Variability of the Gravitational Constant G

Vasile Turcu[1]

[1]Independent researcher

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Abstract

A brief history of the gravitational constant G is made and the current value is given. G for the Earth is calculated at perihelion and aphelion, resulting in different values. G is variable on a revolution in all cosmic bodies. Variable G is calculated for the solar system. The physical consequences with variable G are explained, which are valid for all cosmic bodies. Measurements of G are suggested to be made on Earth at perihelion and aphelion, on the International Space Station, and on the Moon.

VARIABILITY OF THE GRAVITATIONAL CONSTANT G

Vasile Turcu

515800-Sebes, str. M. Eminescu 30, Alba, Romania, EU

vturcu1@yahoo.com

Abstract. A brief history of the gravitational constant G is made and the current value is given. G for the Earth is calculated at perihelion and aphelion, resulting in different values. G is variable on a revolution in all cosmic bodies. Variable G is calculated for the solar system. The physical consequences with variable G are explained, which are valid for all cosmic bodies. Measurements of G are suggested to be made on Earth at perihelion and aphelion, on the International Space Station, and on the Moon.
Keywords: Dynamic astronomy - variable G - solar system - ISS - Halley comet.

1 Introduction

Newton's law of universal attraction is

$$\boldsymbol{F} = -G\frac{M \cdot m}{r^2}\frac{\boldsymbol{r}}{r}\ N \ \ with \ \ \frac{\boldsymbol{r}}{r} = 1 \ \ in\ module \ \ or \ \ F = G\frac{M \cdot m}{r^2}\ N \tag{1}$$

where M is the mass of the central body, m is the mass of the satellite body, $\boldsymbol{F}$ is the force of attraction between M and m, r is the distance between M and m, and G is the constant of universal attraction equal to $6.67430(15) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ (CODATA, 2019). The negative sign comes from the inverse orientations of the $\boldsymbol{F}$ and $\boldsymbol{r}$ vectors. The orbital motion of a cosmic body of mass m is due to the dynamic balance between the gravitational force (1) (centripetal force) and the centrifugal inertia force.

$$-G\frac{M \cdot m}{r^2}\frac{\boldsymbol{r}}{r} = \frac{m \cdot \boldsymbol{v}^2}{r} \ \ with \ \ \frac{\boldsymbol{r}}{r} = 1 \ \ in\ module \ \ or$$

$$G\frac{M \cdot m}{r^2} = \frac{m \cdot v^2}{r} \ \ results \ \ G = \frac{\boldsymbol{v}^2 \cdot r}{M} \ \ or \ \ G = \frac{v^2 \cdot r}{M}\ m^3\ kg^{-1}\ s^{-2} \tag{2}$$

with $\boldsymbol{v}$ = orbital linear velocity of the satellite body with mass m. The gravitational influence of M on m is G. The gravitational constant G is defined by (2), where M = constant. The value of G depends on $\boldsymbol{v}$ = orbital velocity of the satellite body m and r = distance between M and m. If the orbit of m is circular then the product $\boldsymbol{v}^2 r$ is constant and G = constant. But all cosmic bodies don't have a circular orbit, they have an elliptical orbit, etc. So the value of G is not as constant as Newtonian and Einsteinian mechanics postulate. The value of G in (1) is variable according to (2). In (Popescu, 1982) if the law of gravitovortex forces is given by Newton's law (1), then G automatically becomes variable. In the scalar-tensor theory of gravity by P.Jordan (1959), R.H.Dicke, and C.H.Brans (1961), if Newton's field equation (1) remains valid, G becomes variable. On the other hand, if we consider G as a fixed constant, then Newton's field equation (1) is no longer valid. If (1) remains valid then G is variable according to (2) from place to place in orbit and from one orbit to another.

2 Background

The variability of the gravitational constant G suggested by a series of empirical data was observed as early as A.Eddington (1906), but the one who explicitly formulated it and argued it as such was P.A.M.Dirac in 1937. The problem with variable G was attacked head-on by several scientists in Europe and the United States, including H.Thiry, P.Kaluza, W.Klein, Veblen, B.P.Jordan, Ehlers, Kundt, Demming, R.H.Dicke, C.Brans and P.J.Peebles. Continuous raising of the precision of determining G, a continuous and increasingly rapid process, in the period of over 220 years since such experiments are made, as well as the correct understanding of the significance of the results thus obtained, an understanding that has become possible in the last decades is the key to solving this difficult problem first addressed by G.Cavendish in 1798. From (Popescu, 1982; Dbachmann, 2020) the following table results.

