainage system as a large impact on the model results. Figure 2 shows the result for the *Coarse* simulation with only 50 moulins which translates to a mean catchment area of 13 km<sup>2</sup> (Table 1).

The first obvious difference with Figure 1 is the drastically smaller amplitude of 190 the velocity changes for the simulation with a lower number of moulins. An other clear 191 result is the diminution in the spread of the computed velocities with the ensemble with 192 only 50 moulins showing almost no spread between its members (Figure 2). The reduc-193 tion of the number of moulins as presented on the *Coarse* simulation (Figure 2) leads 194 to the disappearance of the end of summer acceleration at all elevations. The velocity 195 response of the *Coarse* simulation is entirely explained by the changes in the evolution 196 of the effective pressure during the melt season. While the evolution at higher altitudes 197 is quite comparable for the Uniform (Figure 1i-j) and Coarse (Figure 2i-j) simulations 198 the patterns are quite different at lower elevations. Closer to the glacier front, the effec-199 tive pressure of the *Coarse* simulations (Figure 2g-h) show an earlier rebound of the pres-200 sures towards a higher summer value after the initial drop than the one in the Uniform 201

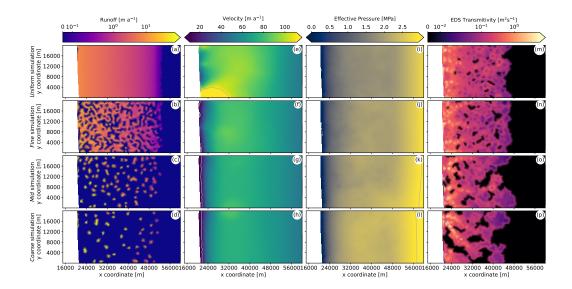


Figure 3. Annual mean values of Runoff (a-d), Velocity (e-h), Effective Pressure (i-l) and efficient drainage system (EDS) Transmitivity (m-p) for the different simulations (*Uniform, Fine, Mid, Coarse*). The ice flow is from right to left and we only show here the lower region of the glacier which experiences melt. The grey lines show the boundaries out of which the colorbar is saturated. We present here the values for a single member of the ensemble and all other ensemble members show a similar pattern.

ensemble (Figure 1g-h). It is this difference in effective pressure that explains why the
 end of summer acceleration events are mostly absent from the simulations with a smaller
 number of moulins.

A map view of the mean annual value of the different variables of the model give a better insight into the sources of the differences that are observed in the effective pressure and velocity evolution. Figure 3 presents map views of the region of the glacier where the runoff takes place leaving out the uppermost region of the glacier where the differences due to a change in recharge distribution are less pronounced.

Figure 3 compares the mean annual runoff, velocity, effective pressure and efficient 210 drainage system transmitivity for a given member of the four ensembles from Uniform 211 at the top to *Coarse* at the bottom. Note here that as the number of moulins reduces, 212 the intensity of the recharge at each of those points increase as the moulins are drain-213 ing a larger area of the glacier and so funnelling a larger amount of water toward the ice 214 base (see Table 1 for statistics on catchment area). In term of velocities Figures 3(e-h) 215 show the large difference that appears between the simulation fed by every node or the 216 ones that receive water through a decreasing number of moulins that was already shown 217 on Figures 1 and 2. The observation of the temporal evolution of velocities are confirmed 218 by the mean annual values with notably faster velocities if the water input is spread over 219 a larger number of moulins. The upstream shift of the maximum velocity region which 220 was noticeable on the temporal evolution is also very clear here with only the Uniform 221 simulation that presents its fastest velocity towards the front of the glacier while the sim-222 ulation with a lower number of moulins have their maximum velocities roughly 12km up-223 stream from the front. This is driven by the effective pressure presented on Figures 3(i-224 1) where we observe higher effective pressure with a lower number of moulins but also 225 a more grainy pattern for the effective pressure which is driven by the coarser water in-226 put. Those fields are explained by the way in which the efficient drainage system devel-227

<sup>228</sup> ops, on Figures 3(m-p) we show the mean transmitivity of the efficient drainage system <sup>229</sup> which describes its efficiency. We see for the *Uniform* simulation a smooth transmitiv-<sup>230</sup> ity pattern for the efficient drainage system with a decrease from high efficiency at the <sup>231</sup> front to lower efficiency at the top of the ablation zone. As the number of moulins is re-<sup>232</sup> duced the efficient drainage system shows a more localised pattern where the regions of <sup>233</sup> the efficient drainage system that are active are also more efficient than they were when <sup>234</sup> a large area of the bedrock was occupied by the efficient drainage system.

## 235 4 Discussion

Our experiments show that the distribution of injection points to the subglacial drainage
 system leads to substantial changes in both the geometry of the efficient drainage system and its overall impact on ice velocity.