Author	Year	Type of measuring scale	$G \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$	Mean quadratic error
G.Cavendish	1798	torsion	6.75	±0.025
F.Reich	1838	torsion	6.64	0.03
F.Heyl	1843		6.63	0.035
A.Cornu	1873	torsion	6.64	0.0085
F.Jolly	1878	vertical	6.47	0.055
J.Wilsing	1889	vertical	6.594	0.075
T.Boys	1889-1894	torsion	6.6576	0.002
K.Braun	1887-1896	torsion	6.655	0.002
G.Poynting	1878-1896	vertical	6.6984	
R.Eötvös	1896	torsion	6.657	0.001
Köning	1884-1897		6.685	0.011
F.Richartz	1898	vertical	6.683	0.011
P.Heyl	1930	torsion	6.678	0.003
P.Heyl and P.Chrzanovschi	1942	torsion	6.673	0.0015
L.Facy	1969	torsion	6.66598	
R.Rose and H.Parker	1969	torsion	6.674	0.002
J.Renner	1970	torsion	6.67	0.004
L.Facy	1971	torsion	6.673	uncertainty
G.Pontikis	1971	torsion	6.671	standard x10^{-11}
Luther and Towler	1982	torsion	6.6726	0.0005
Karagioz and Izmailov	1996	torsion	6.6729	0.0005
Bagley and Luther	1997	torsion	6.674	0.0007
Gundlach and Mercowitz	2000	torsion	6.674215	0.000092
Quinn et al.	2001	torsion	6.67559	0.00027
Armstrong and Fitzgerald	2003	torsion	6.67387	0.00027
Tu et al.	2010	torsion	6.67349	0.00018
Quinn et al.	2013	torsion	6.67545	0.00018
Newman et al.	2014	torsion	6.67433	0.00013
Kleinevoss	2002	two pendulums	6.67422	0.00098
Schlamminger et al.	2006	beam balance	6.674252	0.000122
Parks and Faller	2010	two pendulums	6.67234	0.00014
Rosi et al.	2014	atomic interferometry	6.67191	0.00099

Table 1

In August 2018, a group of Chinese researchers announced new torsion balance measurements, which obtained the values $6.674184(78) \cdot 10^{-11}$ and $6.674484(78) \cdot 10^{-11}$. It is noted that the first decimal place also differs. However, the measurements reached the sixth decimal place and the relative standard uncertainty reached $2.2 \cdot 10^{-5}$ in 2019. The value of G is obtained through these experiments, and theoretically with (2). The following table shows the CODATA values recommended for G by the National Institute of Standards and Technology (NIST).

Year	$G \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$	Relative standard uncertainty $\cdot 10^{-6}$
1969	6.6732(31)	460
1973	6.6720(49)	730
1986	6.67449(81)	120
1998	6.673(10)	1500
2002	6.6742(10)	150
2006	6.67428(67)	100
2010	6.67384(80)	120
2014	6.67408(31)	46
2018	6.67430(15)	22

Table 2

From (CODATA,2019) the Newtonian gravitational constant G

The numerical value	$6.67430 \cdot 10^{-11}$	$m^3\, kg^{-1}\, s^{-2}$
Standard uncertainty	$0.00015 \cdot 10^{-11}$	$m^3\, kg^{-1}\, s^{-2}$
Relative standard uncertainty	$2.2 \cdot 10^{-5}$	
Concise form	$6.67430(15) \cdot 10^{-11}$	$m^3\, kg^{-1}\, s^{-2}$

3 Earth

From (nssdc./earthfact,2020) the Earth has at perihelion
$v = 3.029 \cdot 10^4\, m\, s^{-1}$
$r = 1.47092 \cdot 10^{11}\, m$
With the mass of the Sun $M = 1.9885 \cdot 10^{30}\, kg$ (nssdc./sunfact,2018) and replacing in (2) we obtain
$G_p = 6.78675 \cdot 10^{-11}\, m^3, kg^{-1}\, s^{-2}$
where G_p = gravitational constant of the Earth at perihelion. From (nssdc./earthfact,2020) at aphelion
$v = 2.929 \cdot 10^4\, m\, s^{-1}$
$r = 1.52099 \cdot 10^{11}\, m$
Replacing in (2) we obtain
$G_a = 6.562 \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$
where G_a = gravitational constant of the Earth at aphelion. At the Earth, G varies in the closed range
$G = [6.562 - 6.78675]\, 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$
From Table 1 it is noted that all the values obtained from 1798 (Cavendish) to the present, fall within this range. The exception is the measurement of F.Jolly in 1878. Results fall within the annual average
$G = 6.674375 \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$

With the average value $v = 2.978 \cdot 10^4 \ m\,s^{-1}$ and $r = 1.495978707 \cdot 10^{11} \ m$ (AU), replacing in (2) we obtain

$G = 6.671895 \cdot 10^{-11} \ m^3\,kg^{-1}\,s^{-2}$

as Rosi et al. 2014. So the value (CODATA, 2019) valid today is the annual average of the constant of the gravitational attraction G. G has the maximum value at perihelion (January 2-5) and the minimum value at aphelion (July 4-6). Consequently, the gravitational acceleration g has the maximum value at perihelion and the minimum one at aphelion, for the same place on the Earth. This leads to the maximum weight at perihelion and the minimum one at aphelion, for a material body located at the same place on the Earth.

In Newton and Einstein's theory of gravity, the gravitational force depends on the value of G. At perihelion, we have the maximum gravitational force, which leads to the minimum volume of the Earth. At aphelion, we have the minimum gravitational force, which leads to the maximum volume of the Earth. This is an orbital motion with expansion-contraction of the planet Earth (Popescu, 1982). This expansion-contraction of the planet Earth results in the phenomenon of tectonic plates, earthquakes, volcanoes, and the configuration of the crust with all forms of relief, which leads to the formation of land and water-covered areas. This is the varying physical phenomenon G, the foundation of the planet Earth and every cosmic body. It is best to observe variable G in comets, which have the minimum volume at perihelion (G max) and the maximum volume at aphelion (G min). G varies seasonally in all cosmic bodies, between the maximum value at perihelion and the minimum value at aphelion. The rate of G growth and decrease on the Earth in a year is

$$\frac{\Delta G}{\Delta t} \ = \ G_p \ - \ G_a = 2.2475 \cdot 10^{-12}/year$$

From (Mould, Uddin, 2014) a decrease rate of G (Kaspi et al., 1994) was detected in binary pulsars.

$$\frac{\Delta G}{\Delta t} = (4 \pm 5) \cdot 10^{-12}/year$$

With lunar laser beam (Muller, Biskupek, 2007)

$$\frac{\Delta G}{\Delta t} = (2 \pm 7) \cdot 10^{-13}/year$$

From (Mould, Uddin, 2014) Li et al., 2013, a decrease in G has been detected with the Planck Space Telescope

$$\frac{\Delta G}{\Delta t} = 1.42 \cdot 10^{-12}/year$$

WMAP

$$\frac{\Delta G}{\Delta t} = 2.48 \cdot 10^{-13}/year$$

BAO

$$\frac{\Delta G}{\Delta t} = 2.27 \cdot 10^{-12}/year$$

Other observations have been made on pulsars, big-bang nucleosynthesis, helioseismology,neutron mass of the star, etc., and the results are falling within these values. ΔG at the Earth roughly falls within these limits. The Brans-Dicke scalar-tensor theory (1961), initiated by Jordan (1959), provides for a decrease in G of

$$\frac{\Delta G}{\Delta t} \approx 10^{-10}/year \;\; result \;\; t \approx \frac{1}{10^{-10}} \approx 10^{10} \; years$$

with $t =$ age of the universe. In (Mould, Uddin, 2014) G decreases with the rate of

$$\frac{\Delta G}{\Delta t} = (3 - 0.73)10^{-10}/year \;\; result \;\; t = (3.3 - 13.7)10^{9} \; years$$

In (Popescu, 1982) gravitovortex theory, (Mould, Uddin, 2014) supernova stars, the scalar-tensor theory by Brans-Dicke (1961) ,et al., G decreases steadily and evenly over time from 1 at the time of birth of the universe (Big-Bang) to today's value. This is how the age of the universe is calculated about 13.7 billion years. But as we have theoretically demonstrated on Earth, ΔG is a seasonal variable between perihelion with G maximum and aphelion with G minimum. G has no differences between years, G has differences only during the year. G decreases and grows with the same value at each revolution. So G does not permanently decrease at a fixed rate from the Big Bang until today and consequently the age of the universe is not this.

But G varies periodically and over time. In (Anderson et al., 2015) the period of variation of G is $P_1 = 5.9$ years (5 years and 11 months), i.e. from 5.9 years to 5.9 years we have a minimum or maximum and G varies in the range of

$G = (6.672 - 6.676)10^{-11} \; m^3 \, kg^{-1} \, s^{-2}$

In (Schlamminger et al., 2015) we have several periods of G variation.

$P_2 = 6.17$ years (6 years and 2 months)

$P_3 = 0.769$ years (9.3 months)

$P_4 = 0.995$ years (12 months)

$P_5 \approx (13 - 14)$ days of the smallest fluctuations in value

For the period P_1 and P_2 we have g minimum approximately in:

August 2022; August 2016; August 2010; July 2004; July 1998; July 1992.

G maximum is approximately in:

January 2019; January 2013; March 2007; January 2001; January 1995.

The period P_1 and P_2 is between approximately $P \approx 5$ years and 11 months - 6 years and 2 months. From (astrocal 2010-2020) to G minimum we have opposition with Mars and Jupiter. We don't have the lower opposition with Venus. In 2010 the lower opposition with Venus was after the minimum and in 2022 it is well after the minimum. Before G maximum we do not have opposition to Mars and Jupiter. We don't have the lower opposition with Venus in 2013 and 2019. When we have, it does not significantly influence the maximum value of G. So at the value of G minimum the orbital parameter v^2r of (2) is minimal and at the value of G maximum the orbital parameter v^2r of (2) is maximum. That is, the other nearby planets disrupt the orbital motion of the Earth, which is reflected in the value of the variable G. The period P_5 is explained by the period of revolution of the Moon in $P = 27.3217$ days. But there must be a period of 1 year with a maximum at perihelion and a minimum at aphelion. Period P_4 corresponds, but the values are small. It is possible to correspond to P_3 or combined P_3 with P_4.

The v and r values of the cosmic satellite body are obtained by observations, G is obtained by experiments, and the mass of the central body M results from (2). v,r and G of m are variable from one time to another in the orbit of m and from one orbit to another. M is constant. From (nssdc./moonfact,2020) the Moon has

$v = 1.0226 \cdot 10^3 \; m\, s^{-1}$ average orbital velocity
$r = 3.844 \cdot 10^8 \; m$ average distance