This conclusion is in line with the study Scholzen et al. (2021) that showed that 239 a more localised water input into the subglacial drainage system leads to a faster devel-240 opment of the efficient subglacial drainage system. It is however quite different from the 241 results presented by A. Banwell et al. (2016) who found that a higher moulin density causes 242 an earlier onset of channelization and overall more widespread efficient drainage system. 243 We argue that the differences here are mostly due to the change in timing in their recharge 244 scenario but also on the definition of the efficient drainage. A. Banwell et al. (2016) con-245 sider any channel that opens as an efficient drainage system, but looking at their Fig-246 ure 4 we see that their simulations present the same patterns as ours with a lower moulin 247 density leading to less widespread efficient drainage system but with a higher efficiency. 248 This leads to a more efficient drainage of the inefficient drainage system and higher ef-249 fective pressures which in turns causes slower ice flow than in experiments with a more 250 homogeneous water recharge. 251

In our model, the specific localisation of water input induces a more stable configuration of the subglacial drainage system. This is due to the fact that the moulins act as anchor points for the subglacial drainage system to develop. This contrasts with the more random development of the efficient drainage system for simulations with a more uniform input which lead to large spread of effective pressure within a given model ensemble (see *Uniform* ensemble and simulations from (de Fleurian et al., 2022).

The observed response can be compared to the results of our preceding study com-258 paring the intensity vs. length of the melt season (de Fleurian et al., 2022). There, a more 259 intense melt season was driving a slower ice flow as it allowed the faster development of 260 the efficient drainage system and as such an overall higher effective pressure. That com-261 pares well with the results that we show here, were a lower number of moulins lead to 262 a more intense and localised water input. This triggers a faster development of the ef-263 ficient drainage system at these locations which help to raise the effective pressure on 264 the whole domain and such lead to a slower glacier. 265

The change in the distribution of the water sources also shows an upstream shift 266 in the maximum velocity values. This is well illustrated in Figures 3(e-h) but also com-267 paring the velocities on Figures 1, S1, S2, and 2 where we see an upstream migration of 268 the fastest velocities. The pattern of the lower density moulin simulations are more co-269 herent with the velocity patterns that have been observed in Greenland (Fitzpatrick et 270 al., 2013; Sole et al., 2013). This change is driven by the concentration of subglacial wa-271 ter in a small number of pathways at the front of the glacier. The large efficiency of the 272 drainage system close to the front leads to a quick rebound of the effective pressure at 273 the beginning of summer towards a high effective pressure. This, in turns, reduces the 274 intensity of the spring speed-up for the simulations with a low moulin density (*Mid* and 275 *Coarse.* The recurrence of late summer acceleration is also reduced when the water in-276 put is achieved through a network of moulins rather than a uniform input. Again that 277

is due to the localisation of the water input which leads to more localised and efficient
subglacial drainage system. Those well developed system tend to be active longer through
the season and avoids the low effective pressure that was observed with more widespread
water input.

## <sup>282</sup> 5 Conclusions

On a synthetic geometry and with an idealised meltwater forcing, we show that a 283 change in the distribution of the meltwater input to the basal drainage system as a large 284 impact on the velocity of the overlying glacier. Our experiments show that as the num-285 ber of moulins diminishes, and the intensity of the recharge at these points increases, the 286 velocity at the surface of the glacier greatly decreases. We associate this decrease in ve-287 locity to a faster, and more localised development of the efficient drainage system in our 288 model with a higher efficiency of this system. The more localised water input into the 289 subglacial hydrology model also leads in this particular model to a better physical sta-290 bility of the model due to the "anchoring" effect of the more localised input which con-291 strains the location of the efficient drainage system. The results of this study show the 292 importance of moving away from uniform water input into subglacial hydrology mod-293 els. This poses the question of what is the real distribution of moulins and warrants more 294 studies to allow a better characterisation of the supraglacial drainage system that would 295 be usable for subglacial hydrology models. 296

## <sup>297</sup> Open Research Section

The Ice-sheet and Sea-level System Model is freely available at https://issm.jpl .nasa.gov/ this specific study uses the development branch of the code at the revision number 27528 last updated on January 19 2023. The model set-up, outputs and post treating scripts corresponding to this study are available on zenodo (doi to come). The figures in this manuscript were generated with the script in the archive above and with Matplotlib v3.5 (Hunter, 2007).

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# Moulin density impacts the effect of subglacial hydrology on ice dynamics

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## Key Points:

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•	The density	of moulins	changes	the	impact	of sub	glacial	water	drainage	on	ice	dy-
	namics.											

• Localised water inputs to the subglacial hydrological system helps with the stability of the system.