With Earth mass $M = 5.9724 \cdot 10^{24} \; kg$ (nssdc./earthfact,2020), replacing in (2) we obtain
$G = 6.73048 \cdot 10^{-11} \; m^3\, kg^{-1}\, s^{-2}$
the average gravitational constant of the Moon. At perigee
$v = 1.082 \cdot 10^3 \; m\; s^{-1}$
$r = 3.633 \cdot 10^8 \; m$
With (2) we obtain
$G_p = 7.12149 \cdot 10^{-11} \; m^3\, kg^{-1}\, s^{-2}$
the gravitational constant of the Moon at the perigee. At apogee
$v = 9.7 \cdot 10^2 \; m\, s^{-1}$
$r = 4.055 \cdot 10^8 \; m$
With (2) we obtain
$G_a = 6.3883 \cdot 10^{-11} \; m^3\, kg^{-1}\, s^{-2}$
the gravitational constant of the Moon at apogee. On the surface of the Moon G varies in the closed range
$G = [6.3883 - 7.12149]10^{-11} \; m^3\, kg^{-1}\, s^{-2}$

On the Moon average G is higher than on the Earth. The explanation is that v of the Moon is greater than a stable dynamic equilibrium resulting in the constant distance from the Earth. I suggest that measurements of G be made on the Moon and with (2) we find out the exact mass of the Earth. This is how G is calculated for all cosmic bodies. That is, G in (1) is variable and this variability is defined by (2). G is not constant as Newton and Einstein postulate.

From (Peat Chris, 2018) the International Space Station (ISS) launched on November 20, 1998, operable today, has the average orbital values:
$v = 7.65 \cdot 10^3 \; m\, s^{-1}$
Perigee height is $4.18 \cdot 10^5 \; m$. With the average radius of the Earth $r = 6.371 \cdot 10^6 \; m$ results in
$r = 6.789 \cdot 10^6 \; m$
Replacing in (2) we obtain
$G = 6.65242 \cdot 10^{-11} \; m^3\, kg^{-1}\, s^{-2}$

The gravitational constant at the perigee, and therefore the mean on the ISS is lower than on the Earth. The explanation is that v of the ISS is lower than a stable dynamic equilibrium resulting in the constant proximity to the Earth. I suggest that measurements of G should be made on the ISS.

4 Solar System

The Sun

From (Shen et al., 2010; P.J.McMillan, 2017; Gillessen et al., 2016; Kaffe et al., 2014) the Sun in the Galaxy has approximately
$v = 2.2 \cdot 10^5 \; m\, s^{-1}$
$r = 2.57 \cdot 10^{20} \; m \;\; or \;\; 27200 \; light\ years$
Galactic bulb mass with bar (Courteau et al., 2014) is approximately
$M = 1.9 \cdot 10^{41} \; kg$

Replacing in (2) we obtain

$G = 6.55 \cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

the approximate value of the gravitational constant of the Sun. Observations need to be improved.

Planets

Using orbital parameters from (nssdc./mercuryfact,2018; venusfact,2018; marsfact,2018; jupiterfact,2018; saturnfact,2019; uranusfact,2018; neptunefact,2018; plutofact,2019), the mass of the Sun in (sunfact, 2018), replacing in (2) the following table results.

Name	Average G $\cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$	G at perihelion $\cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$	G at aphelion $\cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$
Sun	6.55		
Mercury	6.532	8	5.3
Venus	6.6738	6.7199	6.63
Earth	6.671895	6.78675	6.562
Mars	6.64	7.29	6
Jupiter	6.678	7	6.35
Saturn	6.755	7	6.29
Uranus	6.679	6.96	6.36
Neptune	6.665	6.76	6.59
Pluto	6.4778	8.3	5.1

Table 3. G in the Solar System

So orbital parameters need to be improved. With general relativity Mercury's perihelion advance is $\delta\theta = 42", 98/century$. In the calculations we used G of the Earth. If we use the average G value from Table 3, Mercury has the perihelion advance $\delta\theta = 42", 04/century$. So Mercury's perihelion advance is not a conclusive test for general relativity.

From (1P/Halley,2003) Halley's comet has orbital parameters

$v = 7.022 \cdot 10^3\ m\, s^{-1}$ average orbital velocity

$r = 2.66 \cdot 10^{12}\ m$ or $17.8\, AU$ large semi-axis

With (2) average G

$G = 6.59 \cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

At perihelion

$v = 5.42 \cdot 10^4\ m\, s^{-1}$

$r = 8.766 \cdot 10^{10}\ m$ or $0.586\, AU$ results with (2)

$G_p = 1.295 \cdot 10^{-10}\ m^3\, kg^{-1}\, s^{-2}$

At aphelion

$v = 9.095 \cdot 10^2\ m\, s^{-1}$

$r = 5.25 \cdot 10^{12}\ m$ or $35.1\, AU$ results with (2)

$G_a = 2.18 \cdot 10^{-12}\ m^3\, kg^{-1}\, s^{-2}$

On February 9, 1986, Halley's comet was at perihelion. After the transition to perihelion, on March 14, 1986, the European spacecraft Giotto studied Halley's comet from a distance of 596 kilometers and obtained

$m \approx 2.2 \cdot 10^{14}\ kg$ mass of the comet's head

$r \approx 5.5 \cdot 10^3\ m$ radius of the comet's head

$\rho \approx 3 \cdot 10^2\ kg\, m^{-3}$ density at perihelion result in

$g_p \approx 9.4 \cdot 10^{-4}\ m\, s^{-2}$ surface gravitational acceleration with G_p

With G_a resulting in aphelion radius (we use g), or $\approx G_p/G_a$

$r \approx 3.4 \cdot 10^5\ m$

So Halley's comet head has at aphelion the diameter of about 61 times the diameter at perihelion. This is an orbital movement with expansion-contraction of the cosmic body, the cause being variable G.

5 Conclusions

The gravitational constant G of each cosmic body in orbit is variable and this variability is given by (2). In orbit G has the maximum value at perihelion and the minimum value at aphelion. The mean value of G is constant over time for a cosmic system in dynamic equilibrium. G is not fixed as Newton's and Einstein's theories of gravity state. G does not decrease permanently over time as provided by the scalar-tensor theory of gravity, gravitovortex theory and others. G increases and decreases by the same value from perihelion to aphelion and vice versa. G is variable is due to the fact that all orbits of cosmic bodies are not circular, they are ellipses, etc. Only for a circular orbit do we have fixed G. To demonstrate the variable G I suggest making measurements on the Earth at perihelion and aphelion, on the International Space Station and on the Moon.

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Vasile Turcu

515800-Sebes, str. M. Eminescu 30, Alba, Romania, EU

vturcu1@yahoo.com

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where M is the mass of the central body, m is the mass of the satellite body, $\boldsymbol{F}$ is the force of attraction between M and m, r is the distance between M and m, and G is the constant of universal attraction equal to $6.67430(15) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ (CODATA, 2019). The negative sign comes from the inverse orientations of the $\boldsymbol{F}$ and $\boldsymbol{r}$ vectors. The orbital motion of a cosmic body of mass m is due to the dynamic balance between the gravitational force (1) (centripetal force) and the centrifugal inertia force.

$$-G\frac{M \cdot m}{r^2}\frac{\boldsymbol{r}}{r} = \frac{m \cdot \boldsymbol{v}^2}{r}\ \ with\ \ \frac{\boldsymbol{r}}{r} = 1\ \ in\ module\ \ or$$

$$G\frac{M \cdot m}{r^2} = \frac{m \cdot v^2}{r}\ \ results\ \ G = \frac{\boldsymbol{v}^2 \cdot r}{M}\ \ or\ \ G = \frac{v^2 \cdot r}{M}\ m^3\ kg^{-1}\ s^{-2} \tag{2}$$

with $\boldsymbol{v}$ = orbital linear velocity of the satellite body with mass m. The gravitational influence of M on m is G. The gravitational constant G is defined by (2), where M = constant. The value of G depends on $\boldsymbol{v}$ = orbital velocity of the satellite body m and r = distance between M and m. If the orbit of m is circular then the product $\boldsymbol{v}^2 r$ is constant and G = constant. But all cosmic bodies don't have a circular orbit, they have an elliptical orbit, etc. So the value of G is not as constant as Newtonian and Einsteinian mechanics postulate. The value of G in (1) is variable according to (2). In (Popescu, 1982) if the law of gravitovortex forces is given by Newton's law (1), then G automatically becomes variable. In the scalar-tensor theory of gravity by P.Jordan (1959), R.H.Dicke, and C.H.Brans (1961), if Newton's field equation (1) remains valid, G becomes variable. On the other hand, if we consider G as a fixed constant, then Newton's field equation (1) is no longer valid. If (1) remains valid then G is variable according to (2) from place to place in orbit and from one orbit to another.

2 Background

The variability of the gravitational constant G suggested by a series of empirical data was observed as early as A.Eddington (1906), but the one who explicitly formulated it and argued it as such was P.A.M.Dirac in 1937. The problem with variable G was attacked head-on by several scientists in Europe and the United States, including H.Thiry, P.Kaluza, W.Klein, Veblen, B.P.Jordan, Ehlers, Kundt, Demming, R.H.Dicke, C.Brans and P.J.Peebles. Continuous raising of the precision of determining G, a continuous and increasingly rapid process, in the period of over 220 years since such experiments are made, as well as the correct understanding of the significance of the results thus obtained, an understanding that has become possible in the last decades is the key to solving this difficult problem first addressed by G.Cavendish in 1798. From (Popescu, 1982; Dbachmann, 2020) the following table results.

Author	Year	Type of measuring scale	$G \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$	Mean quadratic error
G.Cavendish	1798	torsion	6.75	±0.025
F.Reich	1838	torsion	6.64	0.03
F.Heyl	1843		6.63	0.035
A.Cornu	1873	torsion	6.64	0.0085
F.Jolly	1878	vertical	6.47	0.055
J.Wilsing	1889	vertical	6.594	0.075
T.Boys	1889-1894	torsion	6.6576	0.002
K.Braun	1887-1896	torsion	6.655	0.002
G.Poynting	1878-1896	vertical	6.6984	
R.Eötvös	1896	torsion	6.657	0.001
Köning	1884-1897		6.685	0.011
F.Richartz	1898	vertical	6.683	0.011
P.Heyl	1930	torsion	6.678	0.003
P.Heyl and P.Chrzanovschi	1942	torsion	6.673	0.0015
L.Facy	1969	torsion	6.66598	
R.Rose and H.Parker	1969	torsion	6.674	0.002
J.Renner	1970	torsion	6.67	0.004
L.Facy	1971	torsion	6.673	uncertainty
G.Pontikis	1971	torsion	6.671	standard x10^{-11}
Luther and Towler	1982	torsion	6.6726	0.0005
Karagioz and Izmailov	1996	torsion	6.6729	0.0005
Bagley and Luther	1997	torsion	6.674	0.0007
Gundlach and Mercowitz	2000	torsion	6.674215	0.000092
Quinn et al.	2001	torsion	6.67559	0.00027
Armstrong and Fitzgerald	2003	torsion	6.67387	0.00027
Tu et al.	2010	torsion	6.67349	0.00018
Quinn et al.	2013	torsion	6.67545	0.00018
Newman et al.	2014	torsion	6.67433	0.00013
Kleinevoss	2002	two pendulums	6.67422	0.00098
Schlamminger et al.	2006	beam balance	6.674252	0.000122
Parks and Faller	2010	two pendulums	6.67234	0.00014
Rosi et al.	2014	atomic interferometry	6.67191	0.00099