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## 12 Abstract

The current increase in temperature over Greenland and other glaciated regions allows 13 for more surface melt, which poses the question of the impact of this extra amount of 14 meltwater on ice dynamics. As subglacial hydrology models evolve they are now easier 15 to apply to realistic scenarios to quantify the effect of an increase in melt on the dynam-16 ics of glaciers. However, a number of processes linking the surface melt to the water pres-17 sure at the base of glaciers are still overlooked in models due to a lack of knowledge or 18 an excess of complexity. Here, we apply a subglacial hydrology model coupled to an ice 19 dynamics model to a synthetic geometry to investigate the impact of moulins distribu-20 tion on the dynamics of the glacier. Our results show that a sparser distribution of moulins 21 leads to the faster development of the efficient drainage system and greatly slows down 22 the glacier. 23

## <sup>24</sup> Plain Language Summary

As climate warms, a larger amount of meltwater is produced at the surface of ice 25 sheets and glaciers. Most of this water makes its way through cracks and other passage-26 ways of the ice to end up at the interface between the glacier itself and the underlying 27 rock. Once at the base of the glacier this water acts as a lubricant and as its pressure 28 increase it has the potential to speed-up the overlying ice. The relationship between ice 20 velocity and water amount is however not straightforward as the drainage system at the 30 base of the glacier can reconfigure and potentially has a large impact on water pressure. 31 Due to this complex interactions, numerical models of subglacial drainage system are needed 32 to get a better idea of the effect of an increase in meltwater production on ice dynam-33 ics. Using such a model in a conceptual set-up, we show that reducing the number of in-34 jection points into the subglacial drainage system in models can lead to a substantial de-35 celeration of the glacier above. 36

## 37 1 Introduction

Since the first measurement of the impact of meltwater on the dynamics of glaciers 38 in Greenland by Zwally et al. (2002) the question of the long term impact of an increase 39 of Greenland surface melt on its dynamics has been debated within the community. Fur-40 ther observations (e.g., Sole et al., 2013; Meierbachtol et al., 2013; Doyle et al., 2014; 41 Tedstone et al., 2015) have revealed a complex interaction between the amount of runoff 42 at the ice surface and the observed accelerations or lack thereof. A large part of the com-43 plexity of the system resides in the way the subglacial water drainage system operates, 44 with the capacity to enhance its efficiency when the water volume injected into the sys-45 tem increases (e.g., Chandler et al., 2013; Andrews et al., 2014). These complex inter-46 actions lead to a decoupling between the volume of available runoff water and the sub-47 glacial water pressure that drives glacier sliding (Bartholomew et al., 2010; Schoof, 2010; 48 Fitzpatrick et al., 2013; Sole et al., 2013; van de Wal et al., 2015). The complexity of this system warrants the use of fully coupled subglacial hydrology ice dynamics models to 50 evaluate if the increase of meltwater production will have a notable effect on ice dynam-51 ics as the temperatures continue to rise (M. Hoffman & Price, 2014; Stevens et al., 2018; 52 Davison et al., 2019). 53

Important efforts have been made to improve the representation of the subglacial 54 hydrological drainage system in models This lead to the development of a new gener-55 ation of multi component models able to compute the water pressure at the base of glaciers 56 (e.g. Pimentel et al., 2010; Werder et al., 2013; de Fleurian et al., 2014; M. J. Hoffman 57 et al., 2018). These models have recently been coupled to ice dynamics models in order 58 to investigate the meltwater lubrication feedback on various timescales (Gagliardini & 59 Werder, 2018; de Fleurian et al., 2022), Given the complexity of the system these stud-60 ies have focused on synthetic designs to isolate the effect of a specific component of the 61

system. Here, we continue on this trend and investigate the impact of the water supply distribution on the subglacial water pressure and its impact on ice dynamics. This question have been under the scope of different studies so far (A. Banwell et al., 2016; Scholzen et al., 2021) but those studies focused on the effect on subglacial water pressure without taking the final step of assessing the effect on ice dynamics.

The distribution intensity and timing of meltwater input to the subglacial drainage 67 system is controlled by the intensity of the surface melt and the efficiency of the supraglacial 68 and intraglacial drainage systems. Models are emerging to represent these components 69 (e.g. A. F. Banwell et al., 2012; Clason et al., 2015; Yang et al., 2022) but the complex-70 ity of each subsystem means that there is still no simulations that can introduce in a re-71 alistic manner all the component of the system. This is a drawback for the modelling 72 of subglacial water pressure as the intensity of water input at the base of glaciers has a 73 large impact on the development of an efficient subglacial drainage system (Colgan et 74 al., 2011; Bartholomew et al., 2012; Tedesco et al., 2013; de Fleurian et al., 2022) and 75 hence on the subglacial water pressure. A recent study by Yang et al. (2020) showed that 76 changing the supraglacial drainage model that was feeding into the subglacial drainage 77 system did not yield large changes in pressure on timescales longer than a day. Mejia 78 et al. (2022) drew similar conclusion after monitoring supraglacial drainage catchment 79 with different characteristics showing again that a change in the supraglacial drainage 80 can lead to lags in subglacial water peak pressure on the order of a few hours. Here we 81 focus on longer timescales and investigate the impact of a changes in the distribution of 82 meltwater input into the subglacial drainage system on a yearly timescale. 83