Table 1

In August 2018, a group of Chinese researchers announced new torsion balance measurements, which obtained the values $6.674184(78) \cdot 10^{-11}$ and $6.674484(78) \cdot 10^{-11}$. It is noted that the first decimal place also differs. However, the measurements reached the sixth decimal place and the relative standard uncertainty reached $2.2 \cdot 10^{-5}$ in 2019. The value of G is obtained through these experiments, and theoretically with (2). The following table shows the CODATA values recommended for G by the National Institute of Standards and Technology (NIST).

Year	$G \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$	Relative standard uncertainty $\cdot 10^{-6}$
1969	6.6732(31)	460
1973	6.6720(49)	730
1986	6.67449(81)	120
1998	6.673(10)	1500
2002	6.6742(10)	150
2006	6.67428(67)	100
2010	6.67384(80)	120
2014	6.67408(31)	46
2018	6.67430(15)	22

Table 2

From (CODATA,2019) the Newtonian gravitational constant G

The numerical value	$6.67430 \cdot 10^{-11}$	$m^3\, kg^{-1}\, s^{-2}$
Standard uncertainty	$0.00015 \cdot 10^{-11}$	$m^3\, kg^{-1}\, s^{-2}$
Relative standard uncertainty	$2.2 \cdot 10^{-5}$	
Concise form	$6.67430(15) \cdot 10^{-11}$	$m^3\, kg^{-1}\, s^{-2}$

3 Earth

From (nssdc./earthfact,2020) the Earth has at perihelion

$v = 3.029 \cdot 10^4\, m\, s^{-1}$

$r = 1.47092 \cdot 10^{11}\, m$

With the mass of the Sun $M = 1.9885 \cdot 10^{30}\, kg$ (nssdc./sunfact,2018) and replacing in (2) we obtain

$G_p = 6.78675 \cdot 10^{-11}\, m^3, kg^{-1}\, s^{-2}$

where G_p = gravitational constant of the Earth at perihelion. From (nssdc./earthfact,2020) at aphelion

$v = 2.929 \cdot 10^4\, m\, s^{-1}$

$r = 1.52099 \cdot 10^{11}\, m$

Replacing in (2) we obtain

$G_a = 6.562 \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$

where G_a = gravitational constant of the Earth at aphelion. At the Earth, G varies in the closed range

$G = [6.562 - 6.78675]\, 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$

From Table 1 it is noted that all the values obtained from 1798 (Cavendish) to the present, fall within this range. The exception is the measurement of F.Jolly in 1878. Results fall within the annual average

$G = 6.674375 \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$

With the average value $v = 2.978 \cdot 10^4 \ m\,s^{-1}$ and $r = 1.495978707 \cdot 10^{11} \ m$ (AU), replacing in (2) we obtain

$G = 6.671895 \cdot 10^{-11} \ m^3\,kg^{-1}\,s^{-2}$

as Rosi et al. 2014. So the value (CODATA, 2019) valid today is the annual average of the constant of the gravitational attraction G. G has the maximum value at perihelion (January 2-5) and the minimum value at aphelion (July 4-6). Consequently, the gravitational acceleration g has the maximum value at perihelion and the minimum one at aphelion, for the same place on the Earth. This leads to the maximum weight at perihelion and the minimum one at aphelion, for a material body located at the same place on the Earth.

In Newton and Einstein's theory of gravity, the gravitational force depends on the value of G. At perihelion, we have the maximum gravitational force, which leads to the minimum volume of the Earth. At aphelion, we have the minimum gravitational force, which leads to the maximum volume of the Earth. This is an orbital motion with expansion-contraction of the planet Earth (Popescu, 1982). This expansion-contraction of the planet Earth results in the phenomenon of tectonic plates, earthquakes, volcanoes, and the configuration of the crust with all forms of relief, which leads to the formation of land and water-covered areas. This is the varying physical phenomenon G, the foundation of the planet Earth and every cosmic body. It is best to observe variable G in comets, which have the minimum volume at perihelion (G max) and the maximum volume at aphelion (G min). G varies seasonally in all cosmic bodies, between the maximum value at perihelion and the minimum value at aphelion. The rate of G growth and decrease on the Earth in a year is

$$\frac{\Delta G}{\Delta t} \ = \ G_p \ - \ G_a = 2.2475 \cdot 10^{-12}/year$$

From (Mould, Uddin, 2014) a decrease rate of G (Kaspi et al., 1994) was detected in binary pulsars.