## $\mathbf{^{84}}$ 2 Methods

## 2.1 Model Description

In order to investigate the influence of moulins density on the effective pressure and 86 velocity evolution of glaciers we carry out coupled ice dynamic, subglacial hydrology sim-87 ulations within the Ice-sheet and Sea-level System Model (ISSM, Larour et al., 2012). 88 Within ISSM we use the Double Continuum (DoCo) approximation for the hydrology 89 model as described in de Fleurian et al. (2014, 2016). The ice flow is then resolved with 90 a Shallow Shelf Approximation (SSA, Morland & Zainuddin, 1987; MacAyeal, 1989) and 91 the coupling is achieved through a friction law in which the effective pressure (N), de-92 fined as the difference between the water pressure at the bed and the ice overlying pres-93 sure, is a key parameter. We elected to use a non-linear friction law described by Schoof 94 (2005) and Gagliardini et al. (2007) which ties sliding velocities  $(\vec{u_b})$  and basal shear stress 95  $(\vec{\tau_b})$ : 96

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$$\vec{\tau_b} + \frac{CN|\vec{u_b}|^{(1/n-1)}}{(|\vec{u_b}| + C^n N^n A_s)^{(1/n)}} \vec{u_b} = 0,$$
(1)

where C is Iken's bound (Iken, 1981) which sets the maximum value taken by  $\tau_b/N$  while A<sub>s</sub> is the sliding parameter without cavitation and n is the rheological exponent in Glen's flow law (Glen, 1958) taken here as (n = 3).

As for most of the model set-up, the subglacial hydrology model version and parameters used in the present study are exactly the same as the one described in de Fleurian et al. (2022). As in de Fleurian et al. (2022) the model is initialised with a parabolic function which resembles a west Greenland land terminating glacier surface elevation. The glacier is initially 150 km long and 20 km wide with a flat bedrock at an elevation  $z_b = 465$ m The surface elevation (z) then follows:

$$z_s(x,y) = 4.5 \times \sqrt{x} + 4000 + 186 \tag{2}$$

The geometry is then relaxed within the coupled model framework to achieve a pseudo steady-state geometry.

**Table 1.** Catchment area characteristics for every simulation, the area taken into account for the average catchment area is the area that experiences melt at anytime during the year. The number of moulins corresponds to the number of catchment areas as there is a single moulin per catchment area.

Simulation name	number of moulins	Average catchment area	smallest catchment area
Uniform	844	$0.7 \ \mathrm{km^2}$	$0.1 \ \mathrm{km}^2$
Fine	467	$1.4 \text{ km}^2$	$0.3 \ \mathrm{km^2}$
Mid	99	$6.5 \ \mathrm{km^2}$	$1.2 \ \mathrm{km^2}$
Coarse	50	$13 \ \mathrm{km^2}$	$2.1 \ \mathrm{km^2}$

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## 2.2 Meltwater Forcing

The surface mass balance is applied through an idealised method following Hewitt (2013), where the surface temperature is described at a reference elevation through the length of the melt season  $(\Delta_m)$ , a positive degree day  $(r_m)$  at the reference elevations, the day of the year when the melt season starts  $(t_{spr})$  and the duration that the temperature takes to reach its maximum value  $(\Delta_t \text{ in days})$ . The reference temperature  $(T_{ref})$ then reads :

$$T_{ref}(t) = \frac{r_m}{\Delta_m} \times \left(\frac{1}{2} \tanh\left(\frac{t - t_{spr}}{\Delta_t}\right) - \frac{1}{2} \tanh\left(\frac{t - (t_{spr} + \Delta_m)}{\Delta_t}\right)\right)$$
(3)

From that temperature, the runoff (r) at the surface of our synthetic geometry is computed through a lapse rate  $(r_s)$  and a given degree day factor (ddf):

$$r(s,t) = \max\left\{0, T_{ref}(t) \times (z_s - 465) \times r_s\right\} \times ddf \tag{4}$$

This setup for the mass balance computation uses the parameters of the reference 121 simulation in de Fleurian et al. (2022) with  $\Delta_t=141$  days, a maximum temperature at 122 the reference elevation  $(r_m/\Delta_m)$  of 5.85 °C and  $\Delta_t=10$  days. The major difference here 123 is that the injection in the subglacial hydrology model is slightly different. Our reference 124 simulation (Uniform in Table 1) uses the set-up that was previously described where ev-125 ery model node is an injection point in the subglacial hydrology model. However we also 126 use some setup where a smaller number of injection point is used. For simplicity we will 127 further call those injection points moulins even if we do not try to actually model the 128 supraglacial and intraglacial components of the drainage system, implying a direct trans-129 fer of surface melt to the base of the glacier at each moulin. The procedure to define those 130 moulins is as follow. First, a given number of nodes are drawn randomly within the nodes 131 of the model that fall within the region that experiences runoff at any time during the 132 year. From this initial draw, we define a Voronoi diagram in which each polygon asso-133 ciated to a given model node represents one drainage basin at the surface of the ice. The 134 moulin for each of those basins is then placed on the lowest elevation node contained in 135 the catchment. From there on, the runoff is integrated over the whole catchment area 136 and the given water discharge is then injected at the location of the moulin through a 137 vertical shaft into the subglacial hydrology model. This procedure insures that for all 138 the simulations the same volume of water is injected in the subglacial drainage system 139 and only the location of the injection changes. Table 1 gives an overview of the differ-140 ent simulations with a few statistics on the catchment areas and number of moulins. 141