$$\frac{\Delta G}{\Delta t} = (4 \pm 5) \cdot 10^{-12}/year$$

With lunar laser beam (Muller, Biskupek, 2007)

$$\frac{\Delta G}{\Delta t} = (2 \pm 7) \cdot 10^{-13}/year$$

From (Mould, Uddin, 2014) Li et al., 2013, a decrease in G has been detected with the Planck Space Telescope

$$\frac{\Delta G}{\Delta t} = 1.42 \cdot 10^{-12}/year$$

WMAP

$$\frac{\Delta G}{\Delta t} = 2.48 \cdot 10^{-13}/year$$

BAO

$$\frac{\Delta G}{\Delta t} = 2.27 \cdot 10^{-12}/year$$

Other observations have been made on pulsars, big-bang nucleosynthesis, helioseismology,neutron mass of the star, etc., and the results are falling within these values. ΔG at the Earth roughly falls within these limits. The Brans-Dicke scalar-tensor theory (1961), initiated by Jordan (1959), provides for a decrease in G of

$$\frac{\Delta G}{\Delta t} \approx 10^{-10}/year \ \ result \ \ t \approx \frac{1}{10^{-10}} \approx 10^{10} \ years$$

with $t =$ age of the universe. In (Mould, Uddin, 2014) G decreases with the rate of

$$\frac{\Delta G}{\Delta t} = (3 - 0.73)10^{-10}/year \ \ result \ \ t = (3.3 - 13.7)10^{9} \ years$$

In (Popescu, 1982) gravitovortex theory, (Mould, Uddin, 2014) supernova stars, the scalar-tensor theory by Brans-Dicke (1961) ,et al., G decreases steadily and evenly over time from 1 at the time of birth of the universe (Big-Bang) to today's value. This is how the age of the universe is calculated about 13.7 billion years. But as we have theoretically demonstrated on Earth, ΔG is a seasonal variable between perihelion with G maximum and aphelion with G minimum. G has no differences between years, G has differences only during the year. G decreases and grows with the same value at each revolution. So G does not permanently decrease at a fixed rate from the Big Bang until today and consequently the age of the universe is not this.

But G varies periodically and over time. In (Anderson et al., 2015) the period of variation of G is $P_1 = 5.9$ years (5 years and 11 months), i.e. from 5.9 years to 5.9 years we have a minimum or maximum and G varies in the range of

$G = (6.672 - 6.676)10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

In (Schlamminger et al., 2015) we have several periods of G variation.

$P_2 = 6.17$ years (6 years and 2 months)

$P_3 = 0.769$ years (9.3 months)

$P_4 = 0.995$ years (12 months)

$P_5 \approx (13 - 14)$ days of the smallest fluctuations in value

For the period P_1 and P_2 we have g minimum approximately in:

August 2022; August 2016; August 2010; July 2004; July 1998; July 1992.

G maximum is approximately in:

January 2019; January 2013; March 2007; January 2001; January 1995.

The period P_1 and P_2 is between approximately $P \approx 5$ years and 11 months - 6 years and 2 months. From (astrocal 2010-2020) to G minimum we have opposition with Mars and Jupiter. We don't have the lower opposition with Venus. In 2010 the lower opposition with Venus was after the minimum and in 2022 it is well after the minimum. Before G maximum we do not have opposition to Mars and Jupiter. We don't have the lower opposition with Venus in 2013 and 2019. When we have, it does not significantly influence the maximum value of G. So at the value of G minimum the orbital parameter v^2r of (2) is minimal and at the value of G maximum the orbital parameter v^2r of (2) is maximum. That is, the other nearby planets disrupt the orbital motion of the Earth, which is reflected in the value of the variable G. The period P_5 is explained by the period of revolution of the Moon in $P = 27.3217$ days. But there must be a period of 1 year with a maximum at perihelion and a minimum at aphelion. Period P_4 corresponds, but the values are small. It is possible to correspond to P_3 or combined P_3 with P_4.

The v and r values of the cosmic satellite body are obtained by observations, G is obtained by experiments, and the mass of the central body M results from (2). v,r and G of m are variable from one time to another in the orbit of m and from one orbit to another. M is constant. From (nssdc./moonfact,2020) the Moon has

$v = 1.0226 \cdot 10^3\ m\,s^{-1}$ average orbital velocity
$r = 3.844 \cdot 10^8\ m$ average distance

With Earth mass $M = 5.9724 \cdot 10^{24}\ kg$ (nssdc./earthfact,2020), replacing in (2) we obtain
$G = 6.73048 \cdot 10^{-11}\ m^3\,kg^{-1}\,s^{-2}$
the average gravitational constant of the Moon. At perigee
$v = 1.082 \cdot 10^3\ m\ s^{-1}$
$r = 3.633 \cdot 10^8\ m$
With (2) we obtain
$G_p = 7.12149 \cdot 10^{-11}\ m^3\,kg^{-1}\,s^{-2}$
the gravitational constant of the Moon at the perigee. At apogee
$v = 9.7 \cdot 10^2\ m\,s^{-1}$
$r = 4.055 \cdot 10^8\ m$
With (2) we obtain
$G_a = 6.3883 \cdot 10^{-11}\ m^3\,kg^{-1}\,s^{-2}$
the gravitational constant of the Moon at apogee. On the surface of the Moon G varies in the closed range
$G = [6.3883 - 7.12149]10^{-11}\ m^3\,kg^{-1}\,s^{-2}$

On the Moon average G is higher than on the Earth. The explanation is that v of the Moon is greater than a stable dynamic equilibrium resulting in the constant distance from the Earth. I suggest that measurements of G be made on the Moon and with (2) we find out the exact mass of the Earth. This is how G is calculated for all cosmic bodies. That is, G in (1) is variable and this variability is defined by (2). G is not constant as Newton and Einstein postulate.