## 142 2.3 Ensemble Design

The experiments of de Fleurian et al. (2022) showed that the model presented a physical instability that needed to perform an ensemble of simulations in order to inves-

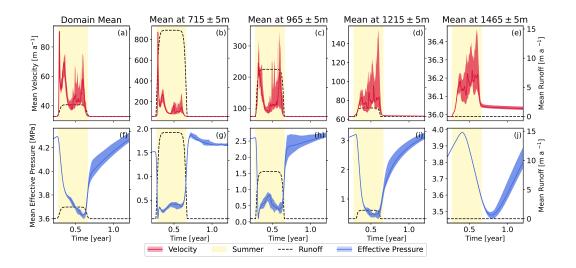


Figure 1. Evolution of velocities (a to e) and effective pressures (f to j) presented as a mean value for the whole domain (a and f) or a given elevation band (b-e and g-j) for the *Uniform* ensemble. The red line and shading show the mean ensemble and spread of the velocity respectively. In the same way, the blue colour represents the effective pressure while the dashed black line is the runoff presented on the right axis. Note that the runoff axis is the same for every plot but that this is not true for the velocity and effective pressure axes. The yellow shading represents the summer period from day 100 to 241.

tigate the impact of the different forcings. Here we expect those instability to be similar for the simulations with a large number of moulins (*Uniform* and *Fine* in Table 1), to cope with this issue we will again perform a ensemble of simulations with this time only 20 members per ensembles. We use the same method as was used in de Fleurian et al. (2022) to produce the ensemble with all parameters of the simulations identical but the starting time which is delayed by one extra second for every simulation.

## 151 **3 Results**

We take as a reference the *Uniform* simulation in which each model node is an injection point for the subglacial drainage system. In this case, the models counts 844 moulins that are active at one time or an other during the simulation. That translates to a mean drainage basin area of 0.7 km<sup>2</sup> (Table 1). The results of this ensemble of simulations are presented in Figure 1 where we show the evolution of the surface velocity and effective pressure as a mean value over the whole domain and at given altitudes.

The mean velocity over the domain (Figure 1a) presents a typical pattern for a glacier 158 with a marked and short-lived spring speed-up event followed by a second acceleration 159 event before the velocities drop down to a lower level. At the end of the melt season we 160 see a large spread in the evolution of the velocities with some of the ensemble members 161 showing a strong re-acceleration while other tend to stay at a more reasonable summer 162 velocity level. The velocity patterns are driven by the evolution of the effective pressure, 163 as shown on Figure 1f the initial speed up is related to a sharp drop in effective pres-164 sure at the beginning of the melt season. Then the activation of the efficient drainage 165 system leads to a gentler slope in the decrease of the effective pressure which in turn al-166 lows the velocity to slow down to their summer level. Finally at the end of the season, 167 the end of the melt leads to a fast increase of the effective pressure which is mediated 168 by the pace at which the efficient drainage system collapses and explains why some mem-169

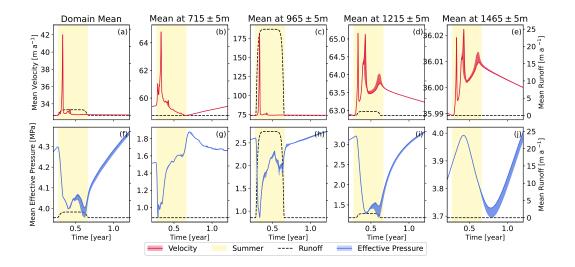


Figure 2. Same as Figure 1 but for the *Coarse* ensemble. Note that the runoff is null at 715 m as there is no moulins in this elevation band

bers have a strong end of summer acceleration while other don't. The velocity and ef-170 fective pressure pattern is quite different depending on the altitude that we consider. At 171 low elevations (bellow 1000m, Figures 1b-c and 1g-h) the velocity and effective pressure 172 evolution are very similar to the one described for the whole domain with a slight dif-173 ference in the fact that the effective pressure rebounds to slightly higher values once the 174 efficient drainage system is activated. At higher elevation but below the no melt zone 175 (Figures 1d and i) the most obvious change is the absence of a spring speed-up event. 176 At this elevation, the velocity evolution is characterised by a gradual increase in veloc-177 ities from the beginning of the melt season all the way to its end. This acceleration is 178 driven by a decrease in effective pressure that first drops at the beginning of the melt 179 season and then levels out but without the rebound that was observed at lower eleva-180 tions. Above the melt region (Figures 1e and j) the velocity pattern is similar as described 181 above but with a much smaller amplitude (note the changes in y-axis range in Figure 1). 182 Here the small amplitude of the velocity changes, the decoupling with the effective pres-183 sure evolution, and the absence of meltwater input points towards a velocity change that 184 is due to the downstream evolution of velocities. 185

Changing the number of moulins that are used to inject water into the subglacial drainage system as a large impact on the model results. Figure 2 shows the result for the *Coarse* simulation with only 50 moulins which translates to a mean catchment area of 13 km<sup>2</sup> (Table 1).

The first obvious difference with Figure 1 is the drastically smaller amplitude of 190 the velocity changes for the simulation with a lower number of moulins. An other clear 191 result is the diminution in the spread of the computed velocities with the ensemble with 192 only 50 moulins showing almost no spread between its members (Figure 2). The reduc-193 tion of the number of moulins as presented on the *Coarse* simulation (Figure 2) leads 194 to the disappearance of the end of summer acceleration at all elevations. The velocity 195 response of the *Coarse* simulation is entirely explained by the changes in the evolution 196 of the effective pressure during the melt season. While the evolution at higher altitudes 197 is quite comparable for the Uniform (Figure 1i-j) and Coarse (Figure 2i-j) simulations 198 the patterns are quite different at lower elevations. Closer to the glacier front, the effec-199 tive pressure of the *Coarse* simulations (Figure 2g-h) show an earlier rebound of the pres-200 sures towards a higher summer value after the initial drop than the one in the Uniform 201

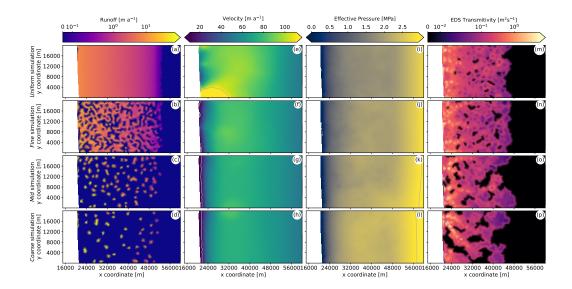


Figure 3. Annual mean values of Runoff (a-d), Velocity (e-h), Effective Pressure (i-l) and efficient drainage system (EDS) Transmitivity (m-p) for the different simulations (*Uniform, Fine, Mid, Coarse*). The ice flow is from right to left and we only show here the lower region of the glacier which experiences melt. The grey lines show the boundaries out of which the colorbar is saturated. We present here the values for a single member of the ensemble and all other ensemble members show a similar pattern.

ensemble (Figure 1g-h). It is this difference in effective pressure that explains why the
 end of summer acceleration events are mostly absent from the simulations with a smaller
 number of moulins.

A map view of the mean annual value of the different variables of the model give a better insight into the sources of the differences that are observed in the effective pressure and velocity evolution. Figure 3 presents map views of the region of the glacier where the runoff takes place leaving out the uppermost region of the glacier where the differences due to a change in recharge distribution are less pronounced.

Figure 3 compares the mean annual runoff, velocity, effective pressure and efficient 210 drainage system transmitivity for a given member of the four ensembles from Uniform 211 at the top to *Coarse* at the bottom. Note here that as the number of moulins reduces, 212 the intensity of the recharge at each of those points increase as the moulins are drain-213 ing a larger area of the glacier and so funnelling a larger amount of water toward the ice 214 base (see Table 1 for statistics on catchment area). In term of velocities Figures 3(e-h) 215 show the large difference that appears between the simulation fed by every node or the 216 ones that receive water through a decreasing number of moulins that was already shown 217 on Figures 1 and 2. The observation of the temporal evolution of velocities are confirmed 218 by the mean annual values with notably faster velocities if the water input is spread over 219 a larger number of moulins. The upstream shift of the maximum velocity region which 220 was noticeable on the temporal evolution is also very clear here with only the Uniform 221 simulation that presents its fastest velocity towards the front of the glacier while the sim-222 ulation with a lower number of moulins have their maximum velocities roughly 12km up-223 stream from the front. This is driven by the effective pressure presented on Figures 3(i-224 1) where we observe higher effective pressure with a lower number of moulins but also 225 a more grainy pattern for the effective pressure which is driven by the coarser water in-226 put. Those fields are explained by the way in which the efficient drainage system devel-227

<sup>228</sup> ops, on Figures 3(m-p) we show the mean transmitivity of the efficient drainage system <sup>229</sup> which describes its efficiency. We see for the *Uniform* simulation a smooth transmitiv-<sup>230</sup> ity pattern for the efficient drainage system with a decrease from high efficiency at the <sup>231</sup> front to lower efficiency at the top of the ablation zone. As the number of moulins is re-<sup>232</sup> duced the efficient drainage system shows a more localised pattern where the regions of <sup>233</sup> the efficient drainage system that are active are also more efficient than they were when <sup>234</sup> a large area of the bedrock was occupied by the efficient drainage system.