From (Peat Chris, 2018) the International Space Station (ISS) launched on November 20, 1998, operable today, has the average orbital values:
$v = 7.65 \cdot 10^3\ m\,s^{-1}$
Perigee height is $4.18 \cdot 10^5\ m$. With the average radius of the Earth $r = 6.371 \cdot 10^6\ m$ results in
$r = 6.789 \cdot 10^6\ m$
Replacing in (2) we obtain
$G = 6.65242 \cdot 10^{-11}\ m^3\,kg^{-1}\,s^{-2}$

The gravitational constant at the perigee, and therefore the mean on the ISS is lower than on the Earth. The explanation is that v of the ISS is lower than a stable dynamic equilibrium resulting in the constant proximity to the Earth. I suggest that measurements of G should be made on the ISS.

4 Solar System

The Sun

From (Shen et al., 2010; P.J.McMillan, 2017; Gillessen et al., 2016; Kaffe et al., 2014) the Sun in the Galaxy has approximately
$v = 2.2 \cdot 10^5\ m\,s^{-1}$
$r = 2.57 \cdot 10^{20}\ m$ *or* 27200 *light years*
Galactic bulb mass with bar (Courteau et al., 2014) is approximately
$M = 1.9 \cdot 10^{41}\ kg$

Replacing in (2) we obtain

$G = 6.55 \cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

the approximate value of the gravitational constant of the Sun. Observations need to be improved.

Planets

Using orbital parameters from (nssdc./mercuryfact,2018; venusfact,2018; marsfact,2018; jupiterfact,2018; saturnfact,2019; uranusfact,2018; neptunefact,2018; plutofact,2019), the mass of the Sun in (sunfact, 2018), replacing in (2) the following table results.

Name	Average G $\cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$	G at perihelion $\cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$	G at aphelion $\cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$
Sun	6.55		
Mercury	6.532	8	5.3
Venus	6.6738	6.7199	6.63
Earth	6.671895	6.78675	6.562
Mars	6.64	7.29	6
Jupiter	6.678	7	6.35
Saturn	6.755	7	6.29
Uranus	6.679	6.96	6.36
Neptune	6.665	6.76	6.59
Pluto	6.4778	8.3	5.1

Table 3. G in the Solar System

So orbital parameters need to be improved. With general relativity Mercury's perihelion advance is $\delta\theta = 42", 98/century$. In the calculations we used G of the Earth. If we use the average G value from Table 3, Mercury has the perihelion advance $\delta\theta = 42", 04/century$. So Mercury's perihelion advance is not a conclusive test for general relativity.

From (1P/Halley,2003) Halley's comet has orbital parameters

$v = 7.022 \cdot 10^3\ m\, s^{-1}$ average orbital velocity

$r = 2.66 \cdot 10^{12}\ m$ or $17.8\, AU$ large semi-axis

With (2) average G

$G = 6.59 \cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

At perihelion

$v = 5.42 \cdot 10^4\ m\, s^{-1}$

$r = 8.766 \cdot 10^{10}\ m$ or $0.586\, AU$ results with (2)

$G_p = 1.295 \cdot 10^{-10}\ m^3\, kg^{-1}\, s^{-2}$

At aphelion

$v = 9.095 \cdot 10^2\ m\, s^{-1}$

$r = 5.25 \cdot 10^{12}\ m$ or $35.1\, AU$ results with (2)

$G_a = 2.18 \cdot 10^{-12}\ m^3\, kg^{-1}\, s^{-2}$

On February 9, 1986, Halley's comet was at perihelion. After the transition to perihelion, on March 14, 1986, the European spacecraft Giotto studied Halley's comet from a distance of 596 kilometers and obtained

$m \approx 2.2 \cdot 10^{14}\ kg$ mass of the comet's head

$r \approx 5.5 \cdot 10^3\ m$ radius of the comet's head

$\rho \approx 3 \cdot 10^2\ kg\, m^{-3}$ density at perihelion result in

$g_p \approx 9.4 \cdot 10^{-4}\ m\, s^{-2}$ surface gravitational acceleration with G_p

With G_a resulting in aphelion radius (we use g), or $\approx G_p/G_a$

$r \approx 3.4 \cdot 10^5\ m$

So Halley's comet head has at aphelion the diameter of about 61 times the diameter at perihelion. This is an orbital movement with expansion-contraction of the cosmic body, the cause being variable G.

5 Conclusions

The gravitational constant G of each cosmic body in orbit is variable and this variability is given by (2). In orbit G has the maximum value at perihelion and the minimum value at aphelion. The mean value of G is constant over time for a cosmic system in dynamic equilibrium. G is not fixed as Newton's and Einstein's theories of gravity state. G does not decrease permanently over time as provided by the scalar-tensor theory of gravity, gravitovortex theory and others. G increases and decreases by the same value from perihelion to aphelion and vice versa. G is variable is due to the fact that all orbits of cosmic bodies are not circular, they are ellipses, etc. Only for a circular orbit do we have fixed G. To demonstrate the variable G I suggest making measurements on the Earth at perihelion and aphelion, on the International Space Station and on the Moon.

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