## 235 4 Discussion

Our experiments show that the distribution of injection points to the subglacial drainage
 system leads to substantial changes in both the geometry of the efficient drainage system and its overall impact on ice velocity.

This conclusion is in line with the study Scholzen et al. (2021) that showed that 239 a more localised water input into the subglacial drainage system leads to a faster devel-240 opment of the efficient subglacial drainage system. It is however quite different from the 241 results presented by A. Banwell et al. (2016) who found that a higher moulin density causes 242 an earlier onset of channelization and overall more widespread efficient drainage system. 243 We argue that the differences here are mostly due to the change in timing in their recharge 244 scenario but also on the definition of the efficient drainage. A. Banwell et al. (2016) con-245 sider any channel that opens as an efficient drainage system, but looking at their Fig-246 ure 4 we see that their simulations present the same patterns as ours with a lower moulin 247 density leading to less widespread efficient drainage system but with a higher efficiency. 248 This leads to a more efficient drainage of the inefficient drainage system and higher ef-249 fective pressures which in turns causes slower ice flow than in experiments with a more 250 homogeneous water recharge. 251

In our model, the specific localisation of water input induces a more stable configuration of the subglacial drainage system. This is due to the fact that the moulins act as anchor points for the subglacial drainage system to develop. This contrasts with the more random development of the efficient drainage system for simulations with a more uniform input which lead to large spread of effective pressure within a given model ensemble (see *Uniform* ensemble and simulations from (de Fleurian et al., 2022).

The observed response can be compared to the results of our preceding study com-258 paring the intensity vs. length of the melt season (de Fleurian et al., 2022). There, a more 259 intense melt season was driving a slower ice flow as it allowed the faster development of 260 the efficient drainage system and as such an overall higher effective pressure. That com-261 pares well with the results that we show here, were a lower number of moulins lead to 262 a more intense and localised water input. This triggers a faster development of the ef-263 ficient drainage system at these locations which help to raise the effective pressure on 264 the whole domain and such lead to a slower glacier. 265

The change in the distribution of the water sources also shows an upstream shift 266 in the maximum velocity values. This is well illustrated in Figures 3(e-h) but also com-267 paring the velocities on Figures 1, S1, S2, and 2 where we see an upstream migration of 268 the fastest velocities. The pattern of the lower density moulin simulations are more co-269 herent with the velocity patterns that have been observed in Greenland (Fitzpatrick et 270 al., 2013; Sole et al., 2013). This change is driven by the concentration of subglacial wa-271 ter in a small number of pathways at the front of the glacier. The large efficiency of the 272 drainage system close to the front leads to a quick rebound of the effective pressure at 273 the beginning of summer towards a high effective pressure. This, in turns, reduces the 274 intensity of the spring speed-up for the simulations with a low moulin density (*Mid* and 275 *Coarse.* The recurrence of late summer acceleration is also reduced when the water in-276 put is achieved through a network of moulins rather than a uniform input. Again that 277

is due to the localisation of the water input which leads to more localised and efficient
subglacial drainage system. Those well developed system tend to be active longer through
the season and avoids the low effective pressure that was observed with more widespread
water input.

## <sup>282</sup> 5 Conclusions

On a synthetic geometry and with an idealised meltwater forcing, we show that a 283 change in the distribution of the meltwater input to the basal drainage system as a large 284 impact on the velocity of the overlying glacier. Our experiments show that as the num-285 ber of moulins diminishes, and the intensity of the recharge at these points increases, the 286 velocity at the surface of the glacier greatly decreases. We associate this decrease in ve-287 locity to a faster, and more localised development of the efficient drainage system in our 288 model with a higher efficiency of this system. The more localised water input into the 289 subglacial hydrology model also leads in this particular model to a better physical sta-290 bility of the model due to the "anchoring" effect of the more localised input which con-291 strains the location of the efficient drainage system. The results of this study show the 292 importance of moving away from uniform water input into subglacial hydrology mod-293 els. This poses the question of what is the real distribution of moulins and warrants more 294 studies to allow a better characterisation of the supraglacial drainage system that would 295 be usable for subglacial hydrology models. 296

## <sup>297</sup> Open Research Section

The Ice-sheet and Sea-level System Model is freely available at https://issm.jpl .nasa.gov/ this specific study uses the development branch of the code at the revision number 27528 last updated on January 19 2023. The model set-up, outputs and post treating scripts corresponding to this study are available on zenodo (doi to come). The figures in this manuscript were generated with the script in the archive above and with Matplotlib v3.5 (Hunter, 2007).

## 304 Acknowledgments

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# Supporting Information for "Moulin density impacts the effect of subglacial hydrology on ice dynamics"

B. de Fleurian<sup>1</sup>, P. M. Langebroek<sup>2</sup>

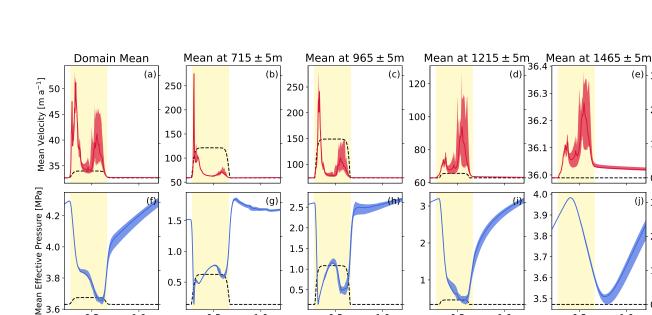
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## Contents of this file

- 1. Figures S1 and S2
- 2. Table S1

**Introduction** This document presents two figures following the one produced in the main manuscript but for the ensembles that are not shown in the manuscript. We also present here a table containing the main parameters of the model and the values that were used for this study.



2.0

1.5

1.0

0.5

Summer

0.5

Time [year]

1.0

Runoff

1.0

1.5

1.0

0.5

0.5

Time [year]

Velocity

1.0

0.5

Time [year]

Figure S1. Evolution of velocities (a to e) and effective pressures (f to j) presented as a mean value for the whole domain (a and f) or a given elevation band (b-e and g-j) for the *Fine* ensemble. The red line and shading show the mean ensemble and spread of the velocity respectively. In the same way, the blue colour represents the effective pressure while the dashed black line is the runoff presented on the right axis. Note that the runoff axis is the same for every plot but that this is not true for the velocity and effective pressure axes. The yellow shading represents the summer period from day 100 to 241.

0

3.9

3.8

3.7

3.6

3.5

0.5

Time [year]

1.0

2

1

0.5

Time [year]

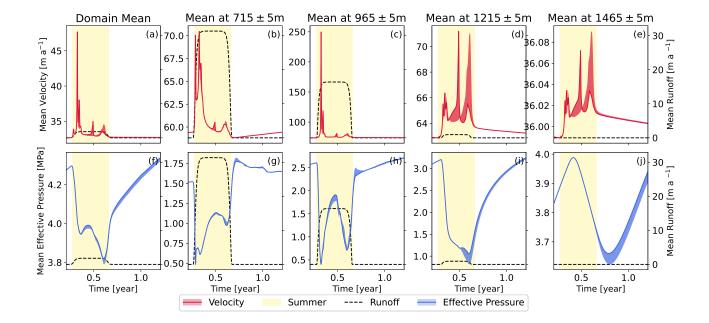
**Effective Pressure** 

(j) - 30 [[] - 20 [] -

0

1.0

March 16, 2023, 11:28am



**Figure S2.** Evolution of velocities (a to e) and effective pressures (f to j) presented as a mean value for the whole domain (a and f) or a given elevation band (b-e and g-j) for the *Mid* ensemble. The red line and shading show the mean ensemble and spread of the velocity respectively. In the same way, the blue colour represents the effective pressure while the dashed black line is the runoff presented on the right axis. Note that the runoff axis is the same for every plot but that this is not true for the velocity and effective pressure axes. The yellow shading represents the summer period from day 100 to 241.

Table S1.Values of the model parameters.

Symbol	Parameter	Value
$\overline{e_s}$	IDS thickness	20 m
$e_e$	EDS initial thickness	$5.0 \times 10^{-3} \mathrm{~m}$
$K_s$	IDS conductivity	$2.0 \times 10^{-3} \text{ ms}^{-1}$
$K_e$	EDS conductivity	$9.0 \times 10^1 \text{ m}s^{-1}$
ω	porosity	0.4
$\gamma$	leakage time	$1.0 \times 10^{-9} \text{ s}^{-1}$
$A_s$	Sliding Parameter	$3.2 \times 10^{-21} \mathrm{~m~Pa^{-3}s^{-1}}$
C	Iken's Bound	0.35
$ ho_w$	water density	$1,000 { m kgm^{-3}}$
$ ho_i$	ice density	$910 \ {\rm kgm^{-3}}$
g	gravitational acceleration	$9.8 \ {\rm ms}^{-2}$
L	latent heat of fusion for the ice	$3.34 \times 10^5 \mathrm{~Jkg^{-1}}$
A	Glen's flow law parameter	$6.34 \times 10^{-25} \text{ Pa}^{-1} \text{s}^{-1}$
n	Glen's flow law exponent	3
$\mu$	water viscosity	$1.78 \times 10^{-3} \text{ Nsm}^{-2}$
$\beta_w$	water compressibility	$5.0 \times 10^{-10} \text{ Pa}^{-1}$

: